(i) $P \leq S(1-f)$

(ii) calls are only made when $S < M$.

The second part of the proposition follows from the previous discussion, and the first part is an immediate consequence of the fact that calls are unprofitable for the contributing shareholder.

The pricing restrictions under the different call policies are summarized in Figure 2. In this figure regions 1 and 4 are prohibited by arbitrage considerations; the line OE is the rational price schedule under the no-call policy; only region 2 is consistent with a value maximizing strategy, and only region 3 is consistent with a value minimizing strategy. Under a MM call policy the price must always lie below the line OE.

III

An Explicit Pricing Model

If the call policy of the issuing firm may be written as at most a function of the value of the firm and time, the contributing share may be treated as a simple contingent claim on the value of the firm and valued by the classic techniques of Black and Scholes (1973). Thus, assume that the issuing firm pays no dividends[25], that the interest rate is a constant r, that the firm is financed exclusively by equity, and that its value follows a stochastic process:

$$\frac{dV}{V} = \mu\, dt + \sigma\, dz \qquad (20)$$

where dz is the increment to a Gausss-Wiener process. Then the value of a contributing share may be written as a function of the current value of the firm, P(V). Let $\lambda(V)$ denote the instantaneous probability rate of a call when the firm value is V, and let $\kappa(V)$ denote the value of a contributing share if it is called when the firm value is V. Then, assuming that there is no risk premium associated with the event of a call, standard arbitrage arguments may be used to

[25] This is not a very restrictive assumption in the current context. Out of 381 Mining and Oil companies listed on the Australian Stock Exchanges as at February 1986, only 43 had paid dividends in the previous 5 years. Two of these companies were included in the sample used in the empirical study below - they had each paid only a single dividend in the previous five years.

show that the value of a contributing share satisfies the ordinary differential equation:

$$\frac{1}{2}P''V^2\sigma^2 + P'rV + \lambda(V)[\kappa(V) - P] - rP = 0 \qquad (21)$$

If $\kappa(V) = hV - k$ and $\lambda(V) = \lambda$, then the complete solution to equation (21) may be written as

$$P(V) = C_1V^{\alpha_1} + C_2V^{\alpha_2} + aV - \frac{\lambda b}{(r+\lambda)} \qquad (22)$$

where

$$\alpha_1 = \{\frac{1}{2}\sigma^2 - r + [(r - \frac{1}{2}\sigma^2)^2 + 2\sigma(r+\lambda)]^{\frac{1}{2}}\}/\sigma^2 \qquad (23)$$

and

$$\alpha_2 = \{\frac{1}{2}\sigma^2 - r - [(r - \frac{1}{2}\sigma^2)^2 + 2\sigma(r + \lambda)]^{\frac{1}{2}}\}/\sigma^2 \qquad (24)$$

and C_1 and C_2 are constants chosen to satisfy the boundary conditions. Note that $\alpha_1 > 1$ and $\alpha_2 < 0$.

Under a value minimizing strategy $\kappa(V) = \lambda(V) \equiv 0$, for $V > k/h$, so that $C_1=0$ and the value of a contributing share is given by:

$$P(V) = C_2V^{\frac{-2r}{\sigma^2}} \qquad (25)$$

where C_2 is a constant chosen to satisfy the boundary condition that the value of the share be given by expression (12), when the call is made at V = v:

$$hv - k = C_2 v^{\frac{-2r}{\sigma^2}} \tag{26}$$

Since the call policy is chosen to minimize the value of the share, it is immediately seen that the optimal policy is to call when V = k/h, and that this policy implies that the value of the assessable shares is equal to zero.

Under a value-maximizing strategy a call is never made[26] and $\kappa(V) - \lambda(V) \equiv 0, C_2 = 0$ [27], so that

$$P(V) = C_1 V \tag{27}$$

The condition that $S \geq P \geq Max[S - fM, 0]$ then implies that $C_1 = h$, so that P = S, and the value of a contributing share is equal to the value of a fully paid share.

We defined a Myers-Majluf call policy as one in which a call is made only if S < M and

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[26] Since no dividends are paid the contributing shareholder loses nothing by postponing the date of his contribution indefinitely.

[27] Otherwise the conditions that $0 \leq P \leq V$ will be violated for sufficiently small values of V.

the asset-investment pair falls into the acceptance region. The condition that $S < M$ following a call is equivalent to the condition that $V < [n_1(1 - f) + n_2]M \equiv \hat{V}$ immediately prior to the call. If the probability that the asset investment pair falls into the acceptance region is independent of V, then the probability of a call will be constant for $V > \hat{V}$. Therefore define the following generalization of a Myers-Majluf call policy:

$$\begin{aligned} \lambda(V) &= \lambda_M \text{ for } V < V^* \\ \lambda(V) &= \lambda_0 \text{ for } V > V^* \end{aligned} \tag{28}$$

Under the generalized Myers-Majluf call policy, the value of a contributing share is given by the solution to equation (21) where $\lambda(V) = \max[hV - k, 0]$, and $\lambda(V)$is given by (28):

$$P_1(V) = C_{11} V^{\alpha_{1M}} + C_{12} V^{\alpha_{2M}}, \qquad V < k/h \tag{29}$$

$$P_2(V) = C_{21} V^{\alpha_{1M}} + C_{22} V^{\alpha_{2M}} + hV - \frac{\lambda_M k}{r + \lambda_M} \qquad k/h \le V \le \hat{V} \tag{30}$$

$$P_3(V) = C_{31} V^{\alpha 10} + C_{32} V^{\alpha_{20}} + hV - \frac{\lambda_0 k}{r + \lambda_0} \qquad \hat{V} < V \tag{31}$$

where α_{iM} and α_{i0} (i=1,2) are defined by equations (22) and (23) with $\lambda = \lambda_M$ and λ_0

respectively and the C_{ij} are constants determined as follows. Since $P < V$ and $\alpha_{2M} < 0$ $C_{12} = 0$; similarly, since $\alpha_{10} > 1$, $C_{31} = 0$. The remaining four constants are determined by the set of four linear equations implied by the conditions that the valuation function be smooth and continuous at $V = k/h$ and $V - \hat{V}$:

$$P_1(k/h) - P_2(k/h) \tag{32}$$

$$P_1'(k/h) - P_2'(k/h) \tag{33}$$

$$P_2(\hat{V}) - P_3(\hat{V}) \tag{34}$$

$$P_2'(\hat{V}) - P_3'(\hat{V}) \tag{35}$$

IV

Empirical Evidence on Contributing Shares

The original sample consists of 30 No Liability firms whose fully paid and contributing shares were traded simultaneously on the Associated Australian Stock Exchanges for a period of at least one year between July 1981 and March 1986; three firms were dropped because they were very thinly traded. For each class of shares the last sale price of the day was taken as the closing price. Details of the issues were taken from Personal Investment and its predecessor Australian Stock Exchange Journal and were verified against the company files held at the Sydney Stock Exchange.

In order to determine whether the pricing of contributing shares violated the arbitrage conditions (14) and (15) the closing prices of contributing and non-contributing shares were compared for all days on which the trading volume in each class was at least 10,000 shares. This minimum volume requirement was imposed in order to minimize the problems caused by non-synthronicity of the closing prices for the two classes of shares. Violations were observed on 49 out of 2004 firm-days. All the violations related to the same firm, and all were violations of the

lower bound condition (15). No explanation is known for these violations.[28]

Table 1 reports the frequency of violations of the boundary conditions imposed by the different call policies described in Section II for 10,000 share days. Approximately 80% of the price observations are inconsistent with the boundary condition imposed by a value-maximizing call policy. Moreover, the higher is the number of available observations for an individual firm, the higher is the proportion that are inconsistent with value maximization. Only firm 27 had no observations that were inconsistent with value maximization[29]; however, firm 17 had only 9.5% of its observations inconsistent with value maximization, and this fell to 7.1% when all 168 matching daily trades were included; firm 12 had only 12% of its observations inconsistent with value maximization, and this falls to 4.4% of all matching daily trades. It should be noted of course that even those observations which are consistent with a value-maximizing policy do not imply that the shares are priced consistent with such a policy. Thus there is strong evidence that, except for three firms, investors did not believe that the issuers of contributing shares were following call policies designed to maximize the value of the contributing shares.

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[28] The low price of the contributing shares of Energy Oil and Gas was commented on by the firm's directors who, in a telex to the Perth Stock Exchange dated February 21, 1983, stated that "The directors of Energy Oil and Gas NL have noted that a discrepancy exists between the market price of fully paid shares and the market price of contributing shares and options. Whilst the directors are unable to substantiate any reason for the market preference for fully paid shares they wish to clarify the position with regard to options and payments of calls on contributing shares".

[29] While this firm had only 42 trading days that met our minimum trading volume requirement, all 285 days on which matching trades were found were consistent with value maximization (and therefore inconsistent with value minimization or a MM call policy).

The other extreme possibility is that firms should pursue call policies to maximize the value of the fully paid shares or to minimize the value of the contributing shares. No firm has fewer than 60% of its observations inconsistent with value-minimization. Thus it seems that investors do not expect firms to pursue policies that will minimize the value of the contributing shares. Since a value minimizing policy would generally mean making a call when the prospects for the company are bad, it is possible that the absence of a value minimizing policy can be attributed to a management reluctance to invest in riskless zero NPV projects.

Under a Myers-Majluf call strategy, calls are made only if $S < M$ and the asset investment opportunity pair falls within the acceptance region. Thus calls are always unprofitable for the contributing shareholders, and the contributing shares are worth less than the prorated value of a fully paid share as described in Proposition 4. For five of the firms, more than 90% of the price observations are consistent with a Myers-Majluf call policy; on the other hand, nine of the firms have over 90% of their observations inconsistent with a Myers-Majluf call policy.

Table 2 provides further detail on the pricing of contributing shares along with information about the calls that actually were made[30]. Columns 3 of the table reports the fraction of price observations for which the fully paid shares traded below par. Columns 4-7 of the table show the proportion of price observations for which the fully paid shares were above

[30] We attempted to estimate the parameters of the valuation model developed under a MM call policy, equations (28)-(30), but for most firms were unsuccessful, either because the call probability parameters were unstable or because there were insufficient observations for $S > M$ and $S < M$ to permit identification of the parameters.

(below) par and the contributing shares were above (below) the no call value of S(1 - f). If calls are expected to be made when the fully paid shares are above (below) par then we should expect the contributing shares to trade above (below) their no call value. There is a strong tendency for the contributing shares to trade above (below) the no call value when the fully paid shares are trading above (below) par - note the preponderance of observations in columns 3 and 6 - suggesting that investors expect calls to be made both above and below par. In particular it is quite rare for the contributing shares to trade below the no call value when the fully paid are above par. For nine firms over 90% of the contributing share prices are below the no call value and four of these firms actually made calls below par. Five firms had over 90% of their contributing share prices above the no call value - only one of these firms made a call on the contributing shares (above par). The average ratio of the contributing share price to its no call value given in column 7 was 126%; for firms that made calls that were unprofitable for the contributing shareholders the average ratio was 78%, while for firms that made no calls the average ratio was 140%. For the two firms that made profitable calls the average price ratio was 202%.

Ten firms actually made calls during our sample period, three firms making two calls. Columns 9 and 10 show the fraction of par value remaining unpaid before and after the calls. Calls were typically for 10-20% of the par value, and in no case was a call made for the whole of the outstanding unpaid balance. This is inconsistent with a policy of maximizing or minimizing the value of the contributing shares, but is consistent with what we have described as a Myers-Majluf policy of calling only when there is a profitable use for the funds. Column

12 reports the loss realized by the contributing shareholders for each call, as a fraction of the no call value. This loss is calculated as the difference between the cash subscribed and the value of the increased ownership share as measured by the corresponding fraction of a fully paid share at the first price observation after the contribution. Of the fourteen calls, eleven were unprofitable for the contributing shareholders while the remaining three were profitable. One of these (firm 8) raised a negligible amount of capital, and one yielded only a 4% profit to contributing shareholders. However the third (firm 24) made a call for contributions when its shares were trading at a 40% premium to par. This is clearly inconsistent with what we have called a Myers-Majluf call strategy.

Figure 3 plots the average price to par value ratio for the fully paid shares around the time of a call for contributions. In this figure trading days refer to the days on which both classes of shares traded. The figure is drawn both with and without firm 8 which has a disproportionate effect on the data since at the time of its call it was trading at over ten times the par value, yet the call raised only a small amount of money. Excluding this firm, there is no pronounced evidence of a price drop prior to the call; the average price ratio was 0.72 at the time of the call which is consistent with the unprofitablity of the calls. Figure 4 plots the average ratio of the contributing share price to its no-call value around the date of a contribution. This ratio is well above unity 100 trading days before the call, implying that investors are expecting a call above par. However, by the time of the call this price ratio has dropped to around 0.70 implying that investors are expecting subsequent calls to be unprofitable to contributing shareholders.

In summary, most calls took place at a loss to contributing shareholders, because the fully paid shares were trading below par; in only one case did our (noisy) measure of the realized loss from contributing exceed 100%, implying that the firms were not attempting to minimize the value of the contributing shares. This is as predicted by what we have called a Myers-Majluf call policy. On the other hand, for 45% of the price observations the contributing share price exceeds its no call value, implying that investors expect future calls to be profitable on average, which is inconsistent with a Myers-Majluf call strategy.

The variation in the market's assessment of the profitability of future calls is puzzling in light of our model. One possibility is that corporate insiders change their relative ownership of contributing and fully paid shares and that this acts as a signal of the profitability of future calls. If this is so, then it seems that contributing shares, while offering a solution for one type of managerial agency problem, give rise to another. A second possibility is that there is an implicit contract that contributing share calls will be made (eventually) if the firm does well, so that management is not maximizing the value of the fully paid shares as our Myers-Majluf call policy assumes. In any event, the findings are of interest in light of the frequent assumption in financial models that management maximizes the interest of one class of investors (stockholders) at the expense of the others (bondholders). That is clearly not the case here, and similar considerations may offer an alternative to the signalling explanation of why convertible bonds are called 'late'.

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TABLE 1

Firm	Numbers of Observations	Violations of		
		Value Maximization	Value Minimization	Myers-Majluf
1	505	97.4%	82.0%	5.3%
2	38	29.0	97.4	97.4
3	176	97.7	81.0	2.3
4	134	91.8	73.1	8.2
5	36	86.1	61.1	58.3
6	138	80.6	92.7	92.0
8	21	100.00	100.00	0.0
10	19	84.2	100.00	100.00
11	18	66.7	83.3	33.3
12	10	10.0	100.0	100.0
13	126	100.00	70.6	0.0
15	27	96.3	96.3	11.1
16	107	77.6	66.1	34.6
17	21	9.5	100.0	100.0
18	108	69.4	81.5	69.5
19	53	41.5	79.2	67.9
20	14	28.6	100.0	100.0
22	14	50.0	100.0	50.0
23	139	99.3	100.0	99.3
26	168	28.6	97.0	92.2
27	42	0.0	100.0	100.0
28	73	93.2	85.0	78.1
WEIGHTED AVERAGE		79.8	84.4	42.6

Proportion of Matching Daily Trades Violating Boundary Conditions imposed by Different Call Policies on days when Volume exceeds 10,000 shares in both Classes of Share.

TABLE 2
Summary of Pricing of Contributing Shares and Calls made.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
			Proportion of Observations					f				Cash Raised	
Firm No.	No. Obs.	P<M %Obs	FP>M P>S(1-f)	FP>M P<S(1-f)	FP<M P>S(1-f)	FP<M P<S(1-f)	Avg P/S(1-f)	Before Call	After Call	Contrib share pr after call	Realized loss/ S(1-f)	Per Share	/Firm
		%	%	%	%	%	%	%	%	$	%	$	%
1	709	94	2	2	3	93	75	40	30	0.23	4	0.05	9.8
								30	20	0.10	20	0.05	10.2
2	179	5	89	2	6	3	129						
3	318	100	0	0	1	99	50	56	38	0.15	14	0.09	39.1
4	301	100	0	0	6	94	68	60	40	0.17	19	0.10	18.4
								40	30		5	0.05	4.0
5	128	23	38	20	16	26	107	60	50	0.20	4	0.05	10.3
6	359	3	90	6	1	4	110						
7	57	100	0	0	0	100	25						
8	210	0	100	0	0	0	133	30	20	3.10	(13)	0.02	0.0
9	113	96	0	1	12	87	85						
10	18	23	33	0	67	0	337						
11	266	77	0	22	0	78	32	50	40	0.10	5	0.05	9.4

1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	30	20	40	40	0	20	88						
3	89	43	38	16	8	38	93	40	38	0.01	108	0.02	15.6
4	249	79	10	4	14	72	90	30	21	0.15	10	0.05	14.4
5	152	51	42	0	58	0	209						
5	345	31	50	12	8	31	102						
7	46	78	2	0	57	41	104						
3	83	31	100	0	0	0	102						
	78	0	100	0	0	0	191						
	37	97	0	0	100	0	212						
	178	100	0	0	1	99	40						
	37	89	0	8	3	89	57						
	24	92	0	0	75	25	115						
	291	26	54	1	30	15	271	90	60	0.09	(86)	0.03	9.9
	252	100	0	0	100	0	358						
	158	14	68	17	0	15	109						
	56	29	61	4	29	7	112						

(TABLE 2 Cont.)

Notes to Table 2

Column 1. Firm identification
Column 2. Number of trading days on which both contributing and fully paid shares traded.
Column 3. Proportion of observations for which price of fully paid shares is less than par value.
Columns 4-7. The proortions of price observations for which the contributing and fully paid shares fall above and below the no-call and par values respectively.
Column 8. The average ratio of contributing share price to the no-call value.
Columns 9-10. The proportions of par value remaining unpaid before and after the call.
Column 11. The contributing share price at the first price observation after the call.
Column 12. The realized loss relative to the pre-call no-call value. The loss is measured as the difference between the value of the appropriate fraction of a fully paid share and the amount contributed.
Columns 12, 13. Cash raised by the call per share and as a proportion of the aggregate value of contributing and fully paid shares.

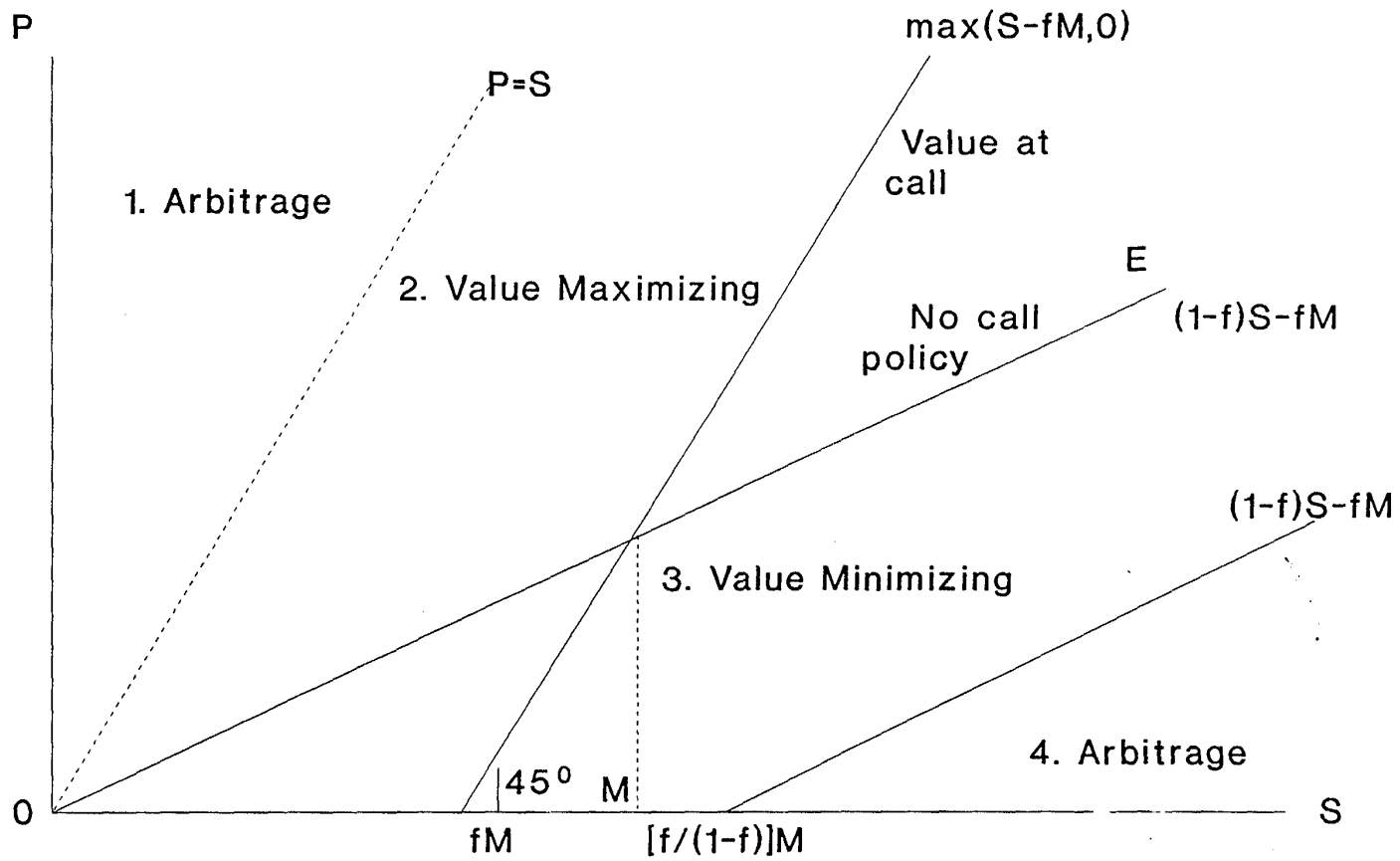

Figure 2: Restrictions on Rational Pricing under Different Call Policies
Under the MM call policy the price of contributing shares must fall below the line OE

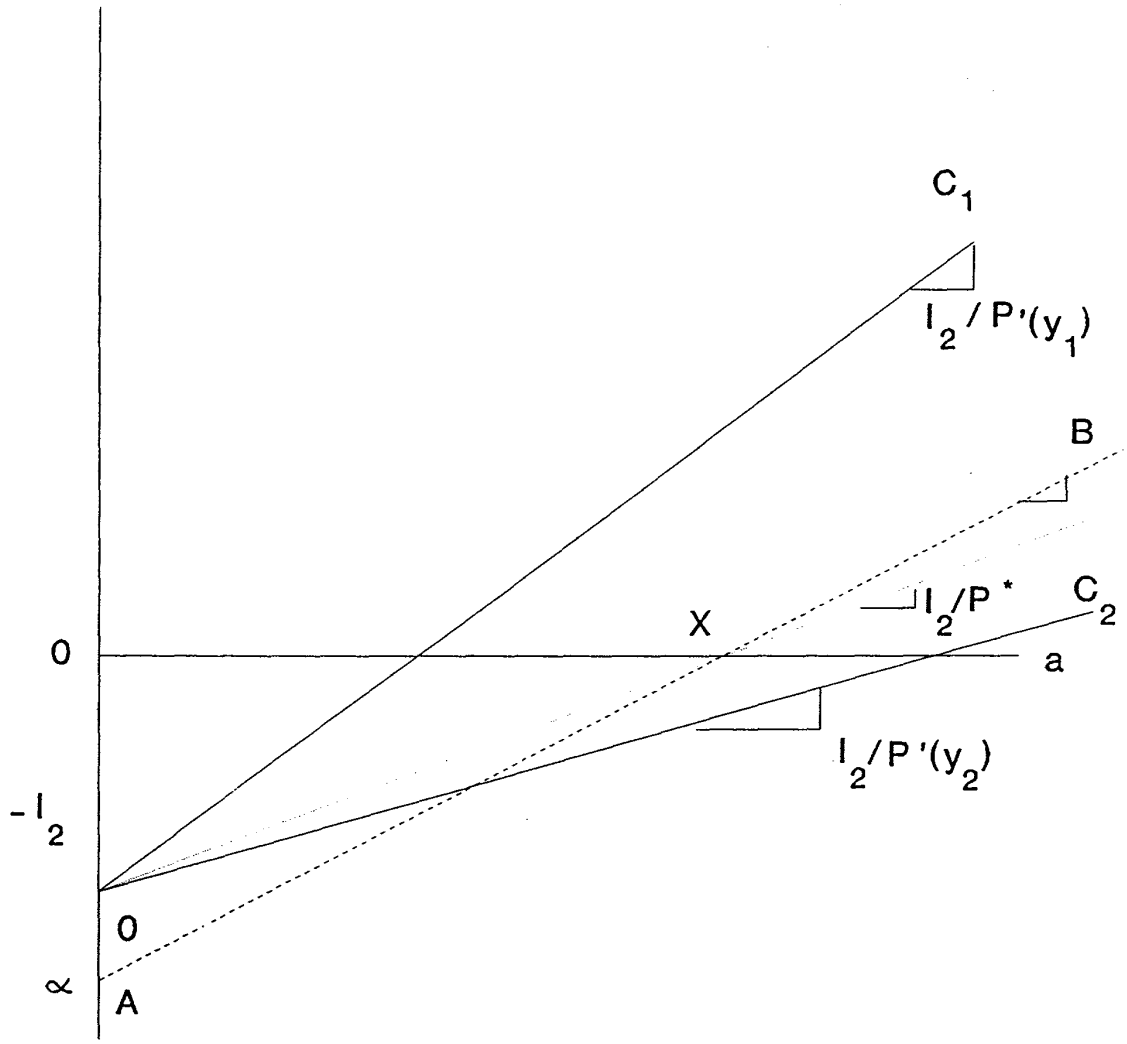

Figure 1: The Myers-Majluf Problem with Perfectly Correlated Asset Values and a Pre-Arranged Stock Sale Price.

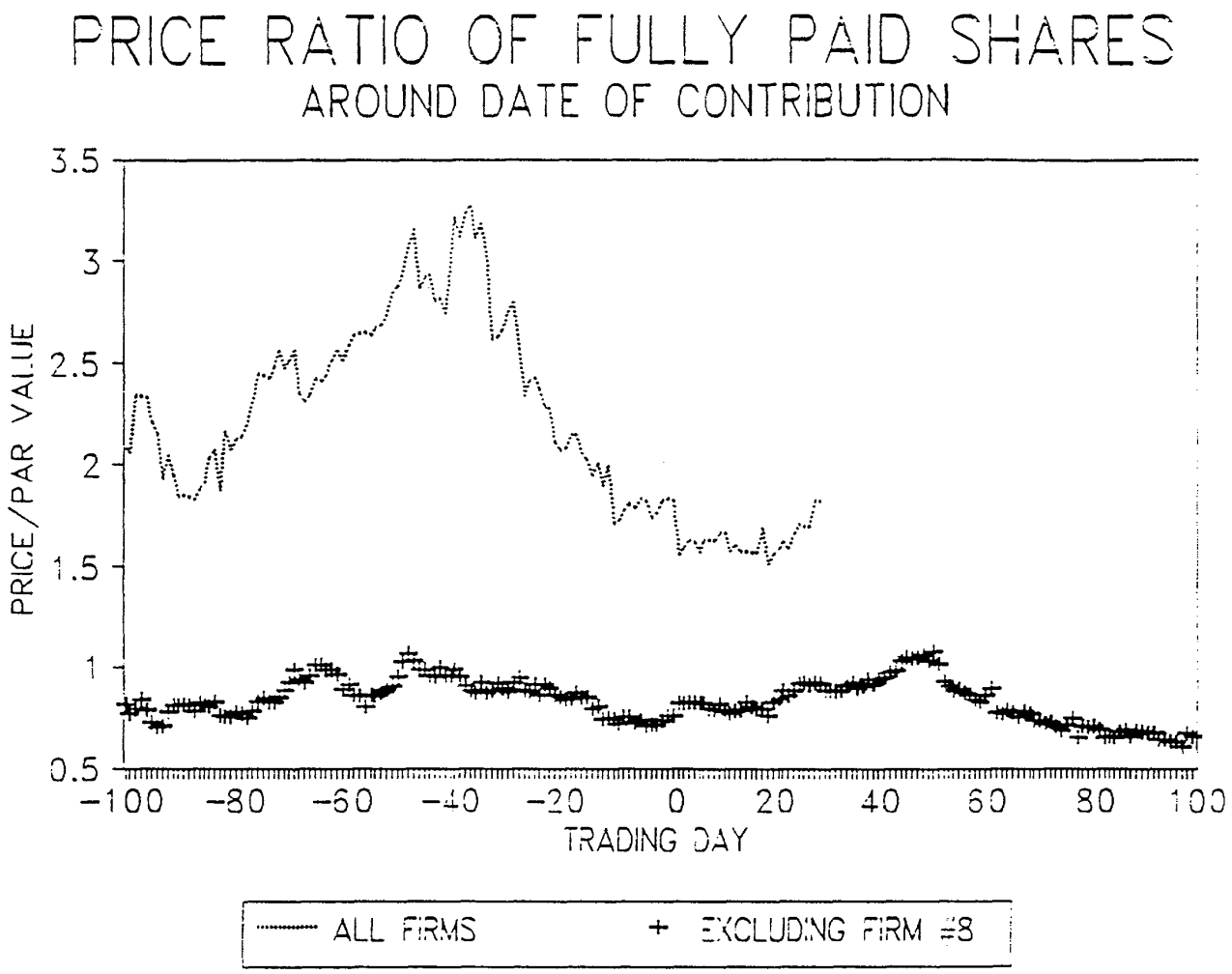
PRICE RATIO OF FULLY PAID SHARES
AROUND DATE OF CONTRIBUTION
PRICE/PAR VALUE
3.5
3
2.5
2
1.5
1
0.5
-100
-80
-60
-40
-20
0
20
40
60
80
100
TRADING DAY
ALL FIRMS
EXCLUDING FIRM #8

FIGURE 3

PRICE RATIO OF CONTRIBUTING SHARES
AROUND DATE OF CONTRIBUTION

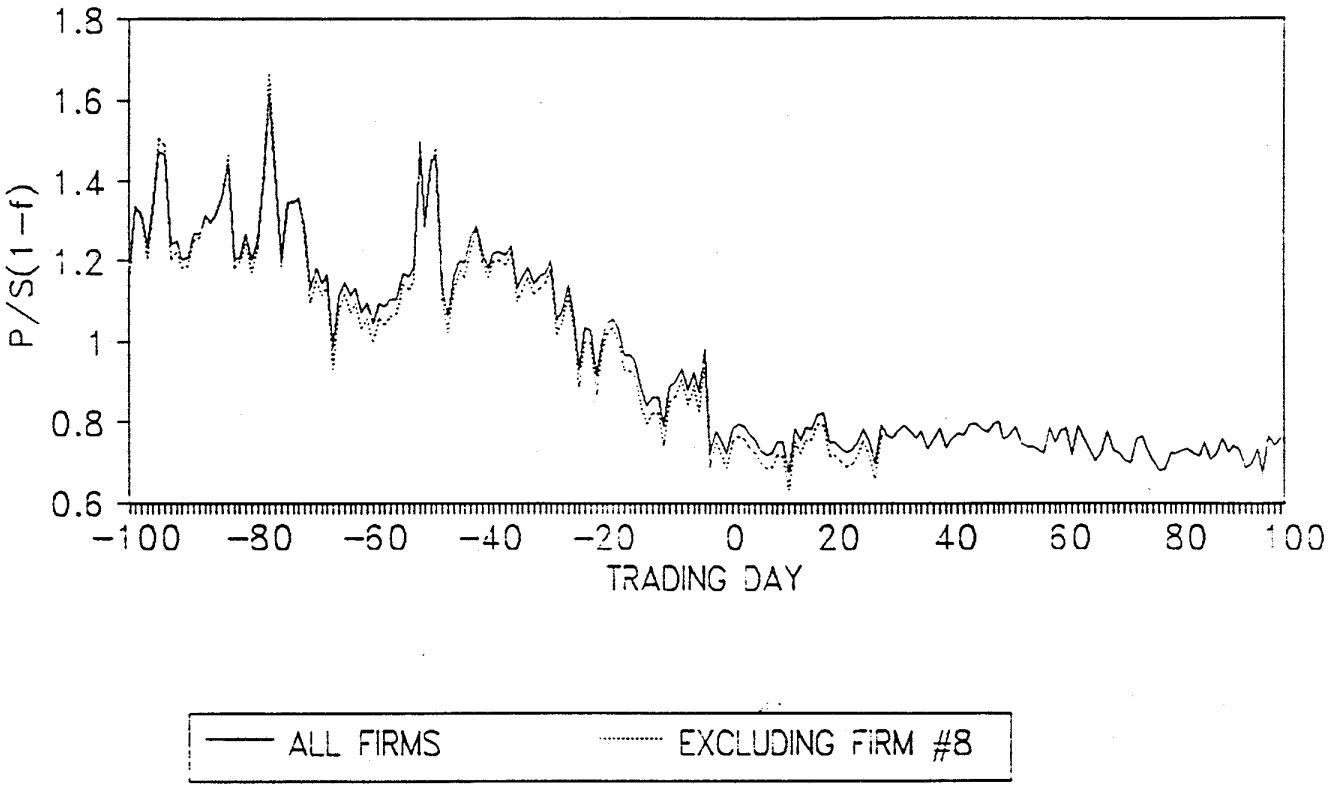

FIGURE 4