Consistent Regulatory Policy under Uncertainty

Michael J. Brennan, Eduardo S. Schwartz

The Bell Journal of Economics, Volume 13, Issue 2 (Autumn, 1982), 506-521.

Stable URL:
http://links.jstor.org/sici?sici=0361-915X%28198223%2913%3A2%3C506%3ACRPUU%3E2.0.CO%3B2-E

Consistent regulatory policy under uncertainty

Michael J. Brennan*

and

Eduardo S. Schwartz**

This article is concerned with the effects of regulation on the risk and value of the regulated firm in a dynamic context. Current regulatory practice is shown to be logically deficient, since it ignores the effect of regulatory policy on the cost of capital and therefore on the appropriate allowed rate of return. A notion of consistency in regulatory policy is developed, and it is shown how consistent regulatory policies may be implemented once the valuation problem is solved.

1. Introduction

■ The problems of equity and of efficiency in resource allocation that arise from the existence of natural monopolies may be dealt with either by socialization or by regulation: the latter approach is especially common in the United States, while the former is favored in most other jurisdictions. In this article we analyze the dynamic effects of regulation on firm risk and value and show how a consistent regulatory policy may be determined. The valuation model is simple and, in the interest of analytic tractability, abstracts from many features of the regulatory environment. Nevertheless, the general approach lends itself to further elaboration and realism, so that the article represents a first step in the theory of the valuation of the regulated firm. No such theory currently exists and yet, as will become apparent below, such a theory is a prerequisite for the determination of a consistent regulatory policy.

A major task of the regulator is to set the prices at which the output of the regulated firm must be sold. His decisions affect the costs borne and the quantity purchased by consumers on the one hand and the returns received by investors on the other. These decisions must, therefore, be tempered by considerations of equity. At the same time, regulatory decisions, by influencing incentives, also affect the behavior of the regulated firm with attendant implications for economic efficiency; it is these efficiency aspects of regulation which have in the main attracted the attention of economists.

Thus there exists an extensive literature concerned with the effects of regulation on firm behavior: most of this literature assumes certainty and a static setting in which only a single regulatory decision is made.[1] Das (1980), Perrakis (1976), and Peles and Stein (1976) have extended this type of analysis to uncertainty, relying on Leland's (1972) model of the expected utility-maximizing firm. This model, however, is unsatisfactory as

* University of British Columbia.

** University of British Columbia.

Previous versions of this article have been presented at seminars at Northwestern, Ohio State, Wisconsin, Yale, New York University, and the London Business School. The comments and suggestions of seminar participants are gratefully acknowledged. We wish to thank especially Fischer Black and the Editorial Board for helpful comments. We retain responsibility for the final contents of the article.

[1] The seminal article here is by Averch and Johnson (1962). Many of the subsequent articles are surveyed in Baumol and Klevorick (1970) and Stein and Borts (1972).

a basis for a theory of the regulated firm, since it ignores the role of the capital market in allocating risk and in providing investment alternatives which affect the incentives of the owners. These capital market alternatives are taken into account by Myers (1973) in his analysis of a value-maximizing regulated firm in a state preference framework. More recently, Marshall *et al.* (1981) have studied the input choices of a regulated value-maximizing firm in the context of the capital asset pricing model. Like us, they treat the risk of the firm as endogenous and affected by the regulatory decision; unlike us, they retain the static framework of the other articles.

Klevorick (1973) has analyzed the dynamic effects of regulation on firm behavior. In his model, regulatory decisions are made at stochastic intervals with the regulator setting output prices according to some known rule. The firm rationally anticipates future regulatory decisions and determines its behavior to maximize the discounted expected value of future cash flows. In keeping with most of the preceding literature on regulation and firm behavior, Klevorick pays no attention to risk, and the discount rate in the firm's objective function is taken as exogenous.[2]

In this article we are concerned, like Klevorick, with the dynamic aspects of regulation. But, while he is concerned with firm behavior and ignores issues of valuation, we are concerned with valuation and therefore suppress the issue of the influence of regulatory policy on firm behavior until Section 6, where we discuss it briefly. The dynamic aspects of regulation arise in our model, as in that of Klevorick, because regulation is an ongoing process in which new regulatory decisions are made as conditions change. In Klevorick's model, rational anticipations of these decisions by the management of the firm affect firm behavior; in our model, rational anticipations of these decisions by investors affect firm risk and value.[3] Consider, for example, the range of possible regulatory responses to the accidental destruction of a plant owned by a regulated firm. At one extreme, the regulatory authority may fail to respond at all, leaving the shareholders of the firm to bear the entire loss; at the other extreme, the plant may be left in the rate base and output prices adjusted so that the regulated firm continues to earn its allowed rate of return on the original investment in the now useless plant. In the latter case, the whole of the loss would be borne by consumers.

This ability of the regulator to allocate stochastic future costs and benefits between consumer and investor means that the investment risk of the regulated firm is endogenous, being a function not only of technological and market uncertainties but also of regulatory policy. It follows that insofar as current regulatory procedures take the investment risk of the regulated firm as exogenous and attempt to determine an allowed rate of return appropriate to this risk, they are conceptually deficient. As Robichek (1978) has remarked, "[F]or a regulated company, the business (and hence, investment) risk depends on the regulatory decision. To require that the rates be set after giving due consideration to 'risk' is circular when such 'risk' is determined to a large extent by the rate-making process." What is required instead is a regulatory procedure or policy that is consistent in the sense that it yields an allowed rate of return which is appropriate for the risk of the firm under that policy.[4] A wide range of regulatory policies are consistent in this sense: they involve different allocations of risk between consumers and investors and, consequently, different allowed rates of return are appropriate for each. The choice between alternative regulatory policies may be made on grounds of efficiency. We shall, however, ignore efficiency considerations by taking the productive decisions of the regulated firm as exogenous.

[2] In a more recent article, Bawa and Sibley (1980) also analyze the dynamic behavior of the regulated firm. In their model, as in the one presented here, the probability that a hearing will be held is endogenous; however, as in Klevorick, the discount rate in the firm's objective function is exogenous.

[3] Sometimes the decisions will follow explicit rules, such as automatic fuel adjustment clauses; more frequently the decision rules are implicit.

[4] We defer until the following section a consideration of what constitutes an appropriate rate of return.

Given these productive decisions, the regulatory authority is able to affect the returns to investors in the firm by changing output prices.

In Section 2, we discuss the nature of the regulatory criterion and its implicit justification which, we argue, is inadequate once attention is given to the effects of uncertainty: we therefore offer an alternative definition of a consistent regulatory policy. Section 3 develops the valuation framework, which is specialized in Section 4 to apply to regulated firms under alternative regulatory policies. In Section 5 we present a numerical example. Finally, Section 6 considers the implications of the model for current procedures used to determine the allowed rate of return, and for the investment incentive implications of regulatory lag.

2. The regulatory criterion and consistent regulatory policy

■ A principal task of the regulatory authority in setting output prices for the regulated firm is to determine the appropriate rate of profit for the shareholders. In this, its objective is to combine equity between investors and consumers with appropriate incentives for the management of the firm. The legal criteria for an appropriate rate of profit are enshrined in the *Bluefield* and the *Hope* decisions of the United States Supreme Court,[5] which define what have become generally known as the "comparable earnings standard" and the "capital attraction standard." Some authorities have maintained that the comparable earnings standard requires that the accounting rate of return of the regulated firm correspond to that for unregulated firms of similar risk, and that this principle also satisfies the capital attraction standard.[6] The modern consensus, however, appears to be that both standards require that the allowed rate of return earned by shareholders on the rate base should be equal to the firm's cost of capital, which is defined as the rate of return an investor could expect to earn on investment in other firms of equivalent risk. Thus, the distinction between the modern consensus and the earlier view is that the former takes as its standard a market determined prospective rate of return, whereas the latter relies on retrospective accounting returns on similar risk firms. The implicit justification of the principle that the allowed rate of return be set equal to the cost of capital appears to be the belief that this will cause the market value of the firm to be equal to the value of the rate base on which the return is allowed. Indeed, this result can be rigorously derived within the quasi-uncertainty valuation model of Miller and Modigliani (1961) in which stochastic cash flows are replaced by their expected values and the effects of uncertainty are assumed to be captured in the discount rate.[7]

But this approach to regulation is, in fact, fundamentally deficient because it neglects entirely the role of future regulatory decisions. This deficiency is not apparent in the Miller and Modigliani quasi-uncertainty valuation model because in this model regulation is a one-shot affair, and it is implicitly assumed that the realized return on the rate base is equal to the allowed return. Nor is the deficiency any more apparent in the models of Myers (1973) and Leland (1974), which do explicitly account for uncertainty in a capital markets context but consider only a single period. In both models a role for regulatory response to future uncertainties is effectively precluded—in the one because there is no uncertainty; in the other because there is no future.

Since regulation is a continuing process, the regulatory policy that is anticipated by investors affects the risk and the value of the regulated firm.[8] For this reason, the principle

[5] *Bluefield Water Works and Investment Co.* v. *Public Service Commission of the State of West Virginia* (262 U.S. 679, 1923) and *Federal Power Commission* v. *Hope Natural Gas Company* (320 U.S. 591, 1944).

[6] See Myers (1972) for relevant citations and for a forceful critique of this "traditional" position.

[7] Davis and Sparrow (1972) argued that all extant models for the valuation of regulated firms rested on the same set of assumptions. This state of affairs appears to be unchanged.

[8] Empirical evidence of this is provided by Clarke's (1980) study of the introduction of fuel adjustment clauses.

that the allowed rate of return be set equal to the cost of capital is logically incomplete, since the cost of capital is a function of the risk of the firm which, in turn, depends *inter alia* upon the anticipated regulatory policy. That policy includes both the timing of regulatory decisions and the rule for setting the allowed rate of return when those decisions are made. To escape fatal circularity this principle must be replaced by one that takes account of the endogeneity of the risk of the firm. We define a *consistent regulatory policy* as a procedure for determining the holding of a rate hearing and setting the allowed rate of return at the hearing such that, when properly anticipated by investors, the procedure causes the market value of the regulated firm to be equal to the value of the rate base at the time the hearing is held.

Implementation of a consistent regulatory policy presupposes that the regulatory authority is able to assess the effect of alternative regulatory policies on the value of the firm. As we have already noted, however, there exists no model for the valuation of regulated firms which takes account of regulatory policy. In the following section, we present a simplified model that captures the essence of valuation under regulation. A more general model is developed in the Appendix.

3. The valuation framework

■ In the interest of simplicity, we restrict our attention to a regulated firm which is financed entirely by equity funds. Then, under the assumptions described below, the value of the firm may be written as a function of the current rate base, B, and the instantaneous rate of return currently earned on the rate base, x: $F(x, B)$. The rate of return on the rate base is defined as the ratio of the instantaneous earnings rate to the rate base. The instantaneous earnings rate, xB, is locally riskless in the sense that it follows a continuous sample path between regulatory hearings. In nontechnical terms, this means that changes in the earnings rate from week to week are small.

As a result of the business risk to which the firm is exposed, the rate of return x evolves stochastically over time. In a complete model, the resulting stochastic process for the rate of return would be derived from fundamental assumptions about demand and cost conditions. But to derive the rate of return from optimizing behavior in the face of stochastic production and demand functions and stochastic input prices would require the introduction of additional state variables and would substantially increase the complexity of the analysis without contributing further insights. It is therefore assumed, for the sake of tractability, that the rate of return on the rate base follows an exogenously determined stochastic process of the general type

$$dx = \mu(x)dt + \sigma(x)dz_x . \tag{1}$$

Here $\mu(x)$ represents the expected rate of change in x and t is calendar time; dz_x is a standard Gauss-Wiener process with mean zero and variance dt. Thus, the business risk of the firm is represented by the variance of the change in x, $\sigma^2(x)$.

The rate of increase in the rate base is equal to the difference between the instantaneous earnings rate, xB, and the aggregate dividend payout rate net of stock issues.[9] This net payout rate, $\delta(\cdot)$, is assumed to be expressible as

$$\delta(x, B) = p(x)B. \tag{2}$$

Then the instantaneous change in the rate base is given by

$$dB = (x - p(x))Bdt. \tag{3}$$

[9] Note that the particular accounting conventions used to determine earnings are irrelevant so long as the same definition is used for x and B and the definition is consistent with the assumed stochastic processes.

In this model, regulatory policy is defined in terms of the rule for holding a regulatory hearing and the rule for determining the outcome of the hearing. It is assumed that the rule for determining a hearing takes the form of a function $\pi(x)$, which is the probability per unit time that a hearing will be held. The motivation for this specification is that a high rate of return will lead to pressure from consumers for a regulatory hearing to reduce the output price of the regulated firm; conversely, should the rate of return fall too low, the firm will press for a hearing to raise output prices.[10]

The outcome of the regulatory hearing is an allowed rate of return $x^*(x)$, which may depend upon the currently earned rate of return, x. The regulator is assumed to be able to adjust output prices instantaneously so that the earned rate of return adjusts immediately to the new allowed rate of return and, starting from this new base, continues to follow the exogenously determined stochastic process (1). We are implicitly assuming that demand for the output of the regulated firm is sufficiently inelastic that there always exists an output price which will yield the allowed rate of return.[11] Furthermore, it should be recognized that, in general, the output price will have implications for the risk of the earnings: in taking the stochastic process (1) as exogenous, we are treating these effects as being of second-order importance.

With the foregoing assumptions, the market value of the regulated firm can be written as $F(x, B)$ and, using equation (A7) in the Appendix, satisfies the partial differential equation:

$$\tfrac{1}{2}\sigma^2(x)F_{xx} + \mu(x)F_x + (x - p(x))BF_B + p(x)B + \pi(x)[F(x^*(x), B) - F(x, B)] = rF + \lambda\sigma(x)F_x . \quad (4)$$

The left-hand side of equation (4) represents the expected return on the firm: the first three terms give the expected change in the value of the firm owing to changes in the state variables x and B, assuming no regulatory hearing takes place. The fourth term, $p(x)B$, is the net dividend received by the owners of the firm, and the last term is the capital gain expected to result from a regulatory hearing. The right-hand side of the equation is the return required on the firm's securities in equilibrium. Here r is the riskless interest rate, which is taken as constant.[12] The second term is a capital asset pricing model type risk premium: $\lambda\sigma(x)$ is the instantaneous covariance between changes in x and the rate of return on aggregate wealth; and an increase in λ represents an increase in the systematic risk of the firm.

The market value of the firm is homogeneous of degree one in the value of the rate base, so that making the substitution $y(x) \equiv F(x, B)/B$, we obtain the following ordinary differential equation for y, the normalized firm value:

$$\tfrac{1}{2}\sigma^2(x)y_{xx} + y_x(\mu(x) - \lambda\sigma(x)) + (x - r - p(x))y + p(x) + \pi(x)[y(x^*(x)) - y(x)] = 0. \quad (5)$$

In this equation the influence of regulatory policy on valuation is captured in the last term, which depends both on the rule for holding hearings, $\pi(x)$, and on the rule for setting the allowed rate of return, $x^*(x)$. It may be noted that if $\partial x^*/\partial x = 0$ and

[10] In a study of regulated electric utilities in Florida, Roberts *et al.* (1978) found that the probability that the regulatory authority would require a decrease in rates was an increasing function of the amount by which the earned rate of return exceeded the previously allowed rate of return. The earned rate of return appeared to have no effect on the probability that the company would seek higher rates, perhaps because company requests were based on prospective rather than current rates of return.

[11] This assumption could be relaxed at the expense of introducing additional variables affecting the value of the firm.

[12] The model can be expanded to incorporate a stochastic interest rate. In general, the resulting partial differential equation will not have a closed-form solution. A restrictive example with a stochastic interest rate for which there exists an analytic solution was included in an earlier draft of this article and is available from the authors. The qualitative results are similar to those of the model presented in this article.

$\partial\pi/\partial x = 0$, then as $\pi \rightarrow \infty$, $y(x) \rightarrow y(x^*)$. This represents a policy of continuous regulation under which the firm always earns the allowed rate of return; consumers bear all of the risk and investors bear none. This is a polar case, the other being that in which $\pi = 0$ and there is no future regulation. To analyze this and intermediate cases, it will be necessary to specify our model further.

4. An explicit model

■ We assume the following:

Assumption 1. The rate of return on the rate base follows an arithmetic Brownian motion, which permits the possibility that, in the absence of regulatory action, the regulated firm may incur losses:

$$dx = \mu dt + \sigma dz. \tag{6}$$

Assumption 2. The output capacity of the firm is proportional to the rate base, B.

Assumption 3. The firm is required by the regulator to maintain capacity equal to potential demand which is growing at the exogenously determined rate g.

Assumptions 2 and 3 imply that the rate base must also grow at the rate g. Equation (3) then defines the dividend payout policy $p(x) = x - g$.

With these substitutions, the differential equation for y is

$$\tfrac{1}{2}\sigma^2 y_{xx} + (\mu - \lambda\sigma)y_x - (r - g)y + (x - g) + \pi(x)[y(x^*(x)) - y(x)] = 0. \tag{7}$$

We shall use equation (7) to discuss the effects of alternative regulatory policies on the value and risk of the equity.

□ **The unregulated case.** The case $\pi(x) \equiv 0$ is of interest not only because it is polar to the case of continuous regulation, but also because it is consistent with current approaches to regulatory issues, which neglect the possibility of future regulatory action. When $\pi(x) = 0$, the complete solution to equation (7) is

$$y(x) = C_1 e^{\gamma_1 x} + C_2 e^{\gamma_2 x} + \frac{x - g}{r - g} + \frac{\mu - \lambda\sigma}{(r - g)^2}, \tag{8}$$

where

$$\gamma_1 = a_1 + a_2,$$

$$\gamma_2 = a_1 - a_2,$$

$$a_1 = -(\mu - \lambda\sigma)/\sigma^2,$$

$$a_2 = [(\mu - \lambda\sigma)^2 + 2\sigma^2(r - g)]^{1/2}/\sigma^2,$$

and C_1 and C_2 are constants to be determined by the boundary conditions.

We restrict our attention to the case $r > g$:[13] then $C_1 = 0$ if $y(x)/x$ is to remain finite as $x \rightarrow \infty$. The constant C_2 is determined by the value of x for which the value of the equity is zero: denote this value by $\hat{x}$. There are in principle two ways in which $\hat{x}$ may be determined. First, if the firm has bonds outstanding, the provisions of the bond indenture may allow bondholders to foreclose when the rate of return drops to a critical level,[14] $\hat{x}$. Even in the absence of debt, shareholders may find it to their advantage to declare bankruptcy voluntarily, since continuing to operate the firm requires them to put

[13] If $r < g$, the present value of future investments in the rate base is infinite.

[14] When the rate of return drops to this level, the interest coverage ratio will have fallen below a critical level as well.

up the funds to finance the exogenously determined expansion of the rate base; for $x < g$ this will involve a negative cash flow. We shall assume that $\hat{x}$ is chosen by the shareholders to maximize $y(x)$ or, equivalently, to maximize $C_2(\hat{x})$, where $C_2(\hat{x})$ is given by the condition that $y(\hat{x}) = 0$:

$$C_2(\hat{x}) = -e^{-\gamma_2 \hat{x}}\left[\frac{\hat{x} - g}{r - g} + \frac{\mu - \lambda\sigma}{(r - g)^2}\right]. \tag{9}$$

Carrying out the maximization of (9) yields

$$\hat{x} = \gamma_2^{-1} + g - \frac{\mu - \lambda\sigma}{r - g}. \tag{10}$$

Then the ratio of the market value of the firm to the rate base is given by expression (8), where $C_1 = 0$ and C_2 is defined by equations (9) and (10).

It is of interest to note that for large x

$$y(x) \rightarrow \frac{x - g}{r - g} + \frac{\mu - \lambda\sigma}{(r - g)^2}. \tag{11}$$

Recalling that $x - g = p$, the instantaneous dividend rate per unit of the rate base, it is seen that the first term of (11) corresponds to the familiar Gordon (1962) growth model when the riskless interest rate is used for discounting; the second term adjusts both for the trend in the rate of return and for risk. The risk adjustment is independent of x for large x because of our assumption that the variance of the stochastic process (6) is independent of x. For small x, the risk adjustment enters the valuation expression (8) in a nonlinear fashion, reflecting its influence on the bankruptcy condition.

We shall contrast this no regulation case with two classes of regulatory policy. In the first class, there is a constant probability rate for the holding of a hearing as in Klevorick's (1973) model of the regulated firm. Under the second class of policy, hearings are held only when the rate of return reaches predetermined upper and lower bounds. Joskow (1974) argues that regulatory reviews are initiated mainly by firms whose profits have dropped below an acceptable level. On the other hand, Roberts *et al.* (1978) found that the probability of a hearing increased as the rate of return became high. Hendricks (1975) and Burness *et al.* (1980) have also constructed models of the regulated firm in which regulatory policy is represented by predetermined bounds on profits. These two classes of regulatory policy were chosen with an eye to tractability and are not intended to be representative of the policies followed by regulatory hearings; nor are they exhaustive of the policies that could be considered within this framework. They are intended, however, to be illustrative of the hitherto largely neglected effects of the possibility of future regulation on the value of the equity of regulated firms and therefore on the choice of regulatory policy.

□ **Stochastic regulatory hearings.** With a constant probability rate for regulatory hearings, the normalized firm value satisfies equation (5) with $\pi(x) \equiv \pi$. We shall assume that the allowed rate $x^*(x)$ is also independent of x. Two cases will be considered: when x^* is arbitrary and when x^* is chosen so that $y(x^*) = 1$. The latter policy is what we have referred to as a consistent regulatory policy.

Under the consistent regulatory policy, equation (7) becomes

$$\tfrac{1}{2}\sigma^2 y_{xx} + (\mu - \lambda\sigma)y_x - (r - g^*)y + (x - g^*) = 0, \tag{12}$$

where $g^* = g - \pi$. Except for the replacement of g by g^*, this equation is identical to that obtained in the unregulated case, the complete solution of which is given in (8). Therefore, the solution in this special case of stochastic regulation and a consistent regulatory policy is obtained from the unregulated case by reducing the exogenously specified growth rate, g, by the probability rate, π. The value of x^* which yields a consistent regulatory policy is obtained by solving the equation $y(x^*) = 1$, where $y(x^*)$ is the valuation function yielded by a particular value of π.

For arbitrary values of the allowed rate, x^*, the solution to equation (7) is more complex. The complete solution to the equation is

$$y(x) = C_1 e^{\gamma_1 x} + C_2 e^{\gamma_2 x} + \frac{x - g + \pi y(x^*)}{r - g + \pi} + \frac{\mu - \lambda\sigma}{(r - g + \pi)^2}. \tag{13}$$

The argument used in the unregulated case implies that $C_1 = 0$. Then, setting $x = x^*$ in equation (13), we may solve for $y(x^*)$. Substituting the resulting expression for $y(x^*)$ into (13) yields

$$y(x) = C_2(\hat{x})\left[e^{\gamma_2 x} + \left(\frac{x - g}{r - g + \pi}\right)e^{\gamma_2 x^*}\right] + \left(\frac{x - g}{r - g + \pi}\right) + \frac{\pi(x^* - g) + \mu - \lambda\sigma}{(r - g)(r - g + \pi)}. \tag{14}$$

In equation (14) we have explicitly shown the dependence of C_2 on $\hat{x}$, which is defined by $y(\hat{x}) = 0$. Setting $x = \hat{x}$ yields the following expression for $C_2(\hat{x})$:

$$C_2(\hat{x}) = -\left[e^{\gamma_2 \hat{x}} + \frac{\pi}{r - g}e^{\gamma_2 x^*}\right]^{-1}\left[\left(\frac{\hat{x} - g}{r - g + \pi}\right) + k(x^*)\right], \tag{15}$$

where $k(x^*) = (\pi(x^* - g) + \mu - \lambda\sigma)/(r - g)(r - g + \pi)$.

Then, assuming again that $\hat{x}$ is chosen by the shareholders to maximize y (and therefore C_2), $\hat{x}$ is given by the solution to the nonlinear equation:

$$\gamma_2 e^{\gamma_2 \hat{x}}\left[\left(\frac{\hat{x} - g}{r - g + \pi}\right) + k(x^*)\right] - \frac{e^{\gamma_2 \hat{x}} + \pi(r - g)^{-1}e^{\gamma_2 x^*}}{r - g + \pi} = 0. \tag{16}$$

Thus, for arbitrary allowed rates under stochastic regulation, the normalized equity value, $y(x)$, is given by equations (14), (15), and (16).

□ **Deterministic regulatory hearings.** Under the particular deterministic policy which we consider, regulatory hearings are held only when the current rate of return reaches predetermined upper and lower bounds, x_u and x_l, respectively. This is represented by setting $\pi(x) = 0$ for $x \neq x_u, x_l$ in equation (7) and $\pi(x_u) = \pi(x_l) = \infty$. Then the normalized firm value satisfies the differential equation,

$$\tfrac{1}{2}\sigma^2 y_{xx} + (\mu - \lambda\sigma)y_x - (r - g)y + (x - g) = 0, \tag{17}$$

subject to the boundary conditions

$$y(x_u) = y(x^*(x_u))$$

$$y(x_l) = y(x^*(x_l)). \tag{18}$$

The complete solution to this equation is given by (8), where the constants C_1 and C_2 are determined by the boundary conditions (18).

If the regulatory policy is consistent so that $y(x^*(x_u)) = y(x^*(x_l)) = 1$, it may be verified that the constant terms are

$$C_1 = \frac{b_u b_{2l} - b_l b_{2u}}{b_{1u} b_{2l} - b_{1l} b_{2u}}$$

$$C_2 = \frac{b_l b_{1u} - b_u b_{1l}}{b_{1u} b_{2l} - b_{1l} b_{2u}}, \tag{19}$$

where $b_{1i} = e^{\gamma_1 x_i}$, $b_{2i} = e^{\gamma_2 x_i}$ and $b_i = 1 - (x_i - g)/(r - g) + (\mu - \lambda\sigma)/(r - g)^2$ for $i = u, l$. Similar expressions may be derived for the general case in which

$$x^*(x_u) = x_u - \epsilon_u, \; x^*(x_l) = x_l + \epsilon_l.$$

TABLE 1 Firm Parameter Values

$\sigma = 0.005$		$r = 0.08$
$\mu = 0.0$		$g = 0.06$
	$\lambda = .14$	

5. A numerical example

■ To illustrate the possible effects of alternative regulatory policies, valuation functions, $y(x)$, were computed for the parameter values given in Table 1, assuming no regulation, stochastic regulatory hearings, and deterministic regulatory hearings. In the examples with regulation, the regulatory policies are assumed to be consistent, so that $y(x^*) = 1$, where x^* is the allowed rate of return.

Table 2 presents some summary statistics for three deterministic and three stochastic policies as well as for the no-regulation case ($\pi = 0$). The allowed rate of return x^* appropriate under the different regulatory policies varies widely as one would expect, and it is noteworthy that the appropriate allowed rate of return may even be less than the riskless interest rate as shown by policy (vii).

The second line of Table 2 indicates the value of $y_x(x^*)$, a scalar measure of the absolute risk borne by shareholders of the firm when the return on the rate base is equal to the allowed rate of return. It is greatest in the unregulated case and decreases monotonically in the stochastic policy case as the probability rate of a hearing rises: in the limit the shareholders bear no risk, since all risk is borne by consumers. The beta coefficient, $\beta(x^*)$, shown on the third line of the table is simply a scalar multiple of $y_x(x^*)$.[15] The last three lines of the table illustrate the dramatically different firm values that may result from the same rate of return on the rate base under different regulatory policies.

Figure 1, which shows the valuation schedules obtained under three different regulatory policies, serves to emphasize the importance of regulatory policy for valuation. In the unregulated case, the schedule resembles that between the value of a call option and the value of the underlying stock.[16] As the probability rate of a hearing increases from zero, the valuation schedule rotates clockwise: the reason for this is that if x is high, there is a probability that it will be subject to a discrete downward adjustment, and conversely

TABLE 2 Alternative Regulatory Policies

	No Regulation	Stochastic Policies			Deterministic Policies		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
		$\pi = 0.1$	0.5	10.0	$x_u = 0.12$	0.18	0.18
					$x_l = 0.01$	0.03	0.06
x^*	.099	.086	.081	.080	.106	.092	.073
$y_x(x^*)$	33.1	8.3	1.9	.1	14.2	25.4	6.2
$\beta(x^*)$	.57	.15	.03	.00[a]	.25	.44	.12
$y(.06)$	.08	.78	.96	1.00	.10	.53	1.00
$y(.08)$	.46	.95	1.00	1.00	.44	.75	1.06
$y(.10)$	1.05	1.12	1.04	1.00	.89	1.23	1.44

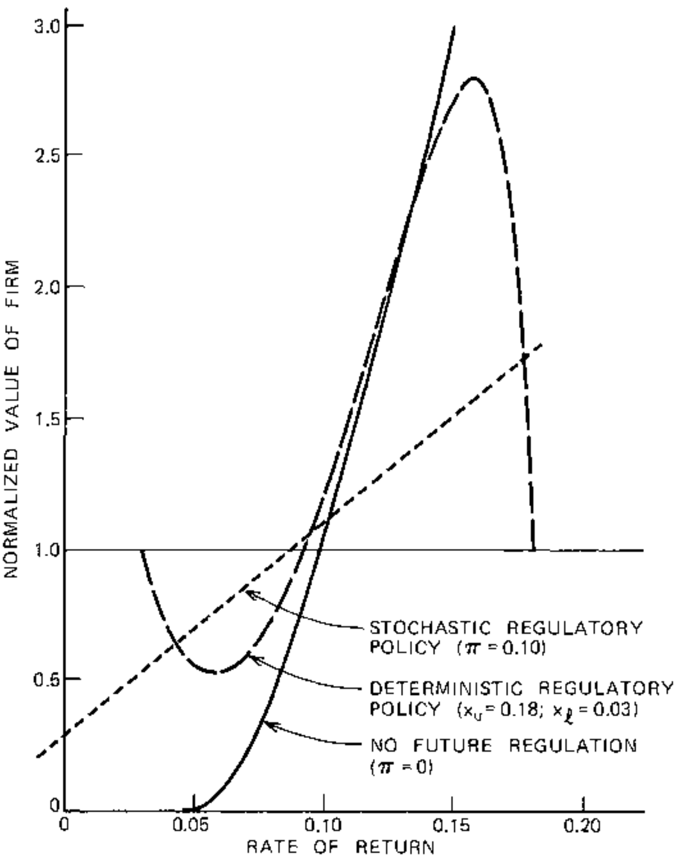
[a] Subject to rounding error.

[15] $\beta(x^*) = y^{-1}y_x(x^*)\lambda\sigma/\sigma_w^2$. The variance of aggregate wealth σ_w^2 is set equal to .2.

[16] Black and Scholes (1973) were the first to point out that the equity in a firm could be regarded as a call option to purchase the firm from the bondholders.

FIGURE 1

NORMALIZED FIRM VALUE FOR DIFFERENT REGULATORY POLICIES



if x is low. These discrete adjustments, which occur when a rate hearing is held, cause capital gains or losses for investors as the normalized equity value reverts to unity. If, as is more realistic, the probability rate of a hearing were an increasing function of the absolute deviation of x from x^*, the valuation schedule would be more *S*-shaped. An extreme example of this is provided by the deterministic regulatory policy: under this policy the equity value actually decreases as the rate of return approaches the upper control limit, x_u, since as soon as the rate of return reaches x_u, it will be reduced to x^*. A similar phenomenon is apparent in the vicinity of x_l.

6. Implications of the model

■ **Setting the allowed rate.** As described in Section 2, the modern consensus approach to regulation implies that the allowed rate of return should be set equal to the cost of capital, and in practice much attention is given at regulatory hearings to determination of the appropriate cost of capital.

Current methods of assessing the cost of capital may be classified according to whether they follow an individual firm approach or a risk class approach. The former method involves forecasting the long-run rate of return that an investor might reasonably expect to earn by purchasing shares in the firm at their current price. Unfortunately, this rate of return will depend upon future regulatory policy and in particular upon the allowed rate of return set as a result of the hearing; but it is the appropriate allowed rate which is at issue. Thus, this individual firm approach is beset by a fatal circularity.

Two variants of the risk class approach may be distinguished. According to the first, the regulated firm's cost of capital is assumed to be equal to that of unregulated firms with similar earnings risk. A reasonable interpretation of the earnings risk approach is that the allowed rate of return should be set at the level which would cause an unregulated firm with similar earnings characteristics to sell at book value.[17] This, however, neglects the fact that the risk of the regulated firm depends not only on the stochastic characteristics of the earnings stream but also on regulatory policy. To illustrate, it may be seen from Table 2 that a return on book value of 10% would cause the market value of an unregulated firm to approximate book value. The same rate of return earned on the rate base of a regulated firm with identical earnings risk[18] may cause the market value to range from 89% to 144% of the value of the rate base, depending on which regulatory policy is followed.

The second variant of the risk class approach assumes that the capital asset pricing model holds and that therefore risk can be measured appropriately by the beta coefficient. Although this is consistent with our valuation model which rests upon an intertemporal version of the capital asset pricing model, the beta coefficient, which is equal to $y^{-1}y_x\lambda\sigma/\sigma^2_{\kappa}$, is stochastic and depends upon x. Therefore, there can be no assurance, and indeed it would be only by coincidence, that the beta coefficient estimated from the nonstationary time series of equity returns would yield a cost of equity capital close to the appropriate allowed rate of return under the consistent regulatory policy.[19]

□ **Investment incentives.** To this point we have neglected the effects of regulatory policy on the incentives of the regulated firm. To illustrate the effects of regulation on investment incentives in the context of this article suppose that, in the absence of regulation, the rate of return on the rate base would follow the same stochastic process as under regulation. Then the effect of regulation on investment incentives may be measured by the difference in the present value of the same investment project to a regulated and an unregulated firm. The investment project is assumed to generate an instantaneous earnings rate ρI, where ρ follows the same stochastic process as, and is perfectly correlated with, x_0, the rate of return on the preexisting rate base, B, and I is the amount invested in the new project. Then the effect of the new investment on the rate of return is given by

$$x(I) = \frac{x_0 B + \rho I}{B + I}. \tag{20}$$

Recalling the expression for the value of the equity of a regulated firm, $F(x, B)$, the gross present value to the stockholders of a marginal dollar invested is given by

$$\frac{dF}{dI} = F_x \frac{dx}{dI} + y(x), \tag{21}$$

where we have used the assumption that the whole investment is added to the rate base so that $dB/dI = 1$. Then, differentiating equation (20) with respect to I, setting $I = 0$, and substituting the result in (21), we obtain the following expression for the gross present value of a marginal investment:

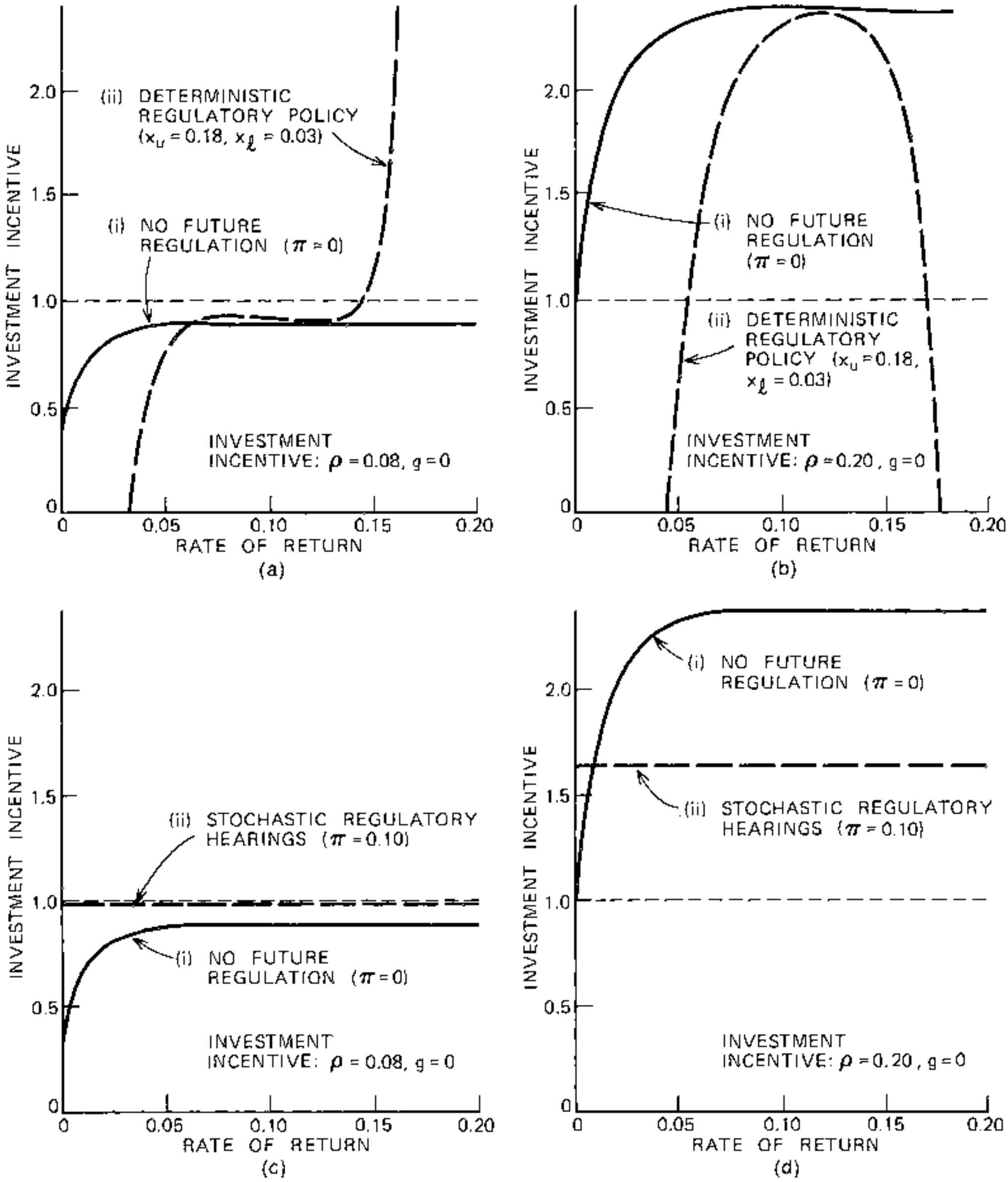
$$\left.\frac{dF}{dI}\right|_{I=0} = y(x) + y_x(x)(\rho - x). \tag{22}$$

[17] This assumes that the book value of the unregulated firm is determined in the same way as the rate base of the regulated firm.

[18] That is, the regulated firm's earnings risk was identical to that of the unregulated firm *before* regulation was imposed on the former.

[19] See Breen and Lerner (1972) and Myers (1972) for a discussion of the problems of using beta in regulatory hearings.

FIGURE 2



Under the standard assumption of stock price maximization, a project will be undertaken only if this expression, which we refer to as the "investment incentive," exceeds unity.

In Figure 2a, the value of the investment incentive is plotted as a function of x for a marginal investment with ρ = .08 for regulatory policy (i), no future regulation, and for policy (vi), a deterministic regulatory policy. The parameter values for these cases are given in Tables 1 and 2, except that g has been set equal to zero. Inspection of the figure reveals that this project would never be accepted by the unregulated firm, but that it becomes highly desirable to the regulated firm as the rate of return approaches x_u, 18%. The explanation of this phenomenon is simple: adoption of the project reduces the currently earned rate of return and thereby postpones the day on which a regulatory hearing takes place, thus prolonging the period for which high returns may be earned. This makes the project desirable despite its low value *per se*. On the other hand, as x approaches x_l, 3%, the project becomes highly undesirable to the regulated firm by postponing a hearing which would raise the overall rate of return. This latter effect is also visible in Figure 2b which depicts the investment incentive for a highly profitable project under the same conditions. However, this profitable project is undesirable to the regulated firm as the rate of return approaches x_u because of its effect in hastening a regulatory hearing. The effects of regulation are proportional to the vertical distance between the schedules in these figures, and it is apparent that under this type of regulatory policy, extreme incentives for the regulated firm may be induced because of the effect of a marginal investment project on the probability that a regulatory hearing will be held.

In Figures 2c and 2d the investment incentives arising from the same projects are illustrated for policy (ii), stochastic regulatory hearings with $\pi = .10$, as well as for the unregulated case. Under these policies adoption of the project has no influence on the probability of regulatory hearings, and the extreme impact of the deterministic regulatory policies is eliminated. Realistic regulatory policies would seem likely to lie between the extremes of our stochastic and deterministic policies, with the probability of a hearing being positively related to the absolute deviation of x from the current allowed rate of return. It seems that even under such policies, regulatory lag will tend to have a substantial effect on investment incentives. Of course, without knowing whether the investment incentives for the unregulated firm are Pareto optimal, it is not possible to make any statements about welfare gains and losses.

7. Conclusion

■ In this article we have analyzed the problem of determining a consistent regulatory policy in a dynamic setting. Contemporary approaches to regulation, which set the allowed rate of return equal to the cost of capital, were shown to be deficient insofar as they take the cost of capital as exogenous, when it is in fact a product of the regulatory policy chosen. To devise a consistent regulatory policy, it is necessary to have a valuation model that explicitly incorporates the effects of regulatory policy. Such a valuation model was constructed, and it was shown how the appropriate allowed rate of return and the risk borne by investors varied as the regulatory policy was changed. Finally, current practices for estimating the appropriate allowed rate of return were analyzed, and a simple analysis was made of the effect of regulatory lag on investment incentives. The models developed in this article are highly simplified, and much work remains to be done in developing more realistic yet tractable models for valuing regulated firms and analyzing the effects of alternative regulatory policies.

Appendix

The general valuation model

■ The valuation model employed in Section 3 is a special case of a general model of the valuation of financial claims. Necessary conditions for equilibrium in the capital market yield a partial differential equation which must be satisfied by the pricing function of any financial claim.

Thus, consider an economy in which:

(1) All investors have time-additive von Neumann-Morgenstern utility functions of the logarithmic form defined over the rate of consumption of a single consumption good.
(2) There are no taxes or transactions costs, trading takes place continuously, and the capital market is always in equilibrium.
(3) The state of the economy is completely described by aggregate wealth, W, and an s-dimensional vector of state variables, X, whose behavior is governed by a system of

stochastic differential equations:

$$dX_j = \mu_j(X, t)dt + \eta_j(X, t)dz_j + (X_j^* - X_j)dq_j\,, \qquad j = 1, \ldots, s, \tag{A1}$$

where t denotes calendar time and dz_j is a standard Gauss-Wiener process. $q_j(t)$ is an independent Poisson process with intensity $\pi_j(X, t)$, and $(X_j^* - X_j)$ is the change in the state variable if the Poisson event occurs.[20] Jumps in the state variables are assumed to be uncorrelated with the return on aggregate wealth.

Merton (1973) has shown, in a related context, that under such assumptions the equilibrium expected rates of return on individual assets will satisfy the specialized version of the intertemporal capital asset pricing model:[21]

$$\alpha_i - r = \sigma_{iw}\,, \tag{A2}$$

where α_i is the expected instantaneous rate of return on asset i, σ_{iw} is the covariance of the rate of return on asset i with the rate of return on aggregate wealth, and r is the instantaneously riskless interest rate.

Cox, Ingersoll, and Ross (1978) have shown that if investors possess rational expectations, so that the price functions they use to make their optimal decisions are the equilibrium price functions that continuously clear the market, then the equilibrium condition (A2) implies a fundamental partial differential equation which must be satisfied by the value of all financial assets.

Thus, define $F_i \equiv F_i(W, X, t)$ as the market value of asset i; then the instantaneous change in the value of the asset is given by:[22]

$$\begin{aligned} dF = [\sum_{j=1}^{s} F_{x_j}\mu_j + F_w(W\alpha_w - C) + F_t + \tfrac{1}{2} \sum_{j=1}^{s}\sum_{k=1}^{s} F_{x_jx_k}\rho_{jk}\eta_j\eta_k \\ + \tfrac{1}{2}\sum_{j=1}^{s} F_{x_jw}\rho_{jw}\eta_j\sigma_w W + \tfrac{1}{2}F_{ww}\sigma_w^2 W^2]dt + \sum_{j=1}^{s} F_{x_j}\eta_j dz_j \\ + F_w\sigma_w W dz_w + \sum_{j=1}^{s} [F(W, X + \Delta X_j, t) - F(W, X, t)]dq_j\,, \end{aligned} \tag{A3}$$

where α_w is the instantaneous expected rate of return on aggregate wealth, C is the rate of aggregate consumption, ρ_{jk} and ρ_{jw} are the instantaneous correlations between the rates of return on asset j and on asset k, and on asset j and on aggregate wealth, respectively. The symbol ΔX_j denotes an s-dimensional vector all of whose elements are equal to zero except element j, which is equal to $(X_j^* - X_j)$.

The expected instantaneous rate of return on asset i, α_i, is the sum of the payout rate on the asset, $\delta_i(W, X, t)$, and the expected price change, divided by the current value of the asset:

$$\alpha_i = F^{-1}[\cdot] + F^{-1}\delta + F^{-1} \sum_{j=1}^{s} [F(W, X + \Delta X_j, t) - F(W, X, t)]\pi_j(X, t), \tag{A4}$$

where $[\cdot]$ is the coefficient of dt in equation (A3).

Since jumps in the state variables are uncorrelated with the return on aggregate wealth, the instantaneous covariance between the return on asset i and the return on

[20] For a detailed discussion of such mixed processes, see Merton (1976).

[21] The specialization arises from the assumption of logarithmic utility, which permits us to omit the additional terms relating to stochastic shifts in the investment opportunity set that would otherwise appear in equation (A2).

[22] See Merton (1976) for the necessary extension of Ito's lemma. The subscript i is omitted for the sake of clarity, and the partial derivatives of F are denoted by the appropriate subscripts.

aggregate wealth is

$$\sigma_{iw} = F^{-1}[\sum_{j=1}^{s} F_{x_j}\eta_j\rho_{jw}\sigma_w + F_w\sigma_w^2 W^2]. \tag{A5}$$

Finally, multiplying the equilibrium condition (A2) by the fraction of aggregate wealth accounted for by asset i, and summing over i, we obtain

$$\alpha_w - r = \sigma_w^2 . \tag{A6}$$

Then, substituting for α_i and σ_{iw} in the equilibrium condition (A2) and using (A6), we obtain the basic partial differential equation which is satisfied by the values of all assets:

$$\sum_{j=1}^{s} F_{x_j}(\mu_j - \eta_j\rho_{jw}\sigma_w) + F_w(rW - C) + F_t + \tfrac{1}{2}\sum_{j=1}^{s}\sum_{k=1}^{s} F_{x_jx_k}\rho_{jk}\eta_j\eta_k + \sum_{j=1}^{s} F_{x_jw}\rho_{jw}\eta_j\sigma_w W$$

$$+ \tfrac{1}{2}F_{ww}\sigma_w^2 W^2 - rF + \delta + \sum_{j=1}^{s} [F(W, X + \Delta X_j, t) - F(W, X, t)]\pi_j = 0. \tag{A7}$$

This equation, which is the basis of our valuation model for the regulated firm, corresponds to equation (25) of Cox, Ingersoll, and Ross (1978) under the assumption of logarithmic utility when there are state variables with discontinuous sample paths. When the appropriate boundary conditions are appended, this equation suffices to determine the value of any security.

References

AVERCH, H. AND JOHNSON, L.L. "Behavior of the Firm under Regulatory Constraint." *American Economic Review*, Vol. 52, No. 5 (December 1962), pp. 1062–1069.

BAUMOL, W.J. AND KLEVORICK, A.K. "Input Choices and Rate of Return Regulation: An Overview of the Discussion." *Bell Journal of Economics and Management Science*, Vol. 1, No. 2 (Autumn 1970), pp. 162–190.

BAWA, V.S. AND SIBLEY, D.S. "Dynamic Behavior of a Firm Subject to Stochastic Regulatory Review." *International Economic Review*, Vol. 21, No. 3 (October 1980), pp. 627–642.

BLACK, F. AND SCHOLES, M. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, Vol. 81, No. 3 (May–June 1973), pp. 637–654.

BREEN, W.J. AND LERNER, E.M. "On the Use of β in Regulatory Proceedings." *Bell Journal of Economics and Management Science*, Vol. 3, No. 2 (Autumn 1972), pp. 612–621.

BURNESS, H., MONTGOMERY, D., AND QUIRK, J. "Capital Contracting and the Regulated Firm." *American Economic Review*, Vol. 70, No. 3 (June 1980), pp. 342–354.

CLARKE, R.G. "The Effect of Fuel Adjustment Clauses on the Systematic Risk and Market Values of Electric Utilities." *Journal of Finance*, Vol. 35, No. 2 (May 1980), pp. 347–358.

COX, J.C., INGERSOLL, J.E., AND ROSS, S.A. "A Theory of the Term Structure of Interest Rates." Stanford University Research Paper No. 468, August 1978.

DAS, S.P. "On the Effect of Rate of Return Regulation under Uncertainty." *American Economic Review*, Vol. 70, No. 3 (June 1980), pp. 456–460.

DAVIS, B.E. AND SPARROW, F.T. "Valuation Models in Regulation." *Bell Journal of Economics and Management Science*, Vol. 3, No. 2 (Autumn 1972), pp. 544–567.

GORDON, M.J. *The Investment, Financing and Valuation of the Corporation.* Homewood, Ill.: Richard D. Irwin, Inc., 1962.

HENDRICKS, W. "The Effect of Regulation on Collective Bargaining in Electric Utilities." *Bell Journal of Economics*, Vol. 6, No. 2 (Autumn 1975), pp. 451–465.

JOSKOW, P. "Inflation and Environmental Concern: Structural Change in the Process of Public Utility Regulation." *Journal of Law and Economics*, Vol. 17 (October 1974), pp. 291–327.

KLEVORICK, A.K. "The Behavior of a Firm Subject to Stochastic Regulatory Review." *Bell Journal of Economics and Management Science*, Vol. 4, No. 1 (Spring 1973), pp. 57–88.

LELAND, H.E. "Theory of the Firm Facing Uncertain Demand." *American Economic Review*, Vol. 62, No. 3 (June 1972), pp. 278–291.

———. "Regulation of Natural Monopolies and the Fair Rate of Return." *Bell Journal of Economics and Management Science*, Vol. 5, No. 1 (Spring 1974), pp. 3–15.

MARSHALL, W.J., YAWITZ, J.B., AND GREENBERG, E. "Optimal Regulation under Uncertainty." *Journal of Finance*, Vol. 36, No. 4 (September 1981), pp. 909–922.

MERTON, R. "An Intertemporal Capital Asset Pricing Model." *Econometrica*, Vol. 41, No. 4 (September 1973), pp. 867–888.

———. "Option Pricing When Underlying Stock Returns Are Discontinuous." *Journal of Financial Economics*, Vol. 3, Nos. 1–2 (January/March 1976), pp. 125–144.

MILLER, M.H. AND MODIGLIANI, F. "Dividend Policy, Growth, and the Valuation of Shares." *Journal of Business*, Vol. 34, No. 4 (October 1961), pp. 411–433.

MYERS, S.C. "The Application of Finance Theory to Public Utility Rate Cases." *Bell Journal of Economics and Management Science*, Vol. 3, No. 1 (Spring 1972a), pp. 58–97.

———. "On the Use of β in Regulatory Proceedings: A Comment." *Bell Journal of Economics and Management Science*, Vol. 3, No. 2 (Autumn 1972b), pp. 622–627.

———. "A Simple Model of Firm Behavior under Regulation and Uncertainty." *Bell Journal of Economics and Management Science*, Vol. 4, No. 1 (Spring 1973), pp. 57–88.

PELES, Y.C. AND STEIN, J.L. "The Effect of Rate of Return Regulation Is Highly Sensitive to the Nature of Uncertainty." *American Economic Review*, Vol. 66, No. 3 (June 1976), pp. 278–289.

PERRAKIS, S. "On the Regulated Price-Setting Monopoly Firm with a Random Demand Curve." *American Economic Review*, Vol. 66, No. 3 (June 1976), pp. 410–416.

ROBERTS, R.B., MADDALA, G.S., AND ENHOLM, G. "Determinants of the Requested Rate of Return and the Rate of Return Granted in a Formal Regulatory Process." *Bell Journal of Economics*, Vol. 9, No. 2 (Autumn 1978), pp. 611–621.

ROBICHEK, A.A. "Regulation and Modern Finance Theory." *Journal of Finance*, Vol. 33, No. 3 (June 1978), pp. 693–705.

STEIN, J.L. AND BORTS, G.H. "Behavior of the Firm under Regulatory Constraint." *American Economic Review*, Vol. 62, No. 5 (December 1972), pp. 964–970.