ASPECTS OF INSURANCE, INTERMEDIATION AND FINANCE

INVITATION LECTURE TO THE 1992 SEMINAR OF THE EUROPEAN GROUP OF RISK AND INSURANCE ECONOMISTS

M.J. BRENNAN*

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* Irwin and Goldyne Hearsh Professor of Finance, University of California, Los Angeles, and Professor of Finance, London Business School.
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The search for knowledge is a continuing struggle to impose patterns or order on the superficial chaos of everyday observations of the physical, social and spiritual world. Such patterns or "laws" allow us to compress our observations about the world into compact formulae or rules, such as the law of gravity or the law of supply and demand. We are driven further to seek order at a deeper level in the form of common unifying principles[1] that apply to superficially disparate phenomena. Thus, the science of the heavens, once the province of a special caste of priests and astrologers, eventually fell under the sway of the physicists, largely through the work of Newton and Kepler. Physicists themselves, having shown in the eighteenth century that heat and sound could also be understood as manifestations of the Newtonian laws of motion, have more recently been pre-occupied in attempting to provide a unified account of the four fundamental forces. Closer to home, students of finance have recognized their discipline as a sub-branch of economics at least since the establishment of the Journal of Financial Economics in 1972[2]. It is therefore curious that insurance and finance should have remained such distinct fields of study for so long, since both disciplines have as their primary concern the pricing and allocation of risk in the economy. Of course, the underlying unity of the fields has been recognized before, most notably by the pathbreaking contributions of the late Karl Borch[3]. Nevertheless, insurance and finance continue largely to be taught as different subjects, in different departments of the business school, and by different professors, and it is only relatively recently that such concepts as market efficiency, asset pricing and option pricing have crossed the disciplinary divide[4]. One reason for the persistence of this divide is

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[1] Barrow (1990) refers to the progressive reduction of the description of facts about the world to simpler and simpler laws as "algorithmic compressibility".

Adam Smith seems to have taken a dim view of this inclination to seek unity in explanation: "Epicurus indulged in a propensity which is natural to all men, but which philosophers are apt to cultivate with a peculiar fondness, as the great means of displaying their ingenuity, the propensity to account for all appearances from as few principles as possible", quoted by Sen (1988, p24).

[2] Perhaps a more significant date in this respect is 1958 when the first of the Modigliani-Miller papers on capital structure was published.

[3] See Boyle (1990) for a survey of Borch's contributions.

[4] I have remarked elsewhere that models of stock price behavior widely accepted by actuaries and insurance specialists in the 1970's cast ne'er a glance in the direction of the finance orthodoxy of efficient markets. One should also note how slow financial theorists were to pick up on the concept of duration developed by Redington (1952).

undoubtedly that the predominantly statistical orientation of the actuary is at odds with the paradigm of rational maximizing behavior that is the stock in trade of the economist. Of course we should not exaggerate the divide - financial economists have learned a lot about moral hazard and adverse selection in financial markets from those who had first observed these phenomena in insurance markets, and the modern insurance specialist is well apppraised of the current state of the major paradigms in finance.

If a distinction is to be made between finance and insurance, it is surely that finance is concerned primarily with aggregate, social or, as we have come to say, non-diversifiable, risks, while insurance is concerned with insurable or diversifiable risks. The death of an individual, while of intense importance to the individual, is of virtually no social consequence - it is the province of the actuary and not the economist, although plagues and famines which cause systematic variation in mortality rates are rightly of concern to the economist also.

This distinction between social risks and insurable risks is mirrored in the institutions that are employed to transfer them. By and large, the former are transferred in organized financial markets whereas the market for the latter consists of insurance and reinsurance companies. A firm that seeks to transfer its interest rate risk will turn to the interest rate futures or options markets, or perhaps to the market for swaptions; to transfer the risk that its factory will burn down it will turn to an insurance company. One reason for this specialisation is that organized security markets trade standardized contracts, whereas insurable risks are, by their very nature, unique, and require individual assessment[5]. Of course, the distinction between social and insurable risks is only a conceptual convenience - most risks share elements of both, and insurance markets ignore the social element at their peril[6]. Thus, an insurance company offering a life contract must bear (or shift through the

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[5] This does not entirely rule out the possibility of trade on organized markets, though it does make it more costly. For examples, consider the reinsurance market or Lloyds. Carter and Diacon (1990) remark that a substantial number of Lloyds insurers "do not attempt to make any detailed underwriting assessment of the risks..relying principally on the reputation of the leader". Such an equilibrium has been described by Bikchandani, Hirshleifer and Welch (1992) as a "cascade".

[6] A case in point may be the UK insurance companies who were rudely surprised by their losses on mortgage insurance in 1991 caused by declines in property values. It is ironical that a futures contract on housing prices should have died for lack of interest in the same year.

financial markets) interest rate as well as mortality risk.[7]

The fact that organized markets trade standardized securities means that they they rarely fit perfectly the idiosyncracies that characterize the risks borne by the individual. This opens a gap for financial intermediaries to bridge, by assuming the individual risk, hedging the systematic element on organized exchanges and bearing the idiosyncratic element in the same way that insurance companies retain mortality risk while shifting interest rate risk[8]. It is striking to the observer that by and large this role has not been performed by insurance companies, but has instead been assumed by banks and other over the counter providers of tailored derivative assets. The explanation for this lost opportunity awaits further analysis.

However, rather than pursue the theme of risk, I shall consider in this paper the insurance company as financial intermediary, and in particular as an intermediary whose products are aimed at the individual investor, in competition with those of other financial intermediaries such as banks and mutual funds. While insurance companies are distinguished in principle from other financial intermediaries in that their liabilities are contingent - on the lives of individuals, on fires and other natural phenomena - in practice this distinction has come to seem less and less relevant. Thus, insurance companies have come to look more like banks in offering fixed rate liabilities such as GIC's while at the same time banks have offered contingent liabilities such as Certificates of Deposit whose payoffs may depend on a stock market index or the rate of inflation in college tuition. Therefore, in what follows I shall not refer explicitly to the actuarial risks inherent in insurance company liabilities, and most of what I shall say in the first two sections of the paper will be equally applicable to other intermediaries. These sections are concerned with the determinants of the intermediary spread, the difference between the rates available on primary securities in the capital market and the rates paid on the liabilities of financial intermediaries.

I define a retail investor as one who does not possess expert knowledge of financial markets[9]. Such individuals face a choice between on the one hand

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[7] It is noteworthy that it was the insurance industry that developed the concept of duration which is an approximation to the interest rate delta of a portfolio of fixed payment assets or liabilities.

[8] For a further development of this theme see Merton (1989).

[9] The importance of knowledge as a determinant of investment choices is emphasized in an article in the **(London) Stock Exchange Quarterly** (1991) which reports the results of a survey that shows "Among non-(stock market)-

venturing unaided into the the treacherous waters of the market for primary securities, guided perhaps by the sometimes unreliable advice of friends or stockbrokers, and on the other hand purchasing the secondary securities issued by financial intermediaries such as mutual funds, insurance companies and banks. The advantages of the intermediary are that it will generally have a reputation to maintain, which provides some level of assurance for the individual; and that it will also typically be willing to provide as a part of its marketing efforts some education to the investor about the product he is purchasing[10]. Of course the cost of these marketing efforts must eventually be recouped from the investor, and the disadvantage of the intermediary is that it will impose an implicit or explicit management and sales fee or spread that creates a wedge between the returns on the primary securities held by the intermediary and the returns realized by the retail investor[11]. This intermediary spread has not received a great deal of attention in the academic literature, perhaps because the annual fee appears small relative to the potential gains and losses that may be experienced on an equity portfolio in the course of a year. Nevertheless, depending on the yield on the underlying portfolio and the holding period of the investor, the present value of this intermediary spread may be very large indeed relative to the wealth of the investor, and regulatory authorities in the United Kingdom have become increasingly concerned about the full disclosure of the spread. In Section I below I show how to compute the present value of the spread under a variety of assumptions, and demonstrate its economic significance.

Having argued for the economic importance of the intermediary spread, I present in Section II a simple equilibrium model of the spread that

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investors there remains a widespread lack of understanding of the market...Even among shareholders there is a very considerable lack of awareness of what they can do with their shares or how best to develop their holdings."

[10] Pauly et al (1986, p70) claim that one of the benefits of whole life insurance is "the investment and retirement counselling and planning services provided both at the time of sale and thereafter".

[11] The distinction between the returns available on primary and secondary securities is evident in the pricing of mutual funds and unit trusts on the one hand which sell at a premium to the underlying asset value if sales charges are taken into account, and investment trusts or closed end investment companies on the other, which typically sell at discount to the underlying net asset value. Mutual funds and unit trusts are (like insurance policies) sold to the investor. Investment trust shares on the other hand are traded like primary securities and do not pay explicit sales commissions except at the time of the initial underwriting.

is equally applicable to a bank, an insurance company or a mutual fund. This model assumes that the demand for intermediary products is inelastic, and specifies the growth rate of the intermediary's liabilities as a (linear) function of the difference between the rate of return it offers and the rate of return offered by the average firm in the industry. The model is shown to imply that the equilibrium spread is an increasing function of the market rate of interest; this appears to accord well with casual evidence on the pricing of life insurance in the U.S.

Section III is concerned with a characteristic of (life) insurance contracts that distinguishes them from the liabilities of most other financial intermediaries: this is the reversionary bonus or dividend that is declared periodically on participating, or with profits, life insurance contracts. Following Wilkie(1987) I interpret a reversionary bonus as a put option on an underlying portfolio of equities. Then, assuming that the insurance company sells these put options at fair value, it is shown that the distribution of the final payoff of the contract is generally suboptimal from the viewpoint of the purchaser/investor. By making explicit assumptions about the investment policy followed by the insurance company, it is possible to quantify the costs of the bonus policy. The cost is measured by the difference between the premium required by the insurance company to produce a given probaility distribution of contract payoff, and the minimum initial investment that would be required to produce the same probability distribution of contract payoff if an optimal investment policy were followed. The analysis assumes that the whole of the investor's wealth is invested in a single premium insurance contract, and, following Wilkie, neglects issues related to mortality. The results suggest that the traditional with profits contract may be subject to significant inefficiency.

I

THE COST OF THE INTERMEDIARY SPREAD

The retail investor who purchases the products of a financial intermediary typically pays an initial sales charge as well as a periodic management fee which may be either explicit or implicit. These fees and charges are avoided by the large investor who is able to purchase primary securities directly, so that the yield on retail bank deposits is less than that on large certificates of deposit, the return on whole life insurance is below the returns on long term bonds[12], and holders of mutual funds or unit trusts are required to pay an explicit management fee. Different financial products involve different combinations of initial sales charge and periodic fees, so that in making comparisons between the costs of these products it is necessary value the periodic fee correctly[13]. In this section we show how to value a periodic explicit management fee such as is charged by most investment managers[14]. The economic significance of this fee or spread is of interest to accountants and actuaries and regulators, as well as to economists and sophisticated individual investors.

Investment management fees are typically expressed as a simple fraction of the value of the portfolio under management[15]. While such fees may appear modest in relation to the assets under management, they are

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[12] Pauly et al (1986, p 70) report that in 1977 the average tax free rate of return earned on the investment element of a whole life policy held for 20 years was 2.71%. This compares with a yield of 5.73% after 25% tax (or 7.64% before tax) on a 20 year Treasury bond.

[13] Rule 12b-1 of the Securities and Exchange Commission, which was adopted in 1980, allows mutual funds in the US to use fund assets to fund sales activities on an ongoing basis. See Trzcinka et al. (1990).

[14] A valuation model such as we develop here is a prerequisite for hedging the present value of management fees; this may be important since management costs are largely fixed while fees depend on the value of funds under management which is influenced by asset returns as well as net fund inflows.

[15] Golec (1992) reports that of the 476 mutual funds listed in Moody's Bank and Finance Manual (1985) only 29 used anything other than a flat fee based on assets. The methods described below are readily applicable to the valuation of incentive fees, provided that each period the present value of the incentive fee is proportional to the value of the managed portfolio at the beginning of the period.

much more significant in relation to the income flow from a typical investment portfolio. Thus the median expense ratio on U.S. equity mutual funds was 1.57% in 1991[16], which compares with a dividend yield on the S&P 500 of approximately 3%. This means that the median management fee is absorbing approximately 50% of the income from the portfolio. Since there is virtually no evidence that the median mutual fund manager is able to outperform the S&P500, the investor in the median fund is surrendering approximately 50% of his income for no benefit beyond what would be available to a large investor investing on his own account. Consider an investor who withdraws each period the net dividend after management fees. As his holding period approaches infinity, the present value of the management fees, C, approaches one half of the value of the portfolio, W, so that the investor's effective wealth[17], net of management fees, V = W - C, is also only one half of the nominal value of the portfolio. More formally, if the periodic management fee is a fraction f of the value of the portfolio and the dividend yield on the portfolio is δ, the effective wealth of an investor who withdraws the net dividend of $(\delta - f)W$ each period and has an infinite holding period is given by:

$$V = (1 - f/\delta)\,W \tag{1}$$

Thus an investor who commits his wealth to a mutual fund intermediary in perpetuity is effectively surrendering close to 50% of his wealth to management fees[18]. Of course the assumption that the holding period is infinite is extreme, and results for finite holding periods may be obtained by using a result of Ross (1978). Thus, PVD, the present value of the dividends receivable over the next T years on a portfolio worth W, whose dividend yield is a constant, δ, is given by

$$PVD = [1 - (1 - \delta)^T]W \tag{2}$$

A management fee at the rate f will absorb a fraction f/δ of the dividend, so that the present value of the management fee over T years, assuming that the investor withdraws the net dividend $(\delta - f)W$, is:

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[16] Business Week, March 23, 1991. Brokerage houses compete directly with mutual funds by offering "wrap accounts" which are increasingly popular and carry annual fees of 2-3% of the value of the account. See Financial Times June 12, 1992.

[17] We define the investor's effective wealth as the difference between gross wealth and the value in perfect markets of the management fee.

[18] This takes no account of the load fees or sales charges that are often charged by mutual funds.

$$C = f/\delta\,[1 - (1 - \delta)^T]\,W \tag{3}$$

Table 1 reports calculations of the present value of the management fees under representative assumptions, and it can be seen that the management fee may account for a substantial fraction of the gross value of the portfolio when the holding period is long[19]. For example, using the median management fee of 1.57% and assuming a dividend yield of 3%, the management fee absorbs over 19% of the value of the portfolio for a holding period of 15 years.

When no funds are withdrawn from the portfolio before maturity, the present value of the management fee is given by $C = [1 - (1 - f)^T]W$. For example, if we take the figures in footnote 12 on a pre-tax basis ($f = 7.64 - 2.71 = 4.93\%$), and apply them to a single premium life insurance policy with a 20 year maturity we would conclude that management fees absorb approximately 33% of the investment component of the policy.

Having demonstrated the economic importance of the intermediary spread, we turn in the following section to consider its determinants.

[19] For shorter holding periods the sales charge, which we have not considered, will be proportionately more important.

II

THE PRICING OF INTERMEDIATED FINANCIAL PRODUCTS[20]

Ever since the pioneering study of Scholes (1972) it has been the conventional wisdom that the demand for traded securities is almost perfectly elastic. If the same were true of the liabilities of financial intermediaries, it is hard to see how such institutions could exist, since with price, or Bertrand, competition they would be unable to recover their fixed costs of operation. Yet we observe that mutual funds, banks and insurance companies are able to survive, by charging fees for intermediation services, either explicitly, or implicitly in the form of a spread between borrowing rates and the rates available on primary securities, that are well in excess of marginal costs. This indirect evidence in favor of the inelasticity in the demand for at least some intermediated products is reinforced by theoretical considerations which suggest that the demand for securities will be less elastic if they are not traded in organized markets, if they have unique characteristics, and if they are directed at the retail market.

First, it is only in organized markets where transaction prices are posted and investors can take short as well as long positions, that uninformed investors are able to free ride off the information contained in security prices[21]; when securities are not traded, individual investors must incur costs of investigation and search which are sufficient to ensure less than perfect elasticity[22]. Moreover, the costs of investigation are likely to be higher the more unique are the securities, for alternatives must then be compared along more different dimensions[23]. Thus most mutual funds[24] offer products that are unique because they depend upon the difficult to describe and validate skills and policies of the management. Insurance companies also offer a

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[20] I am grateful to Douglas Breeden for conversations that led to this formulation.

[21] See Grossman (1976). Note that it takes a degree of financial sophistication, not likely to be possessed by the retail investor, to extract information content from security prices.

[22] Carlson and McAfee (1983) show that consumer search costs lead to demand inelasticity. Pauly et al (1986) report that consumers pay lower prices for insurance in markets where it is easier to be informed about prices and qualities.

[23] Ross(1989) argues that marketing costs will be higher for securities that are unique and are not traded in active markets.

[24] An exception perhaps is the index fund.

bewildering array of products which are difficult to compare[25]. For bank deposits locational factors are likely to play a more significant role in reducing demand elasticity, since the product is relatively easy to understand and compare. Finally, retail purchasers of financial products face major problems in assessing all but the simplest of them[26], so that search costs are likely to be of proportionately greater importance for small retail investors than for large institutional investors[27].

These considerations point to an inelasticity in the demand for retail intermediated products that is not present for primary securities traded in active markets. In this section we develop a simple model of the pricing of intermediated products for which demand is imperfectly elastic; the model yields testable restrictions on the relation between the equilibrium spread and the market rate of interest[28].

In order to consider the pricing policy of an insurance company or other financial intermediary faced with inelastic product demand, define $V(Q,r,m,X)$ as the value of the company in excess of the market value of its assets when it pays the optimal rate of return on its outstanding liabilities and pursues an optimal marketing policy, where Q is the nominal (book) value of its liabilities, r is the market interest rate, m is the average rate paid on liabilities by all firms in the industry, and X is a vector of state variables describing the current state of the economy. $V(.)$ is the net present value of the company's liability stock, which arises because the company pays a rate of return on them which is below the rate available on primary securities. We assume that all liabilities are paid the same rate of return, which is adjusted

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[25] In this respect insurance companies are like automobile dealerships who resrict the degree of competititon between themselves by refusing to quote fixed prices, and forcing consumers to incur search costs to discover the true offer prices.

[26] Patel, Zeckhauser and Hendricks (1991) report that flows of funds into mutual funds are unduly influenced by recent past performance, which suggests that some investors are unable to evaluate the contribution of management well.

[27] Finsinger and Pauly (1986, p5) state that "the only real source of monopoly power in this (insurance) market appears to be imperfect consumer information.."

[28] See Kraus and Ross (1982) for a related model of the pricing of liability insurance. Unlike the model in this paper, those authors assume a regulated market for the products of the insurance company.

continuously. If the intermediary is a casualty insurance company, the rate of return paid on its liabilities would correspond to the excess of claims paid over premiums received. The model would correspond to a life insurance company which continuously adjusted its dividend policy, or to a bank which only accepted short term deposits. We shall refer to the intermediary as simply an insurance company.

The stock of outstanding liabilities is assumed to follow the stochastic differential equation:

$$dQ = Qg(u,m,r,s)dt + Q\eta_Q(X)dz_Q \tag{4}$$

where g(.) is the expected growth rate of liabilities, dz_Q is a Gauss-Wiener process, u is the current rate paid on liabilities, and s is the rate of marketing expenses incurred by the company per dollar of liabilities. We shall assume that the expected growth rate of liabilities can be written as the sum of functions of the difference between the current rate paid on liabilities and the average rate for the industry (u - m), the difference between the average rate paid by the industry and the market interest rate (m - r), and the rate of marketing expenses incurred by the firm per unit of outstanding liabilities, s:

$$g(u,m,r,s) = h(u - m) + k(m - r) + c(s) \tag{5}$$

where $h'(.)$, $k'(.)$,$c'(.) > 0$, and h'', $c'' < 0$, $c'(0) = \infty$. This function, whose form is chosen for analytic tractability, is intended to reflect the fact that a consumer, in deciding whether to purchase insurance from a particular company, will be influenced by the reputation of the insurance industry as a whole for offering value for money, as reflected in the industry spread (r - m)[29], as well as by the relative price charged by the particular firm (u - m)[30]; reflecting our assumption about the need to educate consumers, marketing expenditures will also raise the growth rate. In general, we might expect $h'(\,)$, the sensitivity of the growth rate to the relative rate differential offered by the company, to decrease as the product becomes more complex

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[29] Babbel (1985) finds considerable sensitivity of the net amount of new insurance to an index of industry costs which corresponds closely to our definition of the spread. Moreover, he finds much greater price elasticity for non-participating policies than for participating policies and remarks (p 234) that "such policies are much easier to compare because they have been shown to be less dispersed and because evaluation of dividends is not required".

[30] Carlson and McAfee (1983) develop a model in which uniformly distributed costs of search for consumers lead to firm demand functions which depend as here on the difference between the price charged by the firm and the average price charged by all firms in the industry.

and unique.

The competitive interest rate r evolves according to

$$dr = \mu(r,X)dt + \eta_r(r,X)dz_r \tag{6}$$

and the dynamics of the industry average rate paid on liabilities may be represented as

$$dm = \alpha(m,r,X)dt + \sigma_m(m,r,X)dz_m \tag{7}$$

where $\alpha(\ )$ and $\sigma_{m(\ )}$ are to be determined, and dz_m identified. Finally, the exogenous state variables describing the state of the economy evolve according to

$$dX = \mu_X(X)dt + \eta_X dz_X \tag{8}$$

The Gauss-Wiener processes dz_Q..etc. may be correlated but it will not be necessary for our purposes to parameterize these correlations.

The instantaneous rate of cash flow of the insurance company is

$$Q(r - u) - Qs \tag{9}$$

the difference between the gross margin earned on liabilities and marketing costs[31].

Let $L^{u,s}[V] \equiv E^*_{u,s}[dV]/dt$ represent the differential generator of V under the controls u and s, where expectations are taken with respect to the equivalent martingale measure under which all claims may be priced by their expected discounted values[32]. Then the Bellman equation for V( ) may be written as:

$$\max_{u,s} \{L^{u,s}[V] + Q(r - u) - Qs - rV\} = 0 \tag{10}$$

Since the cash flow rate is homogeneous of degree one in the level of liabilities Q, it follows that V( ) is also homogeneous of degree one:

[31] We are ignoring the costs of operating the company.

[32] See Harrison and Kreps (1979).

$$V(Q,r,m,X) = Q\,v(r,m,X) \tag{11}$$

where v is the value of a unit of liabilities. Then, substituting the partial derivatives of V from (11) in (10), we obtain:

$$\max_{u,s} \{L^{u,s}[v] + vg + (r - u) - s - rv\} = 0 \tag{12}$$

Notice that u and s appear in $L^{u,s}[v] + vg$ only in the term affecting the growth rate of liabilities: vg(u,m,r,s). Therefore, using (5), the first order conditions for a maximum in (12) are:

$$vh'(u - m) - 1 = 0 \tag{13}$$

$$vc'(s) - 1 = 0 \tag{14}$$

These conditions yield determinate values for the controls u and s in terms of v so long as $v > 0$, and the second order conditions will be satisfied at these values since $h''(u - m), v''(s) < 0$. Substitution of these values of the controls into (12) yields a partial differential equation for v. Solving this, an expression for v, the value of the insurance company per dollar of liabilities, is obtained. An explicit expression for the controls u and s in terms of the state variables r, m, and X may be found by substituting for v in conditions (13) and (14).

While this procedure yields the optimal policy of the individual company, it is more enlightening to proceed to an equilibrium analysis in which there are identical competing insurance companies. We consider a symmetric equilibrium in which all insurance companies pay the same rate on their liabilities[33], so that

$$u = m \tag{15}$$

Imposing condition (15) in (13), we find that in equilibrium the value of a unit of liabilities is:

[33] Carlson and McAfee (1983) show in a model with consumer search that equilibrium price dispersion converges to zero as the variance of firms' cost functions goes to zero; this corresponds in our model to all intermediaries having the same capital market opportunities.

$$v = 1/h'(0) = \text{constant} \tag{16}$$

Note that under perfect competition $h'(0) = \infty$ so that, as we should expect, the value of the firm is zero.

The optimal marketing expenditure rate, s^*, is also a constant which is given by the solution to:

$$c'(s) = 1/v = h'(0) \tag{17}$$

We note again that if the industry is perfectly competitive the optimal rate of marketing expenditure is zero. Then, substituting for u, the rate paid on liabilities in (12) from (15), and using the fact that v(r, m, X) is constant, we have

$$\frac{g(m,m,r,s)}{h'(0)} + (r - m) - s^* - \frac{r}{h'(0)} = 0 \tag{18}$$

Equation (18) is an implicit equation for the intermediary spread, (r - m), in terms of the market interest rate, r. In order to obtain an explicit expression for the spread, we shall assume that $k(m - r) = k_0 + k_1(m - r)$ where $k_1 > 0$. Then the average industry spread between the market interest rate r and the rate paid on liabilities, m is given by:

$$(r - m) = [1 - k_1/h'(0)]^{-1}[s^* - \frac{h(0) + k_0 + c(s^*)}{h'(0)} + \frac{r}{h'(0)}] \tag{19}$$

Thus in equilibrium the average industry spread between the rate on primary securities and the rate paid on the liabilities of the insurance company is an increasing function of the market interest rate, r, provided that $k_1 < h'(0)$. But this is just the condition that $\partial g/\partial m < 0$, that the growth rate of an individual company's liabilities be a decreasing function of the average rate paid by all firms in the industry. The spread is proportional to the sum of the equilibrium marketing expense rate, s^*, and a linear increasing function of the market interest rate, r. The intuition for this latter result is that v, the value of a unit of liabilities, is constant in equilibrium: when the interest rate is high the company must earn a higher margin on each unit of liabilities in order to ensure that it earns the (risk-adjusted) interest rate on the value of a unit of liabilities which, as we have seen, is independent of the interest rate. The key to this result is the assumption that in setting u, the rate it pays on liabilities, the individual company affects the growth rate of its liabilities by an amount that depends only on the difference between u and the industry average rate, m. In particular, we are assuming that the number of firms in the industry is sufficient that the effect of u on m can be neglected. The sensitivity of the spread to the market interest rate is a decreasing function of the sensitivity of liability flows to interest rates as measured by $h'(\,)$, because the value of a unit of liabilities is inversely proportional to $h'(\,)$ [See equation (16)].

We do not have direct empirical evidence on the intermediary spread. However, there is some indirect evidence on this quantity for insurance companies. Babbel and Staking (1983) have calculated a time series of the ratio of the present value of the expected costs of life insurance to the expected value of (death) benefits for the period 1950-1979. For a one year contract this would correspond exactly to the spread (r - m)[34]. Figure 1 plots the Babbel/Staking cost benefit ratio against the corporate bond rate annually for their sample period. The positive relation predicted by our model is evident in the plot[35]. While certainly not conclusive, this is encouraging for so simple a model. It would be interesting to gather data on marketing expenses and determine whether this is a constant fraction of liabilities as the model predicts. Of course, we have made a strong assumption that all companies pay the same rate on their liabilities; while this is consistent with all companies facing the same capital market opportunities and having the same (zero) costs of servicing liabilities, differential cost functions are likely to lead to rate dispersion in equilibrium[36].

[34] A one year contract costs \$1 and pays an expected benefit of \$(1 + m); hence the cost benefit ratio is $[(1 + m)e^{-r}] \approx (r - m)$.

[35] The regression equation is (standard error in parenthesis)
cost/benefit ratio = -0.109 + 0.229 bndyld $R^2 = 0.76$, N = 30
(0.024)

[36] Carlson and McAfee (1983).

III
BONUS POLICY

Thus far we have not distinguished between insurance companies and other types of financial intermediary. However, a distinguishing feature of contracts issued by insurance companies is that they frequently contain an element of discretionary bonus or dividend. For example, the classic UK with profits policy allows the policyholder to benefit from the periodic declaration of reversionary bonuses as well as a terminal bonus. These bonuses, which are at the discretion of the directors of the company, are additions to the sum assured and are payable at maturity. One effect of the bonus is to increase the life cover of the policyholder - we shall ignore this in keeping with our general neglect of mortality considerations. The second effect, which we shall focus on, is that the declaration of a bonus increases the minimum final payoff of the policy. The questions we shall address in a simple context are: first, is it efficient to ratchet up the minimum final payoff over time by reversionary bonus declarations? Secondly, if it is not efficient, what is the efficiency cost of the bonus declaration policy?

First, we should note that it is possible that the reversionary bonuses are to some degree cosmetic. That is, the original sum assured may be calculated using an assumed rate of interest that is below market rates, the insurance company may invest in bonds with a maturity corresponding to that of the policy and simply declare bonuses to bring the final payment up to what would originally have corresponded to the prevailing market interest rate. I am going to assume however that the bonus is not cosmetic and predetermined in this fashion, but rather that the insurance company takes investment risk on behalf of the policyholder, and that the declaration of future bonuses is discretionary and dependent on future investment performance. I shall also assume that each generation of policyholders is treated fairly in the sense that its payoff depends upon the returns of an identifiable portfolio so that there is no intergenerational transfer.

Under these assumptions the insurance company should sell a financial claim whose payoff corresponds to the payoff that the policyholder would have received from investing on his own account if he had the necessary expertise; that is, a policy that would maximize his expected utility of final wealth.

In order to see that an individual would not in general wish to ratchet up his minimum final payoff, consider an investor with initial wealth, W_0, who is concerned with maximizing the expected utility of wealth at time T, U(W); let p_s and π_s denote the price at time zero of a dollar to be received in state s at time T and the probability of state s respectively. The first order condition for a maximum in this problem is usually written as $U'(W_s) = \lambda p_s/\pi_s$, where

λ is the Lagrangean multiplier associated with the budget constraint. A guaranteed minimum payment at time T such as is promised by an endowment policy implies that $W_s = M$ for $s \in S^*$, where S^* is some subset of states and M is the face value of the policy. Now, it will generically be the case that the price/probability ratio p/π will differ across states, so that if the marginal utility function is continuous it will not be optimal to have the same wealth in different states. Thus, in the absence of discontinuities in the marginal utility function, the investor's optimal payoff will vary across states. A marginal utility function that is discontinuous at $W = M$ is illustrated in Figure 2. Now the first order conditions must be written as

$$\begin{aligned} U'(W) &> \lambda p_s/\pi_s \quad \text{for } W < W_s \\ U'(W) &< \lambda p_s/\pi_s \quad \text{for } W > W_s \end{aligned} \tag{20}$$

For all states such that $\lambda p_s/\pi_s < U'(M)$, it is optimal to choose the same wealth level M. This corresponds to the situation which makes it optimal to purchase a participating insurance policy with a minimum benefit M. Denote the allocation of state contingent wealth that satisfies (20) (and the budget constraint) by $\{A\}$.

The question to be considered is whether it will be optimal to subsequently ratchet up the minimum final payment as is accomplished by the declaration of a reversionary bonus. This would imply a change in the allocation of state contingent wealth from $\{A\}$ to $\{A'\}$. However, it will never be optimal to make such a shift in a complete market rational expectations equilibrium[37]. The reason is that in a complete market the state-contingent wealth allocations, $\{A\}$ and $\{A'\}$ were both initially available. Since $\{A\}$ was chosen initially, it is revealed to be preferred to $\{A'\}$. Thus, in a complete market at least, it is never optimal to ratchet up the minimum final payment.

Moreover, a policy of ratcheting will be costly or inefficient in the sense that there will exist another policy that does not involve ratcheting, and provides the same distribution of final wealth while requiring a lower initial investment. The efficiency cost of declaring reversionary bonuses is measured by the difference between the premium for the policy with reversionary bonuses and the minimum amount which, invested optimally, would provide the same distribution of final payoff as the policy. Table 2 provides a simple example of a state contingent wealth allocation which is inefficient[38]; an

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[37] For a formal proof of this result see Huang and Litzenberger (1988) Section 7.5.

[38] See Green and Srinivastava (1985).

individual with state independent utility will be indifferent between the allocations {X} and {Y} which both promise 50 or 100 with equal probability; however, allocation {Y} is more expensive than {X} and we would say that it has an efficiency cost of 45 - 30 = 15. In the same way, we shall quantify the efficiency cost of the wealth allocation yielded by an insurance contract with reversionary bonuses under certain assumptions.

In order to evaluate the possible order of magnitude of the cost of the reversionary bonus system we shall assume the simplest possible setting. We consider a single payment policy and ignore mortality risk. We assume that there is a single risky asset which follows the stochastic process:

$$\frac{dP}{P} = \mu\, dt + \sigma\, dz \tag{21}$$

where dz is the increment to a standard Gauss-Wiener process, and the interest rate is a constant, r[39]. We then have the following results which are proven in the Appendix:

<u>Result 1:</u> For an investor with a state independent utility function defined over final wealth, U(W), the optimal dollar investment in the risky asset is a determinate function of current wealth and time, x(W, t). The number of units of the risky asset to be held at time t is z(W, P, t) = x(W, t)/P.

<u>Result 2:</u> Under the optimal investment strategy, the investor's final wealth is a determinate function of the value of the risky asset, h(P).

<u>Result 3:</u> Let G(P) denote the distribution function for the price of the risky asset at the horizon, and let F(W) denote the distribution of final wealth under a given arbitrary investment strategy. Then the same distribution of final wealth, F(W), can be obtained by purchasing (or pursuing an investment policy that replicates) a contingent claim with payoff h(P), where

$$F[h(P)] = G(P) \tag{22}$$

<u>Result 4:</u> The value at time t of a contingent claim that pays of h(P) at time

[39] Cox and Leland (1982) were the first to show that policies such as ratcheting the minimum payoff were inefficient under these assumptions. Dybvig (1988a, 1988b) shows how to measure the inefficiency of a given portfolio strategy under more general assumptions. Brennan and Solanki (1981) derive the optimal payoff function under similar assumptions.

$Q(P,t)$, is given by

$$Q(P,t) = e^{-r(T-t)} E^{*}\{h[P(T)]\} \tag{23}$$

where $E^{*}\{\ \}$ denotes expectations with respect to the equivalent martingale measure that prices all financial claims in this market[40].

The foregoing results allow us to determine the cost of following any particular inefficient investment strategy for a given level of initial wealth, W_0. We first determine the distribution of final wealth under the strategy, $G(W)$; we then use Result 3 to determine the contingent claim that yields the same distribution of final wealth as the strategy: $h(P) = F^{-1}[G(P)]$. Finally, we determine the initial value of this contingent claim, $Q(P,0)$, using Result 4, and compare it with W_0.

In order to apply this technique to a with-profits or participating life insurance policy we shall follow David Wilkie (1987) and consider a single premium policy. The policy is assumed to correspond to the purchase of units in a mutual fund or unit trust, together with an equal number of put options on the units, with exercise price chosen so that a minimum payoff at maturity is assured. Let P_t denote the price of units of the unit trust at time t, n_t the number of units of the trust and the number of put options on the trust purchased, E_t the exercise price of the puts, M the face value of the policy and B_t the accumulated bonuses declared up till time t. The payoff at maturity T on the portfolio held at time t is

$$n_t[P_T + \max(E_t - P_T, 0)] = n_t E_t + n_t \max(P_T - E_t, 0) \tag{24}$$

The minimum payoff must correspond to the face value of the policy plus accumulated bonuses, so that the number of puts and their exercise price is related to the accumulated bonus by:

$$n_t E_t = M + B_t \tag{25}$$

Let $g(P_t, E_t, \tau)$ denote the value at time t of a European put on one unit with exercise price E_t, where $\tau \equiv T - t$. Then the choice of exercise price and number of fund units and puts must satisfy the budget constraint:

$$n_t[P_t + g(P_t, E_t, \tau)] = W_t \tag{26}$$

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[40] Under the equivalent martingale measure the expected return on all assets is equal to the riskless interest rate. See Harrison and Kreps (1979).

where W_t is the value of the policyholder's aggregate portfolio at time t. This is determined by the current value of the units and the portfolio choice at time t-1:

$$W_t = n_{t-1}[P_t + g(P_t, E_{t-1}, \tau)] \tag{27}$$

Given this setting, the bonus policy can be regarded as the procedure for determining B_t or, equivalently, n_tE_t, the guaranteed minimum payout at maturity (face value plus accumulated bonuses).

We shall focus first on a constant bonus rate policy which raises the guaranteed minimum payout at a constant rate each year so long as it is feasible to do so. Depending on the asset returns, a point may be reached at which there is insufficient wealth to declare the regular bonus, because the available wealth is less than the present value of the projected minimum payout if the regular bonus is declared, $M + B_t$. Under these circumstances, the highest final payout that can be guaranteed is equal to the compounded value of the current wealth, and this is achieved by investing all available wealth in the riskless security up to the horizon. We assume that when there is insufficient wealth to declare the regular bonus, the highest possible bonus is declared and there are no further bonus declarations. Again following Wilkie (1987), we assume that the put options are priced according to the Black-Scholes (1973) model. For our basic example we assume that the initial investment is \$1000, the effective annual interest rate is 5%, and that the logarithm of the annual return on the units is normally distributed with mean and standard deviation of 8% and 20% respectively; this implies that the expected rate of return on the units is 10% per year.

To evaluate the cost of a given bonus policy we then proceed as follows, using Monte Carlo simulation. We assume an initial investment W_0 = \$1000 which corresponds to the single premium paid at contract initiation, a 20 year contract, and a given policy face value, M. Then the initial number of units purchased, n_0, and the initial exercise price of the puts, E_0, are determined by solving equations (25) and (26) - the initial price of a unit is set arbitrarily at 100. The lognormally distributed return on the fund units for the year is generated by using a normal random number generator. Equation (27) is used to calculate the wealth at the end of the year, and the new number of units and put exercise price are determined using equations (25) and (26) for the new wealth level, time to maturity, and planned bonus level. The procedure is continued for 20 years producing a final contract payoff equal to the terminal wealth under the strategy. This procedure is repeated 2500 times to yield an estimated distribution of the contract payoff or terminal wealth, G(W). The payoff function of a contingent claim on the unit price, h(P), that yields the same distribution of terminal wealth is calculated using expression

(22), noting that the unit price at maturity is lognormally distributed[41]. Finally the initial cost of the contingent claim is estimated by evaluating the expectation in (23) using Monte Carlo simulation; in these simulations μ, the expected return on the unit trust, is set equal to the risk free rate in keeping with the martingale assumption underlying expression (23).

Figure 3 summarizes the results of the Monte Carlo simulations of the bonus and investment strategy described above when the initial face value of the policy is \$1500 and the annual reversionary bonus rate, b, is 5% of the sum of the face value and previous bonuses; the figure shows the payoff of the policy at the end of 20 years and the associated terminal value of the unit trust units, whose initial value is \$100. While there is a tendency for the payoff to increase with the terminal value of the fund units, the relationship is not simple; the reason for this is that the policy payoff depends not only on the terminal value of the fund unit, but also on the time path of the fund unit value; for example, if the fund return is sufficiently low in the early years, the projected bonus will fail to be met and the final payoff will be independent of the returns on the fund units during the later years. If the fund does well in the later years, this will not benefit the policyholder, so that good long term performance of the fund may be accompanied by either high or low payoffs for the policyholder. The risk of the policy payoff may be thought of as the sum of the risk associated with the terminal value of the fund units and the "residual" risk of the payoff given the terminal value of the fund units. In the setting described here there is no reward associated with this latter risk which therefore makes the contract inefficient. Figure 4 presents the estimated cumulative distribution of the policy payoff at the end of the 20 years, G(W), while Figure 5 shows the contingent claim h(P) whose payoff has the same marginal distribution as the policy payoff; h(P) is determined from expression (22). We note that, unlike the policy payoff, h(P) is a monotone increasing function of P, so that the contingent claim has no unrewarded "residual" risk; the payoff can be approximated by a bond plus two call options on the fund unit with different exercise prices. Finally the payoff function h(P) is valued, by evaluating expression (23) using Monte Carlo simulation. The estimated value of the contingent claim is \$913.39 (with a standard error of \$11.23). This compares with the initial investment in the insurance policy of \$1000. Thus the inefficiency due to the assumed bonus declaration and associated investment policy amounts to approximately 8.7% of the initial investment. Table 3 reports the efficiency costs of a 20 year \$1000 single premium contract under different assumptions about the face value of the policy and the annual bonus rate, b. It is apparent that the efficiency cost is an increasing function of the

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[41] Under our assumptions, $\ln(P_T/100) \sim N(\mu T - 0.5\sigma^2 T, \sigma^2 T)$.

bonus rate within this range[42].

If the with-profits insurance contract is inefficient, as the above analysis suggests, then it is natural to ask why it has survived. We should note the restrictive assumptions under which we have demonstrated inefficiency - first among these is the assumption of complete markets. If the market is incomplete, no such conclusion of inefficiency can be drawn. In particular, the insurance contract may be viewed as helping to complete the market. One significant way in which capital markets have been incomplete until recent times has been the lack of a real risk free asset. Our analysis has been implicitly cast in real terms, whereas insurance contracts are written in nominal terms. It is possible that an advantage of the participating contract in the absence of real bonds is that the face value of the policy plus bonuses could be implicitly tied to the price level - an explicit tie would be inappropriate since in the absence of a real riskless asset the insurance company would have no way of hedging the price level risk. If this account is correct then we would expect the popularity of these contracts to diminish as indexed bonds become available, facilitating the sale of real insurance contracts.

Secondly, we have implicitly assumed that the investor/insurance purchaser has a state-independent von Neumann-Morgenstern utility function defined over terminal wealth. It is possible that insurance purchasers have more complex utility functions that depend not only on the terminal wealth, but also on the time path of wealth over the life of the contract. If so, it is possible that a contract in which the payoff depends on the path of the values of the underlying investment is optimal.

Thirdly, since these contracts are sold to the retail investors we have characterized as uninformed, it is possible that the inefficiency of the contracts we have described has no effect on their marketability. Indeed the ability of the insurance salesman to point to a long history of stable reversionary bonuses may provide a selling point that would be absent in the time series of returns on the contingent claim that we have described as offering the same terminal payoff distribution at lower cost.

Finally, it should be emphasized that the magnitude of the efficiency losses we have described depends upon the model of the reversionary bonus process and associated investment strategy that we have described. It is

[42] The relation will not be monotonic, because for a sufficiently high bonus rate, the investment will be almost 100% in bonds and this will involve no efficiency cost.

possible for example, that the risk of the underlying unit trust or mutual fund is altered in response to past investment results whereas we have assumed that it is constant. The effect of varying the risk of the fund on efficiency will depend upon the precise strategy for varying it. Since the bonus process is discretionary, there are no formal rules, and this impedes a definitive assessment of its implications.

IV
CONCLUSION

Insurance, and particularly life insurance, is a product that is sold predominantly to what I have described as retail investors - those who have only limited knowledge of financial markets. This limited knowledge causes the demand for retail financial claims to be relatively inelastic, in contrast to the demand for primary securities which are traded in active secondary markets. As a result, retail investors earn returns which may be significantly below those available to sophisticated investors on primary securities, the difference corresponding to the intermediary spread. On the assumption that the search costs of retail investors cause the demand for the product of a single company to depend on the difference between the return offered by the company and the average return offered by all companies in the industry, we have shown that the intermediary spread is an increasing function of the interest rate; casual examination of the costs of life insurance in the United States bears out this hypothesis.

Secondly, we have considered an aspect of life insurance contracts that is particular to them, the dependence of the contract payoff on the declaration of periodic reversionary bonuses through the life of the contract. We have argued that under certain assumptions the declaration of reversionary bonuses leads to an inefficient distribution of wealth at maturity, and have quantified the efficiency cost under simplifying assumptions. However, our conclusions here must be tempered by our lack of knowledge of the investment policies of insurance companies and their relation to bonus declarations, as well as by the simplifying assumptions we have made about capital markets.

APPENDIX

Proof of Result 1

The optimal investment strategy is the solution to the following control problem:

$$\max_{x} \{ V_{WW}\sigma^2 W^2 + V_W[r + x(\mu - r)] + V_t \} = 0 \qquad \text{(A-1)}$$
$$V(W,T) = U(W)$$

It is immediate that the optimal control is a function of current wealth and time, x(W,t).

Proof of Result 2:

Lemma: The value of the wealth accumulated under a portfolio strategy in which the number of units invested in the risky security is z(W,P,t) and the balance of the portfolio is invested in riskless securities may be written as Q(P,t), and

i) Q(P,t) satisfies the Black-Scholes (1973) equation:

$$\frac{1}{2}Q_{PP}\sigma^2P^2 + rPQ_P + Q_t - rQ = 0 \qquad \text{(A-2)}$$

ii) For each state $\omega \in \Omega$ at time t

$$z(W(\omega),P(\omega),t) = Q_P(P(\omega),t) \qquad \text{(A-3)}$$

Proof Let $W(\omega,t)$ denote the wealth accumulated under the strategy z(W,P,t) in state ω at time t. Then

$$W(\omega,t) = z[W(\omega),P(\omega),t]\,P(\omega) + B(\omega,t) \qquad \text{(A-4)}$$

where $B(\omega,t)$ is the amount held in the riskless security, and

$$dW = zdP + rBdt \qquad \text{(A-5)}$$

Consider the value function Q(P,t). Ito's Lemma implies that:

$$dQ = Q_P dP + [Q_t + \frac{1}{2}\sigma^2P^2Q_{PP}]dt \qquad \text{(A-6)}$$

If $W(\omega,0) = Q(\omega,0)$, then, comparing coefficients in (A-5) and (A-6), $W(\omega,t) = Q[P(\omega),t]$ for all $t > 0$, iff

$$z[W(\omega),P(\omega),t] = x[Q(P(\omega),t),t]P(\omega,t) - Q_P(P(\omega),t) \qquad \text{(A-7)}$$

$$Q_t + \frac{1}{2}\sigma^2 P^2 Q_{PP} - rB \qquad \text{(A-8)}$$

(A-7) is equivalent to ii), and (i) follows from substituting for B from (A-4) in (A-8).■

Result 2 follows by defining $h(P) \equiv Q(P,T)$.

Proof of Result 3: We wish to show that h(P) has the same distribution function as W.

$$Pr[h(P) < x] = Pr[P < h^{-1}(x)] = G[h^{-1}(x)]$$
$$Pr[W < x] = F(x)$$

But $G[h^{-1}(x)] = F(x)$, so $Pr[h(P) < x] = Pr[W < x]$.

Proof of Result 4: See Harrison and Kreps (1979).

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	$\delta = 2\%$			$\delta = 3\%$			$\delta = 4\%$		
f=	1%	1.57%	2%	1%	1.57%	2%	1%	1.57%	2%
T=1 year	1%	1.57%	2%	1%	1.57%	2%	1%	1.57%	2%
5	4.80	7.54	9.61	4.71	7.39	9.42	4.62	7.24	9.23
10	9.14	14.36	18.29	8.75	13.74	17.51	8.38	13.16	16.76
15	13.07	20.52	26.14	12.22	19.19	24.45	11.45	17.97	22.90
20	16.62	26.09	33.24	15.21	23.87	30.41	13.95	21.90	27.90
100	43.37	68.09	86.74	31.75	49.84	63.50	24.58	38.59	49.16

Present Value of the Management Fees on a Portfolio held for a Fixed Period (T) as a function of the Portfolio Yield (δ) and the Management Fee Rate (f).

TABLE 1

State	1	2
Probability	1/2	1/2
State Price: P_J	0.1	0.4
Allocation {X}	100	50
Allocation {Y}	50	100

$$\text{Cost of } \{X\} = 0.1 \times 100 + 0.4 \times 50 = 30$$

$$\text{Cost of } \{Y\} = 0.1 \times 50 + 0.4 \times 100 = 45$$

Example: The Efficient and Inefficient State Contingent Wealth Allocations.

TABLE 2

Face Value of Policy: M	Annual Reversionary Bonus Rate: b	Efficiency Cost
$1000	1%	$ 7.02
	3	25.90
	5	52.44
	10	128.94
$1250	1	10.76
	3	42.53
	5	77.60
	10	128.52
$1500	1	16.01
	3	57.88
	5	86.59
	10	112.36

Efficiency Cost of Annual Reversionary Bonus for 20 years
$1000 Single Premium Contract

$\mu = 0.08, \sigma = 0.20, r = .05$

TABLE 3

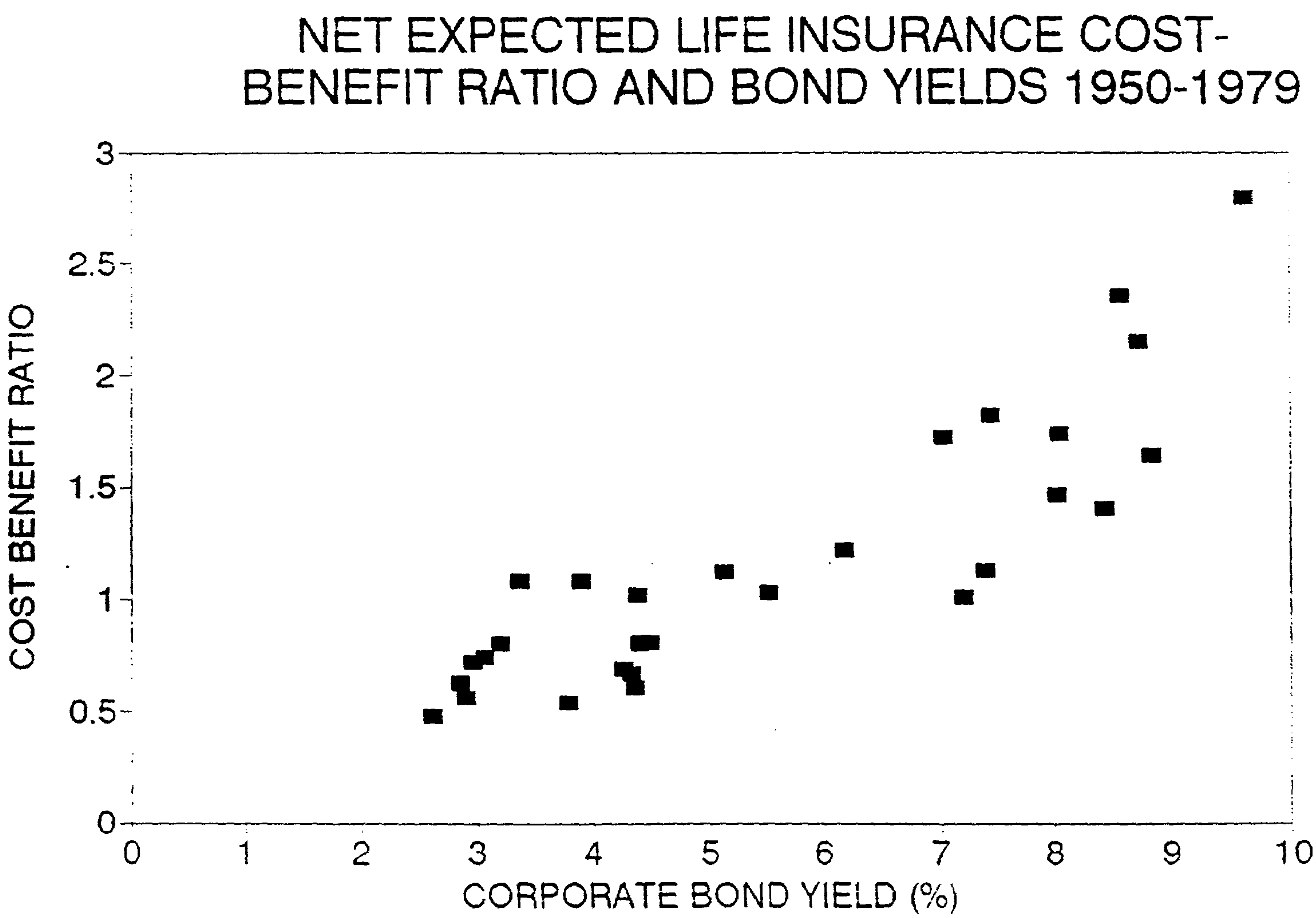
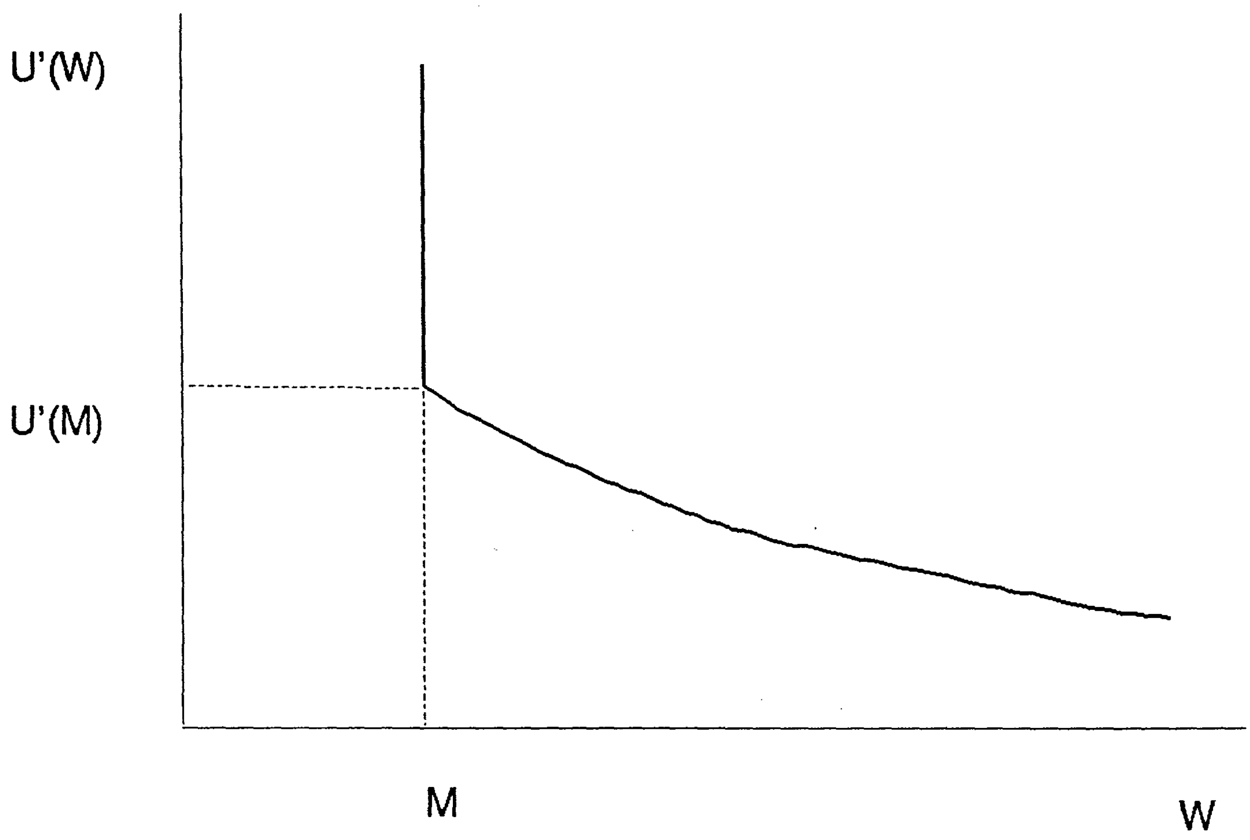

NET EXPECTED LIFE INSURANCE COST-BENEFIT RATIO AND BOND YIELDS 1950-1979
COST BENEFIT RATIO
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
CORPORATE BOND YIELD (%)

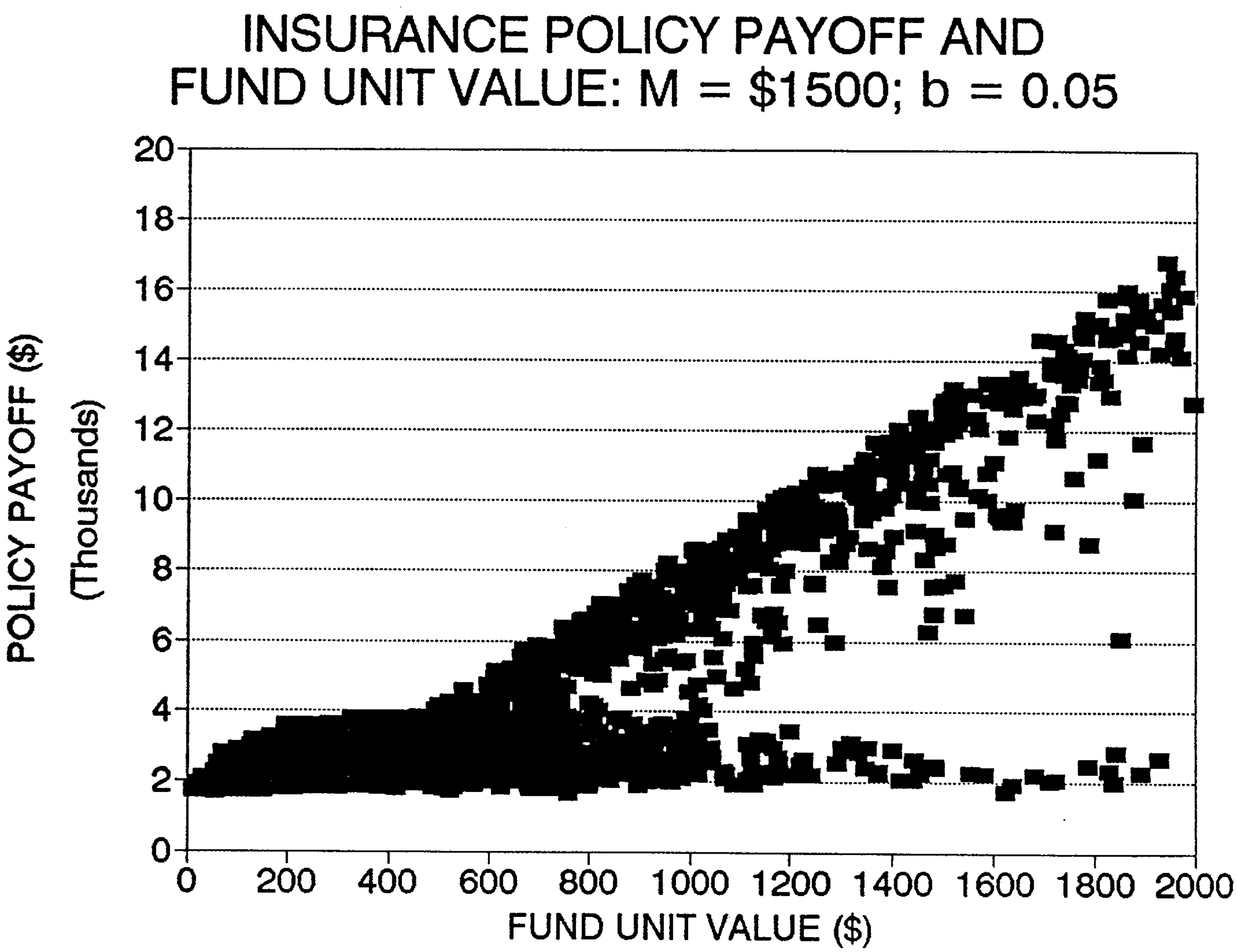
FIGURE 1



A Discontinuous Marginal Utility Function

The wealth level M is chosen for all states such that $\lambda p_s/\pi_s < U'(M)$.

FIGURE 2



Payoffs on 20 year Single Premium Contract and associated terminal values of Mutual Fund Units: the results of 25 simulations. $\mu = 0.08$, $\sigma = 0.20$, $r = 0.05$

FIGURE 3

CUMULATIVE DISTRIBUTION OF PAYOFF UNDER INSURANCE POLICY: M = $1500; b = 0.05

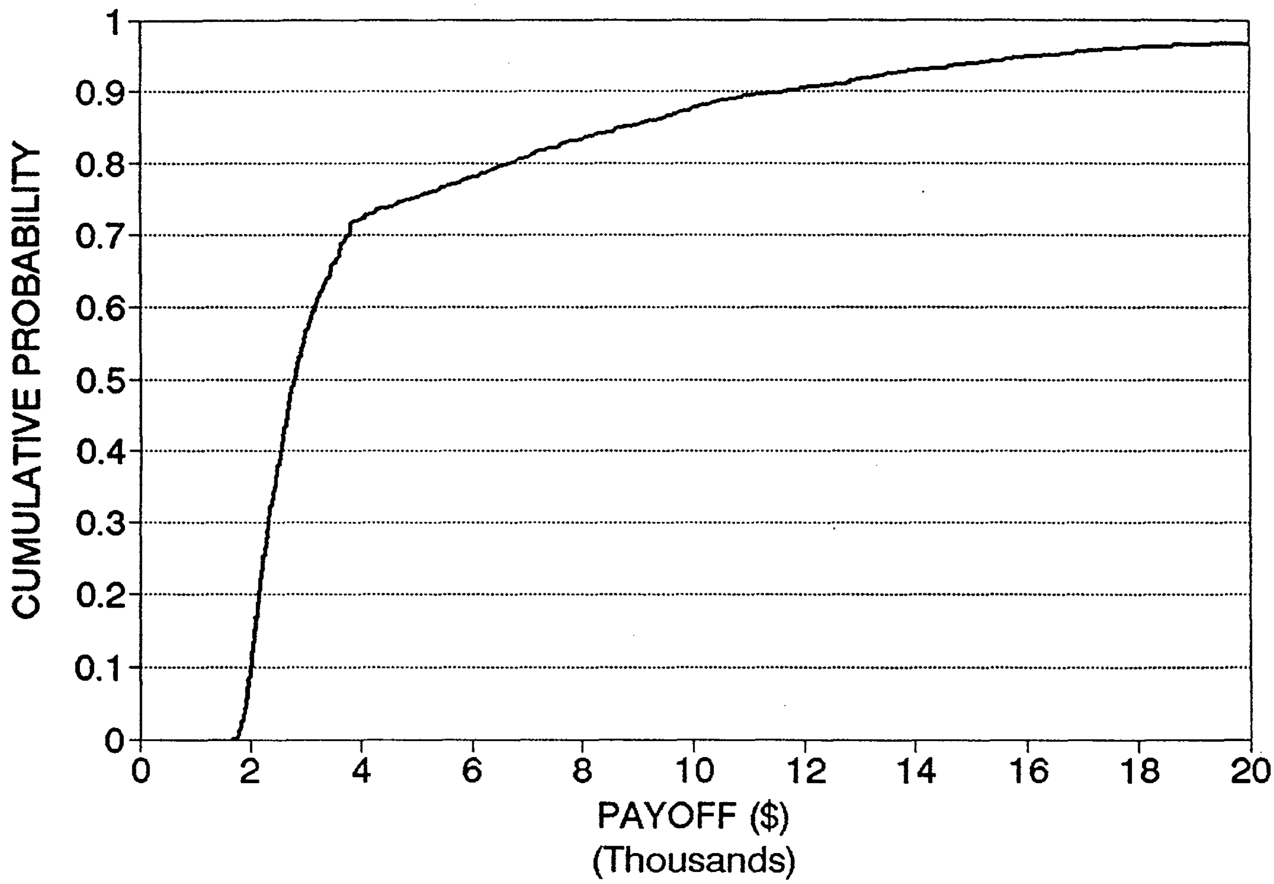


FIGURE 4

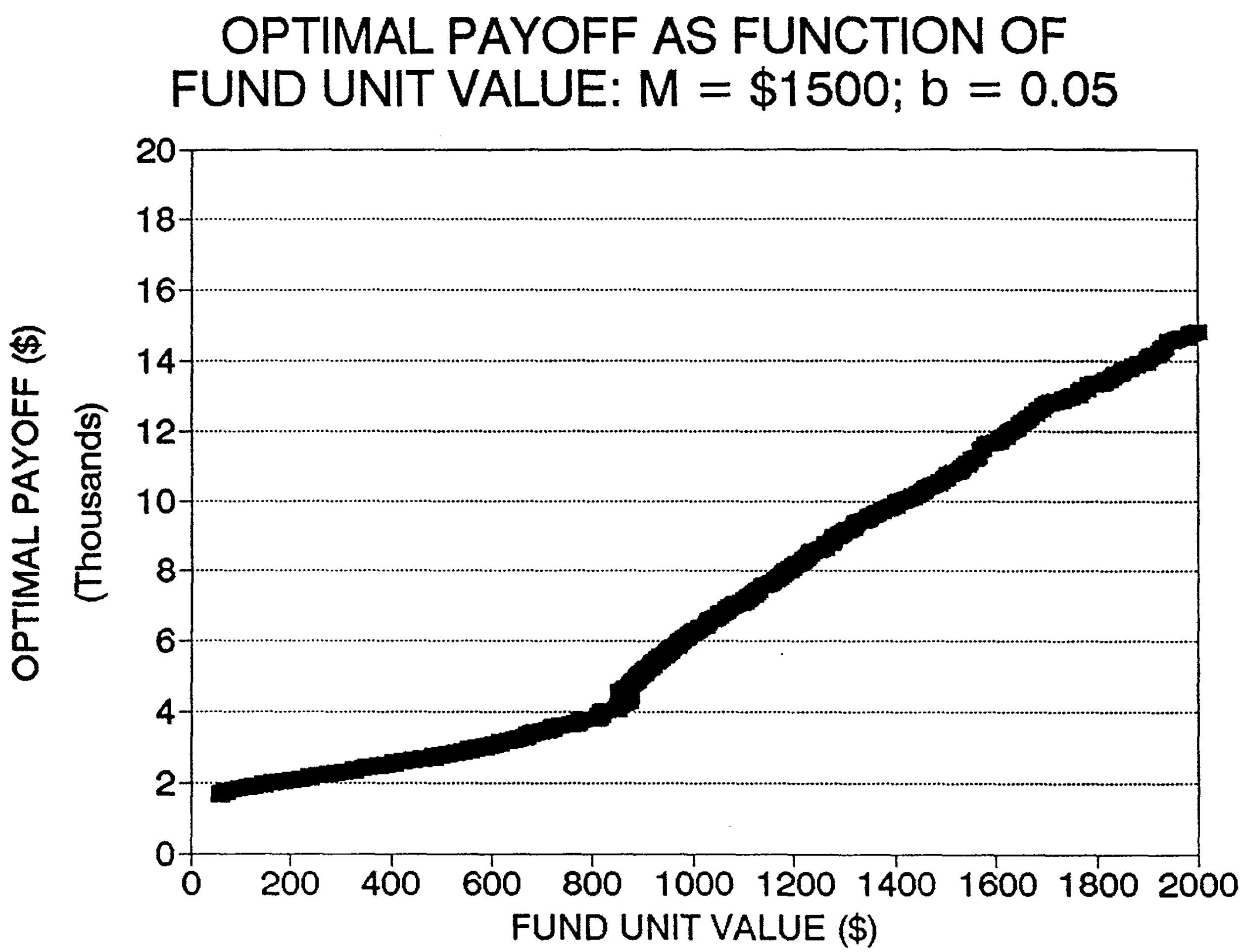

OPTIMAL PAYOFF AS FUNCTION OF
FUND UNIT VALUE: M = $1500; b = 0.05
OPTIMAL PAYOFF ($)
(Thousands)
0
2
4
6
8
10
12
14
16
18
20
0
200
400
600
800
1000
1200
1400
1600
1800
2000
FUND UNIT VALUE ($)

FIGURE 5