Vol. 56, No. 2, February 2010, pp. 286–301

ISSN 0025-1909 | EISSN 1526-5501 | 10 | 5602 | 0286



DOI 10.1287/mnsc.1090.1113 © 2010 INFORMS

# Predicting Utility Under Satiation and Habit Formation

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We introduce a modification of the discounted utility model that accounts for both satiation and habit formation in intertemporal choice. Preferences depend on the satiation level and the habitual consumption level. These two state variables, together with the shape of the value function, drive the properties of the model. One unique feature of our model is that it addresses the trade-off between seeking variety and maintaining acquired habits. We examine several properties of our model, such as the nontrivial patterns of desirability (willingness to pay) for an additional unit of consumption, or the effect of abstaining from consumption (craving). We explore the shape of optimal consumption patterns in discrete and continuous choice settings. If subjects underestimate the changes in satiation and habituation levels, as occurs under projection bias, our model explains why people buy more when hungry, or prefer variety in advance of consumption but stay with the same consumption good in actual use.

Key words: time preference; discounted utility; habit formation; satiation; variety seeking; projection bias History: Received February 18, 2007; accepted October 22, 2009, by David E. Bell, decision analysis. Published online in Articles in Advance January 12, 2010.

#### 1. Introduction

Present consumption influences the utility of future consumption in two important ways. First, it creates *satiation*, thereby reducing the satisfaction derived from consumption in the near future. A rich meal for dinner seems less attractive after a heavy lunch. A trip to Hawaii may seem less attractive if one had traveled to Hawaii the previous year. Second, present consumption contributes to *habit formation* and, therefore, increases future marginal utility. Luxury goods, technological devices, and narcotics are examples of habit-forming goods.

In the discounted utility (DU) model, consumption independence is assumed whereby the utility of consumption in each period is computed afresh and is independent of past consumption. Several researchers have examined the influence of current consumption on future tastes (Duesenberry 1952, Stigler and Becker 1977, Bell 1977). Becker (1996), Pollak (1970), Ryder and Heal (1973), and Wathieu (1997) propose models that modify the DU model to account for habit formation (HA model). McAlister (1982) and Baucells and Sarin (2007) present a modification of the DU model that accounts for satiation (SA model).

In psychology, both habit formation and satiation have been studied and observed in animals and humans (Groves and Thompson 1970, Solomon 1980). (Read et al. 1999, p. 183) state that "The most important taste change effects are habit formation and satiation." Becker (1992) argues that both satiation and habit formation are relevant in predicting future utility. Satiation and habit formation capture the basic effects of *diminishing sensitivity* and *adaptation*. Diminishing sensitivity is a fundamental property of perception and satisfaction, and adaptation is a fundamental property of living organisms subject to evolutionary forces.

In this paper, we propose the habit formation and satiation (HS) model that jointly incorporates both habit formation and satiation in the evaluation of consumption streams. In this model, the carrier of utility in each period is v(y - r + x) - v(y). Positive utility is obtained if the consumption (x) is higher than the habitual consumption level or reference level (r). The amount of utility is moderated by the satiation level (y). When x = r, a neutral utility level of zero is achieved. Consumption in excess of r induces positive per-period utility, but also contributes to satiation level (y).

In §2, we introduce our HS model. Section 3 explores some of its properties. In a constant consumption case, we study the desirability (willingness to pay),

satisfaction, and withdrawal from consumption. For the HS model, desirability decreases at first (satiation phase), increases in the interim (sensitization phase), and finally decreases (habituation phase). In contrast, abstaining from consumption results in negative satiation levels and produces craving. Consumption then satisfies the unmet need by creating a high utility experience. This is observed when one enjoys lunch when hungry or a cold glass of water after a long bike ride.

One unique feature of the HS model is that it addresses the trade-off between seeking variety and maintaining acquired habits. In §4, we show that our HS model predicts both variety seeking and habit formation. First, we explore the optimal plan under unit per-period consumption. Later, in §5, we allow for variable consumption quantity, and show that in the optimal plan, consumption is increasing and alternates between goods over time. The predictions of the HS model are distinctly different from those obtained by the HA or SA models alone. For the case of a single good, both U-shaped and increasing consumption plans can be optimal for the HS model.

In §6, we examine the impact of projection bias (Loewenstein et al. 2003) on choices. Projection bias leads an individual to believe that the future will be similar to the present (current r and y will change more slowly than they actually do). Under projection bias, our model is consistent with the empirical observation that people prefer variety in advance of consumption but stick with the same consumption good in the actual future choice (Gilbert 2006).

We conclude, in §7, with the implications of applying our analysis to consumer choice and rational decision making.

## 2. A Model of Habit Formation and Satiation

Let  $(x_1, x_2, ..., x_T)$  be a consumption stream. What is the total utility that a consumer obtains from such a stream? The DU model proposes to evaluate the total utility as

$$V_{\rm DU}(x_1,\ldots,x_T) = \sum_{t=1}^{T} \delta^{t-1} v(x_t),$$

where  $v(x_t)$  is the per-period utility of consumption  $x_t$  in period t, and  $\delta^{t-1}$  is the discount factor associated with period t. The DU model was first proposed by Samuelson (1937) and subsequently axiomatized by Koopmans (1960) and Koopmans et al. (1964) for countable infinite streams. A key feature of the DU model is the separability over time. Thus, the utility derived from present consumption is not affected by past consumption. Hence, it does not account for habit formation or satiation.

Throughout, we interpret the per-period utility as the experienced utility in that period (Kahneman et al. 1997), with v(0) = 0. In all subsequent models, we set the discount factor  $\delta$  to one. We do this for two reasons. First, for simplicity, as impatience is not salient to the main contribution of the paper. Second, it can be argued on normative grounds that experienced utility should not be discounted. Discounting can be easily incorporated, and we shall remark on how discounting would modify some of our results.

Many goods are habit forming; for these goods, present consumption increases the marginal utility of future consumption. Utility derived from current consumption depends on the habitual level of consumption (reference level). Kline (2006) reports that when monkeys were offered raisins and not the customary apple, their neurons fired strongly in response to the welcome change. After a few repetitions, the euphoria stopped as the animals had adapted to the better food. People also get accustomed to eating in higher priced restaurants or staying in better quality hotels as their income increases. This adaptation does not happen instantly, but over time their reference level relative to which utility is evaluated moves upward.

Several modifications of the DU model have been proposed to incorporate the phenomenon of habit formation. Of these modifications, we adopt Wathieu (1997). Wathieu's model captures the key aspect of habit formation (i.e., consumption today increases the marginal utility of future consumption) by introducing a reference level. Wathieu's finite-time model successfully explains the preference for increasing consumption sequences. Reference levels, denoted as habitual consumption levels, are psychologically sound (Kahneman and Tversky 1979, Rabin 1998). In Wathieu's habit formation model (HA), zero utility can be interpreted as a neutral state of neither satisfaction nor dissatisfaction, and positive utility is obtained when the consumption level exceeds the habitual level of consumption. The HA model replaces the per-period utility v(x) by v(x-r):

$$V_{\text{HA}}(x_1, \dots, x_T) = \sum_{t=1}^{T} v(x_t - r_t), \text{ where}$$

$$r_t = r_{t-1} + \alpha(x_{t-1} - r_{t-1}), \quad t = 2, \dots, T, \ r_1 \text{ given.}$$

Here,  $0 \le \alpha \le 1$  is the *speed of habituation* and  $r_t$  is the corresponding habituation level. When  $\alpha = 0$ , the HA model reduces to the DU model. When  $\alpha = 1$ , the reference level in the current period is simply the previous period's consumption. The speed of habituation,  $\alpha$ , could be different for different goods.

Besides habit formation, a second important way that consumption today affects future preferences is satiation. Satiation decreases the marginal utility of future consumption. de Graaf et al. (2004) review the relationship between physiologic and subjective measures in quantifying satiation from food intake. Smale et al. (2001) show that satiation leads to changes in brain activity. The degree of satiation depends on the consumption good. Heatherington et al. (2002) found that the desire to eat chocolate declined significantly over time, but not for bread and butter.1 For most foods though, pleasantness of taste declines over time with the intake. People find sweets less pleasant if the amount consumed in the recent past was large. The same ethnic meal on the third consecutive evening seems less appealing. The pleasure derived from a sport or entertainment (movie) diminishes with frequent repetitions. Even socializing is subject to satiation effects, as one may not want to meet the same friend too frequently. Satiation could last a year or more, for example, from a vacation focused on a theme such as camel rides or museums. The concept of diminishing marginal utility has been a cornerstone of economic theory. We simply extend this concept to a dynamic setting in which satiation due to recent past consumption influences the utility of current consumption.

Baucells and Sarin (2007) introduce a satiation model in which the contribution of current consumption to experienced utility is over the satiation level achieved due to previous consumption. Thus, the carrier of utility is the increment from current satiation. Suppose that the satiation level is y at the beginning of a period. Consumption of x in this period yields a per-period utility of v(y + x) - v(y), or

$$V_{\text{SA}}(x_1, \dots, x_T) = \sum_{t=1}^{T} v(y_t + x_t) - v(y_t), \text{ where}$$

$$y_t = \gamma(y_{t-1} + x_{t-1}), \quad t = 2, ..., T, y_1 \text{ given,}$$

where  $0 \le \gamma \le 1$  is the satiation retention factor or *speed of satiation*, and  $y_t$  is the corresponding satiation level. Satiation wanes with the passage of time and has a halflife that depends on the nature of the good consumed. Indeed, the satiation level is given by the cumulative discounted consumption in previous periods; that is,  $y_t = \sum_{s=1}^{t-1} \gamma^{t-s} x_s + \gamma^{t-1} y_1$ . If  $\gamma$  is zero, then  $y_t = 0$  in all periods, and the SA model reduces to the DU model. If  $\gamma$  equals one, then the satiation level is equal to the previously accumulated consumption.

In the DU model, the incremental utility of larger intakes of x decreases because of the diminishing sensitivity of v. The satiation level y is a stock of recent

consumption that acts as if we had consumed y just before consuming x; hence, x's utility is calculated starting at v(y). The SA and HA models are separate departures of the DU model, and neither collapses into the other.

Our next goal is to present a hybrid model, called the HS model, for habit formation and satiation. The HS model is strongly reference dependent: the carrier of utility is consumption in excess of the habituation level and the driver of satiation is also consumption in excess of the habituation level. The per-period utility in the HS model is given by v(y + x - r) - v(y), or

$$V_{\text{HS}}(x_1, \dots, x_T) = \sum_{t=1}^{T} v(y_t - r_t + x_t) - v(y_t),$$
 (1)

$$y_t = \gamma (y_{t-1} - r_{t-1} + x_{t-1}),$$

$$t = 2, \ldots, T, y_1$$
 given, and (2)

$$r_t = r_{t-1} + \alpha(x_{t-1} - r_{t-1}), \quad t = 2, ..., T, r_1 \text{ given.}$$
 (3)

As before,  $\gamma$  and  $\alpha$  are the speed of satiation and of habituation, respectively. Recall that v(0) = 0. Moreover, we assume that v is S-shaped: strictly concave above zero, and convex below zero. We also assume that it is steeper for losses than it is for gains (Kahneman and Tversky 1979), and assume some form of loss aversion. Unless stated otherwise, we set  $r_1 = y_1 = 0$ .

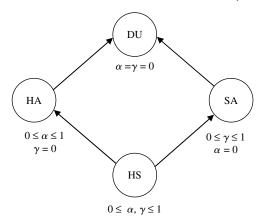
The HS model has the following distinct features:

- It is a simple extension of the SA model, where the absolute level of consumption is replaced by the consumption over the reference level.
- It is psychologically appealing. For a given habituation level, consumption gives higher pleasure when the satiation level is low and lower pleasure when it is high. A warm shower is a welcome routine, but feels especially pleasurable after a long hike in the cold.
- It inherits from the SA model the *local substitu*tion property. This property simply says that eating a slice of cake now and another slice a few seconds later should yield about the same utility as eating two slices of cake now. This property is lacking in the DU and HA models.<sup>2</sup>

 $<sup>^1</sup>$  In animals, time scales in satiation depend on energy densities of diets. Sea lions were offered either herring (H) or capelin (C) each day. When offered herring day after day (HHHH···), they consumed 8.3 kg per day (high energy); and, when offered capelin each day (CCCC···), they ate 14 kg per day (low energy). The daily consumption was higher when the food was made available on alternate days (H – H – ···) or (C – C – ···) (Rosen and Trites 2004).

<sup>&</sup>lt;sup>2</sup> More formally, local substitution requires that the two-period consumption stream (2,0) should yield the same utility as the stream (1,1) when the time separation between periods 1 and 2 approaches zero. Let  $\Delta$  be the length of the time interval between periods. If  $\gamma > 0$ , we claim that  $U_{\rm HS}(x_1,x_2) \to U_{\rm HS}(x_1+x_2,0)$  as  $\Delta \to 0$ . To see this, notice that as  $\Delta$  goes to zero, one should expect  $\alpha(\Delta)$  to go to zero and  $\gamma(\Delta)$  to go to one. One natural way to maintain the satiation halflife and the habituation rate approximately constant would be to set  $\gamma(\Delta) = \gamma^{\Delta}$  and  $\alpha(\Delta) = 1 - (1 - \alpha)^{\Delta}$ . Letting  $\Delta \to 0$ , it is easy to check that both  $U_{\rm HS}(x_1,x_2)$  and  $U_{\rm HS}(x_1+x_2,0)$  tend to  $v(y_1-2r_1+x_1+x_2)-v(y_1)$ . The local substitution property has been analyzed by Hindy and Huang (1992) and Bell (1974).

Figure 1 The HS Model Has the Other Three Models as Special Cases



• As we will show, the HS model explains some phenomena that cannot be explained by either the HA or the SA models alone. Still, the HS model particularizes to the HA model if  $\gamma = 0$ , to the SA model if  $\alpha = 0$ , and to the DU model if both  $\gamma$  and  $\alpha$  are zero (see Figure 1).

### 3. Desirability, Satisfaction, and Withdrawal

In the HS model, the *desirability* of consumption is determined by the anticipated *satisfaction* derived from consumption and the agony of *withdrawal* experienced by foregoing consumption. Let *y* and *r* be the satiation and habituation levels at the beginning of a period.

How desirable is it to consume x? The answer is given by the difference between the utility of consuming, v(y-r+x)-v(y), and the utility of not consuming, v(y-r)-v(y). Formally,

$$Desirability = v(y - r + x) - v(y - r). \tag{4}$$

Desirability is intimately related to the willingness to pay for x, a fundamental concept in marketing.<sup>3</sup> But desirability and satisfaction are not the same thing. How much will the consumer enjoy x? The satisfaction is given by the per-period utility, or

$$Satisfaction = v(y - r + x) - v(y). \tag{5}$$

Finally, if the consumer were not to consume, what would be the dissatisfaction experienced? We call this withdrawal, which is given by

$$With drawal = -[v(y-r) - v(y)]. \tag{6}$$

It is clear that

$$Desirability = Satisfaction + Withdrawal.$$
 (7)

In psychology, there is considerable evidence that both satisfaction and withdrawal depend on the habitual level of consumption and satiation. Tolerance is the diminished effects of a given dose after repeated use. Thus, larger doses are needed to produce equal hedonic sensations. Withdrawal is the unpleasant physical sensations experienced when the consumption is interrupted. Tolerance and withdrawal are hallmarks of drugs such as cocaine, nicotine, and alcohol (Korsmeyer and Kranzler 2009). By treating r and y as state variables that influence utility, we are simply modeling these well-established psychological phenomena.

The components of satisfaction, withdrawal, and desirability can be seen in Figure 2. In the figure, the starting point is to mark the satiation level y on the horizontal axis. Next, we subtract the reference level r, and mark y-r. Finally, we add consumption x and mark y-r+x. This produces three utility points, namely, v(y), v(y-r), and v(y-r+x). The differences between these points determine *Desirability*, *Satisfaction*, and *Withdrawal*.

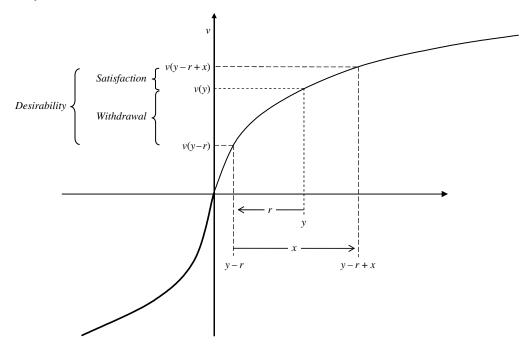
The following remarks can be made by observing Figure 2:

- The desirability of x is completely determined by the difference between y and r, and the particular values of y and r determine how desirability decomposes into satisfaction and withdrawal.
- The value function has a kink at zero and is steeper for losses than for gains. For consumption, this means that if satiation *y* is low relative to *r*, then the withdrawal effect will take place in the negative domain and will be quite strong (Wathieu 2004).
- In the DU and SA models (r = 0), there is no withdrawal. In the HA models (y = 0), satisfaction occurs in the positive zone of the value function and withdrawal in the negative zone of the value function. In the HS model (y might be positive or negative), both satisfaction and withdrawal can occur in any part of the value function.
- Increasing the satiation level, y, while keeping r and x constant, shifts the three points v(y), v(y-r), and v(y-r+x) to the right in parallel. If  $y-r \ge 0$ , then the three levels are in the concave region, and increasing y has two effects. First, it lowers the satisfaction from consumption. Second, it has the positive effect of mitigating withdrawal. That satiation mitigates withdrawal can be compared to the reaction "I don't mind skipping tennis today, because I have been playing all week."

In our model, desirability and satisfaction may not always go hand-in-hand. Desirability can be high

 $<sup>^3</sup>$  Assuming a quasi-linear utility for money, the desirability is equal to the willingness to pay for x. In general, willingness to pay is an increasing function of desirability.

Figure 2 Desirability Is the Sum of the Satisfaction and Withdrawal



because of an urge to avoid the pain of withdrawal and not necessarily because the satisfaction from the consumption is high. Desirability and satisfaction are closely related to wanting and liking. It has been shown that wanting and liking exert their influences through separate neural circuits (Berridge 2001, Berridge and Robinson 2003). Kahneman and Snell (2006) observed a positive correlation between wanting and liking, but not a perfect correlation (as the DU model implies). Our equation is consistent with this observation.

### 3.1. Three Phases of Desirability Under Steady Consumption

Consider a constant unit consumption  $x_t = 1$ , t = 1, 2, ..., T. Desirability is not necessarily monotonic over time, and may exhibit up to three phases (see Figure 3). With repeated consumption, desirability may decrease first (*satiation phase*), increase in the middle (*sensitization phase*), and decrease later (*habituation phase*).

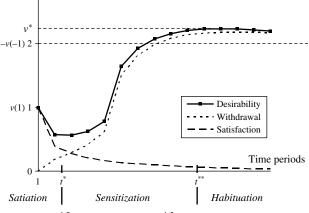
Let  $D(t) = v(y_t - r_t + 1) - v(y_t - r_t)$  be the desirability in period t. In the early periods, the habituation level is relatively low; therefore, increasing satiation levels lowers desirability. Once habituation levels increase, desirability increases because of an increase in withdrawal. Desirability and withdrawal may later decrease because of the diminishing sensitivity of v for losses. In the long run, the habituation level tends to one, satisfaction tends to zero, and desirability tends to -v(-1).

PROPOSITION 1. Assume that  $0 < \gamma$ ,  $\alpha < 1$ , and that v is S-shaped and exhibits loss aversion in the form of

 $v'(0^-) > v'(0^+)$ . Under constant unit consumption there exists  $0 < t^* < t^{**}$  such that D(t+1) < D(t) for  $t < t^*$  (satiation), D(t+1) > D(t) for  $t^* < t < t^{**}$  (sensitization), and D(t+1) < D(t) for  $t > t^{**}$  (habituation).

The values of  $t^*$  and  $t^{**}$  depend on the relative values of  $\gamma$  and  $\alpha$ . Not all the phases are visited. If  $\gamma < \alpha$ , then  $t^* < 1$  and we skip the satiation phase, and may only have sensitization and habituation (these two phases were introduced by Wathieu (2004) in the HA model, in which  $\gamma = 0$ ). In contrast, if  $\alpha$  is sufficiently close to zero or  $\gamma$  is sufficiently close to one, then  $t^*$  is surely larger than T, and we are left with only the satiation phase (see the proof in §A.1 in the appendix). In the SA model ( $\alpha = 0$ ), all consumption occurs in

Figure 3 Comparison of the Desirability for One Unit of Consumption
After Repeated Consumption of One Unit



*Note.*  $v(x) = x^{1/2}$ ,  $x \ge 0$ ;  $v(x) = -2|x|^{1/2}$ , x < 0; y = 0.6;  $\alpha = 0.25$ .

the satiation phase, desirability is equal to satisfaction, and both decrease with repeated consumption. Finally, if  $v'(-1) \ge v'(0^+)$ , then  $t^{**}$  is infinite and we stay in the sensitization phase, with D(t) monotonically increasing toward -v(-1).

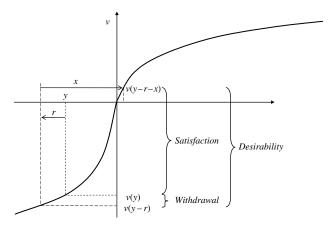
#### 3.2. Craving

We now explore the effects of abstaining from consumption on desirability, satisfaction, and withdrawal. If a consumer abstains from consumption, then he builds a stock of negative satiation. As shown in Figure 4, a resumption of consumption will provide a high level of satisfaction. The consumer will experience *craving* if the satisfaction obtained from a unit consumption is higher than at neutral levels; that is, v(y-r+1)-v(y) is higher than v(1).

Craving happens only when y < 0. An interpretation of y < 0 is the accumulation of unmet need. Anton (1999) argues that craving arises because of an imbalance in brain activity that results from nonconsumption. Craving is typically measured by self-reports of subjects on multi-item questionnaires (Flannery et al. 1999). In empirical work, craving is often researched for food, drugs, alcohol, and smoking. Chocolate, for example, is the most craved food, especially among females (Rozin et al. 1991, Yanovski 2003). Craving can be induced by environmental factors, for example, the first sign of snow for a ski enthusiast. Satisfaction of the unmet need creates a high amount of pleasure (i.e., experienced utility) at the moment consumption is resumed.

In the HS model, if we stop consuming a good that we are habituated to (r > 0 and y close to zero), then negative satiation will start to build up. Scitovsky (1976) argues that pleasure results from intermittent satisfaction of desires. We note that with long absences (e.g., giving up tennis for a few years), both the reference level and the satiation level revert to zero. Consumption after such long absence will not

Figure 4 Craving Occurs After Accumulating Negative Levels of Satiation



cause craving. In our model, for craving to occur, abstinence from consumption should be neither too short nor too long.

PROPOSITION 2. Assume v is S-shaped and exhibits loss aversion in the form of  $-v(-x) \ge \lambda v(x)$ , x > 0,  $\lambda > 1$ . If  $y \ge 0$ , then craving is not possible (DU, HA, and SA models). If y < 0,  $\lambda > 1 + r/|y|$  and  $y - r + 1 \ge 0$ , then the subject will experience craving, i.e., satisfaction higher than v(1).

One consumption sequence that leads to craving is abstaining, for a sufficiently large number of periods, from a good to which one has become habituated. Assume that one is habituated to unit consumption so that the initial habituation level is one and the initial satiation level is zero. Let period 1 be the first period when abstinence from consumption begins. It can be shown that the satiation and habituation levels evolve, for  $t \ge 1$ , as follows:<sup>4</sup>

$$y_t = -\frac{\gamma (1 - \alpha)^{t-1} - \gamma^t}{1 - \alpha - \gamma}$$
 and  $r_t = (1 - \alpha)^{t-1}$ . (8)

Initial satiation level is zero, but from period two onward, satiation levels take negative values. In particular, negative satiation peaks at some period  $t^*$  and then returns to zero as t increases. In contrast, habituation levels,  $r_t$ , decay monotonically from one to zero. It can be shown that maximum satisfaction occurs at or after  $t^*$ . The period that maximizes *Satisfaction*,  $v(y_t - r_t + 1) - v(y_t)$ , solves<sup>5</sup>

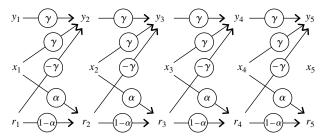
$$[v'(y_t - r_t + 1) - v'(y_t)] \partial y_t / \partial t = v'(y_t - r_t + 1) \partial r_t / \partial t. \quad (9)$$

Marketers use promotional tools, such as a brief stay in a luxury resort, to create desire. Similarly, an outing to celebrate an anniversary is made more enjoyable if one chooses a simple and routine life for a few days before the big event. Fasting before a fancy meal exacerbates hunger, but heightens the pleasure from subsequent eating.

<sup>4</sup> Using Equations (2) and (3), note that if  $x_t = 0$ ,  $t = 1, 2, 3, \ldots$ , then we can write  $y_{t+2} - r_{t+2} = (\gamma - \alpha + 1)(y_{t+1} - r_{t+1}) + (1 - \alpha)\gamma(y_t - r_t)$ . This is a second-degree homogeneous recursion with solution  $y_t - r_t = c_1\gamma^t + c_2(1 - \alpha)^t$ , and initial conditions  $y_1 - r_1 = -1$  and  $y_2 - r_2 = -\gamma - 1 + \alpha$ . The solution is then  $y_t - r_t = -((1 - \alpha)^t - \gamma^t)/(1 - \alpha - \gamma)$ , from which we can easily derive (8).

<sup>5</sup> One can check that  $y_t$  is single peaked, with a minimum at  $\bar{t} = \bar{t}^* + 1$ , where  $\bar{t}^*$  is given by (21). Hence, if the solution to (9) were at a satiation level with  $\partial y_t/\partial t < 0$ , then there would be a later period with the same satiation level but smaller habituation level, and this would certainly produce higher satisfaction. Hence,  $\partial y_t/\partial t > 0$ , and because the right-hand side of (9) is negative, it has to be the case that  $v'(y_t - r_t + 1) - v'(y_t) < 0$ . Because v is convex for losses, this necessarily implies that  $y_t - r_t + 1 \ge 0$ . Because  $\bar{t}^*$  goes to infinity as  $\gamma$  goes to one, craving will necessarily occur if  $\lambda > 1$  and  $\gamma$  is sufficiently large.

Figure 5 Direct and Indirect Effect of Consumption on Future Levels of Habitual Consumption and Satiation



#### 3.3. Habituation Mitigates Satiation

Consider the effect that current consumption,  $x_s$ , has on the habituation and satiation levels in period t > s. In Figure 5, one can visualize the dynamic effects of Equations (2) and (3). According to these equations, both the satiation level and the habitual consumption level are weighted sums of past consumption. The coefficient that  $x_1$  has in the expression for  $y_3$ , for instance, is the sum of the coefficients across all directed paths that connect  $x_1$  with  $y_3$ ; and the coefficient of a given path is the product of coefficients along this path. Thus,  $x_1$  appears in the expression for  $y_3$  multiplied by  $\gamma^2 - \alpha \gamma$ . The impact of  $x_1$  on  $x_2$  is straightforward  $\alpha$ 

PROPOSITION 3. Both  $r_t$  and  $y_t$  are linear functions of  $x_s$ , s = 1, ..., t - 1. If t > s, then the coefficient of  $x_s$  in  $r_t$  and  $y_t$ , respectively, is given by

$$\partial r_t / \partial x_s = \alpha (1 - \alpha)^{t - s - 1}, \tag{10}$$

$$\partial y_t/\partial x_s = [\gamma(1-\gamma)\gamma^{t-s-1} - \gamma\alpha(1-\alpha)^{t-s-1}]/(1-\alpha-\gamma)$$

if 
$$\alpha + \gamma \neq 1$$
, and (11)

$$\partial y_t / \partial x_s = \gamma^{t-s-1} (\gamma - (1-\gamma)(t-s-1))$$
 otherwise. (12)

One can distinguish three effects of current consumption,  $x_s$ , on future utility. First, current consumption increases the future habitual consumption level as given by (10). The increase in reference level produced by the first effect always reduces future experienced utility, but increases the marginal utility of future consumption. Second, current consumption increases future satiation level via the positive coefficient  $\gamma^{t-s}(1-\gamma)$  in (11). Third, the interaction between satiation and habit formation lowers future satiation via the negative coefficient  $\gamma \alpha (1-\alpha)^{t-s-1}$  in (11). The net result of these two effects is always positive and equal to  $\gamma$  in the subsequent period (t = s + 1); however, if  $\alpha > 0$ , then  $\partial y_t / \partial x_s$  eventually becomes negative as t - s grows larger. In fact, if  $\alpha > \gamma$  and  $t \ge s + 2$ , then  $\partial y_t / \partial x_s < 0$ . This interaction effect, which may produce  $\partial y_t/\partial x_s < 0$ , shows that habituation mitigates satiation. Because satiation is driven by consumption in excess of the habituation level, current consumption increases future habituation levels,

which has the indirect effect of reducing the satiation level two or more periods ahead. Hence, current consumption increases satiation level in the next (and possibly some subsequent) period(s), but reduces the satiation level of the remaining periods. For those periods where  $\partial y_t/\partial x_s < 0$ , if v is concave, then the marginal utility of  $x_t$  unambiguously increases with  $x_s$ .

The fact that habituation mitigates satiation implies that, once a habit is formed, a consumer is able to sustain large intakes without experiencing satiation. One gets satiated with surfing or listening to classical music when the habitual level of consumption is low, but enjoys larger "doses" as one gets accustomed to these activities. Somebody used to classical music may listen to it for an entire day without experiencing satiation. In *The Wealth of Nations*, Adam Smith (1776, p. 183) argues that "The desire of food is limited in every man by the narrow capacity of human stomach, but desire of the conveniences and ornaments of buildings, dress, equipage, and household furniture, seems to have no limit or certain boundary."

Craving (y < 0) and the subsequent realization of high utility from consumption, or occasional abstinence from consumption without significant reduction in utility when satiation level is high (y > 0) are realistic features of the HS model. These two features are the result of the interaction between habit formation and satiation and, therefore, are not available in any of the other three models.

#### 4. Discrete Choice Problem

The HS model addresses the conflict between habit maintenance and variety seeking. Habit formation produces the effect "the more you get, the more you want." In contrast, satiation produces the opposite marginal effect "the more you get, the less you want" in the subsequent period. This introduces a desire to seek variety. Roughly speaking, we would switch goods to avoid the satiation phase, but this change is not permanent because of the sensitization phase. Our model incorporates the tension between these two forces in determining the optimal allocation of consumption.

Of course, to illustrate this conflict we need to introduce a multigood version of the HS model. We choose an additively separable model in which each good contributes to utility independently of the consumption of other goods, and having good specific reference points (Bleichrodt et al. 2009). To gain insight into the variety seeking problem, we first consider the situation in which a consumer is restricted, in each time period, to the choice of one out of *K* goods. Initially, we set the consumption amount to one unit. An example of such a choice is having Indian food

or American food for dinner, or a choice of sport for Sunday morning (tennis, hiking, or golf).

Formally, our discrete unit choice problem is

$$\max \sum_{t=1}^{T} \sum_{k=1}^{K} v^{k} (y_{t}^{k} - r_{t}^{k} + x_{t}^{k}) - v^{k} (y_{t}^{k})$$
 (13)

s.t. 
$$y_t^k = \gamma^k (y_{t-1}^k + x_{t-1}^k - r_{t-1}^k),$$
  
 $t = 2, \dots, T, \forall k, y_1^k \text{ given, and } (14)$ 

$$r_t^k = r_{t-1}^k + \alpha^k (x_{t-1}^k - r_{t-1}^k),$$

$$t = 2, ..., T, \forall k, r_1^k \text{ given, and } (15)$$

$$x_t^k \in \{0, 1\} \quad \forall t, \forall k, \tag{16}$$

$$\sum_{k=1}^{K} x_t^k \le 1, \quad \forall t. \tag{17}$$

Our model is not to be seen as restricted to food or leisure. The model gives general insights into how to enhance the enjoyment of life. In traveling, for example, sequencing visits to places of interest is an important determinant of enjoyment. Changing the ordering may have a profound effect on the overall evaluation of the sequence. For the HS model, timing and sequencing are the key decision variables in managing satisfaction and withdrawal under habit formation and satiation.

#### 4.1. Habit Formation and Variety Seeking

The discrete unit consumption problem is complex. There are  $(K+1)^T$  possible consumption sequences, including the possibility of no consumption in some periods. A rational consumer will choose a sequence that maximizes total utility (13) over the T periods. To ease notation, we write BAAC, for instance, to denote that  $x_1^2 = x_2^1 = x_3^1 = x_4^3 = 1$ , and all the other values of  $x_t^k$  are equal to zero. We will use a dash "—" in position t to denote no consumption ( $x_t^k = 0$  for all t in period t).

To illustrate the trade-off between habit formation and variety seeking, consider the case of two periods (T=2) and a menu with two choices per period (K=2). To gain insight, we make these goods *symmetric*: they share the same utility function and the same speed of habituation and satiation  $(v^k=v, \alpha^k=\alpha, \text{ and } \gamma^k=\gamma, \text{ for all } k)$ . Because of symmetry, we will assume that A is the first good to be consumed.

Table 1 lists the utility of all the possible sequences. In the table, we add the superscripts A or B in v to help trace the sources of utility. In this simple case, the habituation and satiation levels in the second period are simply  $\alpha$  and  $\gamma$ , respectively. The last three sequences, -A, A-, and --, are weakly dominated by AA. Hence, only AB and AA are candidates for optimal consumption. It is instructive to consider carefully the utility components that contribute to total utility. In the first period, we consume A

Table 1 Utility from the Discrete Choice Between A and B in Two Periods

No.	Sequence	Utility first period	Utility second period
1	AB	<i>v</i> <sup>A</sup> (1)	$V^{B}(1) + V^{A}(\gamma - \alpha) - V^{A}(\gamma)$
2	AA	$V^A(1)$	$V^A(\gamma - \alpha + 1) - V^A(\gamma)$
3	-A	0	<i>ν</i> <sup>A</sup> (1)
4	A-	<i>∨</i> <sup>A</sup> (1)	$V^A(\gamma-\alpha)-V^A(\gamma)$
5		0	0

afresh in both AB and AA. As expected, when we initiate consumption with no previous effects of habit formation or satiation, we obtain  $v^A(1)$ .

Consider the utility of AB in the second period, which has two components. The first is the effect of consuming B afresh,  $v^B(1)$ . The second is the withdrawal effect from not consuming A,  $v^A(\gamma - \alpha) - v^A(\gamma)$ ; a good for which we now have a habit. The utility of AA in the second period has one component, which is less than  $v^A(1)$  because of habit formation and satiation.

The question of which is best, *AB* or *AA*, boils down to comparing the desirability of *B* and *A* in the second period. *AB* is at least as good as *AA* if and only if

$$v(1) \ge v(1 + \gamma - \alpha) - v(\gamma - \alpha). \tag{18}$$

Intuitively (see Figure 2), it is clear that if  $y - r = \gamma - \alpha > 0$ , then  $v(1 + \gamma - \alpha) - v(\gamma - \alpha)$  is less than v(1) because of the concavity of v. In contrast (see Figure 4), if y - r < 0, then loss aversion ensures that v(1) is smaller than  $v(1 + \gamma - \alpha) - v(\gamma - \alpha)$ .

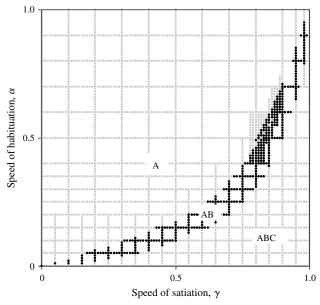
PROPOSITION 4. Assume v(x) is S-shaped and satisfies loss aversion in the form of  $-v(-x) \ge v(x)$ , x > 0. If  $\gamma \ge \alpha$ , then AB is optimal. Conversely, if  $\gamma \le \alpha$ , then AA is optimal.

The result is intuitive. If  $\gamma$  is large, then satiation dominates and we want to choose AB and seek variety. If  $\alpha$  is large, habit formation dominates and we want to choose AA and maintain the habit.

#### 4.2. Optimal Sequences

Proposition 4 shows that  $\gamma$  drives variety seeking and  $\alpha$  drives habit formation. But is this also true for more than two periods and two goods? This investigation is challenging because, as shown in §3.3, satiation and habituation levels are a complex function of  $\alpha$  and  $\gamma$ . Further, the optimal solution for the integer program with the S-shaped value function cannot be easily identified because of the presence of local optima. To guarantee optimality, we use a piecewise linear form for the value function. As shown in §A.2 in the appendix, the discrete unit choice problem with piecewise linear value function can be formulated as a mixed integer optimization problem. We use the simple form  $v(x) = \max\{-\lambda, \lambda x, \min\{1, x\}\}$ ,  $\lambda = 2$ , which

Figure 6 Optimal Amount of Variety as a Function of the Common Speed of Satiation and Habituation



*Notes.* In the first region (upper left gray zone), we only initiate the habit of A. In the second region (middle black zone), we consume goods A and B. In the third region (bottom right gray zone) we acquire habits for A, B, and C.

we call the *canonical value function*, for all goods. We choose T=12 periods and K=3 goods to illustrate our results. We also set  $\gamma$  and  $\alpha$  to be symmetric across the goods. We use XPRESS solver to find the optimal sequences, for a grid of values of  $\gamma$  and  $\alpha$  in increments of 0.05 or 0.1, and then some fine search in regions of interest (increments of 0.01). The results are illustrated in Figure 6.

The  $(\gamma, \alpha)$  plane is divided into three regions. The computational results portray the delicate balance between seeking variety and starting a new habit. If the speed of habituation is small, "ABC" region, then satiation is the prominent factor and the optimal sequence is to seek variety among the three goods for a wide range of values of  $\gamma$  (Figure 6). As  $\alpha$ increases, variety seeking is still optimal, but for a narrower range of values of  $\gamma$ . If the speed of habituation is high, and the speed of satiation low, then it is optimal to stay with a single good, and not initiate the habit for the other two. There is an intermediate "AB" region where the optimal plan is to seek variety between just two of the three available goods. By symmetry, these can be any two of the three goods. Hence, starting with homogeneous preferences for a population, one may observe heterogeneous outcomes, e.g., people choosing and liking different consumption goods based on some arbitrary initial selection. Examples include type of cuisine, favorite sports team, and hobbies.

Figure 6 shows that higher speed of satiation is accompanied by a greater variety (number of different

Table 2 Our Computational Results of Optimal Sequences Show That Variety Decreases When  $\gamma$  Increases from 0.81 to 0.82, and Increases When  $\alpha$  Increases From 0.47 to 0.48

γ	α	1	2	3	4	5	6	7	8	9	10	11	12
0.81	0.5	_	_	_	_	_	Α	В	Α	В	Α	В	Α
0.82	0.5	Α	-	Α	-	Α	_	Α	Α	Α	Α	Α	Α
8.0	0.47	Α	_	Α	_	Α	_	Α	Α	Α	Α	Α	Α
8.0	0.48	_	_	_	_	_	Α	В	Α	В	Α	В	Α

goods consumed). Similarly, higher speed of habituation is accompanied by a consumption of fewer number of goods. This is generally true, but not always. In Table 2, for  $\alpha = 0.5$ , the optimal sequence contains less variety as  $\gamma$  increases. Similarly, for  $\gamma = 0.8$ , more goods are consumed as  $\alpha$  increases. This result is a bit disconcerting but it illustrates that the optimal sequences for alternative values of  $\gamma$ and  $\alpha$  are complex. For example, optimal sequences may contain periods of no consumption (if  $\gamma$  is large, the withdrawal is mitigated by satiation, and abstinence increases future satisfaction by lowering habituation levels). Furthermore, even in the AB and ABC regions, the optimal sequences do not alternate perfectly between two or three goods. A bit of insight into this complex pattern of the optimal sequences is provided by recognizing that in spite of the assumed symmetry of  $\gamma$  and  $\alpha$  across goods, the good chosen in the first period already induces an asymmetry in the subsequent habituation and satiation levels.

#### 4.3. Life Simplicity

An important qualitative conclusion from Figure 6 is that total utility may be maximized by deliberately choosing only a few habituating goods. Withdrawal is zero for goods for which consumption, and thus habit, has not been initiated. Hence, initiating a habit for a good that we do not plan to consume with sufficient frequency may be a bad idea, because it will produce many periods of withdrawal. This observation points to the optimality of *life simplicity*. To gain further insight, let us consider rotating sequences, that is, a particular permutation of l goods is repeated again and again. Because only one good is consumed in a period, each good will necessarily have l-1 periods of withdrawal when it is not consumed.

We now formalize the idea that the optimal l is finite. That is, even with plenty of available periods, goods, and activities, it is optimal to consume a limited number of goods, and set consumption for the rest of the goods to zero in all periods.

PROPOSITION 5. Assume v is S-shaped and exhibits loss aversion in the form of  $-v(-x) \ge v(x)$ , x > 0. Let  $0 < \gamma$ ,  $\alpha < 1$ . Consider a consumption sequence in which a particular permutation of l goods is repeated again and again. For any given good, and for sufficient large l, the l-1 periods of withdrawal do not compensate the one period of

Table 3 **Optimal Consumption Sequence with Asymmetric Goods** 7 4 5 6 8 12 0.05 0.13 0.5 0 В В Α R Α Α R Α Α Α Α Α В 0.25 0 R R В Α Α Α Α

satisfaction from consumption. Hence, the optimal permutation contains less than l goods.

The optimal amount of variety (length of the rotating sequence) depends on the curvature of v and the values of  $\gamma$  and  $\alpha$ . Variety may be the spice of life, but too much variety reduces overall satisfaction.

#### 4.4. Exploration and Exploitation

If goods are nonsymmetric, that is, they have different speeds of habituation and satiation, then the model can engender a large number of patterns. We have calculated optimal sequences for some parameter combinations. In our numerical results, we frequently observe that the optimal pattern has a phase of exploration (variety seeking) followed by a phase of exploitation (habit formation). Exploration in our setup is not motivated by the desire to reduce uncertainty or to learn about tastes; instead, as the person becomes satiated with a recent consumption, he seeks out other alternatives (Lattin 1987, Inman 2001). At the beginning of the sequence, habituation levels are low, satiation levels are therefore high, and it is optimal to seek variety. Later, habituation levels increase and it becomes optimal to concentrate consumption on fewer goods (see Table 3), those with large  $\alpha$  and for which it was optimal to initiate consumption. In Table 3, we fix all parameters except for  $\alpha_1$ . When  $\alpha_1$ increases from 0.13 to 0.25, a higher speed of habituation leads to a delay in the initiation of the consumption of A.

The pattern in Table 3 is frequently observed in practice. An example may be switching between playing golf and tennis, playing golf with increasing frequency until ultimately switching solely to golf. More broadly, casual observation indicates that young people seek variety in their activities and relationships, whereas with age, people tend to settle on a few, very established habits.

#### 5. Optimal Consumption Levels

For better insight into the nature of optimal solutions, we now allow the possibility of varying the consumption quantity. We take the discrete unit choice problem and eliminate the binary and unit consumption constraints (16) and (17). Now, we add a budget constraint:

$$\sum_{t=1}^{T} \sum_{k=1}^{K} x_t^k \le I, \tag{19}$$

where I is the available budget under unit price. Finally, we impose the constraint that only one type

Table 4 Optimal Consumption Sequences in a Discrete Choice with Variable Quantity

γ	α	1	2	3	4	5	6	7	8	9	10	11	12
0.5	0.1	Α	Α	В	Α	В	Α	В	Α	В	Α	В	A
Total	A = 6.9	0.53	0.79	_	0.93	_	1.02	_	1.11	_		_	1.28
Total	B = 5.1	-	-	1	-	0.89	-	0.99	-	1.08	-	1.17	-
0.4	0.1	_	Α	В	Α	В	Α	В	Α	В	Α	В	Α
Total	A = 6.5	_	0.8	_	0.98	-	1.05	-	1.15	_	1.24	-	1.32
Total	B = 5.5	-	-	1	-	0.97	-	1.07	-	1.17	-	1.26	-
0.3	0.1	_	Α	Α	Α	Α	Α	Α	Α	Α	Α	Α	Α
Total	A = 12	_	0.9	0.82	0.86	0.93	1	1.07	1.14	1.21	1.28	1.35	1.42
Total	B = 0	_	-	-	-	-	-	-	-	-	-	-	-
0	0.1	_	_	_	Α	Α	Α	Α	Α	Α	Α	Α	Α
Total	A = 12	_	_	_	0.67	1.07	1.17	1.27	1.37	1.47		1.67	1.77
Total	B = 0	_	-	-	-	-	-	-	-	-	-	-	_

of good is consumed in each period. See §A.2 in the appendix for a formulation of this *discrete choice problem with variable quantity* as a mixed integer problem. To make the results comparable to the discrete unit choice problem, we set I equal to the number of time periods, T.

Table 4 presents the numerical results for optimal sequences as  $\gamma$  varies for a fixed level of  $\alpha$ . In this example, goods A and B are symmetric in their speed of satiation and habituation. The pattern of the first two sequences in Table 4 corresponds to an individual's budget for meals, which increases over time, but alternates between the type of food or restaurant. This is consistent with the observation of Lyubomirsky et al. (2005) that the pernicious effects of habit formation can be attenuated by attending to the timing and variety of consumption. Scitovsky (1976) similarly argues that pleasure can be enhanced through novelty and variety.

For the HA model, the optimal consumption would be to choose one of the goods, say *A*, and then have an increasing budget over time (the fourth sequence in Table 4). For the SA model, the optimal consumption is to alternate between *A* and *B* with a stable budget. The pattern obtained by the HS model (variety seeking with increasing budget) is more realistic. People enjoy a mix of entertainment (plays, concerts, operas), but over time spend more on the activities to which their aesthetic pleasures become accustomed.

Life simplicity may also be optimal: In the case that  $\gamma$  is small ( $\gamma \le 0.3$  in our example), we obtain that the optimal plan is to stay with one good. Note that the marginal utility of consumption in any period  $s \le T - 1$ , is given by

$$\frac{\partial V}{\partial x_s} = v'(y_s - r_s + x_s) + \sum_{t=s+1}^{T} \left[ v'(y_t - r_t + x_t) - v'(y_t) \right] \frac{\partial y_t}{\partial x_s} - v'(y_t - r_t + x_t) \frac{\partial r_t}{\partial x_s}.$$
(20)

Besides the direct effect produced in the period of consumption, one needs to account for the impact on future utility. The impact of increased habituation levels is always negative, whereas the impact of satiation levels is negative first and eventually becomes positive. The net effect is mostly negative and carries over all the subsequent periods. Hence, it might be optimal to postpone the initiation of certain habits.

Suppose it is optimal to stay with one good. In this case, it is possible to characterize the optimal consumption plan, which maximizes  $\sum_{t=1}^{T} v(y_t + x_t - r_t) - v(y_t)$  subject to constraints (2), (3), and (19).

PROPOSITION 6. Assume  $0 < \gamma < 1$ , and that v is continuous, strictly concave, twice differentiable except at zero, and for  $x \ge 0$ , exhibits nondecreasing relative risk aversion.<sup>6</sup> The optimal consumption sequence has nondecreasing satiation levels,  $y_1 \le y_2 \le \cdots \le y_T \le y_{T+1}$ . Furthermore, the optimal plan is either increasing or U-shaped. If  $\alpha \ge \gamma$ , then  $x_1 \le x_2 \le \cdots \le x_T$ . Alternatively, if  $\hat{v}''' \ge 0$ , then for any  $\alpha$  and  $\gamma$ ,  $x_2 \le x_3 \le \cdots \le x_T$  (here,  $\hat{v}(y) \equiv \gamma[v(y/\gamma) - v(y)]$ ).

We remark the following:

- Under constant consumption (§3), the satiation levels decline and go to zero. In contrast, the optimal consumption sequence calls for *increasing satiation levels*. Hence, constant sequences are far from optimal.
- If  $\alpha \geq \gamma$ , then habituation is stronger than satiation. It is better to plan for *increasing consumption levels*. This agrees with the results of Wathieu (1997), who shows that the optimal sequence for the HA model ( $\gamma = 0$ ) is increasing (assuming no discounting). In the presence of discounting, we expect the optimal sequences to be U-shaped.
- If  $\gamma > \alpha$ , then a reasonable sequence is that consumption is either increasing or U-shaped (the third sequence in Table 4). Consumption might be high in the first period because we begin with a zero satiation level ( $y_1 = 0$ ). This agrees with the results of Baucells and Sarin (2007), who show that the optimal sequences in the SA model are U-shaped.

#### 6. Projection Bias

The HS model is normative for a consumer experiencing habit formation and satiation. But do consumers act rationally in predicting the future effects of current choices? There is ample evidence that subjects often fail to predict future experienced utility (Gilbert et al. 1998). Loewenstein et al. (2003) provide evidence that, although people qualitatively understand that they

Table 5 Utility from the Discrete Choice Between A and B in Two Periods Under Projection Bias

Sequence	Predicted utility
AB	$V^{A}(1) + V^{B}(1) + V^{A}(\pi + (1 - \pi)(\gamma - \alpha)) - V^{A}(\pi + (1 - \pi)\gamma)$
AA	$V^{A}(1) + V^{A}(\pi + (1 - \pi)(\gamma - \alpha) + 1) - V^{A}(\pi + (1 - \pi)\gamma)$

will get adapted or satiated, they underestimate the magnitude of these changes. Essentially, under projection bias, a subject predicts that the future satiation and habituation levels will be similar to current levels.

Simonson (1990) and Gilbert (2006) observe that in advance of consumption, subjects prefer variety, but in the actual choice, they prefer the same consumption good, presumably because the actual satiation level has waned. In other words, subjects express a preference for the peach yogurt tomorrow after consuming the strawberry yogurt today, but end up actually consuming the strawberry yogurt again tomorrow.

For simplicity, we consider the case of two periods and introduce the projection bias parameter  $0 \le \pi \le 1$ . The consumer correctly predicts the utility he will experience from consumption in period 1. However, when forecasting the per-period utility of period 2, the consumer incorrectly predicts  $y_2$  and  $r_2$ . From the viewpoint of period 1, the prediction of  $y_2$  and  $r_2$ , denoted by  $\hat{y}_{2,1}$  and  $\hat{r}_{2,1}$ , respectively, soon after consuming  $x_1$  is (we omit the superscript k)

$$\hat{y}_{2,1} = \pi(x_1 + y_1) + (1 - \pi)y_2$$
 and 
$$\hat{r}_{2,1} = \pi r_1 + (1 - \pi)r_2.$$

When the projection bias parameter  $\pi$  is equal to zero, the consumer correctly predicts the future satiation and habituation levels. When  $\pi$  is equal to one, then the consumer falsely believes that the current levels of satiation and habituation will apply to the future period.

Setting  $y_1 = r_1 = 0$ , and considering the discrete unit choice problem with two periods and two goods, we have that predicted utility of AB and AA is given in Table 5.

Using Proposition 4, it is straightforward to show that the predicted preference of AB over AA is driven by the sign of  $\hat{y}_{2,1} - \hat{r}_{2,1}$ . We have the following:

PROPOSITION 7. If  $\gamma \geq \alpha$ , then the optimal choice is AB, and the consumer will correctly predict this choice for all  $0 \leq \pi \leq 1$ . In contrast, if  $\alpha > \gamma$ , then the optimal choice is AA, but if  $\pi > (\alpha - \gamma)/(1 - \alpha + \gamma)$ , then the subject will incorrectly predict that AB is better (and make the correct prediction otherwise).

In the discrete case and two periods, the consumer could revise his choice in period 2 and implement the optimal sequence, say *AA*. However, in practice, the

<sup>&</sup>lt;sup>6</sup> The HARA class of linear risk tolerance with positive intercept satisfies this assumption. This includes as special cases both the power and the exponential value functions. Holt and Laury (2002) show that subjects do exhibit increasing relative risk aversion and use an expo-power class to model such preferences.

consumer may have committed to *AB*. In problems with variable quantity choice and budget constraint, projection bias leads subjects to spend too much too soon, making it impossible to revert to the optimal sequence (Baucells and Sarin 2008).

It has been well documented that shoppers who are hungry overbuy food (Nisbett and Kanouse 1968, Gilbert et al. 2002). In our model, such behavior is easily represented with variables such as the anticipated satiation level. When hungry (low y), the anticipated satiation level is lower than what it will turn out to be. This effect applies to other consumption goods, and it is often used as a marketing tool. For instance, at the beginning of the ski season, when people are "hungry" for skiing, people predict that they may go skiing more often than they actually will. Hence, they may find an offer of 10 ski passes attractive. As the season unfolds and the satiation level for skiing increases, the consumer may end up not taking advantage of the package. Similarly, theme parks offer seasonal passes at a premium, and these end up having no extra value for a consumer who after visiting the park a few times is satiated for the season. Finally, consumers may overpurchase vacation days on cruise ships or resorts because one does not anticipate correctly the satiation associated with staying in the same place for several days.

#### 7. Conclusions

This paper introduces the HS model, a utility model that incorporates both satiation and habit formation into evaluating time streams of consumption. Present consumption creates satiation, but also contributes to habit formation, thereby influencing the utility of future consumption. In the HS model, satiation is influenced by parameter  $\gamma$  and habit formation by parameter  $\alpha$ . Thus, the evaluation of a consumption stream and the optimal consumption plan depends on the relative values of  $\alpha$  and  $\gamma$ .

We show that under constant consumption, the desirability (willingness to pay) of a unit consumption goes through three distinct phases. It decreases first (satiation), then increases (sensitization), and again decreases (habituation). Abstinence from consumption creates craving and produces heightened pleasure (satisfaction) when consumption resumes. Fasting before a festive meal will produce such an experience.

A prediction of the HS model in discrete choice situations is that a consumer will seek variety in the early periods to mitigate the effects of satiation, but will gradually shift consumption toward highly habitforming goods. A rational consumer will limit the amount of habit-forming goods consumed by not initiating consumption of some goods. Consumers may start with identical preferences, but because of some arbitrary initial choice may gravitate toward different choices. For example, people initially may choose a sport activity such as golf, tennis, or running on some social or circumstantial basis, but then get habituated and continue to indulge in the same sport.

The optimal consumption sequence obtained by using the HS model is either U-shaped or increasing under some reasonable forms of the value function. In the optimal plan, satiation levels are always increasing. Total utility or well-being is maximized by controlling reference and satiation levels in early periods and then gradually increasing consumption over time. Hence, a rational consumer facing a budget constraint will pay careful attention to the optimal moment to initiate a new habit.

Our aim in this paper has been to present a two-parameters model that allows us to mimic and rationalize a large number of preference and consumption patterns that are beyond the patterns compatible with usual utility models. We have shown that the patterns generated by our model are consistent with those empirically observed by psychologists or documented by other decision researchers. We offer a relatively parsimonious model that has realistic empirical implications in some domains of contemporary debate such as happiness or life simplicity in a consumer society.

We suggest a few directions for future research. The first is to examine the implications of the HS model in the context of time horizon uncertainty. Second, it would be interesting to explore the full implications of the HS model to well-being and happiness research. Here, one could think of total utility as measuring lifetime happiness and experienced utility as measuring moment happiness. Third, an axiomatic study of the HS model is worth exploring. Finally, empirical research is needed to test the predictions of the HS model.

The speed of habituation ( $\alpha$ ) could be sign dependent. For certain goods, one could imagine having  $\alpha^+ > \alpha^-$  (i.e., habitual levels of consumption adapt to current consumption faster when consumption exceeds the reference level and diminishes slowly when consumption is below the habitual level of consumption). Similarly,  $\gamma$  could be different as satiation increases faster with consumption than it diminishes with no consumption. More generally, satiation and habituation levels may be more complex functions of past and future consumption (Koszegi and Rabin 2006). Empirical research may clarify the evolution of satiation and habituation levels.

We have shown that the flexibility of the HS model is needed to account for well-documented preferences and behaviors. Becker (1992) argues that both satiation and habit formation are relevant in analyzing the influence of past consumption on present behavior. Heaton (1995, p. 681) finds "evidence for a preference structure in which consumption at nearby dates is substitutable and where habit over consumption develops slowly." Our model shows that sophisticated patterns of variety seeking, selective formation of habits, and increasing consumption sequences are consistent with the rational choice of a consumer that experiences satiation and habit formation.

#### Acknowledgments

The authors are grateful to the associate editor and two anonymous referees for their helpful comments, and to Peter Kolesar and German Loewe for their suggestions.

#### Appendix

#### A.1. Proofs

PROOF OF PROPOSITION 1. For  $t \ge 1$ , let  $X_t = y_t - r_t + 1$ . Under unit consumption,  $x_t = 1$ ,  $t \ge 1$ , we can use Equations (2) and (3) and write  $X_{t+2} = (\gamma - \alpha + 1)X_{t+1} + (1 - \alpha)\gamma X_t$ . This is a second-degree homogeneous recursion with solution  $X_t = c_1\gamma^t + c_2(1-\alpha)^t$ , and initial conditions  $X_1 = 1$  and  $X_2 = \gamma - \alpha + 1$ . Solving the recursion yields

$$X_t = \frac{(1-\alpha)^t - \gamma^t}{1-\alpha - \gamma},$$
 
$$y_t = \gamma X_{t-1} = \frac{\gamma (1-\alpha)^{t-1} - \gamma^t}{1-\alpha - \gamma}, \quad \text{and}$$
 
$$r_t = 1 - (1-\alpha)^{t-1}.$$

As a function of a continuous  $t \ge 0$ ,  $X_t$  is single peaked: begins at  $X_0 = 0$ , increases until some  $\tilde{t}^* > 0$ , and then decreases, tending again to zero as t increases to infinity. To see that the peak is unique, solve the first-order condition,  $(1-\alpha)^t \ln(1-\alpha) = \gamma^t \ln(\gamma) = 0$ , to obtain

$$\tilde{t}^* = \frac{1}{\ln((1-\alpha)/\gamma)} \ln \frac{\ln \gamma}{\ln 1 - \alpha}.$$
 (21)

The solution is unique and strictly positive. This single peak is a maximum because  $X_1=1$  and  $X_0=X_\infty=0$ . Note that  $\tilde{t}^*$  can either be small  $(\alpha \to 1 \text{ or } \gamma \to 0)$  or arbitrarily large  $(\alpha \to 0 \text{ or } \gamma \to 1)$ . We define  $t^*$  as the unique continuous value between  $\tilde{t}^*-1$  and  $\tilde{t}^*$  for which  $X_{t^*+1}=X_t^*$  and hence  $D(X_{t^*+1})=D(X_{t^*});\ t^*$  is strictly positive because  $X_1>X_0$ . If  $\gamma<\alpha$ , then  $X_2=\gamma-\alpha+1<1$  and  $t^*<1$ . If  $\gamma\geq\alpha$ , then  $X_2>1$  and  $t^*\geq 1$ .

Because v is S-shaped, D(X)=v(X)-v(X-1) increases for  $X<X^{**}$  and decreases for  $X>X^{**}$ , where  $X^{**}$  is the solution to the first-order condition v'(X-1)=v'(X). If  $v'(-1)\leq v'(0+)$ , then  $X^{**}$  exists and is unique. If v'(-1)>v'(0+), then set  $X^{**}=0$ . Because  $v'(0^-)>v'(0^+)$ ,  $X^{**}<1$  and, as  $X_1=1$ , there is a  $\tilde{t}^{**}>1$  for which  $X_{\tilde{t}^{**}}=X^{**}$ . As  $X_{\tilde{t}^*}>1$ , we have that  $\tilde{t}^{**}>\tilde{t}^*$ . Let  $t^{**}$  be the time period larger than  $t^*$  for which  $D(X_{t^{**}})=D(X_{t^{**}+1})$ . Because D is a single-peaked function of X, if  $t^{**}$  exists, then it is unique. If  $X^{**}=0$  or  $t^{**}$  does not exist, then we can set  $t^{**}=\infty$ . By construction,  $t^{**}>t^*$ .

Initially,  $X_1 = 1$  is above  $X^{**}$ . Hence,  $D(X_t)$  and  $X_t$  move in opposite directions. For  $t < t^*$ ,  $X_t$  increases and D(t)

decreases; for  $t^* < t < t^{**}$ ,  $X_t$  decreases and D(t) increases; and for  $t > t^{**}$ , both  $X_t$  and D(t) decrease.  $\square$ 

PROOF OF PROPOSITION 2. In the SA model, satiation levels are nonnegative, and in the DU and HA models, satiation levels are zero. If  $y \ge 0$ , then the incremental utility of satisfaction is always evaluated in the positive part of the value function, and by concavity, v(y-r+1)-v(y) cannot be higher than v(1). If y < 0,  $y-r+1 \ge 0$ , and  $\lambda > 1+r/|y|$ , then

$$v(y-r+1) - v(y) \ge v(y-r+1) + \lambda v(-y)$$
  
  $\ge [1-r-(\lambda-1)y]v(1) > v(1). \quad \Box \quad (22)$ 

PROOF OF PROPOSITION 3. Inspection of Figure 5 shows that  $x_1$  increases  $r_2$  in  $\alpha x_1$ , increases  $r_3$  in  $\alpha (1-\alpha)x_1$ , increases  $r_4$  in  $\alpha (1-\alpha)^2 x_1$ , etc., showing that  $\partial x_t/\partial x_s$  is given by (10). Regarding Equation (11), note that  $x_s$  increases  $y_{s+1}$  in  $\gamma x_s$ . The effect of  $x_s$  on  $y_{s+2}$  is a combination of a direct effect ( $\gamma^2$ ) and an indirect effect ( $-\alpha \gamma$ ) because of the mitigation effect that habit formation has on satiation. The final effect of  $x_s$  on  $y_t$ , t > s, is given by

$$\begin{split} \partial y_t/\partial x_s &= \gamma^{t-s} - \alpha \gamma^{t-(s+1)} \\ &- \alpha (1-\alpha) \gamma^{t-(s+1)-1} - \dots - \alpha (1-\alpha)^{t-s-2} \gamma \\ &= \gamma^{t-s} - \alpha (1-\alpha)^{t-s-1} \sum_{q=1}^{t-s-1} \left[ \frac{\gamma}{1-\alpha} \right]^q \\ &= \frac{\gamma (1-\gamma) \gamma^{t-s-1} - \gamma \alpha (1-\alpha)^{t-s-1}}{1-\alpha-\gamma}. \quad \Box \end{split}$$

PROOF OF PROPOSITION 4. Inequality (18) can be written as  $v(z) \geq v(1+z) - v(1)$ , with  $z = \gamma - \alpha$ . We argue that the direction of inequality (18) is driven by the sign of z. If  $z \geq 0$ , then the concavity of v for gains readily implies (18), as  $v(z) \geq v(x+z) - v(x)$ ,  $x, z \geq 0$ , and AB is optimal. If  $z \leq 0$ , then by loss aversion and concavity, respectively, we have that  $v(z) \leq -v(-z) \leq v(1+z) - v(1)$ . Therefore, either AA is optimal.  $\square$ 

PROOF OF PROPOSITION 5. Consider a consumption sequence that rotates regularly among l goods. As utility is separable across goods, we consider the contribution to utility of one of them. We consider a period in which we happen to consume such a good, and relabel that as period 1, which is preceded and followed by l-1 periods of abstinence from this good. During abstinence from the good,  $x_t = 0$ , t = 2, 3, ..., l, we use Equations (2) and (3) to write  $y_{t+2} - r_{t+2} = (\gamma - \alpha + 1)(y_{t+1} - r_{t+1}) + (1 - \alpha)\gamma(y_t - r_t)$ . Solving this second-degree homogeneous recursion produces

$$y_t - r_t = y_2 \gamma^{t-2} - r_2 \frac{(1-\alpha)^{t-1} - \gamma^{t-1}}{1-\alpha - \gamma},$$
  
 $y_t = y_2 \gamma^{t-2} - r_2 \frac{\gamma (1-\alpha)^{t-2} - \gamma^{t-1}}{1-\alpha - \gamma},$  and  $r_t = r_2 (1-\alpha)^{t-2}, \quad t = 2, 3, \dots.$ 

For l sufficiently large,  $y_{l+1}$  and  $r_{l+1}$  tend to zero. Hence, both  $y_1$  and  $r_1$ , which follow after l periods of abstinence, are close to zero (and equal to zero if it is the first occasion of consumption). Therefore, satisfaction from consumption,  $v(y_1-r_1+1)-v(y_1)$ , can be assumed to be less than  $v(1)+\epsilon$ .

By the same token,  $y_2$  and  $r_2$  are arbitrarily close to  $\gamma$  and  $\alpha$ , respectively, and  $\sum_{\tau=2}^{l} r_{\tau} = r_2[1-(1-\alpha)^{l-1}]/\alpha$  tends to one.

We now calculate the total withdrawal, and argue that, for l sufficiently large, it strictly exceeds v(1). Hence, replacing the consumption of the good under consideration for abstinence will increase utility. During the proof, we will use the fact that if d > 0 and  $y - r + d \ge 0$ , then

$$v(y) - v(y - r) > v(y + d) - v(y - r + d).$$
 (23)

If  $y-r \ge 0$ , then (23) holds by the concavity of v. If  $y-r < 0 \le y$ , then combining  $v(y) \ge v(y+d) - v(d)$  and  $-v(y-r) \ge v(r-y) \ge v(d) - v(y-r+d)$  yields (23). Finally, if y < 0, then combining  $v(y) \ge -v(y) \ge v(y+d) - v(d)$  and  $-v(y-r) \ge v(d) - v(y-r+d)$  yields (23).

Case  $\alpha > \gamma$ . If l is sufficiently large, then  $y_2 - r_2$  is close to  $\gamma - \alpha$ , which is strictly negative. Set  $d = r_2 - y_2 > 0$  and use (23) to conclude that  $v(y_2) - v(y_2 - r_2) = \epsilon_2 + v(r_2)$  for some  $\epsilon_2 > 0$ . For  $t \geq 3$ , let  $d_t = \sum_{\tau=2}^t r_\tau - y_t$ , which is strictly positive because  $r_2 > y_2$  and  $y_t$  is decreasing. Note that  $y_t - r_t + d_t = \sum_{\tau=2}^{t-1} r_\tau \geq 0$ , so that we can use (23) to show that

$$\begin{split} \sum_{t=2}^{l} v(y_t) - v(y_t - r_t) &= \epsilon_2 + v(r_2) + \sum_{t=3}^{l} \left[ v(y_t) - v(y_t - r_t) \right] \\ &\geq \epsilon_2 + v(r_2) + \sum_{t=3}^{l} \left[ v(y_t + d_t) - v(y_t - r_t + d_t) \right] \\ &= \epsilon_2 + v(r_2) + \sum_{t=3}^{l} \left[ v\left(\sum_{\tau=2}^{t} r_{\tau}\right) - v\left(\sum_{\tau=2}^{t-1} r_{\tau}\right) \right] \\ &= \epsilon_2 + v\left(\sum_{\tau=2}^{l} r_{\tau}\right) \to \epsilon_2 + v(1). \end{split}$$

Case  $\alpha \leq \gamma$ . If l is sufficiently large, v strictly concave for gains ensure that  $v(y_2)-v(y_2-r_2)=\epsilon_3+v(1)-v(1-r_2)$  for some  $\epsilon_3>0$ . For  $t\geq 3$ , let  $d_t=1-\sum_{\tau=2}^{t-1}r_\tau-y_t$ , which is strictly positive because either  $y_t$  is negative or, otherwise, for  $t\geq 3$ ,  $y_t=\gamma(y_t-r_t)< y_t-r_t$ , which we can recursively use to show that  $y_t< y_2-\sum_{t=2}^{t-1}r_\tau<1-\sum_{\tau=2}^{t-1}r_\tau$ . Moreover,  $y_t-r_t+d_t=1-\sum_{\tau=2}^{t}r_\tau\geq 0$ , so that we can use (23) to show that the total withdrawal exceeds v(1):

$$\begin{split} &\sum_{t=2}^{l} v(y_{t}) - v(y_{t} - r_{t}) \\ &= \epsilon_{3} + v(1) - v(1 - r_{2}) + \sum_{t=3}^{l} \left[ v(y_{t}) - v(y_{t} - r_{t}) \right] \\ &\geq \epsilon_{3} + v(1) - v(1 - r_{2}) + \sum_{t=3}^{l} \left[ v(y_{t} + d_{t}) - v(y_{t} - r_{t} + d_{t}) \right] \\ &= \epsilon_{3} + v(1) - v(1 - r_{2}) + \sum_{t=3}^{l} \left[ v\left(1 - \sum_{\tau=2}^{t-1} r_{\tau}\right) - v\left(1 - \sum_{\tau=2}^{t} r_{\tau}\right) \right] \\ &= \epsilon_{3} + v(1) + v\left(1 - \sum_{\tau=2}^{l} r_{t}\right) \rightarrow \epsilon_{3} + v(1). \quad \Box \end{split}$$

PROOF OF PROPOSITION 6. Our key observation is that, given satiation levels  $y_1, y_2, \dots, y_T, y_{T+1}$ , both the consumption and reference levels can be determined using (3)

and (2). Indeed, noting that  $r_{t+1} - r_t = \alpha(x_t - r_t) = \alpha(y_{t+1}/\gamma - y_t)$  and  $x_t = r_t + y_{t+1}/\gamma - y_t$ , t = 1, ..., T, both  $r_{t+1}$  and  $x_t$  can be directly calculated as follows:

$$r_{t+1} = r_1 + \sum_{\tau=1}^{t} (r_{\tau+1} - r_{\tau}) = r_1 + \alpha \sum_{\tau=1}^{t} (y_{\tau+1}/\gamma - y_{\tau}),$$

$$t = 1, \dots, T - 1 \quad \text{and} \quad (24)$$

$$x_t = r_1 + (y_{t+1}/\gamma - y_t) + \alpha \sum_{\tau=1}^{t-1} (y_{\tau+1}/\gamma - y_{\tau}),$$

$$t = 1, \dots, T. \quad (25)$$

As a consequence, the optimization program can be reformulated as one of choosing  $(y_2, \ldots, y_T, y_{T+1})$  to find

$$\max \sum_{t=1}^{T} v(y_{t+1}/\gamma) - v(y_{t})$$
s.t.  $Tr_{1} + \sum_{t=1}^{T} (1 + \alpha(T - t))(y_{t+1}/\gamma - y_{t}) \le I$ .

Here, the budget constraint is obtained by adding (25) across all periods. If we define  $\hat{v}(y) \equiv \gamma [v(y/\gamma) - v(y)]$ , then the first-order conditions are

$$\hat{v}'(y_t) = \lambda[(1 - \gamma)(1 + \alpha(T - t)) + \alpha], \quad t = 2, ..., T$$
 (26)

and

$$v'(y_{T+1}/\gamma) = \lambda. \tag{27}$$

Note that the right-hand side of (26) is positive and decreasing in t. If v is concave in the positive domain, then  $\hat{v}'$  is positive and decreasing near zero. (Note that  $\hat{v}'(0) = v'(0)(1-\gamma) > 0$  and  $\hat{v}''(0) = v''(0)(1/\gamma - \gamma) < 0$ .) To have a unique solution, it is sufficient to have  $\hat{v}'$  decreasing in the domain in which  $\hat{v}'$  is positive. To ensure this, it suffices to assume that v exhibits nondecreasing relative risk aversion. To see this, note that

$$-y\frac{v''(y/\gamma)}{\gamma^2v'(y)} > -y\frac{v''(y/\gamma)}{\gamma v'(y/\gamma)} \ge -y\frac{v''(y)}{v'(y)},$$
 (28)

where the first inequality follows from  $\hat{v}'(y) = v'(y/\gamma) - \gamma v'(y) > 0$ , and the second from nondecreasing relative risk aversion. This implies that  $-v''(y/\gamma) > -\gamma^2 v''(y)$ , or that  $\hat{v}''(y) < 0$ .

The right-hand side of (26) is positive and nonincreasing in t. The left-hand side is positive and strictly decreasing. Therefore, a unique solution exists and satisfies  $0 \le y_2 \le y_3 \le \cdots \le y_T$ . To see that  $y_T \le y_{T+1}$ , note that

$$v'(y_T/\gamma) - \gamma v'(y_T) = \lambda (1 - \gamma + \alpha) \ge \lambda - \gamma \lambda$$
$$= v'(y_{T+1}/\gamma) - \gamma v'(y_{T+1}/\gamma)$$
$$\ge v'(y_{T+1}/\gamma) - \gamma v'(y_{T+1}).$$

Regarding the properties of consumption, we have that  $x_1 = y_2/\gamma - y_1$  and, for t = 1, ..., T - 1,

$$x_{t+1} - x_t = \alpha(r_t - x_t) + (y_{t+2}/\gamma - y_{t+1}) - (y_{t+1}/\gamma - y_t)$$
  
=  $(y_{t+2} - y_{t+1})/\gamma - (y_{t+1} - y_t) + \alpha(y_{t+1}/\gamma - y_t).$ 

If  $\alpha \geq \gamma$ , then the third term cancels the second, and  $x_{t+1} - x_t \geq 0$ , t = 1, ..., T, and increasing consumption from  $x_1$  and on follows. And if  $\hat{v}'' \geq 0$ , then  $y_{t+1} - y_t \leq y_{t+2} - y_{t+1}$  and  $x_{t+1} \geq x_t$ , t = 2, ..., T - 1, and increasing consumption

from  $x_2$  and on follows. If consumption levels are increasing, then so are the habitual levels of consumption.

The program can be solved explicitly for the power utility function,  $v(x) = x^{\beta}$ . In this case, experienced utility in each period, as well as the indirect utility for money, is proportional to  $I^{\beta}$ .  $\square$ 

PROOF OF PROPOSITION 7. It is straightforward to show, as done in Proposition 4, that the predicted preference of *AB* over *AA* is driven by the sign of

$$\hat{y}_{2,1} - \hat{r}_{2,1} = \pi + (1 - \pi)(\gamma - \alpha). \tag{29}$$

Because  $\pi$  is positive, if  $\gamma-\alpha$  is positive, then so is  $\hat{y}_{2,1}-\hat{r}_{2,1}$ . Hence, when AB is optimal, then it is always perceived to be optimal. But if  $\gamma-\alpha$  is negative and  $\pi>(\alpha-\gamma)/(1-\alpha+\gamma)$ , then  $\hat{y}_{2,1}-\hat{r}_{2,1}$  is positive and AB is perceived to be optimal, a perception that is wrong because it is AA that maximizes total utility.  $\square$ 

### A.2. Mixed Integer Programming (MIP) Formulation of the Discrete Choice Problem

To find optimal solutions to the discrete unit choice problem, we propose the following piecewise linear value function, which we call the *canonical value function* (see Figure A.1):

$$v(x) = \max\{-\lambda, \lambda x, \min\{1, x\}\}, \quad \lambda \ge 1.$$
 (30)

In this setup, what is the utility of one unit of consumption? If y = r = 0, then the utility is one. If 0 < y - r < 1, then the utility decreases to 1 - y; and if  $-1 \le y - r \le 0$  and y > 0, then the utility is 1 - r. Finally, if -1 < y - r < 0 and y < 0, then the utility is 1 - r - y.

The canonical value function has four segments. In the formulation, we will consider that v has twice as many segments, the first four correspond to the evaluations of v at y-r+x and the last four to the evaluation of v at y. We apply the standard procedure to linearize v: per each good k and period t, besides having the consumption decision variable  $x_t^k$ , we introduce eight continuous and eight binary decision variables, one for each segment of the value function, namely,  $v_{t,s}^k$  and  $b_{t,s}^k$ , respectively.

Here is the complete MIP formulation of the discrete unit choice problem. It can be easily extended to any number

Figure A.1 Canonical Value Function

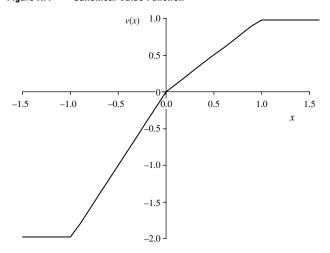


Table A.1 Coefficients for the MIP Representing the Piecewise Linear Value Function

		v(y – 1	(+x)			V(y)					
Segments	1	2	3	4	5	6	7	8			
Slope $c_s$ Intercept $d_s$	0 -2	2	1 0	0	0 2	-2 0	-1 0	0 -1			
Interval begins, $L_s$ Interval ends, $U_s$	−2 −1	-1 0	0 1	1 2	−2 −1	-1 0	0 1	1 2			

of segments, which is conceptually important, because any piecewise differentiable function can be approximated by a piecewise linear function.

$$\max \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{s=1}^{8} c_s v_{t,s}^k + d_s b_{t,s}^k$$
 (31)

s.t. 
$$y_{t+1}^k = \gamma^k (y_t^k + x_t^k - r_t^k)$$
,

$$t = 1, ..., T - 1 \quad \forall k, y_1^k = 0, \quad (32)$$

$$r_{t+1}^k = r_t^k + \alpha^k (x_t^k - r_t^k),$$

$$t = 1, ..., T - 1 \quad \forall k, r_1^k = 0, \quad (33)$$

$$\sum_{s=1}^{4} v_{t,s}^{k} = y_{t}^{k} - r_{t}^{k} + x_{t}^{k} \quad \forall t \ \forall k,$$
 (34)

$$\sum_{s=5}^{8} v_{t,s}^{k} = y_{t}^{k} \quad \forall t \ \forall k, \tag{35}$$

$$\sum_{s=1}^{4} b_{t,s}^{k} = 1, \quad \sum_{s=5}^{8} b_{t,s}^{k} = 1 \quad \forall t \ \forall k,$$
 (36)

$$L_s b_{t,s}^k \le v_{t,s}^k \le U_s b_{t,s}^k \quad \forall t \ \forall k \ \forall s, \tag{37}$$

$$\sum_{k=1}^{K} x_t^k \le 1 \quad \forall t, \tag{38}$$

$$x_t^k \in \{0, 1\} \quad \forall t \ \forall k, \quad \text{and}$$

$$b_{t,s}^k \in \{0,1\} \quad \forall t \ \forall k \ \forall s. \tag{39}$$

The coefficients  $c_s$ ,  $d_s$ ,  $L_s$ , and  $U_s$  are given in Table A.1.

This program can be easily transformed into the discrete choice problem with variable quantity. To do so, we add binary decision variables  $h_t^k$ ,  $\forall t$ ,  $\forall k$ , and replace rows (38) and (39) for the following constrains (M is an arbitrary large constant):

$$\sum_{k=1}^{K} h_t^k \le 1 \quad \forall t, \tag{40}$$

$$0 \le x_t^k \le M h_t^k \quad \forall t \ \forall k, \tag{41}$$

$$\sum_{t=1}^{T} \sum_{k=1}^{K} x_t^k \le I, \tag{42}$$

$$h_t^k \in \{0,1\} \quad \forall t \ \forall k \quad \text{and} \quad b_t^k \in \{0,1\} \quad \forall t \ \forall k \ \forall s.$$
 (43)

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