MACROECONOMIC-DRIVEN PREPAYMENT RISK AND THE VALUATION OF MORTGAGE-BACKED SECURITIES

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Abstract. We introduce a reduced-form modeling framework for mortgage-backed securities in which we solve for the implied prepayment function from the cross section of market prices. From the implied prepayment function, we find that prepayment rates are driven not only by interest rates, but also by two macroeconomic factors: turnover and rate response. Intuitively, turnover represents prepayments for exogenous reasons like employment-related moves, household income shocks, and foreclosures, while rate response reflects frictions faced by borrowers in refinancing into a lower rate. We find that the empirical turnover and rate response measures are both significantly related to macroeconomic measures, suggesting that these factors represent a source of systematic risk. Consistent with this, we find that implied prepayments are substantially higher than actual prepayments, providing direct evidence of significant prepayment risk premia in mortgage-backed security prices. We analyze the properties of the prepayment risk premium and find that it is almost entirely due to compensation for turnover risk. We also find evidence that mortgage-backed security prices were significantly affected by Fannie Mae credit risk and the Federal Reserve's Quantitative Easing Programs.

1. INTRODUCTION

A mortgage-backed security is a securitized claim to the principal and interest payments generated by a pool of fixed-rate mortgages. Mortgage-backed securities have traditionally been issued either by agencies such as Fannie Mae, Freddie Mac, and Ginnie Mae, or by private issuers. Agency mortgage-backed securities have the attractive feature that the timely payment of principal and interest is backed by either an implicit or explicit government guarantee. As of June 2015, the total notional amount of agency mortgage-backed securities outstanding was $7.171 trillion, making this market one of the largest sectors of the global fixed income markets.[1]

Given the importance of the market, it is not surprising that much research has been devoted to the issue of how mortgage-backed securities should be valued. The first generation of pricing models treated mortgage prepayments as the result of a borrower attempting to maximize the value of an implicit interest rate option. Key examples include Dunn and McConnell (1981a, 1981b) and Brennan and Schwartz (1985). The second generation of pricing models is based on detailed econometric models of historical prepayment behavior. In these models, interest rate paths are simulated (under the risk-neutral measure) and the econometric prepayment model (estimated under the actual measure) is used to specify the cash flows along each interest rate path. Key examples include Schwartz and Torous (1989, 1992, 1993) and Richard and Roll (1989).

These types of models, however, have a number of important drawbacks. For example, prepayments are driven exclusively by interest rate changes in these models. Thus, by ruling out other possible sources of prepayments, the only risk premium allowed in these models is an interest rate risk premium—there is no scope for a separate prepayment risk premium. Furthermore, these models tend to produce prices that often diverge significantly from market prices, and can only be reconciled by introducing ad hoc "option-adjusted spreads" into the framework.

This paper presents a reduced-form framework for the valuation of mortgage-backed securities. This framework differs fundamentally from previous work in that we solve for the implied prepayment function rather than imposing an external prepayment model. A key advantage of this approach is that by studying the implied prepayment function, we can identify factors that the market views as important drivers of prepayment risk as well as the risk premia associated with those factors. The implied prepayment function is easily identified using

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[1]See www.sifma.org/research/statistics.aspx.

the information in the cross section of mortgage-backed securities with different coupon rates (the coupon stack). Our approach also allows for the possibility that mortgage cash flows may be discounted at higher rates than Treasuries because of either the perceived credit risk of the agency guaranteeing the mortgage-backed security, or the illiquidity of mortgage-backed securities themselves. We apply the framework to study the valuation of Fannie Mae mortgage-backed securities over the 1998–2014 period. A number of important results emerge from this analysis.

First, we find that implied prepayments are driven by interest rates as well as two additional macroeconomic risk factors: turnover and rate response. The turnover rate reflects prepayments occurring for exogenous reasons unrelated to interest rates, but possibly correlated with macroeconomic fluctuations. Examples include adverse income shocks or unemployment resulting in a move or a foreclosure, negative shocks to housing values resulting in underwater borrowers strategically defaulting on non-recourse loans, or homeowners with appreciated property taking cash-out mortgages to extract home equity. We find that the variation in the empirical turnover rate is strongly related to macroeconomic fundamentals such as consumption growth, changes in unemployment, and mortgage delinquencies. This suggests that turnover risk may be systematic in nature and carry a large risk premium. This is supported by evidence that the implied turnover rate is more closely related to returns in financial market than is the empirical turnover rate. The rate response factor represents the time variation in the sensitivity of refinancing activity to mortgage rate incentives. For example, borrowers may be less able to refinance into a lower mortgage rate after declines in housing prices, during recessions in which the borrowers' income or credit may have been impaired, or during periods in which mortgage lending standards are tightened. The results also provide evidence that this factor may be systematic in nature.

Second, we find that the implied prepayments on agency-guaranteed mortgages behave very differently than actual prepayments. For most mortgage-backed securities, implied prepayment rates are significantly higher than actual prepayment rates. This result provides some of the first direct evidence that the market incorporates significant prepayment-related risk premia into the prices of mortgage-backed securities, and may also explain why previous models based on empirical prepayment functions face challenges in their attempts to match the market prices of mortgage-backed securities.

Third, we study the determinants of the prepayment risk premium by mapping it into the risk premia associated with the turnover and rate response factors. We find that the turnover factor carries a large positive premium throughout the entire sample period, consistent with the systematic nature of turnover risk. In

contrast, the risk premium for the rate response factor is slightly negative before the financial crisis, and becomes slightly positive after the crisis. Taken together, these results indicate that the large majority (84.50 percent) of the prepayment risk premium represents compensation for the risk of systematic or macroeconomic-driven shocks in the turnover rate.

Fourth, we find that cash flows from mortgage-backed securities are discounted at a rate 72 basis points higher on average than are cash flows from Treasuries. This spread varies significantly through time and is strongly correlated with the credit spread between Fannie Mae debt and Treasuries. Furthermore, the spread is significantly related to supply-related factors such as Federal Reserve purchases of mortgage-backed securities during its quantitative easing programs and changes in dealers' inventory positions of mortgage-backed securities. These results provide the first direct evidence that agency credit and liquidity spreads influence the pricing of mortgage-backed securities.

Finally, we show that this simple reduced-form valuation framework fits the market prices of mortgage-backed securities very closely. The median root mean squared error across the entire coupon stack is on the order of 27 cents per $100 notional, which is on the same order of magnitude as the bid-ask spread for mortgage-backed securities. This accuracy compares very well to previous generations of valuation models for mortgage-backed securities. Furthermore, this result provides evidence that the pricing of mortgage-backed securities in the market may be much more rational than is commonly believed among market practitioners.

This paper contributes to the extensive literature on the pricing of mortgage-backed securities. Important recent work in this area includes Gabaix, Krishnamurthy, and Vigneron (2007) who study the pricing of interest-only strips and document that their option-adjusted spread covaries with the moneyness of the market, consistent with a prepayment risk premium and the existence of specialized mortgage-backed security investors. Boyarchenko, Fuster, and Lucca (2014) allow for the possibility that the implied prepayment rate may differ from the actual prepayment rate. They document an option-adjusted spread smile across mortgages with varying rates and use information from the interest-only/principal-only market to decompose the option-adjusted spread into its physical and risk-neutral components. They find evidence of a significant non-interest-rate-related prepayment risk premium in the components of the option-adjusted spread. This paper significantly extends the literature by using the entire cross section of mortgage-backed security prices to identify the implied prepayment function and its components, the associated prepayment risk premia, and the credit/liquidity component of mortgage-backed security prices.

The remainder of the paper is organized as follows. Section 2 provides a

brief introduction to the agency mortgage-backed security market. Section 3 reviews the literature. Section 4 describes the data. Section 5 presents the valuation framework. Section 6 describes the implied prepayment model. Section 7 discusses the empirical methodology. Section 8 presents the empirical results for the implied prepayment function. Section 9 discusses the prepayment risk premium and its components. Section 10 makes concluding remarks.

2. U.S. AGENCY MORTGAGE-BACKED SECURITIES

Agency mortgage-backed securities are issued by Fannie Mae (FNMA), Freddie Mac (FHLMC), or Ginnie Mae (GNMA).[2] Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs), whereas Ginnie Mae is a wholly-owned government corporation. The U.S. agency mortgage-backed securities market is among the largest and most liquid bond markets worldwide. Furthermore, more than 70 percent of the $9.8 trillion U.S. home mortgage market serves as collateral for agency mortgage-backed securities. Immediately prior to the financial crisis of 2007–2008, private financial institutions accounted for more than 50 percent of U.S. mortgage-backed security issuance. Since the crisis, however, "private label" issuance has declined dramatically and now represents less than four percent of total mortgage-related issuance. In contrast, agency mortgage-backed security issuance has grown rapidly; the total notional size of the agency mortgage-backed security market increased 58 percent from 2006 to 2015.[3] In this section, we review the key features of agency mortgage-backed securities.

2.1 Credit Quality

In exchange for monthly fees, the agencies guarantee the timely payment of mortgage interest and principal. The guarantee protects investors from defaults on the underlying mortgages since delinquent mortgages must be purchased out of the trust at par by the issuer. This means that a default appears as a prepayment from an investor's perspective. Because GNMA securities carry the full faith and credit guarantee of the United States, their credit quality should be the same as that of U.S. Treasuries. FNMA and FHLMC securities carry a credit guarantee from the issuing GSE rather the United States. Historically, the GSE guarantee was viewed as an "implicit" government guarantee because investors believed

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[2]Fannie Mae, Freddie Mac, and Ginnie Mae refer to the Federal National Mortgage Association, the Federal Home Loan Mortgage Corporation, and the Government National Mortgage Association, respectively.

[3]See Federal Reserve Board Flow of Funds Table L.217 and www.sifma.org/research/statistics.aspx.

that the government would back the agencies in times of stress. This view was validated in September 2008 when the government placed FNMA and FHLMC in conservatorship and provided them with unlimited access to collateralized funding. Both FNMA and FHLMC are supervised and regulated by the Federal Housing Finance Agency.

2.2 Mortgage-Backed Security Cash Flows

In this paper, we focus on agency mortgage-backed securities backed by pools of fixed-rate mortgages. A fixed-rate mortgage is structured so that the borrower is obligated to make the same payment each month, consisting of interest and principal. In general, fixed-rate mortgages can be prepaid at any time without penalty. Each month, therefore, a pool of mortgages generates cash flows consisting of scheduled interest, scheduled principal, and possibly prepaid principal. A pass-through mortgage-backed security distributes to investors the principal and interest payments from the underlying mortgage loans, less guaranty and servicing fees. Because the guaranty and servicing fees are based on the outstanding balance, these fees decline over the life of the mortgage.[4]

Mortgage servicers collect and aggregate payments from the underlying mortgage loans and pass the payments to the mortgage-backed security trust. Mortgage payments are due on the first of the month (with a grace period determined by state law). Investors, however, receive the payments after a delay of 14, 19, or 24 days, depending on the mortgage-backed security program. If a loan becomes delinquent, servicers advance scheduled principal and interest until either the loan becomes current or is bought out of the trust at par. Servicers retain a monthly fee based on a percentage of the outstanding mortgage balance at the beginning of the month. This fee is often referred to as a "servicing strip" because the cash flows resemble an interest-only strip. In the FNMA, FHLMC, and GNMA II programs, mortgages with different gross coupons can be pooled together as long as the net coupon (gross coupon minus servicing and guaranty fees) is identical among all the loans in the mortgage pool. In the GNMA I program, the gross coupon is always 50 basis points higher than the net coupon.

2.3 Agency Mortgage-Backed Security Trading

Agency mortgage-backed securities trade on either a to-be-announced (TBA) basis or a specified pool basis. The TBA market is a highly liquid forward market and accounts for 90 percent of all mortgage-backed security trading. From 2007 to 2014, the daily trading volume of U.S. agency mortgage-backed securities averaged $276 billion, which compares well with the $525 billion daily trading

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[4]See www.fanniemae.com, www.freddiemac.com, and www.ginniemae.gov for more information about agency securitization programs.

volume for U.S. Treasuries. Typically, pass-throughs are traded as specified pools if they command a premium over TBAs or if they are ineligible for TBA delivery.[5]

Similar to Treasury note and bond futures, a buyer of a TBA agrees to the trade without knowing the exact pools that will be delivered. Instead, the buyer and seller agree to six parameters: price, par amount, settlement date, agency program, mortgage type, and coupon. TBA trades generally settle to a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Nearly all TBA trades occur with settlement dates less than or equal to three months forward. Two days prior to the settlement date of the trade, the seller notifies the buyer of the exact pools that will be delivered (the 48-hour rule). The pools are then exchanged for the cash payment on the settlement date.

Market participants generally adhere to standards referred to as the "Good Delivery Guidelines" maintained by SIFMA. These guidelines specify the eligible collateral for a TBA trade and various operational guidelines such as the number of bonds per million dollars notional of a trade, the allowable variation in the delivery amount, and the costs of failing to deliver. TBA trades may also be executed with stipulations such as production year, weighted average maturity (WAM), weighted average loan age (WALA), FICO score, loan-to-value ratio, or geographic distribution. A stipulated TBA trade, however, would likely occur at a price higher than an unstipulated TBA.

2.4 The Quantitative Easing Programs

Table 1 provides a listing of the major events in the agency mortgage-backed securities market during the study period. Among the most significant of these events are the Federal Reserve's quantitative easing programs, commonly known as QE I, QE II, and QE III. The first program, QE I, was announced on November 25, 2008 and directed the purchase of up to $500 billion of agency mortgage-backed securities and $100 billion of GSE debt. The stated goal of QE I was to reduce the cost and increase the availability of credit for the purchase of houses. QE I was expanded on March 18, 2009 to allow additional purchases of up to $750 billion of agency mortgage-backed securities and $100 billion of agency debt. The QE II program was announced on November 3, 2010 and authorized the purchase of up to $600 billion of longer-term Treasury securities. The QE III program was announced on September 13, 2012 and directed the purchase of up to $40 billion per month of agency mortgage-backed securities and $45 billion

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[5]Trading volume data comes from FINRA TRACE https://www.finra.org/industry/trace/structure-product-activity-reports-and-tables. See Vickrey and Wright (2013) for a discussion of the TBA market. Also see Carlin, Longstaff, and Matoba (2014).

per month of Treasury securities. These programs had large effects on the supply of mortgage-backed securities in the market.[6]

3. LITERATURE REVIEW

Mortgage-backed security valuation is challenging because mortgage terminations (prepayments and defaults) may depend on many factors such as changes in interest rates, housing prices, employment status, family size, etc. For reviews of the literature, see Kau and Keenan (1995), Capone (2001), Hayre (2001), Wallace (2005), and Fabozzi (2006).

The first generation of pricing models was pioneered by Dunn and McConnell (1981a, 1981b) and extended by Brennan and Schwartz (1985). This framework approaches the valuation of mortgage-backed securities from the perspective of contingent claims theory. In particular, this approach models mortgage prepayments as the result of a borrower attempting to maximize the value of an implicit interest rate option. Dunn and Spatt (2005) and Stanton and Wallace (1998) extend the approach to model the prepayment decision as the result of minimizing lifetime mortgage costs in the presence of refinancing costs. These papers, however, imply an upper bound on mortgage prices that is often violated empirically, as demonstrated by Stanton (1995) and Boudoukh, Richardson, Stanton, and Whitelaw (1997). Later papers add frictions to allow for higher mortgage prices and consider the value of the prepayment option jointly with the option to default. Important contributions are Titman and Torous (1989), Kau, Keenan, Muller, and Epperson (1992), and Kau and Slawson (1995), Downing, Stanton, and Wallace (2005), Longstaff (2005), and many others. An important drawback of this modeling approach is that actual borrowers generally do not follow the prepayment strategies implied by these models. Thus, actual mortgage cash flows and mortgage-backed security prices often diverge significantly from those implied by these types of models.

The second generation of mortgage-backed security pricing models takes a more empirical approach. Typically, these models begin with a detailed econometric model of the historical behavior of prepayments, including elements such as geography, seasoning, burnout, seasonality, and other factors. Key examples of this approach include Schwartz and Torous (1989, 1992, 1993), Richard and

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[6]For a detailed discussion of the effects of the quantitative easing programs, see Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011, 2013), Christensen and Rudebusch (2012), Thornton (2014), and Christensen and Gillan (2015).

Roll (1989), and Deng, Quigley, and Van Order (2000). In this framework, interest rate paths are simulated (under the risk-neutral measure) and the econometric prepayment model (estimated under the actual measure) is applied to specify the cash flows along each interest rate path. Important drawbacks of this modeling approach include the fact that, by using an empirical prepayment model, this approach does not allow for the possibility of a prepayment risk premium. In particular, this approach requires that prepayment dynamics be the same under risk-neutral and actual measures. In actuality, market participants disagree significantly about prepayment forecasts (see Carlin, Longstaff, and Matoba (2014)). The net effect of these factors is that the prices implied by market participants' models often diverge significantly from market prices. These differences are typically expressed in terms of a yield spread adjustment to discount rates, known as the option-adjusted spread.

Given the inconsistent use of risk-neutral and actual measures in solving for the option-adjusted spread, it is not surprising that the economic interpretation of the option-adjusted spread is murky, and its values are frequently more volatile than the underlying mortgage-backed security prices. To illustrate this, Figure 1 plots the time series of option-adjusted spreads for FNMA 6.50 percent mortgage-backed securities as implied by the sequence of pricing models used by a major Wall Street mortgage dealer. As shown, the dealer changed its model frequently during the 2007–2015 period, primarily because the prior version of the model was failing to capture current market prices. The plot shows that changes in the model are often associated with large discontinuities in the time series of the option-adjusted spread that can be on the order of 100 basis points or more. This erratic behavior in the option-adjusted spread, even when holding the dealer fixed, provides a strong argument for basing empirical analysis on mortgage-backed security prices directly, rather than on option-adjusted spreads. This point is reinforced further by the evidence in Carlin, Longstaff, and Matoba (2014) that there is significant disagreement across dealers in terms of forecasted prepayment speeds, which would, in turn, translate into large differences in option-adjusted spreads across dealers.

Levin and Davidson (2005) attribute the variability in option-adjusted spreads to a prepayment risk premium and suggest a method to use market prices to adjust the empirical prepayment process for prepayment risk. Gabaix, Krishnamurthy, and Vigneron (2007) study the pricing of interest-only strips and document that their option-adjusted spread covaries with the moneyness of the market, consistent with a prepayment risk premium and the existence of specialized mortgage-backed security investors. Boyarchenko, Fuster, and Lucca (2014) document an option-adjusted spread smile across mortgages with varying rates and use information from the interest-only/principal-only market to decompose option-adjusted spreads into their physical and risk-neutral compo-

nents. Their work includes the important insight that information about the risk-neutral prepayment rate can be extracted from interest-only/principal-only securities. Other important recent contributions in understanding the economics of mortgage-backed security markets include Cheyette (1996), Linetsky (2004), Goncharov (2006), Gorovoy and Linetsky (2007), Malkhozov, Mueller, Vedolin, and Venter (2014), Hanson (2014), and Song and Zhu (2015).

4. THE DATA

The primary data for the study consist of monthly prices (observed at the end of each month) from the TBA market for FNMA mortgage-backed securities with varying coupons. The sample period is January 1998 to September 2014. The data are obtained from a proprietary data set compiled by a major Wall Street mortgage-backed security dealer. However, we have cross validated the proprietary data with prices publicly available in the Bloomberg system and found the two sources to be very similar. To insure that we include only prices for actively traded mortgage-backed securities, we limit the data set to mortgage-backed securities with coupon rates that are within 300 basis points of the current coupon mortgage rate. The data set also includes three-month horizon conditional prepayment rate (CPR) information for each coupon.

Table 2 presents summary statistics for the data. As shown, the sample includes mortgage-backed securities with coupons ranging from 2.50 percent to 9.50 percent. Of course, not all coupons are actively traded throughout the entire sample period. The higher coupon mortgage-backed securities appear during the early part of the sample period when mortgage rates are considerably higher, and vice versa for the lower coupon mortgage-backed securities.

We also collect data for a wide variety of macroeconomic, mortgage market, and financial variables that will be used in the analysis throughout the paper. The Appendix provides a description of each of these variables and the sources of the data.

Finally, we collect historical data on Treasury constant maturity rates from the Federal Reserve H.15 release. We use a standard cubic spline approach to bootstrap the prices of zero-coupon bonds $D(t)$ for maturities ranging up to 30 years for each month during the sample period (methodology is described in the Appendix).

5. THE REDUCED-FORM FRAMEWORK

In valuing mortgage-backed securities, we use a reduced-form framework in which an instantaneous prepayment process p_t plays the central role. Specifically, p_t is the fraction of the remaining notional balance of the underlying mortgage pool that is prepaid each instant. Thus, p_t can be viewed as a prepayment intensity or hazard rate.[7] Our approach will be to solve for the implied value of p_t and its dynamics from the cross section and time series of prices of mortgage-backed securities with different mortgage rates.

For expositional clarity, we assume for the present that mortgage cash flows are paid continuously and that the fixed mortgage rate m on the mortgages in the underlying pool is the same as the coupon rate on the mortgage-backed security. Let c denote the payment on a mortgage with an initial principal balance of one. Since the present value of the mortgage equals one at inception,

$$1 = c \int_0^T e^{-mt} \, dt, \tag{1}$$

$$= (c/m)(1 - e^{-mT}), \tag{2}$$

and the mortgage payment c is,

$$c = \frac{m}{1 - e^{-mT}}. \tag{3}$$

Let I_t denote the principal balance of the mortgage at time t. The change in the principal balance is just the difference between the interest on the mortgage balance and the mortgage payment,

$$dI_t = mI_t - c. \tag{4}$$

Solving this first-order differential equation subject to the initial condition implies

$$I_t = \frac{1 - e^{-m(T-t)}}{1 - e^{-mT}}. \tag{5}$$

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[7]The proportional rate p_t at which loans are being prepaid is often referred to as the CPR of the mortgage-backed security by market participants.

Now, consider a mortgage-backed security where the individual mortgages in the underlying pool are all T-year fixed-rate mortgages. Without loss of generality, we normalize the initial notional balance of the pool to be one. We denote the remaining notional balance of the underlying pool at time t as N_t, which, given the definition of p_t, can be expressed as

$$N_t = \exp\left(-\int_0^t p_s \, ds\right). \tag{6}$$

In turn, the remaining principal balance of the underlying pool is given by $N_t I_t$.[8]

Finally, let $F(m, T)$ denote the value of a mortgage-backed security where the underlying mortgages have a mortgage rate of m and maturity of T. The value of the mortgage-backed security at time zero is given formally by

$$F(m, T) = E^Q\left[\int_0^T \exp\left(-\int_0^t r_s + w_s \, ds\right) N_t \left(c + p_t \, I_t\right) dt\right], \tag{7}$$

where $E^Q[\,\cdot\,]$ denotes expectation under the risk neutral measure, r_t is the riskless interest rate, and w_t plays the role of a credit/liquidity spread. The rationale for including w_t in the model is to allow for the possibility that cash flows from agency mortgages may be discounted a higher rate than Treasury cash flows, either because the credit risk of the agency may not be as strong, or because agency mortgages may be less liquid than Treasuries.

6. THE PREPAYMENT FUNCTION

To complete the valuation framework for mortgage-backed securities, we need to specify the prepayment process p_t. Before doing this, however, it is useful to first consider some of the stylized facts about actual prepayment rates.

To illustrate the relation between prepayments and refinancing incentives, Figure 2 plots the prepayment rates for FNMA mortgage-backed securities as functions of the prices of these securities. As shown, there is a strong relation

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[8]It is important to distinguish between the remaining notional amount and the principal balance since mortgage payments are based on the original notional amount of the mortgages while prepayment cash flows are based on the remaining principal balance.

between the prepayment rate and the price of the mortgage-backed securities. When the price of the mortgage-backed security is less than 100, the coupon rate on the mortgage is lower than the current market rate, and the borrower has no incentive to refinance. When the price is greater than 100, however, the borrower may be able to reduce his mortgage costs by refinancing. Interestingly, the relation between prepayment rates and mortgage prices has the appearance of a piecewise linear function similar to that of a call option payoff.

In particular, when the price of the mortgage-backed security is less than 100, the relation is flat, although generally not zero. In fact, the prepayment rates for these mortgage-backed securities can be as high as 10 to 15 percent, because borrowers often prepay mortgages for reasons other than to reduce mortgage costs. For example, borrowers often prepay mortgages even when the market rate is higher than their mortgage rate for exogenous reasons such as a retirement or a career-related move. Also, borrowers may refinance into a higher mortgage rate to extract home equity after an increase in housing prices. During the recent financial crisis, a major source of exogenous prepayments has been the high rate of foreclosures throughout the U.S. (a foreclosure results in the pass through of the entire remaining mortgage balance to the holders of an agency-guaranteed mortgage-backed security).

When the price of the mortgage-backed security is greater than 100, the relation is generally increasing, but spreads out as the price increases. A closer inspection of the data, however, indicates that the relation is actually close to linear at a point in time, but that the slope of the relation varies over time. Thus, the unconditional relation appears spread out. To illustrate this, Figure 3 plots the prepayment rate and price relation for selected dates during the sample period. As shown, the prepayment functions display varying slopes over time.

Motivated by these stylized facts, we use a simple generic specification of the implied prepayment function that allows for both exogenous and rate-related prepayments. Specifically, we model the prepayment function as

$$p_t = x_t \; + \; y_t \; \max(0, \; m \; - \; a \; - \; b \; r_t). \tag{8}$$

In this specification, x_t denotes the exogenous hazard rate at which mortgages are prepaid in the absence of refinancing incentives. Intuitively, x_t captures all the non-interest-rate-related background factors that lead to prepayments. Thus, we will refer to x_t simply as the turnover rate.

The refinancing incentive is determined by the difference between the mortgage rate m and the implied rate at which mortgages can be refinanced. We allow this implied rate to be a general affine function $a + b \; r_t$ of the short-term

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riskless rate, rather than constraining it to be a specific short-term or long-term rate. The values of a and b will be estimated from the data.

The term y_t which multiplies the refinancing incentive term $\max(0,\ m\ -\ a\ -\ b\ r_t)$ in Equation (8) measures how sensitive borrowers are to refinancing incentives. For example, a borrower whose house value was less than the mortgage balance would typically have a very low propensity to refinance, or equivalently, a low value of y_t. After the introduction of the Home Affordable Refinancing Program (HARP) in 2009, however, the same borrower might have been much more likely to refinance given the same level of refinancing incentive. Similarly, the propensity to refinance could also vary with the required loan-to-value standards in the mortgage market. Given the role that y_t plays in the prepayment function, we denote it as the rate response factor.

7. ESTIMATION METHODOLOGY

In this framework, the value of a mortgage-backed security is a function of the three state variables: w_t, x_t, and y_t (in addition to the interest rate). To complete the specification of the model, we assume that the dynamics of the state variables are given by the following system of stochastic differential equations under the risk-neutral pricing measure,

$$dw = (\alpha_w - \beta_w\ w)\ dt\ +\ \sigma_w\ dZ_w, \qquad (9)$$

$$dx = (\alpha_x - \beta_x\ x)\ dt\ +\ \sigma_x\ \sqrt{x}\ dZ_x, \qquad (10)$$

$$dy = (\alpha_y - \beta_y\ y)\ dt\ +\ \sigma_y\ \sqrt{y}\ dZ_y. \qquad (11)$$

where the Brownian motions are independent of each other. The credit/liquidity spread w_t follows a mean-reverting process that can take on both positive and negative values, paralleling the specification used by Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), and many others. The state variables x_t and y_t driving prepayments both follow mean-reverting square-root processes, ensuring that prepayment rates are always nonnegative. Note that this specification of dynamics places this model within the familiar affine framework widely used throughout the financial literature.

To model the evolution of the riskless rate, we assume that r_t follows the single-factor Vasicek (1977) process. This assumption, however, can easily be relaxed to allow for a more general multi-factor specification. The riskless rate is assumed to be independent of the other state variables.

The estimation of the model can be viewed as consisting of three steps. First, we select an initial parameter vector θ, where $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y\}$. Second, conditional on θ and for each month t during the sample period, we solve for the values of w_t, x_t, and y_t that best fit the model to the prices for the coupon stack (the cross section of mortgage-backed securities with different coupon rates) by minimizing the root mean squared error (RMSE). Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. Third, we iterate over alternative values of the parameter vector θ until we find the vector that results in the lowest global root mean square error (RMSE). Table 3 reports the parameter values obtained from the estimation along with their asymptotic standard errors. The outputs are the parameter values and the time series of state variables.

The details of the estimation process are described in the Appendix. As discussed, the parameters for the riskless rate are estimated separately. Specifically, we use an approach in which we fit exactly the ten-year spot rate for each month in the sample period, and adjust the volatility parameter of the Vasicek model each month to match the basis point volatility of a one-year into ten-year swap option. The results, however, are very robust to the specification of the interest rate model.

8. THE IMPLIED PREPAYMENT FACTORS

In this section, we discuss the empirical results and their implications. First, we examine how well the model is able to fit the market prices of mortgage-backed securities. We then study the properties of the three state variables of the model: the credit/liquidity spread w_t, the turnover rate x_t, and the rate response factor y_t.

8.1 Fitting Mortgage-Backed Security Prices

The coupon stack for each month in the sample period typically includes between 6 to 10 mortgage-backed securities with varying coupon rates at 50 basis point increments. The estimation algorithm solves for the values of the three state variables w_t, x_t, and y_t that best fit the model to the coupon stack. Since there are more prices than state variables, it is clear that there will be residual differences between model values and market values. To quantify the magnitude of these differences, we compute the RMSE for each month in the sample period.

Figure 4 plots the time series of the RMSEs. As shown, the model fits the mortgage-backed security prices extremely well. For much of the sample period, the RMSEs range from about 5 to 30 cents for mortgage-backed security prices quoted in terms of a $100 notional position. This range compares reasonably well with the bid-ask spreads of actively-traded mortgage-backed securities, which discussions with traders indicate are typically on the order of three to four ticks, or 32nds of a point. Once the financial crisis begins in 2008, however, the RMSEs tend to become larger in value. Intuitively, this may simply be the result of the massive shocks that the housing and mortgage markets experienced during the financial crisis, as well as a lack of liquidity and risk capital in the markets to arbitrage mispricing among mortgage-backed securities. The median RMSE for the pre-crisis period is 23.8 cents. The median RMSE for the entire sample period is 27.3 cents.

8.2 The Mortgage-Backed Security Pricing Factors

The estimation algorithm solves for the implied values of the three factors driving mortgage-backed securities prices for each month during the sample period: the credit/liquidity spread, the turnover rate, and the rate response factor. Table 4 provides summary statistics for the implied values of these factors. These pricing factors are discussed individually below.

8.3 The Credit/Liquidity Spread

Table 4 shows that the mean value of the credit/liquidity spread is about 72.1 basis points with a standard deviation of 52.2 basis points. This mean value is in relatively close agreement with the average spread on FNMA debt issues during the sample period. For example, the average spread of FNMA ten-year debt over Treasuries during the January 2000 to September 2014 period is 49.8 basis points. We will study the link between the implied spread and FNMA credit spreads in more depth shortly.

Figure 5 plots the time series of the implied credit/liquidity spread values over the sample period, along with the spread for FNMA agency debt. As shown, the majority of the implied spreads are positive. In particular, 178 or 88.6 percent of the 201 estimates are positive. The fact that some of the implied spreads are negative, however, hints that the implied spreads may be reflecting more than the credit risk of FNMA bonds, particularly since FNMA credit spreads are uniformly positive throughout the 2000–2014 period.

This latter observation is reinforced by comparing the spread values shown

in Figure 5 with the key events in the timeline given in Table 1. For example, the downward spike around April 2009 coincides with the large expansion of the QE I program to purchase an additional $750 billion of MBS. The very large downward spike around September 2012 coincides with the announcement of the QE III program to purchase $40 billion of agency MBS per month. Thus, these observations hint that the massive purchases of MBS during QE I and QE III may have had an effect via new production and existing collateral being removed from the market. The potential effect is two-fold: a direct decrease in supply would increase prices and decrease spreads, an indirect effect on liquidity would increase spreads. It appears that the first effect dominates the second.

On the other hand, Figure 5 also shows that the implied spreads appear to be correlated with key events that may impact the credit risk of FNMA. For example, the local minimum in the spread during the early part of 2005 coincides with a period when the mortgage delinquency rate reached historically low values (the calm before the storm). Similarly, the spread attains its largest values during the Lehman crisis period of Fall 2008. However, after FNMA and FHLMC are placed into conservatorship and their credit risk is essentially defeased, the implied spread quickly returns to pre-crisis levels, and subsequently actually reaches historical lows.

To examine the properties of the implied spread in more detail, we regress monthly changes in the spread on a number of explanatory variables reflecting changes in the credit risk and liquidity of the mortgage-backed securities market. First, we include monthly changes in the yield spread between FNMA notes and Treasury notes with similar maturities. The intuition for including this spread is that if FNMA's cost of debt capital were to increase relative to that of the Treasury, then the value of the FNMA guarantee should decline, resulting in lower mortgage-backed security prices, or equivalently, higher implied spreads. Second, we include current and lagged changes in primary dealers' holdings of mortgage-backed securities as reported by the Federal Reserve Bank of New York. The intuition for this measure is that when primary dealers increase their inventories, we would expect that the liquidity of the mortgage-backed securities market would improve, leading to a decline in the implied spread. Third, we include changes in the general collateral repo rate in the regression. Because the repo rate reflects the cost to a mortgage-backed securities dealer of financing inventory, an increase in the repo rate would provide dealers with a strong incentive to reduce inventory, allowing investors to purchase mortgage-backed securities at better prices or lower spreads. Finally, we include several measures relating to the supply of mortgage-backed securities in the market. The first is the net issuance of mortgage-backed securities. The second is the ratio of Federal Reserve purchases of mortgage-backed securities to total net issuance. This ratio reflects how much of the flow of issuance is being absorbed by the Federal Reserve. A

high value for this ratio could imply that dealers are being crowded out of the market, potentially impairing market liquidity. We also include the lagged value of the change in the spread as a control variable.

Table 5 presents the regression results. As illustrated, the change in the FNMA credit spread is strongly related to the change in the credit/liquidity spread implied from the prices of mortgage-backed securities. The regression coefficient is positive and highly significant with a t-statistic of 9.12. Although this result is very intuitive, to our knowledge, this is the first direct evidence that the credit risk of the agency guaranteeing the timely payment of principal and interest is related to the pricing of mortgage-backed securities. The regression coefficient of roughly 0.84 indicates that while the implied spread is closely related to the spread on FNMA debt, the relation is not one-to-one and that there are other drivers of the implied spread.

In particular, Table 5 shows that the coefficient for the lagged change in primary dealers' inventory is negative and significant. The negative signs for both the current and lagged changes are consistent with the view that the implied spread also reflects the overall liquidity of the mortgage-backed securities market. Specifically, that when primary dealers increase their inventory, the liquidity of the market increases, resulting in a decline in the implied spread.

Table 5 also show that there is a significant positive relation between changes in the implied spread and ratio of Federal Reserve purchases to total net issuance of mortgage-backed securities. This result is consistent with the interpretation that the large purchases of mortgage-backed securities during the quantitative easing programs crowded out other players in the market and adversely affected liquidity.

Given the strong empirical relation between the credit/liquidity rate and the FNMA credit spread, a simple estimate of the size of the liquidity component in mortgage-backed securities could be obtained by subtracting the FNMA credit spread from the credit/liquidity spread. Figure 6 plots this estimate of the liquidity spread. As shown, during the pre-crisis period, the liquidity spread is positive with an average value of around 25 basis points. The liquidity spread spikes during the period leading up to the Lehman default and the conservatorship of Fannie Mae and Freddie Mac in September 2008. Afterwards, however, the liquidity spread declines to near zero with downward spikes coinciding with the initiation and extension of the QE I program. The initiation of the QE III program with its massive purchases of agency mortgage-backed securities coincides with the large negative spike in the liquidity spread. Discussions with industry sources suggest that as the Federal Reserve's purchases of agency mortgage-backed securities began to crowd other players out of these markets, the difficulty of finding tradeable collateral made existing supplies of mortgage-backed securities trade

at a premium. The liquidity estimates shown in Figure 6 are consistent with this view.

8.4 The Turnover Rate

The implied turnover rate is based on the risk-neutral measure since its value is inferred from the prices of mortgage-backed securities. Because prepayment rates are directly observable, however, the turnover rate under the actual or empirical probability measure is easily estimated from the data.[9] As part of our analysis, we will contrast the properties of the empirical and implied turnover rates and examine their implications for risk premia.

Table 4 shows that the mean value of the implied turnover rate is about 17.2 percent. As we discuss later, this is higher than the observed refinancing rate for mortgages with coupon rates below the current market mortgage rate.

To explore this further, we regress quarterly changes in both the empirical and implied turnover rates on variables that reflect the degree of financial distress in the mortgage markets and the state of the macroeconomy. As proxies for financial distress, we include the change in the mortgage foreclosure rate and the lagged change in the mortgage deliquency rate (both from the Mortgage Bankers Association National Delinquency Survey). As macroeconomic measures, we include the lagged growth rate in US personal consumption expenditures and the change in the US unemployment rate. The reason for focusing on quarterly changes in the regression is that a number of the explanatory variables are only observed quarterly. We include the contemporaneous value of some variables and the lagged value for others (rather than including both values) simply because the limited number of observations necessitates a parsimonious specification. Thus, we report results including only the value of each variable that appears to have the most explanatory power. We note that the empirical results are largely unaffected by this simplification.

We also include variables that may reflect various dimensions of the wealth and consumption incentives of mortgage borrowers. In particular, we include the return on the Barclays US Aggregate Bond Index, the lagged return on the CRSP value-weighted stock index, and the lagged change in the National Association of Home Builders Market Index. Finally, we also include the change in the fraction of refinanced mortgages where the new balance is more than five percent higher

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[9]We estimate the empirical turnover rate for each month during the sample period by simply taking the average prepayment rate for all mortgage-backed securities trading at a price below 100 that month.

than the balance on the refinanced loan, obtained from Freddie Mac's Cash-out Refinance Report. These types of loans are known as cash-out mortgages since the borrower ends up with additional cash. An increase in the fraction of cash-out mortgages could signal that borrowers are refinancing for consumption reasons (rather than interest-rate-related reasons) such as extracting equity from a home after housing values increase. We also include the lagged change in the turnover rate as a control for time series effects.

Table 6 reports the results from the regression. Focusing first on the regression for changes in the actual turnover rate, the results show that turnover is strongly related to changes in the state of the mortgage market as well as general macroeconomic conditions. In particular, an increase in the delinquency rate is significantly positively related to subsequent turnover. This is intuitive since higher delinquencies forecast future foreclosures. In contrast, the change in the current foreclosure rate is significantly negatively related to actual turnover. This suggests that by removing problem loans from the underlying pool via foreclosure, the average quality of the remaining loan pool improves and uncertainty about credit outcomes is resolved. The coefficient for the change in the unemployment rate is positive and significant, consistent with the interpretation that involuntary turnover increases during economic downturns as borrowers face adverse shocks and distress-related prepayments increase (via foreclosures, employment-related moves, etc.). On the other hand, the coefficient for consumption growth is positive and significant, consistent with the view that voluntary mortgage turnover may increase when borrowers increase their consumption of housing by selling and moving up to more expensive homes, etc. This interpretation is also consistent with the significant positive coefficients for lagged stock returns and the housing index, which suggest that as asset values increase, voluntary turnover also increases.

Turning our attention now to the regression for changes in implied turnover, we see that implied turnover behaves very differently from actual turnover. In particular, none of the macroeconomic and mortgage market financial distress variables are significant in this regression. Instead, changes in implied turnover are strongly related to returns in other markets. For example, the coefficients for bond returns, stock market returns, and changes in the housing market index (at the ten-percent level) are all significant. These results suggest that the implied turnover rate may include financial risk premia correlated with those in other financial markets. Furthermore, changes in the implied turnover rate are significantly positively related to cash-out mortgage activity.

In summary, the relation between actual turnover rates and macroeconomic factors such as consumption growth, unemployment, and financial distress in the housing markets suggests that turnover risk may be very systematic in nature.

If so, it would not be surprising if turnover risk were to carry a large risk premium. This possibility is strengthened by finding that changes in the implied turnover rate are more strongly correlated with financial market returns than with macroeconomic fundamentals. We will explore this issue in depth later in the paper.

8.5 The Rate Response Factor

Table 4 reports summary statistics for the implied rate response factor. As in the previous section, we explore the properties of the rate response factor by regressing quarterly changes in its empirical and implied values on variables that may reflect the frictions borrowers face in refinancing mortgages.[10]

The first set of variables are chosen to proxy for mortgage credit conditions. In particular, we include the change in the bank tightening index which measures the fraction of bank loan officers who describe the trend of credit availability as tightening (from the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices). We also include the change in the average loan-to-value ratio for all conventional mortgages. These loan-to-value statistics are compiled and reported by the Federal Housing Finance Agency. A decrease in the average loan-to-value ratio would be consistent with the scenario in which lenders are tightening mortgage credit, and vice versa. Finally, we include the change in the total amount of nonmortgage consumer credit outstanding as reported by the Federal Reserve Board. Changes in this index may reflect fundamental changes in the availability of credit to borrowers.

Since the credit frictions faced by mortgage borrowers may also depend on macroeconomic conditions, we also include a number of macroeconomic measures in the regression. As before, we include the lagged growth rate in US personal consumption expenditures. We also include the change in the Conference Board's Consumer Confidence Index, and the change in the unemployment rate.

Finally, we again include variables that may be related to the wealth of mortgage borrowers since this could potentially affect their ability to refinance: the return on the Barclays US Aggregate Bond Index, the lagged return on the CRSP value-weighted stock index, and the lagged change in the National

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[10]The empirical rate response factor is estimated for each month during the sample period by regressing the observed prepayment rates for the mortgage-backed securities with prices above 100 on the refinancing incentive (defined as the difference between the mortgage coupon rate and the current coupon mortgage rate).

Association of Home Builders Housing Market Index.

Table 7 reports the results from the regressions. Beginning again with the results for the empirical rate response factor, we find that none of the three mortgage credit condition proxies (the bank tightening index, the average loan-to-value ratio, and the amount of consumer credit) are significant. On the other hand, the results indicate that both the lagged consumption growth rate and change in consumer confidence are significant. The negative signs for these variables, however, are perplexing since they suggest that frictions are higher as macroeconomic conditions improve. One possible interpretation of this is that borrowers pay less attention to refinancing when macroeconomic conditions are strengthening and their individual financial situations improve. The strong link between macroeconomic conditions and the rate response factor again argues that the market may view this factor as a systematic risk which should carry a risk premium. Finally, neither the bond, stock market, nor housing market measures are significant.

The results for the implied rate response factor are again very different from those for the empirical factor. As before, the link between macroeconomic variables and the implied factor is much weaker. Furthermore, there is a strong correlation between the implied rate response factor and both the bond market return and changes in the housing index. This again suggests that a major component of the implied rate response factor may represent a risk premium compensating investors for the systematic nature of the variation in this factor.

9. THE PREPAYMENT RISK PREMIUM

In this section, we examine whether the market prices of mortgage-backed securities incorporate a risk premium for prepayment risk. Since we model prepayment risk as an explicit function of the turnover rate and the rate response factor, our framework also allows us to break down the total prepayment risk premium further into the components related to the turnover rate and the rate response factor.

We note that mortgage-backed securities may also incorporate premia for interest rate risk and agency credit risk. Rather than focusing on these well-known and extensively-researched types of risk premia, however, we will focus exclusively on the prepayment risk premium since there has been relatively little previous research on this topic in the literature.

9.1 Is there a Prepayment Risk Premium?

To address the issue of whether there is a prepayment risk premium, we follow the standard approach of comparing values estimated under the risk-neutral pricing measure with those estimated under the actual or objective measure. Because the implied prepayment function is estimated directly from the market prices of mortgage-backed securities, it represents the prepayment function under the risk-neutral pricing measure. In contrast, prepayments under the actual or empirical measure are directly observable.

It is important to observe that since the prepayment rate represents a risk-neutral hazard rate or probability in this framework, the implied value of the turnover rate need not equal the empirical turnover rate. This follows from Jarrow, Lando, and Yu (2005) who show that if hazard rates or intensities are sensitive to shocks that carry risk premia (for example, such as macroeconomic factors), then their values can differ between the risk-neutral and actual measures. This is analogous to what occurs in reduced-form credit models in which the risk-neutral default probability or hazard rate need not equal the actual default probability. A key difference, however, is that the actual probability of default is extremely difficult to measure given how rare default events are. Thus, it is very challenging to estimate the difference between risk neutral and actual default probabilities.[11] In contrast, empirical prepayment rates are directly observed and differences between the prepayment rate under the risk neutral and objective measures are easily identified.

The implied prepayment rate for each mortgage-backed security is given by simply substituting its weighted average coupon rate into the fitted prepayment function. Observe that in doing this, we are solving for the instantaneous implied prepayment rate which can be compared directly to the three-month realized CPR for the mortgage-backed security.[12]

The upper panel of Figure 7 plots the time series of the monthly averages for both the implied and empirical prepayment rates. The bottom panel plots the time series of the prepayment risk premium which is computed as the difference between the implied and realized prepayment rates. As shown in the upper panel, the implied and realized prepayment rates generally track each other over time, but there are some notable exceptions, particularly during the peak of the financial crisis. This is particularly evident in the lower panel which shows that the prepayment risk premium reached extreme values during late 2008 and early 2009.

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[11]For example, see Huang and Huang (2012) and Giesecke, Longstaff, Shaeffer, and Strebulaev (2011).

[12]To solve for the risk premium over longer horizons, we would need also need to solve for the parameters of the w, x, and y processes under the objective measure.

The upper panel of Table 8 presents summary statistics for the implied prepayment rates, the empirical prepayment rates, and the prepayment risk premium. To make the results more interpretable, we first compute the average implied and actual prepayment rates across all coupons for each month. The summary statistics in Table 8 are then computed using the time series of the monthly averages.

As shown, the average implied prepayment rate across the entire sample of mortgage-backed securities is 37.257 percent. In contrast, the average empirical prepayment rate for the same sample of mortgage-backed securities is 21.241 percent. Thus, the implied prepayment function is clearly very different from the actual prepayment function. The average difference between the implied and actual prepayment rates is 16.015 percent. The hypothesis that this difference is zero is strongly rejected by the data. These results provide direct confirmation that there is a substantial prepayment risk premium incorporated into mortgage-backed security prices. This direct evidence of prepayment risk premia in the mortgage-backed securities market corroborates the evidence of prepayment risk premia in the option-adjusted spreads of interest-only/principal-only securities reported by Gabaix, Krishnamurthy, and Vigneron (2007) and Boyarchenko, Fuster, and Lucca (2014).

On average, the implied prepayment rate is substantially higher than the empirical prepayment rate. This has important implications for the pricing of mortgage-backed securities. In particular, the prices of mortgages with coupon rates below the current market rate are increasing in the prepayment rate while the opposite is true for the prices of mortgages with coupon rates higher than the current market rate. Thus, the positive prepayment risk premium implies that discount mortgages will have higher values than implied by empirical prepayment functions, while the reverse will be the case for premium mortgages. These results are broadly consistent with the empirical evidence provided in Duarte, Longstaff, and Yu (2007).

To provide a cross sectional perspective, Figure 8 presents a scatter diagram of the prepayment risk premium for the individual mortgage-backed securities in the sample against their price. As shown, there is clearly heterogeneity in the prepayment risk premium across mortgages with different coupons. For example, although the risk premium is positive on average across all mortgages, there are clearly more negative values for mortgage-backed securities with prices in excess of 100. The average prepayment risk premium for mortgage-backed securities with prices below 100 is 17.17 percent, while the average for those with prices greater than or equal to 100 is 14.83 percent.

To provide more insight into the heterogeneity in the prepayment risk premia across mortgage-backed securities, it is useful to break down the premium into

its components. In the following sections, we will examine the turnover and rate response risk premia separately.

9.2 The Turnover Risk Premium

As in the previous section, we can identify the turnover risk premium by comparing the implied turnover rate with the empirical turnover rate. The upper panel of Figure 9 plots the time series of the implied turnover rate and the empirical turnover rate.

As illustrated, virtually all of the implied turnover rates are higher than the realized turnover rates. Some of the largest values of the implied turnover rate occur during the first half of 2005. Industry sources suggest that a large fraction of this turnover was motivated by borrowers attempting to "cash out" some of the equity in their homes resulting from the rapid increase in housing values. Thus, the upwards spike in the implied turnover rate during this period may reflect a shift away from the usual interest-rate-related reasons for refinancing towards consumption related incentives for refinancing. The lower panel of Figure 9 plots the turnover risk premium which is computed as the difference between the implied and empirical turnover rates.

The middle panel of Table 8 presents summary statistics for the implied turnover rate, the empirical turnover rate, and the risk premium. The average implied turnover rate is substantially higher than the realized turnover rate. The average implied turnover rate is 17.211 percent, while the average empirical turnover rate is only 3.679. Thus, the average turnover risk premium is 13.532 percent for the sample period. This value is highly statistically significant.

Recall from the previous section that the average prepayment risk premium is 16.015 percent on average. Thus, the average turnover risk premium of 13.532 percent represents 84.50 percent of the entire average prepayment risk, making it by far the primary component. Given the earlier evidence that turnover risk is related to broad trends in the economy, these result suggest that much of the prepayment risk premium in mortgage-backed securities can be linked to the effects of non-interest-rate-related macroeconomic fluctuations on prepayment behavior.

9.3 The Rate Response Risk Premium

The upper panel of Figure 10 plots the time series of the implied rate response factor and the empirical rate response factor. As illustrated, the implied and empir-

ical rate response factors display considerable time series variation and generally track each other closely. Some of the highest values of the rate response factor occur during the 2002–2003 period during which refinancings reached historically high levels. More recent increases in the rate response factor coincide with the rapid expansion of the Home Affordability Refinancing Program (HARP) in which investors with home values below their mortgage balance were allowed to refinance their homes.

The lower panel of Figure 10 shows the rate response risk premium measured as the difference between the implied and empirical rate response factors. Although the risk premium takes both positive and negative values, the time series plot shows that there are generally more negative values than positive values.

The lower panel of Table 8 presents summary statistics for the implied and realized rate response factors along with the risk premium. The average implied rate response factor of 10.432 is slightly lower than the realized rate response factor of 11.907. The average rate response risk premium of −1.475 is relatively small compared to the magnitude of the realized factor, but just crosses the threshold of being statistically different from zero.

The fact that the average rate response risk premium is negative explains why the prepayment risk premia for premium mortgage-backed securities are generally lower than those for discount mortgage-backed security, or are even negative in some cases. Furthermore, the relatively small magnitude of the average rate response risk premium also explains why this risk premium represents only a small portion of the total prepayment risk premium.

10. CONCLUSION

We present a new reduced-form framework for modeling the prices of mortgage-backed securities. Rather than imposing an exogenous prepayment function, our approach solves for the actual prepayment function used by the market in pricing mortgage-backed securities. By studying the properties of the implied prepayment function, our goal is to shed light on the key drivers of prepayment risk as perceived by the market.

The evidence suggests that macroeconomic factors play a large role in driving prepayment risk. In particular, we find that prepayment risk can be broken down into three components: interest-rate risk, turnover risk, and rate response risk. We infer the values of the turnover rate and the rate response factors from the data and show that they are strongly related to macroeconomic factors such as consumption, employment, consumer confidenc, and financial market returns.

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Given the strong link between macroeconomic factors and the drivers of implied prepayment risk, our direct evidence that there is a large prepayment risk premium incorporated into the prices of agency mortgage-backed securities makes intuitive sense. What is surprising, however, is that the large majority of the prepayment risk premium appears to be compensation for turnover risk.

We also provide the first direct evidence that mortgage-backed security prices are also driven by changes in the credit risk of the agency guaranteeing the timely payment of principal and interest as well as by changes in the liquidity of the securities. These results are consistent with findings for other markets.

Although there is an extensive literature on the pricing of mortgage-backed securities, previous pricing models have struggled in matching market prices. By their nature, these models generally do not allow for non-interest-rate prepayment factors or risk premia. The differences between model-implied prices and market prices are generally mapped into what is commonly known as the option-adjusted spread. Our results indicate that much of the option-adjusted spread may be explained by allowing for macroeconomic-driven prepayment risk factors and their associated risk premia and by allowing for agency credit risk in the discounting of cash flows. Given the fundamental role of the housing and mortgage-backed security markets in the macroeconomy, our results have many important implications for researchers, policy makers, and practitioners.

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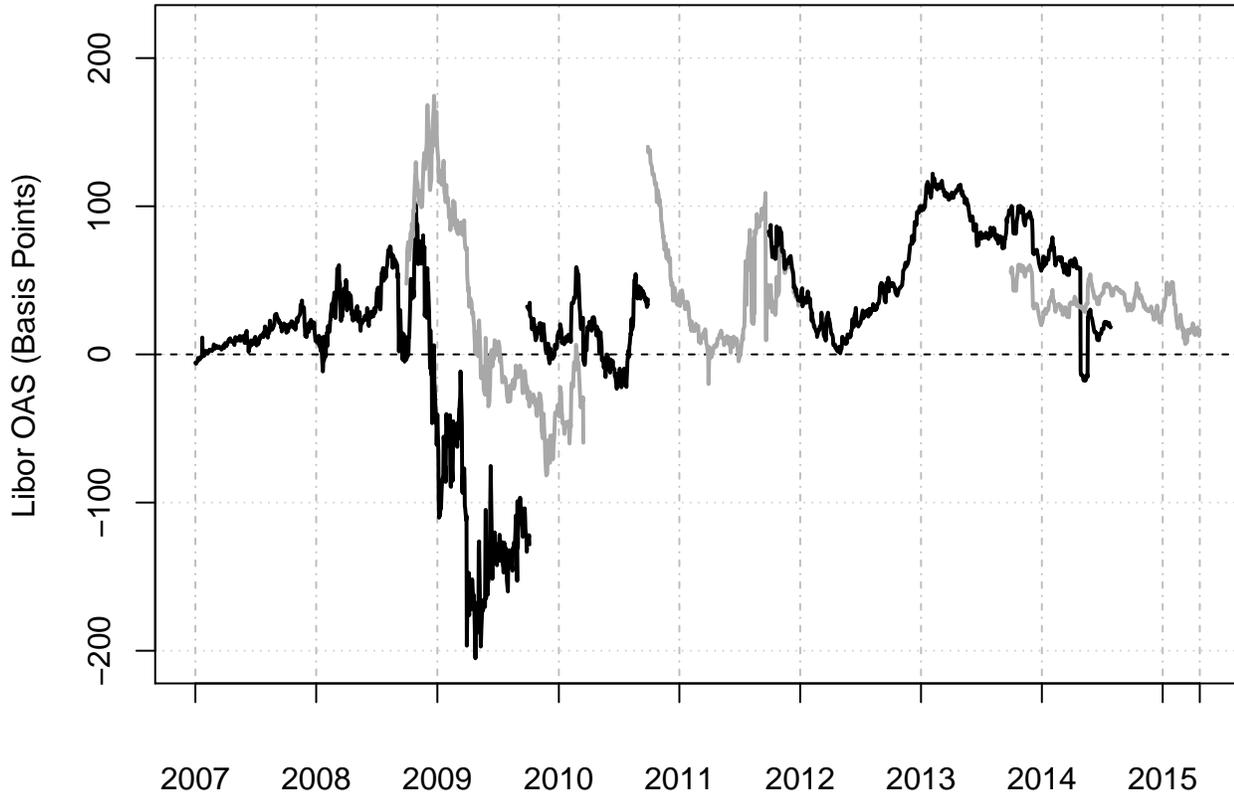
Figure 1. Effects of Prepayment Model Changes on Libor Option-Adjusted Spreads. This figure shows the Libor option-adjusted spread (OAS) in basis points for FNMA 6.50 percent mortgages implied by the series of prepayment models used by a specific major Wall Street dealer. Each line, alternating black and gray, represents a different version of the dealer's prepayment model. During the time period illustrated, the dealer used six different versions of its prepayment model. The Libor option-adjusted spread is highly model dependent, and updates to the prepayment model can lead to large differences in the option-adjusted spread.
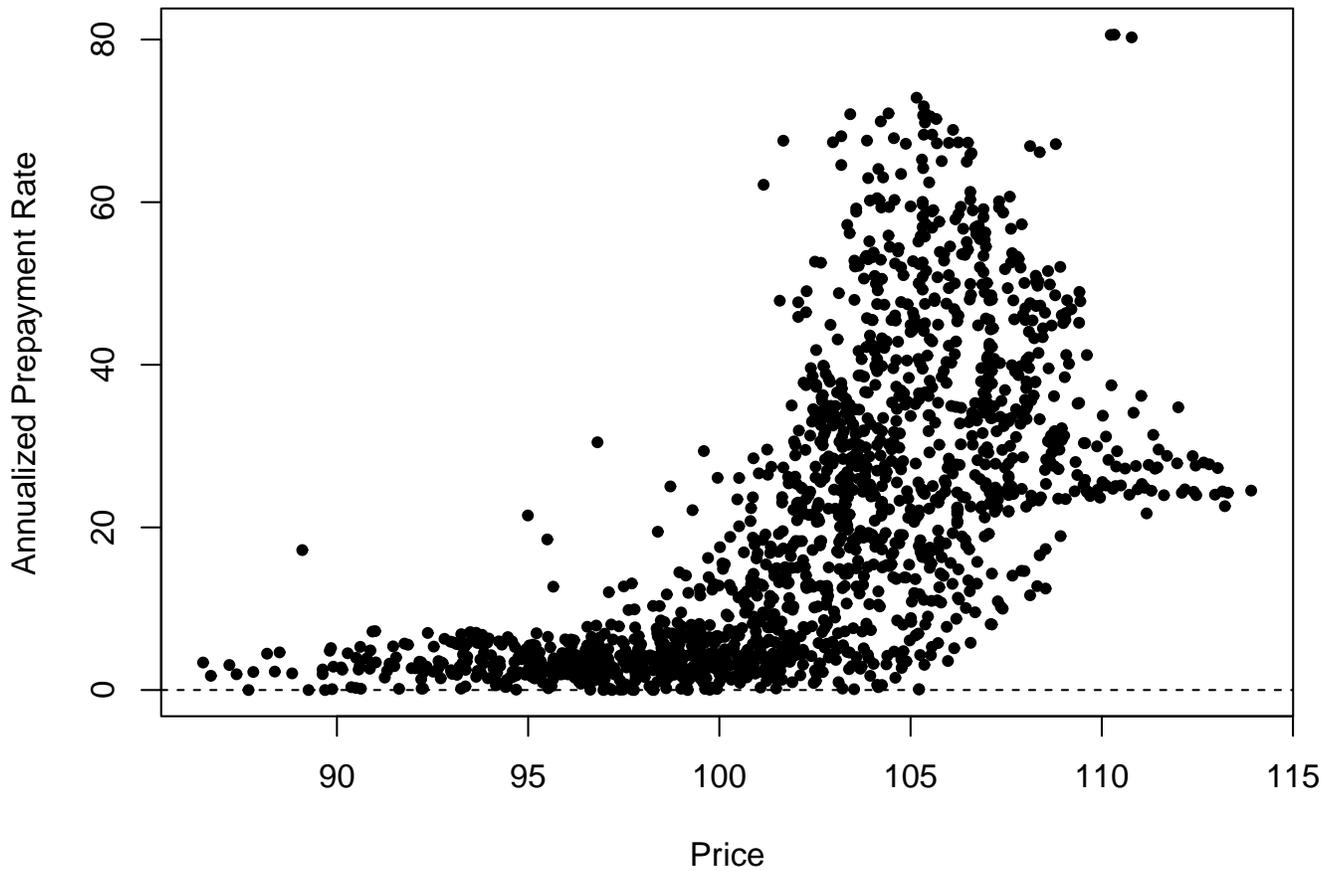
Figure 2. Prepayment Rates for FNMA Mortgage-Backed Securities.
This figure plots the three-month prepayment rates for FNMA mortgage-backed
securities against the prices of mortgage-backed securities. The prepayment rates
are expressed as annualized percentages of the principal balance of the mortgage-
backed security. The data consist of monthly observations for all liquid coupons
over the January 1998 to September 2014 sample period.
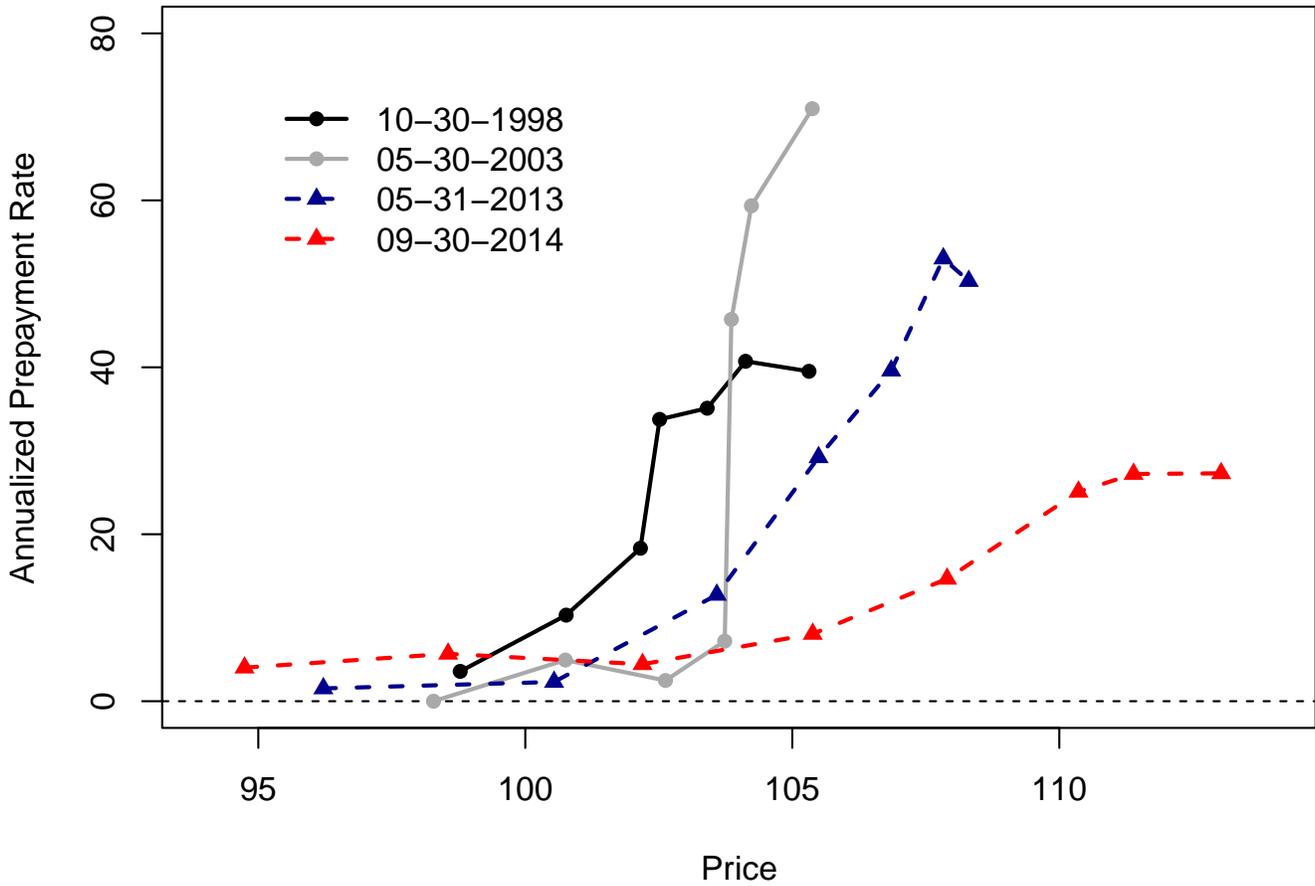
Figure 3. Prepayment Rates for FNMA Mortgage-Backed Securities for Selected Dates. This figure plots the three-month prepayment rates for FNMA mortgage-backed securities against the prices of the mortgage-backed securities for the indicated dates. The prepayment rates are expressed as annualized percentages of the principal balance of the mortgage-backed security.
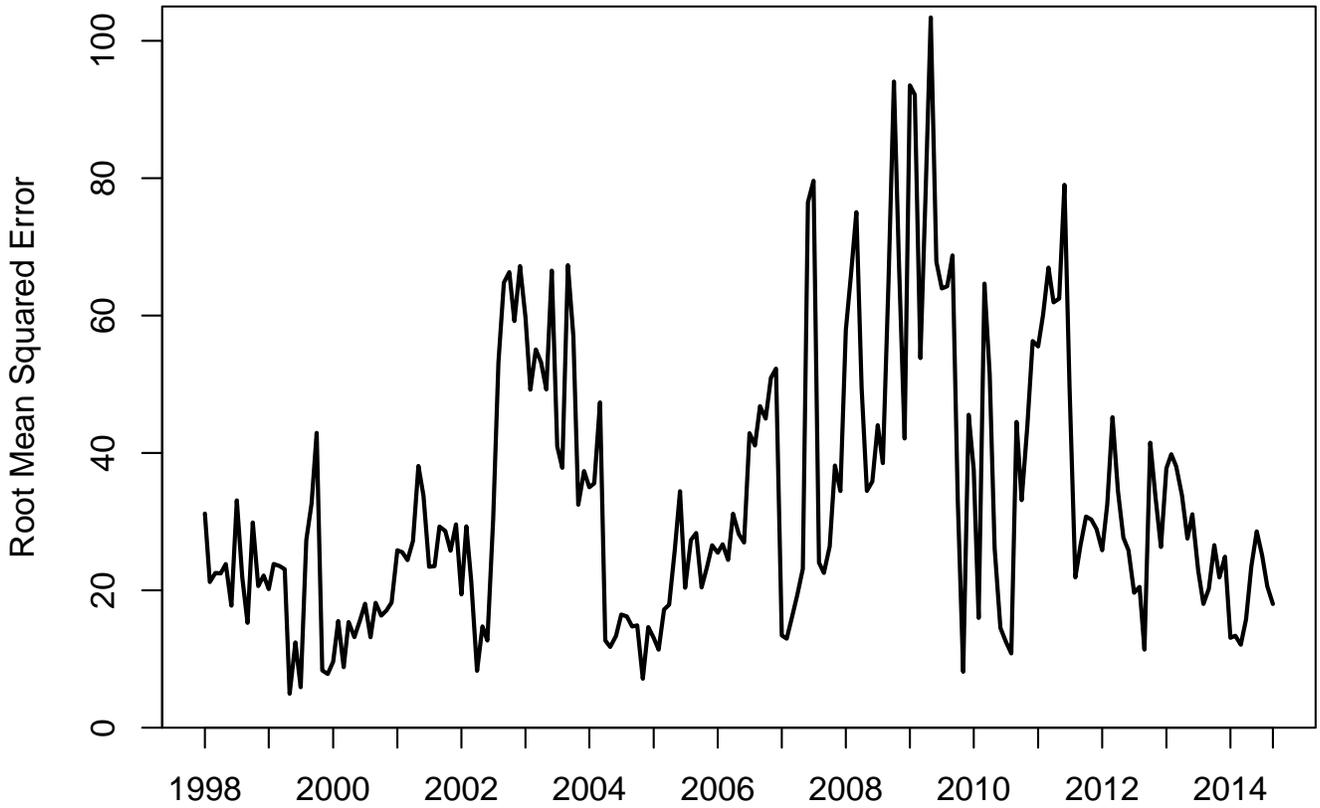
Figure 4. Root Mean Squared Errors from Fitting the Model. This figure plots the time series of root mean squared errors from fitting the model to the cross section of mortgage-backed security prices. The root mean squared error is expressed as cents per $100 notional position.
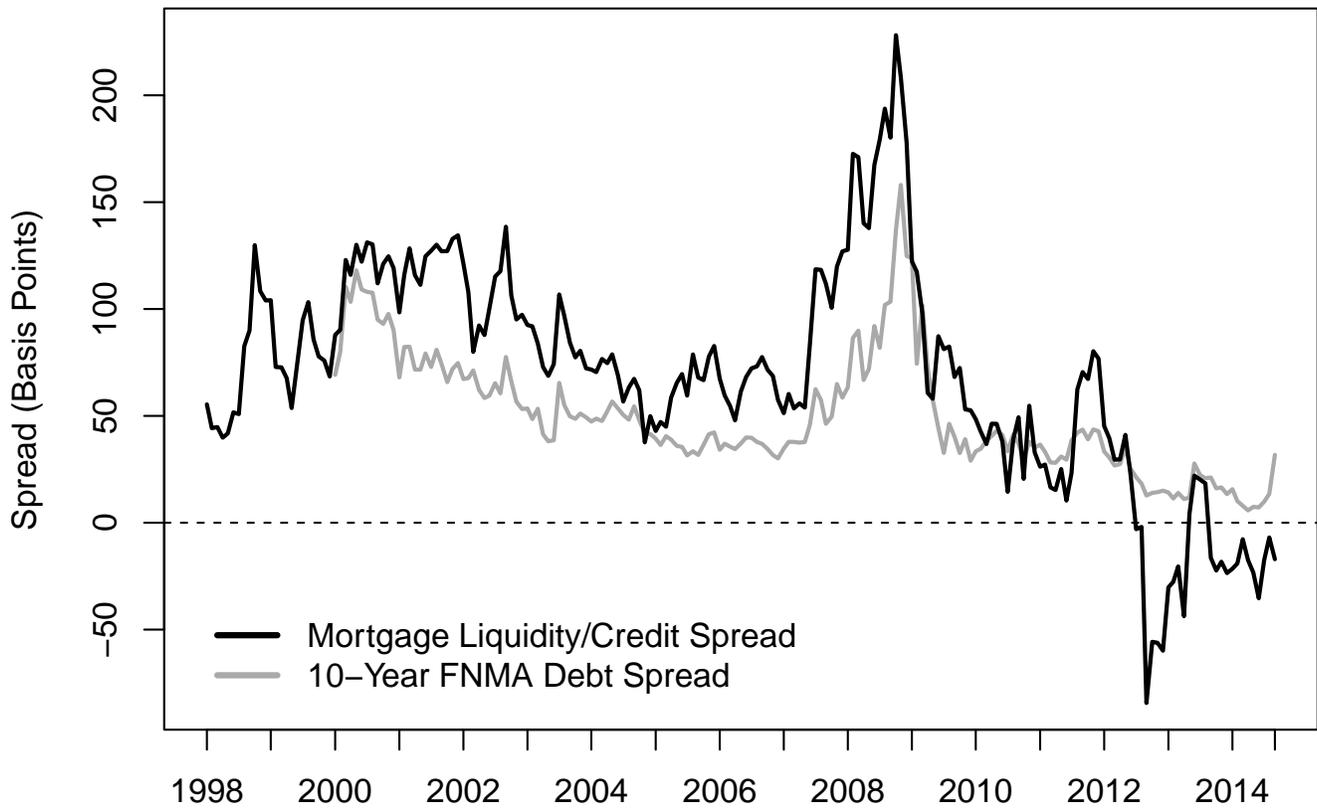
Figure 5. The Implied Credit/Liquidity Spread and the Credit Spread for FNMA Agency Debt. This figure plots the time series of the implied credit/liquidity spread as well as the credit spread for ten-year FNMA agency debt over the ten-year Treasury rate. Both spreads are expressed in basis points.
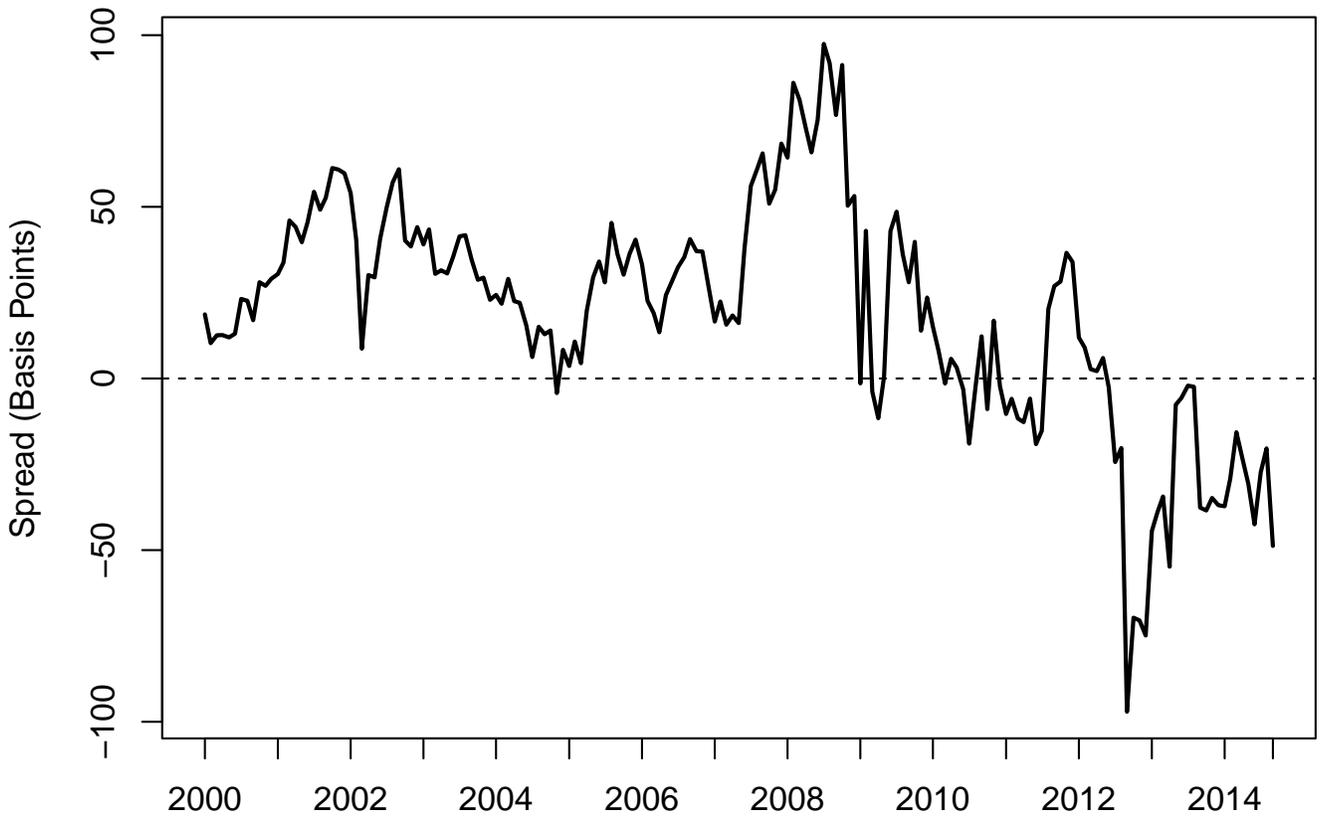
Figure 6. The Estimated Liquidity Spread. This figure plots the time series of the estimated liquidity spread in mortgage-backed securities. The liquidity spread is computed as the difference between the implied credit/liquidity spread and the 10-year FNMA credit spread. The spread is expressed in basis points.
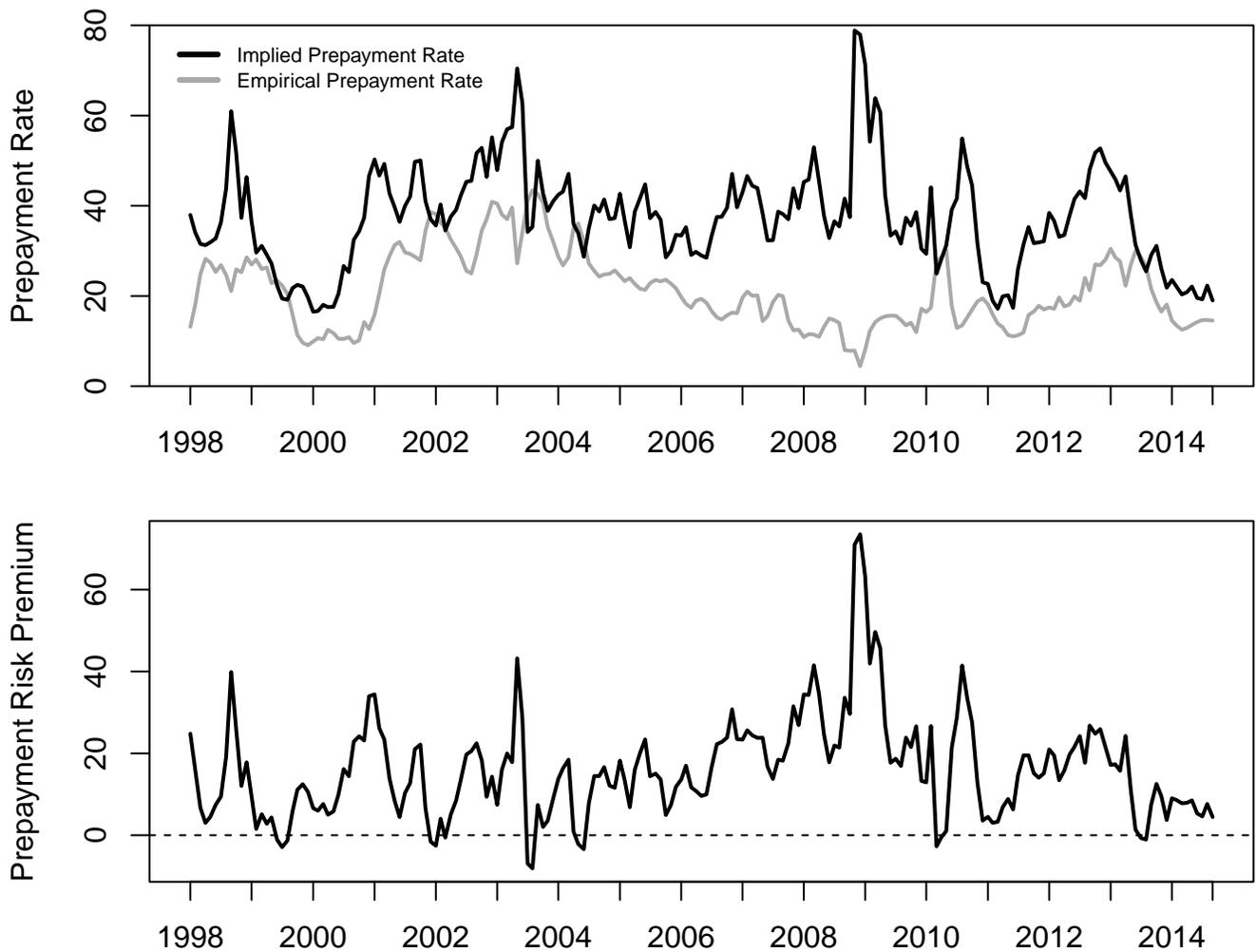
Figure 7. The Implied and Empirical Prepayment Rates and the Prepayment Risk Premium. The upper panel plots the time series of the implied prepayment rate and the empirical prepayment rate (both averaged across all coupon rates for each month). The lower panel plots the time series of the prepayment risk premium defined as the spread between the implied and empirical prepayment rates. The rates and the risk premium are expressed as annualized percentages of the principal balance of the mortgage-backed security.
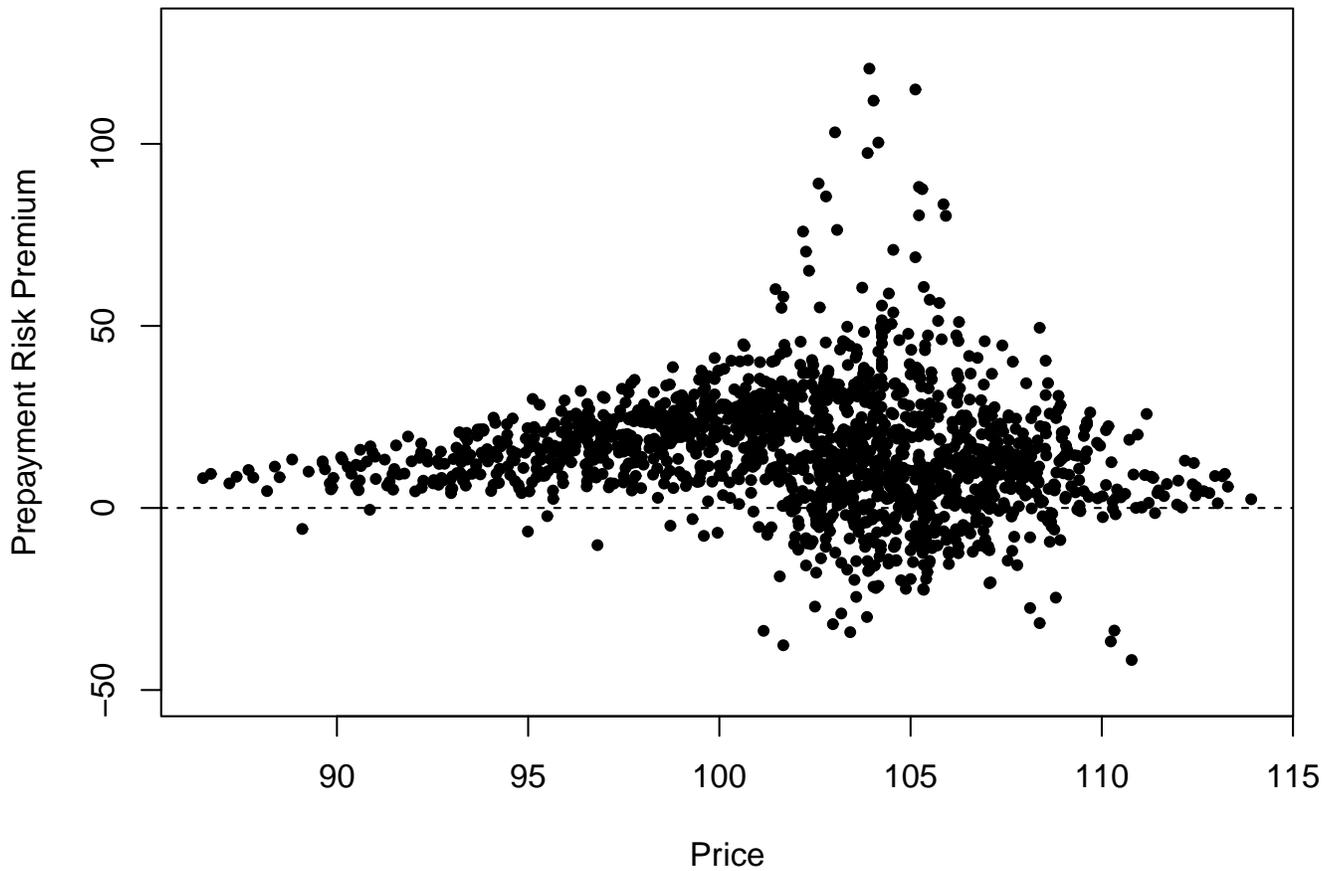
Figure 8. Prepayment Risk Premia. This figure plots the prepayment risk premium for individual mortgage-backed securities as a function of the price of the mortgage-backed security. The risk premia are expressed as annualized percentages of the principal balance of the mortgage-backed security.
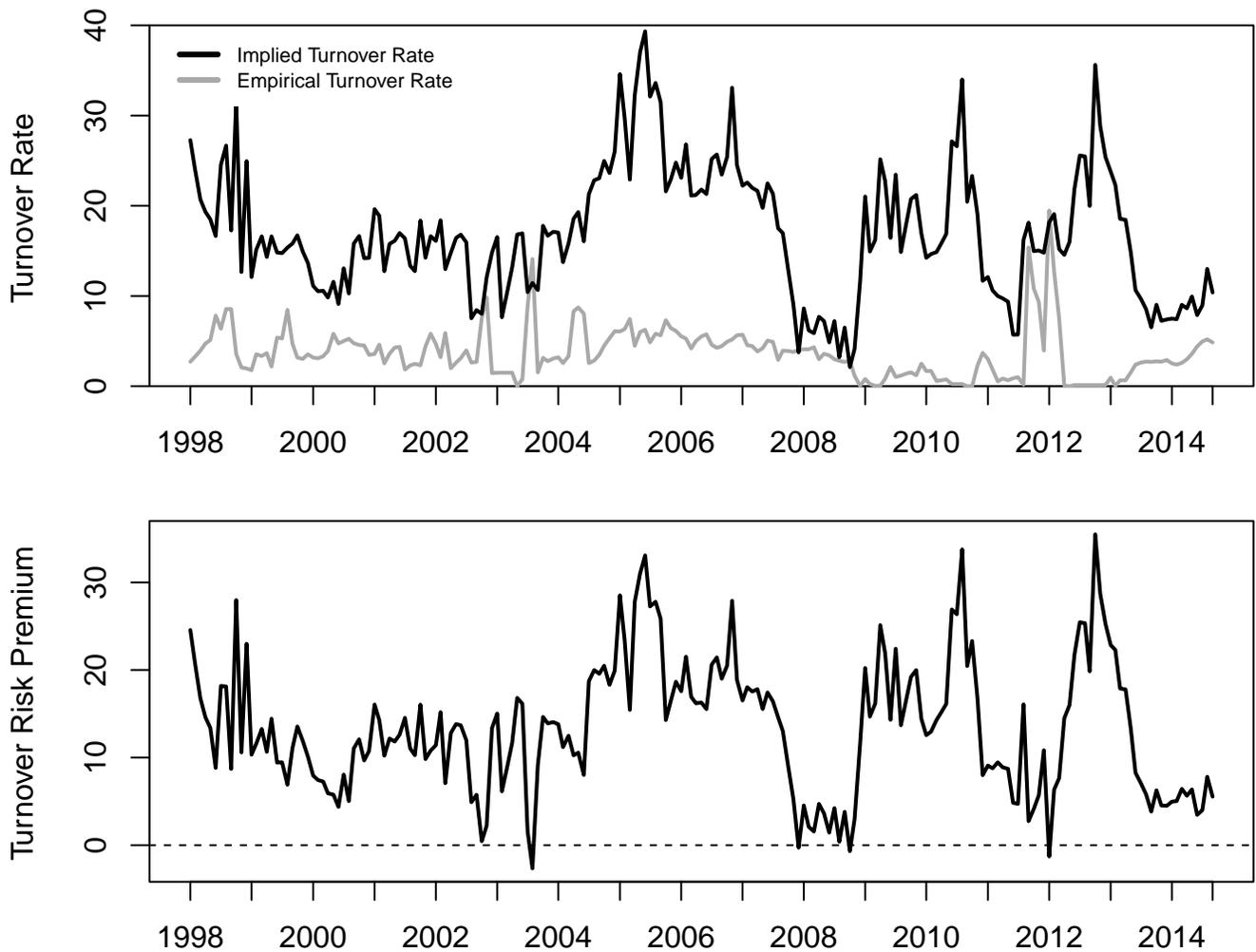
Figure 9. The Implied and Empirical Turnover Rates and the Turn-over Risk Premium. The upper panel plots the time series of the implied turnover rate and the empirical turnover rate. The lower panel plots the time series of the turnover risk premium defined as the spread between the implied and empirical turnover rates. The rates and the risk premium are expressed as annualized percentages of the principal balance of the mortgage-backed security.
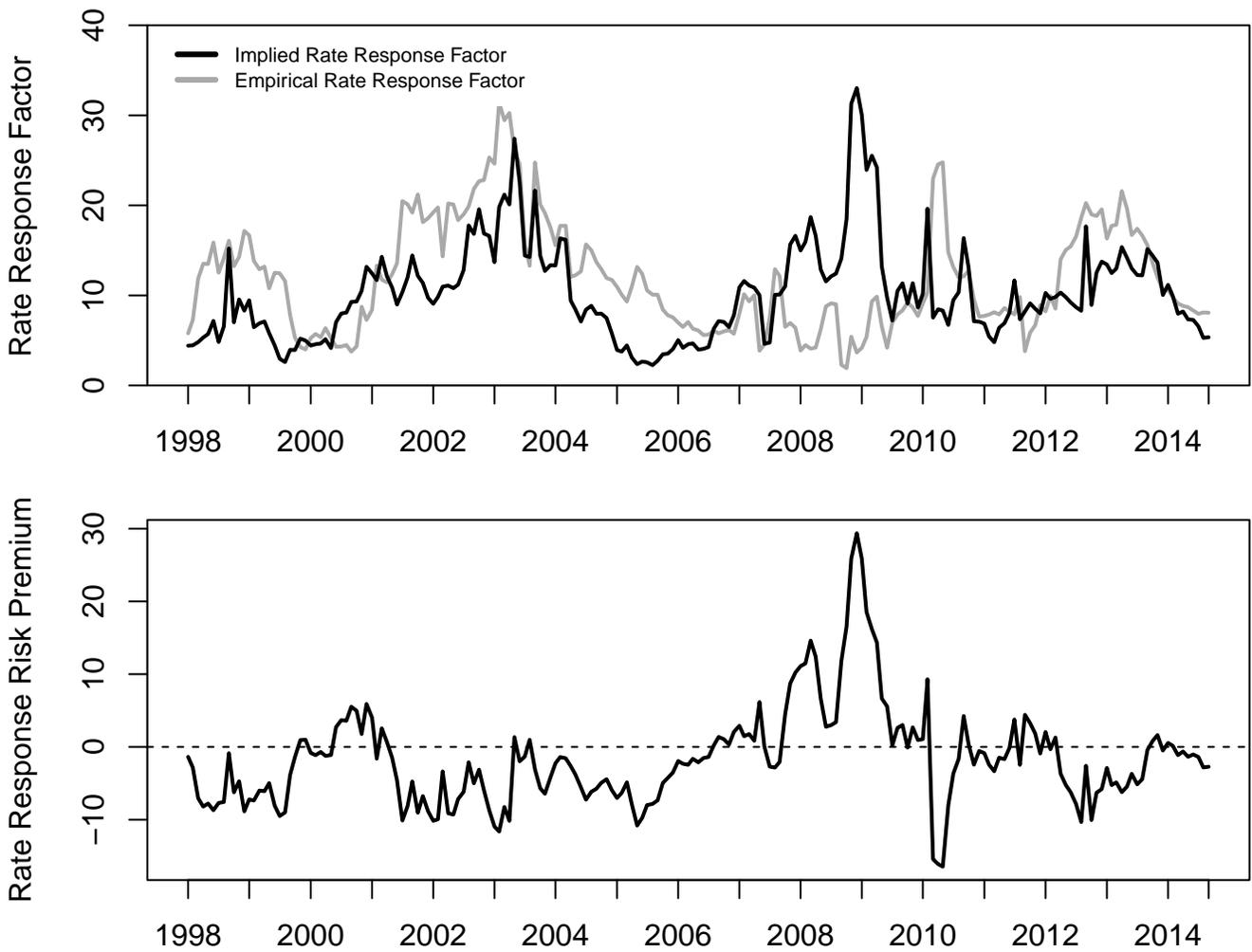
Figure 10. The Implied and Empirical Rate Response Factors and the Rate Response Risk Premium. The upper panel plots the time series of the implied rate response factor and the empirical rate response factor. The lower panel plots the time series of the rate response risk premium defined as the spread between the implied and empirical rate response factors.

Table 1

Major Events in the Agency Mortgage-Backed Securities Market

1	2002	Sep–Dec	High levels of refinancing activity after Federal Reserve lowers interest rates.
2	2003	Jan–Jun	Refinancing activity continues and reaches historically high levels.
3	2005	Jan–Jun	Mortgage delinquency rates reach historically low levels.
4	2007	Jun–Jul	Two Bear Stearns MBS funds suffer large losses and are liquidated. S&P places 612 subprime CDOs on creditwatch.
5	2008	Mar	Financially distressed Bear Stearns avoids bankruptcy by being acquired by JP Morgan.
6		Jul	Federal Reserve Bank of New York is authorized to lend to FNMA and FHLMC if need arises.
7		Sep	FNMA and FHLMC are placed into conservatorship, Lehman Brothers defaults.
8		Dec	Federal Reserve announces QE I program to purchase up to $500 billion of agency MBS.
9	2009	Mar	Home Affordable Refinance Program and Stability Plan announced, making refinancing easier for high LTV loans.
10		Mar	Federal Reserve expands QE I program to purchase up to an additional $750 billion of agency MBS.
11		Dec	Treasury lifts all caps on the amount of FNMA and FHLMC preferred stock it may hold.
12	2010	Mar	QE I purchases of agency MBS ends.
13		Aug	FOMC agrees to keep Fed holdings of securities at constant levels by reinvesting cash flows in Treasuries.
14		Nov	Federal Reserve announces QE II program to purchase up to $600 billion of Treasuries.
15	2011	Jun	QE II purchases of Treasuries ends.
16		Sep	Maturity Extension Program "Operation Twist" announced. Agency MBS cash flows to be reinvested in agency MBS.
17	2012	Sep	Federal Reserve announces QE III program, an open-ended program to purchase up to $40 billion of agency MBS per month.
18	2013	Jun	Ben Bernanke announces "tapering" of QE programs, Dow drops 659 points.
19	2014	Oct	QE III purchases of agency MBS and Treasuries ends.

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http://www.federalreserve.gov/releases/chargeoff/delallsa.htm.

Table 2

Summary Statistics for FNMA Mortgage-Backed Securities. This table reports summary statistics for FNMA mortgage-backed securities with the indicated coupon rates. Average Moneyness denotes the average difference between the coupon rate and the current coupon mortgage rate. Average CPR denotes the average three month constant proportional repayment rate. N denotes the number of observations. The sample consists of monthly observations for the period from January 1998 to September 2014.

Coupon	Average Moneyness	Average CPR	Minimum Price	Average Price	Maximum Price	N
2.50	−0.416	4.794	90.566	96.557	103.219	37
3.00	−0.146	2.449	89.258	98.382	105.555	49
3.50	0.057	6.637	92.250	100.048	107.250	70
4.00	0.470	9.355	87.688	102.255	107.758	74
4.50	0.055	10.244	90.609	99.804	108.313	137
5.00	0.493	15.543	93.484	101.862	111.047	145
5.50	0.345	14.703	86.500	100.816	111.969	184
6.00	0.656	18.082	89.813	102.026	113.031	185
6.50	0.830	21.787	92.531	102.496	113.219	160
7.00	1.208	29.586	94.875	103.632	113.906	150
7.50	1.499	33.953	97.094	103.975	109.563	132
8.00	1.881	34.790	99.188	104.705	108.250	120
8.50	2.104	35.830	101.031	104.974	108.688	91
9.00	2.231	41.759	102.500	105.229	107.563	58
9.50	2.374	22.478	103.000	105.478	107.188	35

Table 3

Estimates of Model Parameters. This table reports the estimate of the model parameters along with their asymptotic standard errors.

Parameter	Value	Std. Error
a	0.00837	0.00151
b	0.90582	0.01256
α_w	0.00196	0.00075
α_x	0.01653	0.00094
α_y	0.02865	0.05145
β_w	0.09795	0.01135
β_x	0.30509	0.00903
β_y	0.02710	0.02357
σ_w	0.00036	0.03092
σ_x	0.01694	0.00629
σ_y	0.07360	0.04235

Table 4

Summary Statistics for the Mortgage-Backed Security Pricing Factors. This table reports summary statistics for the agency credit/liquidity spread (Spread), the turnover rate (Turnover), and the rate response factor (Response). Spread is expressed in basis points. Turnover is expressed as a percentage. The factors are estimated from the cross section of mortgage-backed security prices. The sample consists of monthly observations for the period from January 1998 to September 2014.

Statistic	Spread	Turnover	Response
Mean	72.092	17.211	10.432
Minimum	−85.151	2.123	2.253
Median	70.471	16.430	9.479
Maximum	228.117	39.343	33.032
Standard Deviation	52.213	7.173	5.518
Serial Correlation	0.947	0.799	0.851
Number	201	201	201

Table 5

Results from the Regression of Monthly Changes in the Implied Credit/Liquidity Spread on Explanatory Variables. This table reports the results from the regression of the monthly change in the implied credit/liquidity spread (measured in basis points) on its lagged value, the change in the FNMA credit spread (measured in basis points), the contemporaneous and lagged change in primary dealers' holdings of mortgage-backed securities (measured in $ millions), the change in the general collateral repo rate (measured in basis points), net issuance of mortgage-backed securities (measured in millions), and the ratio of Federal Reserve purchases of mortgage-backed securities to net issuance of mortgage-backed securities. All changes are monthly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with four lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is January 2000 to September 2014.

Variable	Coefficient	t-Statistic
Intercept	0.27457	0.23
Lagged Change in Implied Spread	-0.01131	-0.14
Change in FNMA Spread	0.83522	9.12**
Change in Dealer Inventories	-0.00018	-1.22
Lagged Change in Dealer Inventories	-0.00022	-2.00**
Change in Repo Rate	-5.28683	-1.55
Net MBS Issuance	-0.00004	-1.00
Ratio of Fed Purchases to Net MBS Issuance	0.04702	2.46**
Adj. R^2		0.2441
N		176

Table 6

Results from the Regression of Quarterly Changes in the Empirical and Implied Turnover Rates on Explanatory Variables. This table reports the results from regressions of the quarterly change in the turnover rate on its lagged value, the change in the mortgage foreclosure rate, the lagged change in the mortgage delinquency rate, the lagged growth rate in personal consumption expenditures, the change in US unemployment rate, the return on the Barclays US aggregate bond index, the lagged CRSP value-weighted stock index return, the change in the cash-out mortgage activity index, and the lagged change in the US Home Builders Housing Index. The center panel presents the results for the regression in which the change in the empirical rate response factors is the dependent variable. The right panel presents the results for the regression in which the change in the implied rate response factor is the dependent variable. All variables are measured quarterly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is January 1998 to September 2014.

	Actual Turnover Rate		Implied Turnover Rate	
Variable	Coefficient	t-Statistic	Coefficient	t-Statistic
Intercept	−0.65304	−1.25	−0.43263	−0.39
Lagged Change in Turnover	−0.45168	−4.37**	−0.32543	−3.78**
Foreclosures	−6.35410	−2.94**	1.83879	0.35
Lagged Change in Delinquencies	2.97624	2.18**	−1.73444	−0.79
Lagged Consumption Growth	0.55219	1.96*	−0.48931	−0.71
Change in Unemployment	1.58191	2.18**	0.48101	0.22
Bond Index Return	−0.13608	−0.48	1.02113	2.29**
Lagged Stock Index Return	0.07417	3.65**	−0.21101	−3.18**
Cashout Mortgage Activity	−2.75268	−0.92	20.04234	2.66**
Lagged Housing Index	−0.10529	2.46**	0.22336	1.83*
Adj. R^2		0.303		0.202
N		67		67

Table 7

Results from the Regression of Quarterly Changes in the Empirical and Implied Rate Response on Explanatory Variables. This table reports the results from the regression of the quarterly change in the turnover rate on its lagged value, the change in the bank credit tightening index, the change in the FHLMC loan-to-value ratio for all conventional mortgages, the change in aggregate consumer credit, the lagged growth rate in personal consumption expenditures, the change in the Conference Board consumer confidence index, the change in the US unemployment rate, the return on the Barclays US aggregate bond index, the lagged CRSP value-weighted stock index return, and the lagged change in the US Home Builders Market Index. The center panel presents the results for the regression in which the change in the empirical rate response factors is the dependent variable. The right panel presents the results for the regression in which the change in the implied rate response factor is the dependent variable. All variables are measured quarterly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample period is January 1998 to September 2014.

Variable	Actual Rate Response		Implied Rate Response	
	Coefficient	t-Statistic	Coefficient	t-Statistic
Intercept	2.14403	1.32	−2.84239	−1.80*
Lagged Change in Rate Response	−0.19025	−1.74*	−0.08544	−0.76
Credit Tightening	−0.04499	−1.51	0.02277	0.51
Change in Loan-to-Value Ratio	−0.56175	−0.40	−0.46346	−0.85
Change in Consumer Credit	−0.01208	−1.63	0.00114	0.05
Lagged Consumption Growth	−1.95335	−3.24**	0.63379	0.75
Change in Consumer Confidence	−0.12507	−2.58**	−0.14176	−1.97*
Change in Unemployment	−0.83600	−0.35	−2.48097	−1.21
Bond Index Return	0.34944	1.46	1.30102	3.79**
Lagged Stock Index Return	−0.01787	−0.34	0.03545	0.49
Lagged Housing Index	−0.05454	−0.81	−0.19088	−3.62**
Adj. R^2		0.242		0.489
N		67		67

Table 8

Summary Statistics for Actual and Implied Prepayment Rates, Turnover Rates, and Rate Response Factors. This table reports summary statistics for the indicated variables. The prepayment rates are computed as averages taken over all mortgage-backed securities each month. Prepayment rates and turnover rates are expressed as percentages. Ratio denotes the actual value divided by the implied value. The sample consists of monthly observations for the period from January 1998 to September 2014.

	Average	Standard Deviation	Minimum	Median	Maximum	N
Implied Prepayment Rate	37.257	11.493	16.526	37.239	78.869	201
Actual Prepayment Rate	21.241	8.395	4.443	19.777	43.438	201
Difference	16.015	12.464	−8.104	14.471	73.515	201
Implied Turnover Rate	17.211	7.173	2.123	16.430	39.343	201
Actual Turnover Rate	3.679	2.832	0.000	3.320	19.470	201
Difference	13.532	7.496	−2.653	13.268	35.491	201
Implied Rate Response	10.432	5.518	2.253	9.479	33.032	201
Actual Rate Response	11.907	6.043	1.921	10.562	31.445	201
Difference	−1.475	6.823	−16.462	−2.107	29.375	201

Appendix

In this appendix, we first describe the variables used in the study along with the sources for the data. We then describe the methodology used in the estimation of the model.

A.1 Data Sources

Table A1 presents the definitions and data sources for the variables used in the study.

A.2 Fannie Mae Mortgage-Backed Security Cash Flows

The pricing data are from the to-be-announced (TBA) market for 30-year Fannie Mae (FNMA) mortgage-backed securities (MBS). Before describing how we estimate our model, it is important to consider the timing of cash flows generated by a FNMA TBA trade.

TBA trades settle according to a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Thirty-year FNMA MBS falls into SIFMA's Class A, which typically settles during the second week of the month. Because we select prices at the end of each month, the settlement date corresponding to these observations is around the second week of the following month (the exact settlement dates can be found on Bloomberg). On the notification date, two days prior to settlement, the buyer is notified of the exact pools to be delivered. On the settlement date, the buyer transfers a payment to the seller, which consists of the agreed upon price (which we observe at month end) plus accrued interest on the exact face value of the pools identified on the notification date. On the record date, the last day of the month, Fedwire records the buyer as the new holder of the security. On the fifth or sixth business day of the next month, the pool factors (current balance / original balance) are released. The pool factor determines the new face value of the mortgage after accounting for scheduled principal payments and prepayments from the previous month. Then, on the payment date later that month, the scheduled principal payments, interest payments, and prepayments, less servicing and guaranty fees are passed to the holder of the security. For FNMA MBS, the payment date is the 25th of the month. If the 25th day happens to fall on a bond market holiday or a weekend, the payment is made on the following business day. A timeline for the timing of payments for a hypothetical TBA trade is shown in Table A2.

A.3 Adjustment for Fees

Fixed-rate mortgage pools consist entirely of fixed-rate loans, but the underlying loans may bear different fixed rates of interest. Interest on a fixed-rate pool is set on the issue date of the related certificates and it is equal to the interest

rates less the fee percentages for each loan in the pool. The fee percentage is the sum of the servicing fee and the guaranty fee for that loan. Fixed-rate loans in a single pool have interest rates that are within a two percent range (sometimes a wider range may be allowed). However, the pass-through rate of each loan in fixed rate pool is the same. Therefore, the pass-through rate will not change if prepayments occur.

Consider the cash flow generated by a pass-through of an individual fixed rate mortgage. Prior to either prepayment or default, the owner of the pass-through receives the constant cash flow c generated by the mortgage loan less the servicing and guaranty fees, which are a percentage of the principal balance I_t. Denote the servicing and guaranty fees by the constant g. Then, the cash flow generated by the pass-through security, c_t^{PT}, in absence of prepayment, is

$$c_t^{PT} = c - gI_t. \qquad (A1)$$

Therefore, the value of a FNMA MBS, after accounting for fees, is given by

$$F(m, T) = E^Q \left[\int_0^T \exp\left(-\int_0^t r_s + w_s \, ds\right) \, N_t \, (c + (p_t - g) \, I_t) \, dt \right], \qquad (A2)$$

where

$$p_t = x_t + y_t \, \max(0, \, m - a - br_t). \qquad (A3)$$

A.4 Valuation

Due to the independence of w_t from the other stochastic processes, we can write

$$F(m, T) = \int_0^T S(t) \, E^Q \left[\exp\left(-\int_0^t r_s \, ds\right) \, N_t \, (c + (p_t - g) I_t) \right] \, dt, \qquad (A4)$$

where

$$S(t) = E^Q \left[\exp\left(-\int_0^t w_s \, ds\right) \right]. \qquad (A5)$$

The expression for $S(t)$ is given by

$$S(t) = A(t) \exp(-w_0 \, B(t)), \qquad (A6)$$

where

$$A(t) = \exp\left(\left(\left(\frac{\sigma_w^2}{2} - \frac{\alpha_w}{\beta_w}\right)t + \left(\frac{\alpha_w}{\beta_w^2} - \frac{\sigma_w^2}{\beta_w^3}\right)(1 - \exp(-\beta_w t))\right.\right.$$
$$\left.\left. + \frac{\sigma_w^2}{4\beta^3}(1 - \exp(-\beta_w t))\right), \right. \tag{A7}$$

$$B(t) = \frac{1}{\beta_w}(1 - \exp(-\beta_w t)). \tag{A8}$$

Applying the separation theorem of Longstaff (1990), we can bring the discount factor out of the expectation. Hence,

$$F(m,T) = \int_0^T S(t)\, D(t)\, \hat{E}\left[\, N_t\, (c + (p_t - g)\, I_t)\,\right] dt, \tag{A9}$$

where $D(t)$ is the price of a zero-coupon bond and the expectation $\hat{E}[\,\cdot\,]$ is taken with respect to the forward measure.

A.5 Adjustments For Discrete Cash Flows

In this section, we adjust the mortgage valuation formula to account for the actual cash flows from a TBA trade. Let

$$\tilde{T} \equiv \{0, t_s, t_1, \ldots, t_K\}, \tag{A10}$$

be the set of points in time related to the payments associated with a mortgage-backed security with K months until maturity. The valuation date, or trade date, is $t = 0$. The MBS settles at $t = t_s$, and the MBS investor receives payments on dates t_1 through t_K. Since the settlement dates are fixed by SIFMA, the amount of time from the valuation date $t = 0$ through the settlement date $t = t_s$ varies depending on the trade date. Also, due to holidays and weekends, each time step after settlement, i.e. $\Delta t_i \equiv t_{i+1} - t_i$, $i = 1, \ldots, K - 1$, may vary by a couple of days. Let CF_i be the cash flow received by the mortgage investor at time t_i. In our framework, the value of the mortgage is

$$F(m, K) = \frac{1}{S(t_s)\, D(t_s)} \sum_{i=1}^K S(t_i)\, D(t_i)\, \hat{E}[\,\mathrm{CF}_i\,]. \tag{A11}$$

To determine the cash flow CF_i at t_i, we can apply standard mortgage cash flow formulas (recall that in continuous time, the cash flow is $N_t\,[c + (p_t - g)\, I_t]$). Following Hayre (2001), for each dollar of a mortgage in month i,

$$\text{Monthly payment} = \text{PAY}_i = \frac{m/12}{1 - (1 + m/12)^{-K}}, \qquad (A12)$$

$$\text{Balance (end of month)} = \text{BAL}_i = \frac{1 - (1 + m/12)^{-K+i}}{1 - (1 + m/12)^{-K}}, \qquad (A13)$$

$$\text{Principal portion of payment} = \text{PRIN}_i = \text{PAY}_i \times (1 + m/12)^{-K-1+i}, \quad (A14)$$

$$\text{Interest portion of payment} = \text{INT}_i = \text{PAY}_i - \text{PRIN}_i. \qquad (A15)$$

Let

$$\tilde{T}_{CF} \equiv \{0, \tau_1, \ldots, \tau_K\}, \qquad (A16)$$

be the set of points in time relevant to determine the monthly cash flows of the MBS. This set corresponds to month-ends. For the example in Table A2, $t = 0$ is March 31st, the month end before the settlement date, and τ_1 is April 30th. Note that it is possible that τ_1 is either before or after the trade date depending on whether the trade date and settlement date occur in the same month. However, because the data are observed at each month end, the elements of $\tilde{T}$ and $\tilde{T}_{CF}$ are ordered as

$$0 < t_s < \tau_1 < t_1 < \tau_2 < t_2 < \cdots < \tau_K < t_K, \qquad (A17)$$

as shown in the example in Table A2.

Recall that N_t represents the fraction of the mortgage pool that has not yet prepaid (i.e. a survival factor). To keep track of monthly prepayments, we calculate the single monthly mortality (SMM), a common object in mortgage modeling. SMM is fraction of the pool outstanding at the beginning of the month that is prepaid during the month. Hence,

$$\text{SMM}_i = \frac{N_{\tau_{i-1}} - N_{\tau_i}}{N_{\tau_{i-1}}}. \qquad (A18)$$

Therefore, the prepayments in a given month i, PP_i, can be written as

$$\text{PP}_i = \left(\text{BAL}_{i-1} \times N_{\tau_{i-1}} - \text{PRIN}_i \times N_{\tau_{i-1}} \right) \times \text{SSM}_i. \qquad (A19)$$

Note that the cash flow CF_i received by the investor at t_i reflects the payments (scheduled and prepaid) at τ_i from the underlying mortgage loans, less servicing and guaranty fees g. Therefore,

$$\mathrm{CF}_i = \mathrm{PRIN}_i \times N_{\tau_{i-1}} + \mathrm{PP}_i + \frac{m-g}{m} \times \mathrm{INT}_i \times N_{\tau_{i-1}}. \qquad (A20)$$

Given paths of r_t, x_t, and y_t, we calculate a path of p_t. After integration and exponentiation, we calculate N_t for the relevant time points. Then, the standard mortgage formulas provide the cash flows.

A.6 The Discount Function

We collect historical data on nominal-constant maturity Treasury rates from the Federal Reserve's H.15 statistical release. Then, we use a standard cubic spline to bootstrap the prices of zero-coupon bonds $D(t)$ for the relevant time points for up to 30 years. For a discussion of this methodology, see Longstaff, Mithal, and Neis (2005).

A.7 Estimation of Interest Rate Dynamics

We assume that the interest rate follows the Ornstein-Uhlenbeck process

$$dr_t = (\alpha_r - \beta_r \, r_t) \, dt + \sigma_r \, dZ_r. \qquad (A21)$$

Mortgage rates are most closely related to the 10-year rate. Therefore, we fit the 10-year zero-coupon rate exactly. From the time series of $D(10)$, we estimate α_r, β_r, and σ_r via conditional maximum likelihood. Note that this provides us with parameters under the real-world probability measure. However, the refinance incentive $\max(0, m - a - br)$ involves an affine function of r. Therefore, when we estimate a and b later in the procedure, we are effectively allowing for a constant risk premium.

Conditional on $r(s)$, the interest rate $r(t)$ is normally distributed with mean $r(s)e^{-\beta(t-s)} + \frac{\alpha}{\beta}\left(1 - e^{-\beta(t-s)}\right)$ and variance $\frac{\sigma^2}{2\beta}\left(1 - e^{-2\beta(t-s)}\right)$. Instead of estimating α_r, β_r, and σ_r directly, we estimate the following functions of the parameters: $\mu = \frac{\alpha}{\beta}, \gamma = e^{-\beta\delta}$ and $V^2 = \frac{\sigma^2}{2\beta}\left(1 - e^{-2\beta\delta}\right)$, where δ denotes the time step of the observed 10-year rates $r_0, r_1, \ldots, r_n$ of r. Let $f(r_i|r_{i-1}; \gamma, \mu, V^2)$ denote the probability density function of r_i conditional on r_{i-1}. Hence, it follows that

$$f(r_i|r_{i-1}; \gamma, \mu, V^2) = \frac{1}{\sqrt{2\pi}V} \exp\left(-\frac{(r_i - (\gamma r_{i-1} + \mu(1-\gamma)))^2}{2V^2}\right). \qquad (A22)$$

Therefore, the log-likelihood function for the observed interest rates is

$$\text{LLK}\left(\gamma, \mu, V^2\right) = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log V^2 - \frac{1}{2V^2}\sum_{i=1}^{n}(r_i - \gamma r_{i-1} - \mu(1-\gamma))^2.$$

$$(A23)$$

Summing over all n observations and then maximizing the log-likelihood function gives $\hat{\alpha}_r = 0.008120$, $\hat{\beta}_r = 0.2168$, and $\hat{\sigma}_r = 0.009511$. Rather than use the estimator of the volatility parameter in the simulations, we adjust the volatility parameter of the Vasicek model each month to match the basis point volatility of a one-year into ten-year swap option. The results, however, are robust to the specification of the interest rate model.

A.8 Estimation Methodology

The estimation of the model can be viewed as consisting of three steps.

1. First, we select an initial parameter vector θ, where $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y\}$.

2. Second, conditional on θ and for each month t during the sample period, we solve for the values of w_t, x_t, and y_t that best fit the model to the prices for the coupon stack (the cross section of mortgage-backed securities with different coupon rates) by minimizing the root mean squared error (RMSE).

   Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. The simulation step solves

$$F(m, K) = \frac{1}{S(t_0)\ D(t_0)}\sum_{i=1}^{K} S(t_i)\ D(t_i)\ \hat{E}[\ CF_i\ ], \qquad (A24)$$

   for given values of w_t, x_t, and y_t. Since $D(t)$ is computed from the Treasury yield curve and $S(t)$ has a closed-form solution, we can focus on the expectation in the equation above. We simulate paths of r_t, x_t, and y_t, and then compute a path of p_t since

$$p_t = x_t\ +\ y_t\ \max\left(0,\ m\ -\ a\ -\ b\ r_t\right). \qquad (A25)$$

   We use monthly time steps, and the square-root processes are simulated following the algorithm presented in Shao (2012). From p_t, we compute the survival factors $N(t)$ for each month, and then compute the mortgage cash flows CF_i. The average cash flows along each path provides the estimate of

the expectation. Given the value of the expectation, we then solve for the mortgage price.

The RMSE of the simulated prices and the market prices provides the objective function for the optimization. We use the controlled random search (CRS) algorithm of Kaelo and Ali (2006) to solve for the initial values of w_t, x_t, y_t which minimize the RMSE for each date t.

3. Third, we apply the CRS algorithm to the parameter vector θ to find the vector that results in the lowest global root mean square error (RMSE). The outputs are the parameter values and the time series of state variables.

Table A1

Data Definitions and Sources

	Data	Frequency	Description and Source
1	FNMA MBS Prices	Monthly	Proprietary data set provided by a major Wall Street MBS dealer. Data cross validated with Bloomberg data.
2	FNMA CPR Data	Monthly	Three-month CPR prepayment rate data collected and provided by eMBS Inc.
3	Treasury CMT Data	Monthly	Constant maturity Treasury rates from Federal Reserve H.15 Selected Interest Rates Release.
4	Discount Function D(T)	Monthly	Discount function out to 30 years bootstrapped from Treasury CMT rates using standard cubic spline interpolation algorithm as described in Longstaff, Mithal, and Neis (2005).
5	Interest Rate Volatility	Monthly	Basis point volatility for 1-year into 10-year swaptions. Proprietary data set provided by major Wall Street MBS dealer.
6	FNMA Agency Credit Spread	Monthly	Ten-year FNMA note yield over ten-year Treasury yield. Proprietary data set provided by a major Wall Street MBS dealer.
7	Primary Dealers' MBS Holdings	Monthly	Federal Reserve Bank of New York: http://www.newyorkfed.org/markets/gsds/search.html.
8	Repo Rate	Monthly	General collateral Treasury three-month repo rate provided by Bloomberg.
9	Net MBS Issuance	Monthly	Net MBS issuance in $ millions provided by eMBS Inc.
10	Federal Reserve MBS Purchases	Weekly	Board of Governors of the Federal Reserve System, Mortgage-Backed Securities Held by the Federal Reserve: All Maturities [MBST], retrieved from FRED, Federal Reserve Bank of St Louis, https://research.stlouisfed.org/fred2/series/MBST. Weekly data aggregated to monthly and quarterly frequency.
11	Consumption Expenditures	Monthly	Seasonally adjusted at annual rates. US Bureau of Economic Analysis, Personal Consumption Expenditures [PCE], retrieved from FRED, Federal Reserve Bank of St. Louis, https://research.stlouisfed.org/fred2/series/PCE.
12	Unemployment Rate	Monthly	Seasonally adjusted unemployment rate provided by Bureau of Labor Statistics, http://data.bls.gov/timeseries/LNS14000000.
13	Delinquency Rate	Quarterly	Seasonally adjusted. Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTDLQT Index).
14	Foreclosure Rate	Quarterly	Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTFORE Index).
15	Cash-Out Mortgage Ratio	Quarterly	Freddie Mac, Office of Chief Economist, Cash-Out Refinance Report, 4Q 2014, http://www.freddiemac.com/finance/refinance_report.html.

Table A1 Continued

	Data	Frequency	Description and Source
16	Barclays Bond Index	Monthly	Monthly returns on the Barclays US Aggregate Bond Index.
17	Stock Market Return	Monthly	Monthly valued-weighted return on all NYSE, AMEX, and NASDAQ firms, Center for Research in Security Prices.
18	Housing Market Index	Monthly	National Association of Home Builders Market Index. This index is based on a monthly survey of NAHB members of the single-family housing market.
19	Consumer Credit	Monthly	Seasonally adjusted. The outstanding amount of credit used by consumers excluding mortgage loans. Federal Reserve, provided by Bloomberg (USHBMIDX Index).
20	Conventional LTV Ratios	Quarterly	Loan to value ratios for all conventional loans. Data reported by the Monthly Interest Rate Survey (MIRS) of the Federal Housing Finance Agency (FHFA).
21	Bank Tightening Index	Quarterly	Percentage of banks tightening mortgage lending standards from Board of Governors of the Federal Reserve System, Net Percentage of Banks Tightening Standards for Mortgage Loans [H0SUBLPDHMSNQ], retrieved from FRED, Federal Reserve Bank of St. Louis https://research.stlouisfed.org/fred2/series/H0SUBLPDHMSNQ.
22	Consumer Confidence Index	Monthly	The Conference Board. Provided by Bloomberg (CONCCONF Index).

Table A2

Cash Flow Time Line for a Hypothetical 30-Year Fannie Mae TBA trade. This table shows the key events and cash flows surrounding a 30-year Fannie Mae TBA trade executed on March 31.

Date	Event	Time	Note
March 31	Trade Date	0	Trade parameters: issuer, maturity, coupon, face value, price, settlement date.
April 6	Factor Date		Pool factors are released by FNMA.
April 12	48-Hour Day		The buyer is notified of the pools the seller will deliver to settle the TBA trade.
April 14	Settlement Date	t_s	The buyer wires the payment to the seller.
April 30	Record Date	τ_1	Fedwire records the buyer as the new holder of record.
May 7	Factor Date		Pool factors are released by FNMA, reflecting January prepayments.
May 25	Payment Date	t_1	The buyer receives the first payment from the MBS.
$\vdots$	$\vdots$		$\vdots$
May 31	Month End	τ_2	Payment at t_2 reflects prepayments over τ_1 to τ_2.
$\vdots$	$\vdots$		$\vdots$
June 26	Payment Date	t_2	The buyer receives the second payment from the MBS.
$\vdots$	$\vdots$		$\vdots$
June 31	Month End	τ_3	Payment at t_3 reflects prepayments over τ_2 to τ_3.
$\vdots$	$\vdots$		$\vdots$
July 25	Payment Date	t_3	The buyer receives the third payment from the MBS.
$\vdots$	$\vdots$		$\vdots$