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The Benefits of Advance Booking Discount Programs: Model and Analysis

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Consider a retailer who sells perishable seasonal products with uncertain demand. Due to the short sales season and long replenishment lead times associated with such products, the retailer is unable to update demand forecasts by using actual sales data generated from the early part of the season and to respond by replenishing stocks during the season. To overcome this limitation, we examine the case in which the retailer develops a program called the "advance booking discount" (ABD) program that entices customers to commit to their orders at a discount price prior to the selling season. The time between placement and fulfillment of these precommitted orders provides an opportunity for the retailer to update demand forecasts by utilizing information generated from the precommitted orders and to respond by placing a cost-effective order at the beginning of the selling season. In this paper, we evaluate the benefits of the ABD program and characterize the optimal discount price that maximizes the retailer's expected profit.

Key words: retailing; marketing/manufacturing interfaces; pricing
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1. Introduction

To compete in global markets, many companies launch new products and phase out old products rapidly. For example, Billington et al. (1998) report a list of challenging problems associated with rapid product replacements, which includes accurate demand forecasting and inventory management. As product life cycles shorten, the fundamental issues in managing interacting areas such as pricing, forecasting, and inventory control mimic those of fashion products. For instance, if a company overforecasts and orders more than the actual demand, then it has to reduce prices to sell the leftover inventory at the end of the selling season. According to a study conducted by Pashigan (1988), the average markdown for fashion merchandise in the apparel industry is around 16%. Conversely, out-of-stock products lead to considerable losses in potential revenue. Thus, responding accurately to changing demand patterns by forecasting and translating forecasts into an efficient supply plan are key ingredients for success, especially for products with short life cycles and high demand uncertainty.

Broadly speaking, the strategies retailers use to match supply with demand for these products depend on the duration of the sales season and the length of the replenishment lead time. When the lead time is suitably shorter than the sales season, retailers can use actual sales data in the early part of the season to update forecasts and replenish supply for the later portion. Fisher et al. (1999) provide a model for optimizing replenishment for retail fashion products under this strategy. However, this tactic has two crucial requirements. First, the sales season has to be sufficiently long and the sales at the beginning of the season has to be fairly representative of the sales for the remainder of the season (so that one can update the demand forecast by using the sales data during the early part of the selling season). Second, the replenishment lead time has to be shorter than the selling season (so that one can respond to the updated demand forecast by replenishing stocks within the selling season). These requirements may not be met in various situations. For example, consider the sales of pumpkin pies at a supermarket during the Thanksgiving holiday in the United States. The selling season is very short (approximately five days), which makes it difficult to capture the sales data during the early part of the selling season. Since replenishment lead times are usually long (approximately five to seven days), it is not possible for the bakery to respond to an updated demand forecast. Consequently, to make supply meet demand under these circumstances, one needs to consider other strategies.

When replenishment based on early season sales is not possible, retailers use various other strategies to effectively match supply with uncertain demand. These include merchandise testing (Fisher and Rajaram 2000), postponement of product differentiation (Lee and Tang 1996), backup agreements with manufacturers (Eppen and Iyer 1997), and inventory pooling (Eppen 1979). In this paper, we consider one such strategy in which the retailer develops a program called an "advance booking discount" (ABD) program that entices customers to commit to their orders at a discount price prior to the selling season. However, these precommitted orders are nonrefundable and filled during the selling season. While the origin of the ABD program is unknown, we have observed its practice at the Maxim's bakery in Hong Kong.[1] Maxim's bakery dominates the sales in the baked-goods market in Hong Kong largely due to its reputation for quality and convenience. Approximately six years ago, Maxim's launched the ABD program for the sales of moon cakes. The moon cake, a traditional Chinese cake stuffed with lotus seed paste and egg yolk, is consumed by the Chinese when celebrating the mid-Autumn festival. Maxim's ABD program operates in the following manner: During the month prior to the mid-Autumn festival, customers can precommit their orders at any of the Maxim's cake shops at 25% off the regular price. Customers pay the discounted price when placing their orders in advance and receive redemption coupons for pick up during the week prior to the mid-Autumn festival. No order cancellation or refund is permitted. Maxim's guarantees the availability of the moon cakes only to those customers who participate in the ABD program. If customers do not participate in the ABD program, they can always try to buy the cake during the week prior to the mid-Autumn festival at the regular price. In addition, we have noticed that several Web-based retailers such as Electronics Boutique, Amazon.com, and Movies Unlimited also commonly use ABD programs. At these retailers, customers can typically guarantee availability of new releases of a broad category of products such as popular movies, video games, toys, music CDs, and books by preordering at a discount. The product is then delivered to the customer after they have been released to the broader market.

There are several important benefits associated with the ABD program. First, the ABD program extends the selling season without the need for immediate delivery. This enables the retailer to sell the product over a longer period of time by being less constrained by production capacity. Second, under the ABD program, the placement of the precommitted orders takes place prior to the season while the fulfillment of these precommitted orders takes place during the season. Therefore, the time window between these two events provides an opportunity for the retailer to utilize the advance booking data to generate a better demand forecast prior to the start of the selling season. Such improved forecasts enable the retailer to place a more accurate order at the start of the season, which in turn reduces over-stock and under-stock costs and improves customer service levels. Third, the ABD program reduces financial risks because payments are received from the precommitted orders prior to the selling season. Fourth, the ABD provides the retailer with the ability to carry out price discrimination between their customers. Finally, price formats such as the ABD could increase the shopping frequency of the customer, which in turn could increase the sales of other items. More discussion on the impact of price formats on retail sales can be found in Tang et al. (2001).

In this paper, we model the decisions under the ABD program, which involve how much to discount, how to use the precommitted orders to update forecasts, and how much to order at the beginning of the season to optimize total expected profits. We use this model to explicitly quantify the first two benefits of the ABD program. We also compare the profit associated with this program to that of the traditional sales program with no early promotion. Finally, we characterize the conditions under which the ABD program is beneficial to the retailer and come to better understand how the degree of demand uncertainty, correlation, and market share affect the optimal discount price and the viability of this program.

This paper is organized as follows. Section 2 provides a brief review of the relevant literature. In §3, we first present the base model (for the case with no discount promotion) and then present the ABD model. Section 4 analyzes the properties of the optimal discount price, which allows us to compare the optimal expected profit associated with the base case to that of the ABD model and characterize the conditions under which the ABD program is beneficial. We also provide numerical examples to illustrate our basic results. We present an extension in §5 and provide conclusions and directions for future research in §6.

2. Literature Review

Initiatives such as the ABD program focus on the coordination of operations and marketing decisions. Several researchers (Eliashberg and Steinberg

[1] The model developed in this paper is motivated by a promotion initiated by the Maxim's bakery in Hong Kong. No implication of the actual practice is intended.

1993, Karmarkar 1996) have highlighted the importance of coordination of such decisions. Because the ABD program specifically uses discount promotion in conjunction with an optimal ordering policy to coordinate the marketing/operations interface, it is also useful to examine the research on optimal ordering policy under discount promotion. This research can be classified under two streams. The first stream focuses on the analysis of the optimal ordering policy for a buyer when the demand is constant and the supplier offers a specific discount policy. The second stream examines how a supplier can use a discount policy as a control mechanism to induce a buyer to coordinate the channels of distribution. Weng (1995) presents a model that integrates these two streams of work. The reader is referred to Weng (1995) and the comprehensive references therein.

To our knowledge, Weng and Parlar (1999) is the first paper to present a model in which the retailer offers a price discount to induce customers to commit to their purchases prior to the beginning of the selling season. They determine the optimal order quantity for the retailer and characterize the optimal discount rate. While our paper addresses a similar problem, our model differs from their model in several aspects. First, their model deals with the case in which the customers belong to a single market segment, while our model deals with two segments. We believe that the two-segment model allows us to capture heterogeneous consumer preferences. Second, they assume that the precommitted orders generated by the program are deterministic, while the remaining demand that occurs during the season is stochastic. In our model, we consider a more realistic case in which both the precommitted orders and the demand during the selling season are stochastic. Third, unlike their model, we consider a fixed cost for implementing the program and allow for the case when customers could use this program as an early reservation system, even when there is no discount. Fourth, we consider the case in which the retailer would utilize the precommitted orders to update the probability distribution of the remaining demand that occurs during the season, while Weng and Parlar (1999) do not model the issue of forecast updating. We believe demand forecast updating is of critical importance because updated demand forecasts allow the retailer to place a more accurate order at the beginning of the season. Finally, while Weng and Parlar (1999) focus on the determination of the optimal order quantity and the optimal discount rate, our emphasis is on examining the benefits of the ABD program. Specifically, we are interested in analyzing general conditions under which the ABD program is beneficial, and examining the impact of demand uncertainty, correlation, and market share on the optimal discount price. Our goal is to develop managerial insights as to when such programs should be instituted.

3. The Analysis Framework

Consider a retailer who sells a seasonal product that belongs to Brand A. The unit cost, selling price, and salvage value of this product are c, p, and s, respectively, where $s < c < p$. There are two customer segments: One buys Brand A and the other buys Brand B, where Brand B corresponds to the aggregation of all other brands that compete with Brand A and are not carried by this retailer. We assume that the joint distribution of the demands for Brands A and B, denoted by D_A and D_B, is a bivariate normal distribution with means μ_A and μ_B, standard deviations σ_A and σ_B, and the correlation coefficient $\rho \in (-1, 1)$. (The bivariate normal distribution is degenerate for $\rho = -1$ and $\rho = 1$.) To simplify the exposition of our analysis, we shall assume that D_A and D_B have the same coefficient of variation θ, where $\theta = \sigma_A/\mu_A = \sigma_B/\mu_B$. This seems reasonable, since both products are similar and will consequently have similar degrees of demand uncertainty. Let μ be the expected total market demand, where $\mu = \mu_A + \mu_B$. Let $\alpha \in (0, 1)$ be the market share of Brand A, where $\alpha = \mu_A/\mu$. Given the definition of α and θ, we have $\mu_A = \alpha\mu$, $\mu_B = (1 - \alpha)\mu$, $\sigma_A = \theta\alpha\mu$, and $\sigma_B = \theta(1 - \alpha)\mu$. We summarize the notation used in the paper in Table 1.

Table 1 Notation (in Order of Appearance)

c	Purchase/production cost per unit of Brand A
p	Regular selling price per unit of Brand A
s	Salvage value per unit of Brand A
D_A	Demand for Brand A, a random variable $\sim N(\mu_A, \sigma_A^2)$
D_B	Demand for Brand B, a random variable $\sim N(\mu_B, \sigma_B^2)$
ρ	Correlation coefficient between D_A and D_B
θ	Coefficient of variation for both D_A and D_B; i.e., $\theta = \sigma_A/\mu_A = \sigma_B/\mu_B$
μ	Total expected market demand; i.e., $\mu = \mu_A + \mu_B$
α	Market share of Brand A product; i.e., $\alpha = \mu_A/\mu$
Q	Order quantity of Brand A in the base case
π	Optimum expected profit in the base case
K	Fixed cost of administering the ABD program
x	Discount coefficient under the ABD program so that the discount price is xp
$R_A(x)$	Fraction of Brand A customers who use the ABD program with a discount price of xp
$R_B(x)$	Fraction of Brand B customers who switch to Brand A under the ABD program
$D_1(x)$	Demand for Brand A under ABD prior to the regular sales season, $\sim N(\mu_1, \sigma_1^2)$
$D_2(x)$	Demand for Brand A under ABD during the regular sales season, $\sim N(\mu_2, \sigma_2^2)$
$\hat{\pi}(x)$	Expected profit under the ABD program without demand updating
$\tilde{\pi}(x)$	Expected profit under the ABD program with demand updating
$\widehat{Q}$	Order quantity under ABD without updating (in addition to precommitted orders)
$\widetilde{Q}$	Order quantity under ABD with updating (in addition to precommitted orders)

3.1. The Base Model

Consider the (base) case in which the retailer does not offer the ABD program. Thus, the retailer charges p for each unit during the selling season and charges s for each unit after the season. The retailer needs to determine the optimal order quantity Q^* that maximizes the total expected profit. Let π be the optimal expected profit, where

$$\pi = \max_{Q\geq 0} E_{D_A}\{-cQ + p\min(Q, D_A) + s(Q - D_A)^+\},$$

and Q denotes the order quantity. The above problem is the newsvendor problem with normally distributed demand. It is well known that the optimal order quantity Q^* and the optimal expected profit π are given as:

$$\begin{aligned} Q^* &= \mu_A + k\sigma_A = (1+k\theta)\alpha\mu, \\ \pi &= (p-c)\mu_A - (p-s)\phi(k)\sigma_A \\ &= [(p-c) - (p-s)\phi(k)\theta]\alpha\mu, \end{aligned} \tag{1}$$

where $k = \Phi^{-1}((p-c)/(p-s))$, and $\Phi(\cdot)$ and $\phi(\cdot)$ are the distribution and the density functions of the standard normal distribution, respectively (Silver et al. 1998). It is easy to check from (1) that the optimal expected profit $\pi > 0$ if and only if the coefficient of variation $\theta < \bar{\theta}$, where

$$\bar{\theta} = \frac{p-c}{(p-s)\phi(k)}.$$

To ensure that the retailer's optimal expected profit $\pi > 0$, we shall assume that $\theta < \bar{\theta}$ throughout this paper. Notice from (1) that the term $(p-s)\phi(k)\sigma_A$ can be rewritten as $[(p-c) + (c-s)]\phi(k)\sigma_A$, which corresponds to the sum of the expected over-stock and under-stock costs associated with the optimal order quantity Q^*. Thus, it is desirable for the retailer to consider launching the ABD program to reduce demand variance σ_A^2. We now discuss how the ABD program enables variance reduction.

3.2. The Advance Booking Discount Model

By incurring a fixed promotion cost K, the retailer can launch the ABD program offering a discount price xp per unit of Brand A prior to the beginning of the season, where the discount coefficient is equal to x and $0 \leq x \leq 1$. If customers accept this offer, then they can precommit their orders by prepaying xp per unit prior to the selling season and pick up their orders during the season. If customers decline this offer, they can always purchase the product during the season by paying regular price p per unit; however, the availability of the product will not be guaranteed.

We consider the impact of the ABD program on customer demand. First, among those customers who plan to buy Brand A during the selling season, $R_A(x)D_A$ will precommit their orders at a lower price xp prior to the selling season, where $R_A(x) \in [0, 1]$ corresponds to the fraction of Brand A customers who precommit their orders at discount price xp. $R_A(x)$ can also be regarded as the customer response to the coupon-type discount promotion inherent in the ABD. It follows that $[1 - R_A(x)]D_A$ customers will purchase the product at regular price p during the selling season. Second, among those customers who plan to buy Brand B during the selling season, $R_B(x)D_B$ will switch from buying Brand B to Brand A at a lower price xp prior to the selling season, where $R_B(x) \in [0, 1]$ represents the fraction of Brand B customers who switch to buying Brand A at discount price xp prior to the selling season. Note that in addition to measuring the customer response to the discount promotion of the ABD, $R_B(x)$ also captures the brand-switching behavior of customers of Brand B. It follows that the remaining $[1 - R_B(x)]D_B$ customers will buy Brand B during the selling season as planned. It is conceivable that consumption may increase as a result of the price discount. However, because the products we consider are either durable or perishable in nature and customers receive the product only during the selling season, it seems reasonable to assume that customers do not consume more during the selling season and that consumption does not increase with the level of the discount. This assumption seems reasonable across a wide variety of durable products such as music CDs, books, toys, video games, or perishable food items that are consumed during a special occasion.

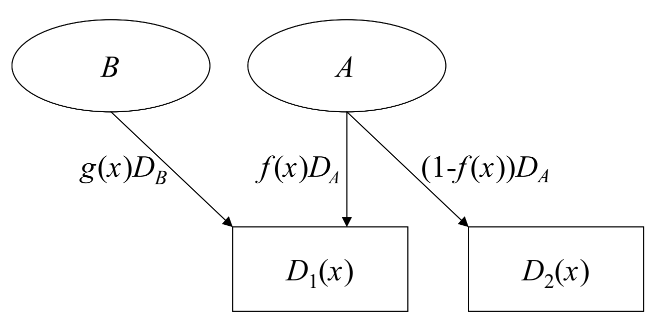
The impact of the ABD program on customer demand is depicted in Figure 1. Based on the above description of the effects of the ABD program, the retailer faces two types of demands: the precommitted orders placed prior to the season $D_1(x)$ and the demand that occurs during the season $D_2(x)$. Specifically, we have:

$$D_1(x) = R_A(x)D_A + R_B(x)D_B \tag{2}$$

$$D_2(x) = [1 - R_A(x)]D_A. \tag{3}$$

Therefore, under the ABD program, the total demand for product A is equal to $D_1(x) + D_2(x) = D_A + R_B(x)D_B \geq D_A$. This implies that the retailer can

Figure 1 The Impact of the ABD Program on Customer Demand



gain additional demand when implementing the ABD program. These additional sales are derived from the customers who switch from buying Brand B at the regular price p to Brand A at the discount price xp prior to the season.

Since $D_1(x)$ and $D_2(x)$ are linear functions of (D_A, D_B) and as (D_A, D_B) follows a bivariate normal distribution, it can easily be shown that $(D_1(x), D_2(x))$ also has a bivariate normal distribution with means μ_1 and μ_2, standard deviations σ_1 and σ_2, and correlation coefficient $\text{Corr}(D_1, D_2)$, where

$$\mu_1 = R_A(x)\mu_A + R_B(x)\mu_B = R_A(x)\alpha\mu + R_B(x)(1-\alpha)\mu \tag{4}$$

$$\mu_2 = [1 - R_A(x)]\mu_A = [1 - R_A(x)]\alpha\mu \tag{5}$$

$$\sigma_1 = [R_A^2(x)\alpha^2\mu^2\theta^2 + R_B^2(x)(1-\alpha)^2\mu^2\theta^2 + 2R_A(x)R_B(x)\alpha(1-\alpha)\mu^2\theta^2\rho]^{1/2} \tag{6}$$

$$\sigma_2 = [1 - R_A(x)]\sigma_A = [1 - R_A(x)]\alpha\mu\theta \tag{7}$$

$$\text{Corr}(D_1, D_2) = \frac{\text{cov}(D_1(x), D_2(x))}{\sigma_1\sigma_2} = \frac{R_A(x) + R_B(x)r\rho}{\sqrt{R_A^2(x) + R_B^2(x)r^2 + 2R_A(x)R_B(x)r\rho}}, \tag{8}$$

and $r = (1-\alpha)/\alpha$. It is also well known (Bickel and Doksum 1977) that the distribution of $D_2(x)$ given $D_1(x)$ (i.e., $(D_2(x) \mid D_1(x) = d_1)$) is normal with mean μ' and standard deviation σ', where[2]

$$\mu' = \mu_2 + \text{Corr}(D_1, D_2)(d_1 - \mu_1)\sigma_2/\sigma_1 \tag{9}$$

$$\begin{aligned}\sigma' &= \sigma_2\sqrt{1 - [\text{Corr}(D_1, D_2)]^2} \\ &= \big\{[R_B^2(x)r^2(1-\rho^2)] \\ &\quad \times \big[R_A^2(x) + R_B^2(x)r^2 + 2R_A(x)R_B(x)r\rho\big]^{-1}\big\}^{1/2} \\ &\quad \times [1 - R_A(x)]\alpha\mu\theta.\end{aligned} \tag{10}$$

We offer two observations based on (10) and (8). First, notice from (10) that $\sigma' \le \sigma_2$. This suggests that the ABD program enables the retailer to utilize $D_1(x)$ to reduce the standard deviation of $D_2(x)$ from σ_2 to σ'. Second, it can be shown that $(\sigma')^2$ is concave in ρ and that $(\sigma')^2$ is decreasing in ρ for $\rho > 0$. This implies that the ABD program enables the retailer to utilize the information about $D_1(x)$ to reduce the standard deviation of $D_2(x)$ even further when the underlying demands D_A and D_B are positively correlated (i.e., when $\rho \ge 0$). This implication can be explained as follows. When $\rho > 0$, it can be seen from (8) that $\text{Corr}(D_1, D_2)$ is positive and it is increasing in ρ. This implies that D_1 has a higher information value about D_2 as ρ becomes more positive. In this case, the retailer can utilize the precommitted order $D_1(x)$ to further improve the accuracy of the forecast for the demand that occurs during the selling season D_2.[3]

These two observations illustrate the basic mechanism by which the ABD program enables the retailer to obtain an improved forecast and place a more accurate order so as to achieve higher expected profits. In the remainder of this section, we shall evaluate the optimal expected profits associated with the ABD program.

3.2.1. Without Demand Forecast Updating. Consider the case when the retailer offers the ABD program with the discount coefficient x. To separate the benefits of variance reduction from the improved forecast due to updating in the ABD program, we first assume that the retailer is unable to utilize the precommitted orders $D_1(x)$ to update the distribution of $D_2(x)$. This scenario is plausible especially when the retailer lacks the infrastructure to capture or analyze sales data.

Because the order is placed at the start of the selling season, the retailer can order the exact amount to fulfill the precommitted orders, $D_1(x)$, observed prior to the selling season. Hence, the profit generated from those precommitted orders is equal to $(xp - c)D_1(x)$. Although the retailer does not use $D_1(x)$ to update the distribution of $D_2(x)$, $D_2(x)$ is still normally distributed with mean μ_2 and standard deviation σ_2 given by (5) and (7), respectively. In this case, the retailer orders quantity $\widehat{Q}$ (in addition to $D_1(x)$) to cover the demand during the selling season. Thus, the profit generated from the demand $D_2(x)$ is equal to $\{-c\widehat{Q} + p\min(\widehat{Q}, D_2(x)) + s(\widehat{Q} - D_2(x))^+\}$. The optimal total expected profit associated with the ABD program *without* demand forecast updating, denoted by $\hat{\pi}(x)$, can be expressed as follows:

$$\hat{\pi}(x) = -K + E_{D_1(x)}[(xp - c)D_1(x)] + \max_{\widehat{Q}\ge 0} E_{D_2(x)}[-c\widehat{Q} + p\min(\widehat{Q}, D_2(x)) + s(\widehat{Q} - D_2(x))^+].$$

By using the standard newsvendor result, one can check that the last term of this equation depends

[2] The bivariate normal distribution allows us to obtain simple expressions for μ' and σ' and to simplify our analysis. To elaborate, if one uses the conjugate prior distributions to determine the posterior distribution of the updated demand, then the mean and the standard deviation of the posterior distribution is quite complex and would complicate the analysis significantly.

[3] When $\rho < 0$, $\text{Corr}(D_1, D_2)$ could be increasing or decreasing in ρ. Thus, the behavior of $(\sigma')^2$ with respect to ρ is inconclusive when $\rho < 0$.

on σ_2, the standard deviation of $D_2(x)$. σ_2 is independent of the correlation coefficient ρ because the retailer does not utilize the precommitted orders $D_1(x)$ to update the distribution of $D_2(x)$. Thus, the optimal order quantity $\widehat{Q}^*$ and the optimal expected profit $\hat{\pi}(x)$ are independent of ρ, where

$$\widehat{Q}^*(x) = \mu_2 + k\sigma_2$$

$$\begin{aligned}\hat{\pi}(x) = -K &+ (xp-c)R_A(x)\alpha\mu \\ &+ [(p-c)-(p-s)\phi(k)\theta][1-R_A(x)]\alpha\mu \\ &+ (xp-c)R_B(x)(1-\alpha)\mu. \qquad (11)\end{aligned}$$

$\hat{\pi}(x)$ consists of the following terms. The first term represents the fixed cost of instituting the ABD program. The second term consists of the expected profits from sales during the early season, while the third term represents the expected profits from sales during the regular season for customers of Brand A. Finally, the fourth term represents the additional profits that are gained because customers of Brand B switch to Brand A due to the promotional effect of the ABD program.

Note that even when there is no demand uncertainty and promotional effects, the ABD program still provides the benefit of brand switching. To elaborate, consider the case in which there is no demand uncertainty (i.e., $\theta = 0$) and the retailer offers the ABD program without discount (i.e., $x = 1$). In this case, it is easy to check from (1) that the profit associated with the base case is given by $\pi = (p-c)\alpha\mu$. Also, it can be seen from (11) that the profit associated with the ABD program without discount is given by $\hat{\pi}(1) = -K + (p-c)\alpha\mu + (p-c)R_B(1)(1-\alpha)\mu$. Therefore, even when there is no demand uncertainty and promotional effects, the ABD program generates additional profit in the amount of $(p-c)R_B(1)(1-\alpha)\mu$. This profit is generated from Brand B's customers, who switch over to Brand A to guarantee availability of the product.

3.2.2. With Demand Forecast Updating. Consider the case in which the retailer offers the ABD program and utilizes the precommitted orders $D_1(x)$ to update the distribution of $D_2(x)$. Similar to the case without demand forecast updating, the profit generated from those precommitted orders is equal to $(xp-c)D_1(x)$. However, unlike the case without demand forecast updating, the retailer now utilizes the information about $D_1(x)$ to update the distribution of $D_2(x)$. Given the updated distribution of $D_2(x)$ (i.e., $D_2(x) \mid D_1(x)$), the retailer would order additional quantity $\widetilde{Q}$ so as to cover the demand during the season. Thus, the profit generated during the season is equal to $\{-c\widetilde{Q} + p\min(\widetilde{Q}, D_2(x)) + s(\widetilde{Q} - D_2(x))^+\}$. The optimal total expected profit associated with the ABD program *with* demand forecast updating, denoted by $\tilde{\pi}(x)$, can be expressed as:

$$\begin{aligned}\tilde{\pi}(x) = -K + E_{D_1(x)}\big\{&(xp-c)D_1(x) \\ &+ \max_{\widetilde{Q}\ge 0} E_{D_2(x)|D_1(x)}[-c\widetilde{Q} + p\min(\widetilde{Q}, D_2(x)) \\ &\qquad + s(\widetilde{Q} - D_2(x))^+]\big\}.\end{aligned}$$

By using the standard newsvendor result, one can check that the last term of this equation depends on σ', the standard deviation of $D_2(x) \mid D_1(x)$. σ' depends on the correlation coefficient ρ since the retailer utilizes the precommitted orders $D_1(x)$ to update the distribution of $D_2(x)$. Thus, the optimal order quantity $\widetilde{Q}^*$ and the optimal expected profit $\tilde{\pi}(x)$ now depend on ρ. By utilizing (9), (10), and the newsvendor result, we can express $\widetilde{Q}^*$ and $\tilde{\pi}(x)$ as:

$$\widetilde{Q}^*(x) = \mu' + k\sigma'$$

$$\begin{aligned}\tilde{\pi}(x) = -K &+ (xp-c)R_A(x)\alpha\mu \\ &+ \big[(p-c)-(p-s)\phi(k)\theta\big(\big\{[R_B^2(x)r^2(1-\rho^2)] \\ &\qquad \cdot [R_A^2(x)+R_B^2(x)r^2+2R_A(x)R_B(x)r\rho]^{-1}\big\}^{1/2}\big)\big] \\ &\cdot [1-R_A(x)]\alpha\mu + (xp-c)R_B(x)(1-\alpha)\mu. \qquad (12)\end{aligned}$$

The interpretation of the terms constituting (12) is similar to that of (11). In addition, recall from §3.2 that σ' is decreasing in ρ for $\rho > 0$. Therefore, the expected profit $\tilde{\pi}(x)$ is increasing in ρ for $\rho > 0$.

We now compare the expected profit $\tilde{\pi}(x)$ given in (12) with the expected profit $\hat{\pi}(x)$ given in (11). For any given discount factor x, it can be shown that

$$\begin{aligned}\tilde{\pi}(x) = \hat{\pi}(x) &+ (p-s)\phi(k)\big[1-\big\{[R_B^2(x)r^2(1-\rho^2)] \\ &\qquad \cdot [R_A^2(x)+R_B^2(x)r^2+2R_A(x)R_B(x)r\rho]^{-1}\big\}^{1/2}\big] \\ &\cdot [1-R_A(x)]\alpha\theta\mu \ge \hat{\pi}(x). \qquad (13)\end{aligned}$$

Therefore, we have proven the following lemma:

Lemma 1. *For any given discount coefficient x, $\tilde{\pi}(x) \ge \hat{\pi}(x)$.*

The above lemma implies that the retailer can realize higher expected profit if the retailer utilizes the precommitted orders $D_1(x)$ to update the demand distribution $D_2(x)$ because the variance of the demand $D_2(x)$ will be further reduced due to updating.

Let $\hat{x}$ and $\tilde{x}$ be the *optimal discount coefficients* that maximize the profit functions $\hat{\pi}(x)$ and $\tilde{\pi}(x)$, respectively. Lemma 1 implies that $\tilde{\pi}(\tilde{x}) \ge \hat{\pi}(\hat{x})$. Combining this fact with the fact that $\hat{\pi}(\hat{x}) \ge \hat{\pi}(1)$, we can conclude that $\tilde{\pi}(\tilde{x}) \ge \hat{\pi}(\hat{x}) \ge \pi$ if $\hat{\pi}(1) \ge \pi$. By comparing $\hat{\pi}(1)$ given in (11) and π given in (1), it can be easily shown that:

Lemma 2. *The optimal expected profits associated with the ABD program ($\tilde{\pi}(\tilde{x})$ and $\hat{\pi}(\hat{x})$) are higher than the* *optimal expected profit associated with the base case* (π) *if*

$$\theta \geq \frac{K-(p-c)R_B(1)\mu_B}{(p-s)\phi(k)R_A(1)\mu_A}.$$

Lemma 2 suggests that it is beneficial to institute the ABD program when the degree of demand uncertainty (measured in terms of the coefficient of variation θ) exceeds a certain threshold. In addition, since $\theta \geq 0$, the condition stated in the lemma always holds when $K \leq (p-c)R_B(1)\mu_B$. This suggests that it is clearly advisable to implement this program when the gain in expected profits purely from brand switching to assure availability exceeds the fixed cost of implementing the ABD program.

To summarize, in this section we have developed expressions for the expected profits associated with the base case, and also the cases without and with demand forecast updating. In addition, we have shown how the retailer can utilize the ABD program to increase sales, improve forecasts, and place a more accurate order so as to achieve higher profit. Finally, we have provided a sufficient condition under which the ABD program enables the retailer to obtain a higher expected profit than that of the base case.

4. The Optimal Discount Coefficient

We now characterize the *optimal discount coefficients* $\hat{x}$ and $\tilde{x}$ that maximize the profit functions $\hat{\pi}(x)$ and $\tilde{\pi}(x)$, respectively. To obtain some structural results, we shall restrict our attention to the case in which the response functions $R_A(x) = 1 - ax^f$ and $R_B(x) = 1 - bx^g$, where $f, g \in (0, \infty)$, and $a, b \in (0, 1]$ are constants.[4] We exclude the case in which $a = b = 0$, because this would imply that $R_A(x) = R_B(x) = 1$ for any value of x, or that all customers will precommit their orders regardless of the discount price. This functional form of the response function can commonly be found in the marketing literature and is known as the deterministic exponential sales response function (Smith and Achabal 1998, Achabal et al. 1990, Narasimhan 1984). When $x = 1$ or there is no discount, the ABD program serves as an early reservation system in which a proportion $1-a$ of Brand A's customers are willing to precommit their orders by paying the regular price p to guarantee the availability of the products during the season. Thus, a can be regarded as the parameter representing the degree of risk aversion of the customers. The lower the value of a, the more averse customers are to the risk of not being able to buy the product during the regular season. The same remark can be made about the Brand B's customers.

In addition, the functional form for $R_A(x)$ and $R_B(x)$ is useful in capturing various types of market response to the ABD program. First, note that $R_A(x)$ is decreasing and bounded between $1-a$ and 1. The parameter f can be regarded as the price sensitivity of the market. For instance, when $f > 1$, $R_A(x)$ customers for Brand A are eager to accept the ABD offer prior to the season and a small discount induces a large proportion to precommit their orders. Conversely, when $0 < f \leq 1$, customers for Brand A are reluctant to accept the ABD offer and a deep discount is required to induce a large proportion to precommit their orders. Similar observations can be made for the response function $R_B(x)$.

We next analyze the optimal discount coefficients. Because Brands A and B are similar, customers can be expected to have similar responses to the ABD program. To capture this possibility, we first explore in this section the case when $R_A(x) = R_B(x) = 1 - ax^f$ analytically. In the next section, we numerically examine the case when $R_A(x) \neq R_B(x)$.

4.1. Without Demand Forecast Updating

We now analyze the difference in profits between the base case given in (1) and the case of no demand forecast updating given in (11), defined by $\hat{\Delta}(x)$, where

$$\begin{aligned}\hat{\Delta}(x) &= \hat{\pi}(x) - \pi \\ &= -K + (xp-c)[R_A(x)\alpha + R_B(x)(1-\alpha)]\mu \\ &\quad - [(p-c)-(p-s)\phi\theta]R_A(x)\alpha\mu. \end{aligned} \tag{14}$$

For convenience, $\phi(k)$ is abbreviated by ϕ in the remainder of this paper.

Prior to presenting the properties of $\hat{x}$, we define a term $\hat{\theta}$ that simplifies our exposition. Let:

$$\hat{\theta} = \frac{p(1-a)}{(p-s)\phi\alpha af} - \frac{(p-c)(1-\alpha)}{(p-s)\phi\alpha}. \tag{15}$$

When $R_A(x) = R_B(x) = 1 - ax^f$, we have:

Proposition 1. *Suppose $\theta < \bar{\theta}$. The optimal discount coefficient $\hat{x}$ has the following properties*:

1. *If $\theta \leq \hat{\theta}$, then $\hat{x} = 1$, or it is optimal not to discount.*
2. *If $\theta > \hat{\theta}$, then $\hat{x} \in (0, 1)$, or it is optimal to offer a discount and $\hat{x}$ is characterized by the first-order condition $\hat{\Delta}'(\hat{x}) = 0$.*

Proof. All proofs are given in the Appendix.

Proposition 1 has the following interpretations. First, when $\theta \leq \hat{\theta}$, the demand is less variable. In this

[4] This functional form is motivated by the following observations. Observe that D_B is the demand for Brand B when no ABD program is launched, while $(1-R_B(x))D_B$ corresponds to the demand for Brand B when an ABD program is launched. If one assumes that the sales response function of Brand B possesses the form $(1-R_B(x))D_B = D_B bx^g$, where bx^g represents the exponential response function with respect to the discount coefficient x, we get $R_B(x) = 1 - bx^g$. The functional form of $R_A(x)$ can be motivated in a similar fashion.

case, Proposition 1 suggests that it is not beneficial for the retailer to introduce any discount into the ABD program (but simply keep the same price) because the variance reduction attained by discounting does not justify the loss of profits due to lowered prices. Next, when $\theta > \hat{\theta}$, the underlying demand is more variable. In this case, Proposition 1 suggests that it is beneficial for the retailer to launch the ABD program by offering a discount price to entice customers to precommit their orders in order to reduce the variance of the demand.

Proposition 2. *Suppose $\theta < \bar{\theta}$. Then $\hat{x}$ has the following properties*:
1. *$\hat{x}$ is decreasing in θ and increasing in α.*
2. *If $f = 1$, then*:

$$\hat{x} = \begin{cases} 1, & \text{if } \theta \le \hat{\theta} \\ (1/2)[1+1/a] - (2p)^{-1}[(p-c)(1-\alpha) \\ \quad + (p-s)\phi\theta\alpha], & \text{if } \theta > \hat{\theta}. \end{cases}$$

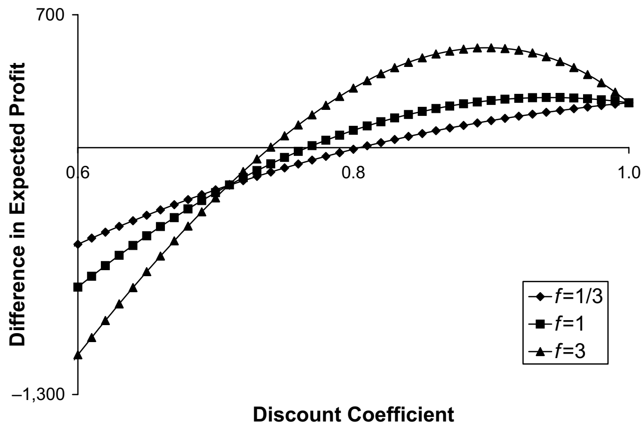
The first part of Proposition 2 suggests that it is optimal to offer a lower discount price as the demand becomes more variable. This is because lowering the discount price will make a larger portion of the demand certain through the precommitted orders, and thus reduce the variance of the demand during the season. In addition, it suggests that it is optimal to offer a higher discount price when the brand market share α is high, because there is only a small gain in additional demand that does not justify lowering the discount price.

4.1.1. Illustrative Example. To better illustrate Propositions 1 and 2, we construct a numerical example with the following parameters: $p = 100$, $c = 50$, $s = 25$ so that $p > c > s$. Also, $K = 200$, $\alpha = 0.5$, $\mu = 100$, $\theta = 0.3$, and $\rho = 0.4$. Since $R_A(x) = R_B(x) = 1 - ax^f$, we set $a = 0.85$, but we will vary $f = 1/3$, 1, and 3. Based on these parameters, it can be shown that $\bar{\theta} = 1.83$. In addition, $\hat{\theta} = 2.05$, -0.54, and -1.40 when $f = 1/3$, 1, and 3, respectively. Notice that $\theta = 0.3 < \bar{\theta}$. Observe that $\theta = 0.3 < \hat{\theta} = 2.05$ when $f = 1/3$, and that $\theta = 0.3 > \hat{\theta}$ when $f = 1$ or 3. In this case, according to Proposition 1, $\hat{x} = 1$ when $f = 1/3$, and $\hat{x} < 1$ when $f = 1$, 3. This result is illustrated in Figure 2, where the optimal discount factor $\hat{x}$ equals 1, 0.94, and 0.90 for the cases when $f = 1/3$, 1, and 3, respectively.

To examine the impact of θ on $\hat{x}$, we vary θ from 0.1 to 1 so that $\theta < \bar{\theta}$. Proposition 2 states that $\hat{x}$ is decreasing in θ. This result is illustrated in Figure 3.

Next, we examine the impact of α on $\hat{x}$, by varying α from 0.1 to 1 (while fixing $\theta = 0.3$). Proposition 2 states that $\hat{x}$ is increasing in α. This result is illustrated in Figure 4.

Figure 2 Expected Profit Difference ($\hat{\Delta}(x)$) vs. Discount Coefficient (x) for ABD Without Updating



4.2. With Demand Forecast Updating

We now analyze the difference in profits between the base case given in (1) and the case with demand forecast updating given in (12). Let $\tilde{\Delta}(x) = \tilde{\pi}(x) - \pi$, where $\tilde{\Delta}(x)$, similar to $\hat{\Delta}(x)$, measures the profit difference between the ABD program with demand forecast updating and the base case. $\tilde{\Delta}(x)$ is given by:

$$\begin{aligned}\tilde{\Delta}(x) = {} & -K + (xp-c)[R_A(x)\alpha + R_B(x)(1-\alpha)]\mu \\ & - (p-c)R_A(x)\alpha\mu + (p-s)\phi\theta \\ & \cdot (1 - \{[R_B^2(x)r^2(1-\rho^2)][R_A^2(x) + R_B^2(x)r^2 \\ & + 2R_A(x)R_B(x)r\rho]^{-1}\}^{1/2}[1 - R_A(x)])\alpha\mu. \quad (16)\end{aligned}$$

Prior to presenting the properties of $\tilde{x}$, we define a term $\tilde{\theta}$ that simplifies our exposition. Let:

$$\tilde{\theta} = \frac{p(1-a)}{(p-s)\phi\nu\alpha a f} - \frac{(p-c)(1-\alpha)}{(p-s)\phi\nu\alpha}, \quad (17)$$

where

$$\nu = \sqrt{\frac{r^2(1-\rho^2)}{1+r^2+2r\rho}} \le 1.$$

Figure 3 Optimal Discount Coefficient ($\hat{x}$) vs. Coefficient of Variation (θ) for ABD Without Updating

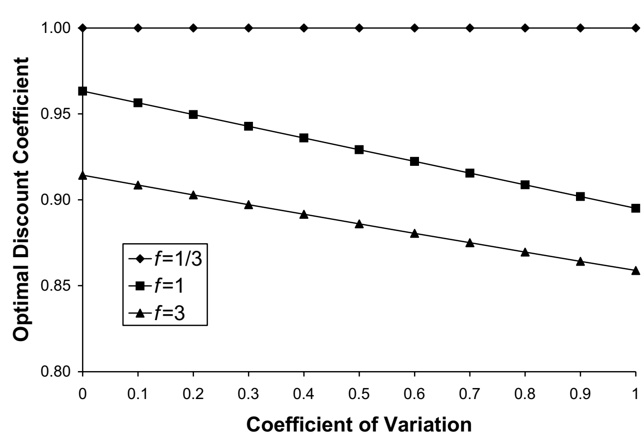
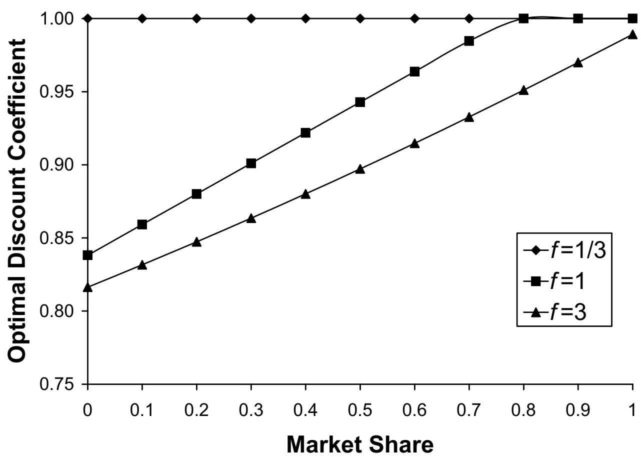


Figure 4 **Optimal Discount Coefficient ($\hat{x}$) vs. Market Share (α) for ABD Without Updating**



When $R_A(x) = R_B(x) = 1 - ax^f$, the following propositions describe the properties of the optimal discount coefficient $\tilde{x}$ that maximizes $\tilde{\Delta}(x)$.

Proposition 3. *Suppose $\theta < \bar{\theta}$. The optimal discount coefficient $\tilde{x}$ has the following properties*:

1. *If $\theta \leq \tilde{\theta}$, then $\tilde{x} = 1$.*
2. *If $\theta > \tilde{\theta}$, then $\tilde{x} \in (0, 1)$ and $\tilde{x}$ is characterized by the first-order condition $\tilde{\Delta}'(\tilde{x}) = 0$.*

Proposition 4. *Suppose $\theta < \bar{\theta}$. Then $\tilde{x}$ has the following properties*:

1. *$\tilde{x}$ is decreasing in θ, and increasing in α (for $\rho \geq 0$).*
2. *$\tilde{x}$ is increasing in ρ when $\rho \geq 0$.*
3. *If $f = 1$, then*:

$$\tilde{x} = \begin{cases} 1, & \text{if } \theta \leq \tilde{\theta} \\ (1/2)[1 + 1/a] - (2p)^{-1}[(p-c)(1-\alpha) \\ \quad + (p-s)\phi\nu\theta\alpha], & \text{if } \theta > \tilde{\theta}. \end{cases}$$

Observe that Propositions 3 and 4 are analogous to Propositions 1 and 2. Thus, they lend themselves to similar interpretations. Essentially, the coefficient of variation θ and the Brand A's market share α have significant impact on the effectiveness of the ABD program, and on the optimal discount coefficient $\tilde{x}$ associated with it.

Proposition 4 suggests that the retailer should offer less discount with a higher value of $\tilde{x}$, when the correlation coefficient ρ becomes more positive. This implication results from the following observations. First, recall from §3.2.2 that the expected profit $\tilde{\pi}(x)$ increases as ρ becomes more positive. Second, as noted in §3.2, when ρ becomes more positive, the retailer can utilize the precommitted order $D_1(x)$ to further improve the accuracy of the forecast for the demand that occurs during the selling season, $D_2(x)$. These two observations imply that as ρ becomes more positive, the retailer can improve the forecast of $D_2(x)$ and increase the expected profit without the need to offer a deeper discount.

The following proposition compares the optimal discount coefficients that maximize the expected profits for the cases with and without demand updating:

Proposition 5. *Suppose $\theta < \bar{\theta}$. Then,*

$$\hat{x} = \tilde{x} = 1, \quad \text{if } \theta \leq \nu\tilde{\theta}$$
$$\hat{x} \leq \tilde{x} \leq 1, \quad \text{if } \theta > \nu\tilde{\theta}.$$

Proposition 5 has the following implication. If the retailer uses the precommitted orders to update the demand forecasts, then the retailer can achieve a higher expected profit by offering a higher discount price so that $\tilde{x}p > \hat{x}p$. This is because the demand variance is further reduced when the retailer updates the demand distribution $D_2(x)$ after observing $D_1(x)$.

4.2.1. Illustrative Example. We now use the same numerical example presented in §4.1.1 to illustrate Propositions 3, 4, and 5. Based on the parameters, it can be shown that $\bar{\theta} = 1.83$. In addition, $\tilde{\theta} = 3.74$, -0.99, and -2.56 when $f = 1/3$, 1, and 3, respectively. Notice that $\theta = 0.3 < \bar{\theta}$. Observe that $\theta = 0.3 < \tilde{\theta} = 3.74$ when $f = 1/3$, while $\theta = 0.3 > \hat{\theta}$ when $f = 1$ or 3. In this case, according to Proposition 3, $\hat{x} = 1$ when $f = 1/3$, and $\hat{x} < 1$ when $f = 1$ or 3. This result is illustrated in Figure 5, where the optimal discount factor $\tilde{x}$ equals 1, 0.95, and 0.91 for the cases when $f = 1/3$, 1, and 3, respectively. In addition, recall from §4.1.1 that $\hat{x}$ equals 1, 0.94, and 0.90 for $f = 1/3$, 1, and 3, respectively. In every single case, we have $\hat{x} \leq \tilde{x}$. This verifies Proposition 5.

To examine the impact of θ on $\tilde{x}$, we vary θ from 0.1 to 1 so that $\theta \leq \bar{\theta}$. Proposition 4 states that $\tilde{x}$ is decreasing in θ. This result is illustrated in Figure 6.

Figure 5 **Expected Profit Difference ($\tilde{\Delta}(x)$) vs. Discount Coefficient (x) for ABD with Updating**

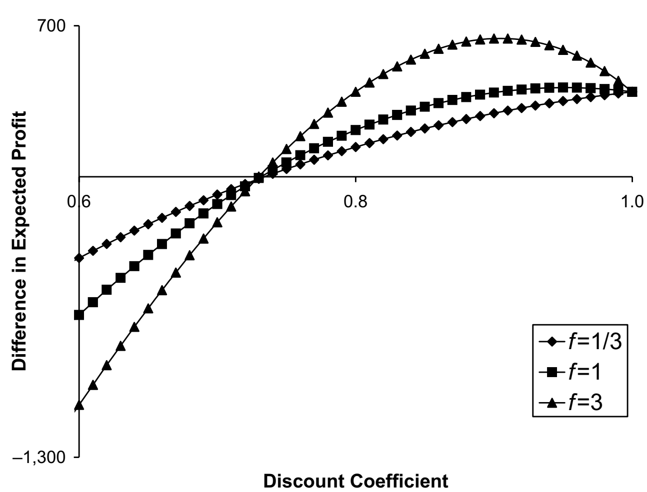
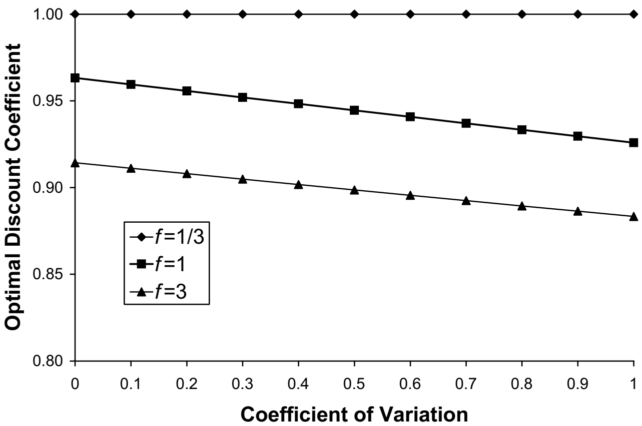


Figure 6 Optimal Discount Coefficient ($\tilde{x}$) vs. Coefficient of Variation (θ) for ABD with Updating



Next, we examine the impact of α on $\tilde{x}$ by varying α from 0.1 to 1 (while fixing $\theta = 0.3$). Proposition 4 states that $\tilde{x}$ is increasing in α. This result is illustrated in Figure 7.

Finally, we examine the impact of ρ on $\tilde{x}$ by varying ρ from -0.9 to 0.9 (while fixing $\alpha = 0.5$ and $\theta = 0.3$). Proposition 4 states that $\tilde{x}$ is increasing in ρ (for $\rho \geq 0$). This result is illustrated in Figure 8.

5. Nonidentical Response Functions

In this section, we consider the impact of nonidentical response functions $R_A(x) = 1 - ax^f$ and $R_B(x) = 1 - bx^g$, where $f \neq g$ and $a < b$. To capture the impact of brand loyalty that could result in a greater response to the ABD from the original customers of Brand A, we set the constants a, b, f, and g, so that $R_A(x) \geq R_B(x)$ for all $x \in (0, 1]$. In particular, without loss of generality, we consider the same parameters as stated in §4.1.1., except that $a = 0.7$, $b = 0.9$, $f = 3$, and $g = 1/3$, 1, and 3. First, consider the case with demand forecast updating. By substituting $R_A(x) = 1 - ax^f$ and $R_B(x) = 1 - bx^g$ into (12), we obtain:

$$\begin{aligned}\tilde{\pi}(x) = & -K + (xp - c)[(1 - ax^f)\alpha + (1 - bx^g)(1 - \alpha)]\mu \\ & + (p - c)a\alpha\mu x^f - (p - s)\phi(k)\theta \\ & \cdot \{[(1 - bx^g)^2 r^2 (1 - \rho^2)][(1 - ax^f)^2 + (1 - bx^g)^2 r^2 \\ & + 2(1 - ax^f)(1 - bx^g) r\rho]^{-1}\}^{1/2} a\alpha\mu x^f. \qquad (18)\end{aligned}$$

The structure of (18) prohibits an exact analysis for the case of the nonidentical response function. However, to better understand how this aspect affects the results in the previous section, we elected to analyze this case numerically. Figures 9 and 10 summarize the first set of numerical results when we vary θ and α from 0.1 to 1, respectively.

Figure 7 Optimal Discount Coefficient ($\tilde{x}$) vs. Market Share (α) for ABD with Updating

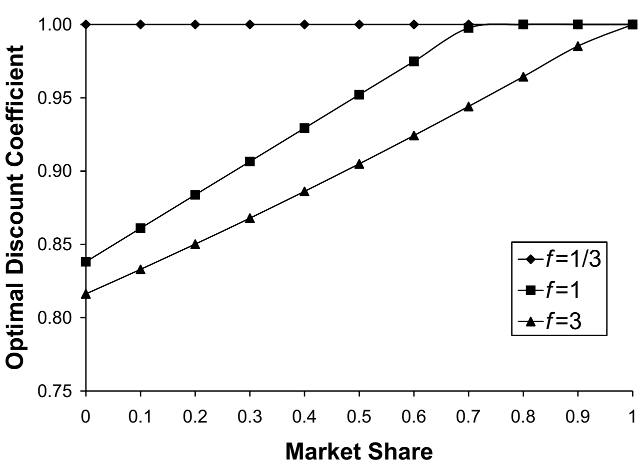


Figure 8 Optimal Discount Coefficient ($\tilde{x}$) vs. Correlation Coefficient (ρ) for ABD with Updating

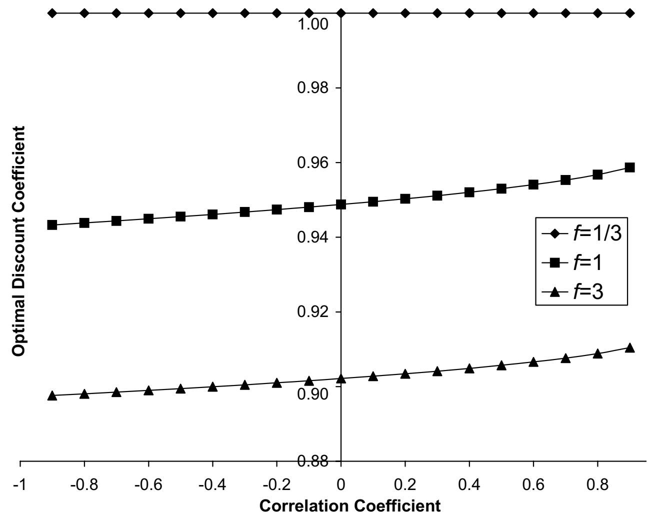


Figure 9 Optimal Discount Coefficient ($\tilde{x}$) vs. Coefficient of Variation (θ) for ABD with Updating when $f = 3$

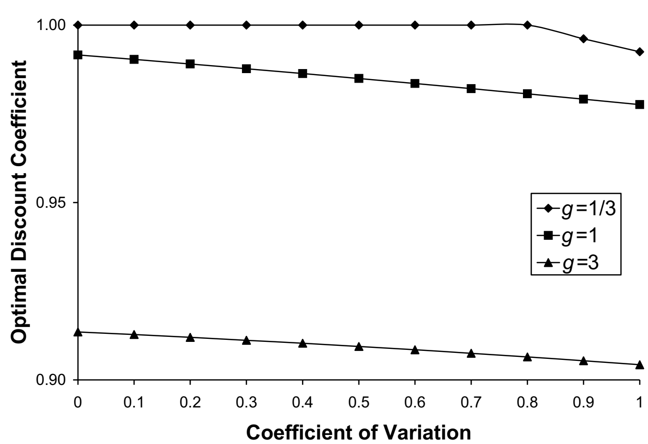


Figure 10 **Optimal Discount Coefficient ($\tilde{x}$) vs. Market Share (α) for ABD with Updating when $f = 3$**

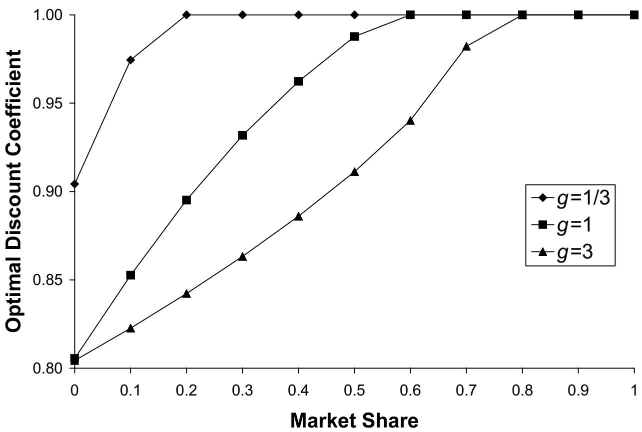


Figure 9 shows that $\tilde{x}$ is decreasing in θ, consistent with Figures 3 and 6, while Figure 10 shows that $\tilde{x}$ is increasing in α, consistent with Figures 4 and 7. In addition, they both corroborate Propositions 2 and 4. In addition to the insights already discussed in §4, Figures 9 and 10 illustrate the impact of different values of g on $\tilde{x}$. For a fixed value of θ (respectively, α), Figure 9 (respectively, Figure 10) shows that $\tilde{x}$ decreases as g increases. These observations can be explained as follows. As g increases, Brand B's customers become more eager to switch to Brand A and precommit their orders; hence, it is beneficial for the retailer to offer a lower discount price to induce more of Brand B's customers to switch to Brand A. For purposes of brevity, we omit a detailed description for the case without demand forecast updating. However, it is important to note that our numerical results were consistent with the results corresponding to Propositions 1 and 2 and Figures 2 through 4.

Next, we analyze the impact of the correlation coefficient ρ under nonidentical response functions. Recall from §3.2.1 that for the case without demand forecast updating the expected profit $\hat{\pi}(x)$ is independent of ρ. Thus, the optimal expected profit $\hat{\pi}(\hat{x})$ and the optimal discount coefficient $\hat{x}$ are independent of ρ. For this reason, it suffices to focus on the case *with* demand forecast updating. Figure 11 summarizes the optimal expected profit $\tilde{\pi}(\tilde{x})$, while Figure 12 reports the optimal discount factor $\tilde{x}$ when we fix $\theta = 0.3$ and vary ρ from -0.9 to 0.9.

Figure 11 shows that as the demands D_A and D_B become more positively correlated (i.e., when $\rho \geq 0$), there is an increase in expected profit. This observation can be explained as follows. When ρ becomes more positive, precommitted orders $D_1(x)$ has a higher information value for predicting $D_2(x)$, the demand during the selling season. As described in §3, this additional information further reduces demand variance, which increases expected profits.

Figure 11 **Optimal Expected Profit ($\tilde{\pi}(\tilde{x})$) vs. Correlation Coefficient (ρ) for ABD with Updating when $f = 3$**

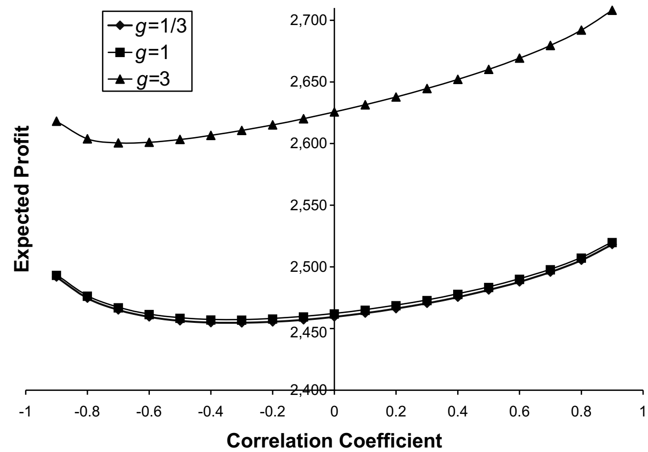
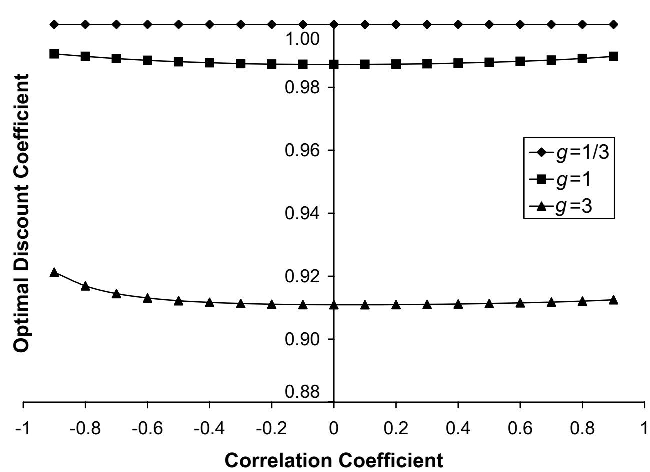


Figure 12 shows that it is optimal for the retailer to offer a higher discount price when the demands D_A and D_B become more positively correlated. This observation can be explained by the fact that the precommitted orders $D_1(x)$ provides useful information for predicting $D_2(x)$, the demand to be observed during the selling season. Thus, the retailer does not need to use a lower discount price to stimulate additional precommitted orders and to improve the demand forecasting accuracy of $D_2(x)$.

Finally, we examine how the optimal expected profit and the optimal discount coefficients are affected by the parameter g. Figure 11 shows that for any fixed value of ρ, the retailer can achieve higher expected profits as g increases, as customers for Brand B become more responsive toward the ABD program. Similarly, for any fixed value of ρ, Figure 12 suggests that it is optimal for the retailer to offer a lower discount price as g increases. This observation can be explained as follows. As g increases, Brand B's

Figure 12 **Optimal Discount Coefficient ($\tilde{x}$) vs. Correlation Coefficient (ρ) for ABD with Updating when $f = 3$**



customers become more eager to switch to Brand A and precommit their orders; hence, it is beneficial for the retailer to offer a lower discount price in order to induce more of Brand B's customers to switch to Brand A.

6. Concluding Remarks

In this paper, we have considered a problem of matching supply with demand for products with short life cycles and highly unpredictable demands. Due to the long replenishment lead time and the short sales season, the retailer is unable to restock during the selling season and respond to market demand. As an alternative strategy, we have considered a scheme called the advance booking discount (ABD) program in which customers can commit to their orders at a discount price prior to the commencement of the sales season, with guaranteed delivery during the season. We have developed a model that enables us to quantify two crucial benefits of the ABD program, including generation of additional sales and better matching of supply with demand through more accurate forecasting and supply planning. In addition, we analyze how the degree of demand uncertainty, correlation, and the level of market share affect when such programs should be instituted and their impact on the optimal discount coefficient.

In practice, it is often difficult to accurately measure parameters such as the degree of demand uncertainty and correlation for a given product. However, it is important to note that in the examples presented in this paper, the changes in the optimal discount factor are relatively small even with radical changes in these parameters. This suggests that optimal discount level is fairly robust with respect to measurement errors of these parameters. Thus, this is an encouraging result from the perspective of implementation of this program.

This paper provides several new avenues for future research. First, it could be useful to examine the ABD program under multiple products with fixed-capacity constraints. Second, it could be helpful to consider a dynamic version of the ABD program, in which the retailer also determines when to start and stop this program, and where discounts vary with time depending upon the level of precommitted orders. Third, it could be instructive to examine the ABD program under retail competition. Finally, it would be important to identify and incorporate the additional financial, operational, and marketing constraints that may be required to implement such programs across different types of product lines.

Several important product lines in retail industry face increased demand uncertainty due to exploding product variety and extended lead times due to global sourcing. In conclusion, we believe that the ABD program can act as an effective tool to match supply with demand for these product lines and this paper provides a useful framework to analyze the benefits of this program.

Acknowledgments
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Appendix

Proof of Proposition 1. Substituting $R_A(x) = 1 - ax^f$ and $R_B(x) = 1 - bx^f$ into (14) and rearranging:

$$\begin{aligned}\hat{\Delta}(x) &= -K + (xp - c)\mu - [(p-c) - (p-s)\phi\theta]\alpha\mu \\ &\quad - \{(xp-c)a - [(p-c)-(p-s)\phi\theta]\alpha a\}x^f\mu. \quad \text{(A1)}\end{aligned}$$

Differentiating with respect to x twice, we obtain:

$$\begin{aligned}\frac{\partial\hat{\Delta}(x)}{\partial x} &= p\mu - (Ax - B)x^{f-1}\mu \\ \frac{\partial^2\hat{\Delta}(x)}{\partial x^2} &= -[fAx - (f-1)B]x^{f-2}\mu,\end{aligned} \quad \text{(A2)}$$

where $A = pa(1+f)$ and $B = caf + [(p-c) - (p-s)\phi\theta]\alpha af$. Note that $A > 0$, $B > 0$ (by the assumption $\theta < \hat{\theta}$), and $\hat{\Delta}''(x) < 0$ if and only if $x > (f-1)B/fA$. For $f \le 1$, $\hat{\Delta}(x)$ is concave over $x \in (0,1]$. For $f > 1$, on the other hand, it can be shown that $0 < (f-1)B/fA < 1$, implying $\hat{\Delta}(x)$ is convex over $x \in (0, (f-1)B/fA]$, and concave over $x \in ((f-1)B/fA, 1]$. Therefore, $\hat{\Delta}(x)$ is quasi-concave over $x \in (0,1]$, and $\hat{x} \in (\max\{0, (f-1)B/fA\}, 1]$.

Notice that zero discount price cannot be optimal, because $\partial\hat{\Delta}(x)/\partial x = p\mu > 0$ at $x = 0$. Hence, given the quasi concavity of $\hat{\Delta}(x)$ over $x \in (0,1]$, the optimal discount coefficient will be either at the boundary ($\hat{x} = 1$) or not ($\hat{x} \in (0,1)$), depending on the sign of $\partial\hat{\Delta}(x)/\partial x$ evaluated at $x = 1$, i.e.,

$$\begin{aligned}&\text{If} \quad \hat{\Delta}'(1) = (p - A + B)\mu \ge 0, \quad \text{then } \hat{x} = 1, \\ &\text{else if} \quad \hat{\Delta}'(1) = (p - A + B)\mu < 0, \quad \text{then } \hat{x} \in (0,1).\end{aligned}$$

Rearranging the terms, we get the following:

$$p - A + B < 0 \quad \text{if and only if}$$

$$\theta > \hat{\theta} = \frac{p(1-a)}{(p-s)\phi\alpha af} - \frac{(p-c)(1-\alpha)}{(p-s)\phi\alpha}.$$

Finally, we observe that in the case where $\hat{\Delta}'(1) < 0$, the first-order condition $\hat{\Delta}'(\hat{x}) = 0$ is necessary and sufficient for optimality, which follows from the quasi concavity of $\hat{\Delta}(x)$ over $x \in (0,1]$. The existence of an optimal discount coefficient follows from $\hat{\Delta}'(0) > 0$, $\hat{\Delta}'(1) < 0$, and the Mean Value Theorem. □

Proof of Proposition 2.

1. When $\theta > \hat{\theta}$, $\hat{x} \in (0,1)$ and it satisfies the first-order condition $\hat{\Delta}'(\hat{x}) = 0$. By the Implicit Function Theorem, we can differentiate the function $\hat{\Delta}'(\hat{x})$ with respect to θ. By considering (A2), it can be shown that:

$$\frac{\partial \hat{x}}{\partial \theta} = -\frac{(p-s)\phi\alpha a f\hat{x}}{fA\hat{x}-(f-1)B}.$$

The denominator, $fA\hat{x}-(f-1)B$, is positive because $\hat{x} \in (\max\{0,(f-1)B/fA\},1]$ (proof of Proposition 1). Hence, $\partial\hat{x}/\partial\theta < 0$. Similarly, differentiating the function $\hat{\Delta}'(\hat{x})$ with respect to α we obtain:

$$\frac{\partial \hat{x}}{\partial \alpha} = \frac{[(p-c)-(p-s)\phi\theta]af\hat{x}}{fA\hat{x}-(f-1)B},$$

where the denominator is positive (by the above argument) and so is the numerator (by the assumption $\theta < \bar{\theta}$). Hence, $\partial\hat{x}/\partial\alpha > 0$.

2. From Proposition 1 we know that $\hat{x} = 1$ when $\theta \le \hat{\theta}$. We also know that when $\theta > \hat{\theta}$, $\hat{x} \in (0,1)$. In this case, $\hat{x}$ satisfies the first-order condition $\hat{\Delta}'(\hat{x}) = 0$, which is necessary and sufficient for optimality. Hence, from (A2):

$$\hat{\Delta}'(\hat{x}) = p\mu - (A\hat{x}-B)\mu = 0 \quad \text{implies} \quad \hat{x} = (p+B)/A$$

where $A = 2pa$ and $B = ca + [(p-c)-(p-s)\phi\theta]\alpha a$. □

Proof of Proposition 3. Substituting $R_A(x) = R_B(x) = 1 - ax^f$ into (16) and rearranging:

$$\tilde{\Delta}(x) = -K + (xp-c)(1-ax^f)\mu - (p-c)(1-ax^f)\alpha\mu + (p-s)\phi\theta[1-\nu a x^f]\alpha\mu. \quad \text{(A3)}$$

Differentiating with respect to x twice, we obtain:

$$\frac{\partial\tilde{\Delta}(x)}{\partial x} = p\mu - (Cx-D)x^{f-1}\mu$$
$$\frac{\partial^2\tilde{\Delta}(x)}{\partial x^2} = -[fCx-(f-1)D]x^{f-2}\mu, \quad \text{(A4)}$$

where $C = pa(1+f)$ and $D = caf + [(p-c)-(p-s)\phi\nu\theta]\alpha af$. Note that $C > 0$, $D > 0$ (by the assumption $\theta < \bar{\theta}$ and the fact that $\nu \le 1$), and $\tilde{\Delta}''(x) < 0$ if and only if $x > (f-1)D/fC$. Analogous to the proof of Proposition 1, whether $\tilde{x}$ is a border or an interior solution depends on the sign of $\tilde{\Delta}'(1)$. The rest of the proof is almost identical, with the basic idea centering around the quasi concavity of $\tilde{\Delta}(x)$ over $x \in (0,1]$. □

Proof of Proposition 4.

1. Similar to the Proof of Proposition 2, we get:

$$\frac{\partial\tilde{x}}{\partial\theta} = \frac{-(p-s)\phi\nu\alpha af\tilde{x}}{fC\tilde{x}-(f-1)D}$$
$$\frac{\partial\tilde{x}}{\partial\alpha} = \frac{[(p-c)-(p-s)\phi(\alpha\,\partial\nu/\partial\alpha+\nu)\theta]af\tilde{x}}{fC\tilde{x}-(f-1)D}.$$

The results follow from an analogous argument. We also observe the following:

$$\alpha\frac{\partial\nu}{\partial\alpha} = \frac{-\sqrt{1-\rho^2}(1+r\rho)}{\alpha(1+r^2+2r\rho)^{3/2}} \le 0 \quad \text{for all } \alpha \in (0,1] \text{ and } \rho \in [0,1).$$

2. Again, by the first-order condition and the Implicit Function Theorem, we get:

$$\frac{\partial\tilde{x}}{\partial\rho} = \frac{-(p-s)\phi\theta(\partial\nu/\partial\rho)\alpha af\tilde{x}}{fC\tilde{x}-(f-1)D}$$
$$\text{where} \quad \frac{\partial\nu}{\partial\rho} = \frac{-r(\rho+r)(r\rho+1)}{(1-\rho^2)^{1/2}(1+r^2+2r\rho)^{3/2}}.$$

The result follows from the observation that $\partial\nu/\partial\rho \le 0$ for $\rho \ge 0$.

3. From Proposition 3, we know that $\tilde{x} = 1$ when $\theta \le \tilde{\theta}$. We also know that when $\theta > \tilde{\theta}$, $\tilde{x} \in (0,1)$. In this case, $\tilde{x}$ satisfies the first-order condition $\tilde{\Delta}'(\tilde{x}) = 0$, which is necessary and sufficient for optimality. Hence, from (A4):

$$\tilde{\Delta}'(\tilde{x}) = p\mu - (C\tilde{x}-D)\mu = 0 \quad \text{implies} \quad \tilde{x} = (p+D)/C$$

where $C = 2pa$ and $D = ca + [(p-c)-(p-s)\phi\nu\theta]\alpha a$. □

Proof of Proposition 5. First, notice from (15) and (17) that $\hat{\theta} = \nu\tilde{\theta}$, where $\hat{\theta} \le \tilde{\theta}$ because $\nu \le 1$. Hence, by Propositions 1 and 3, $\hat{x} = \tilde{x} = 1$ for $\theta \le \hat{\theta}$; and $\hat{x} < 1$, $\tilde{x} = 1$ for $\hat{\theta} < \theta \le \tilde{\theta}$ (when $\nu < 1$). To see how $\hat{x}$ compares to $\tilde{x}$ when $\theta > \tilde{\theta}$, we use a deduction by contradiction. Suppose $\hat{x} > \tilde{x}$ and $\theta > \tilde{\theta}$. Propositions 1 and 3 imply that $\hat{x}$ and $\tilde{x}$ are both in $(0,1)$ and they satisfy the following first-order conditions:

$$p\mu - (A\hat{x}-B)\hat{x}^{f-1}\mu = 0$$
$$p\mu - (C\tilde{x}-D)\tilde{x}^{f-1}\mu = 0,$$

where $A = pa(1+f)$, $B = caf + [(p-c)-(p-s)\phi\theta]\alpha af$, $C = pa(1+f)$, and $D = caf + [(p-c)-(p-s)\phi\nu\theta]\alpha af$. These two conditions can be equivalently expressed as:

$$p\hat{x}^{1-f} = A\hat{x} - B$$
$$p\tilde{x}^{1-f} = C\tilde{x} - D.$$

Dividing the two sides, and then using the relations $A = C$, $B \le D$, $\hat{x} > \tilde{x}$, we obtain:

$$\left(\frac{\hat{x}}{\tilde{x}}\right)^{1-f} = \frac{A\hat{x}-B}{C\tilde{x}-D} \ge \frac{A\hat{x}-B}{A\tilde{x}-B} > \frac{\hat{x}}{\tilde{x}}.$$

But this implies $(\hat{x}/\tilde{x})^f < 1$ or $\hat{x} < \tilde{x}$, which contradicts our initial assumption. Thus, $\hat{x} \le \tilde{x} < 1$, when $\theta > \tilde{\theta}$. □

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