Performance evaluation and benchmark errors (I)*

"True portfolio management ability is not indicated if the measured performance is due to the benchmark's own error."

Richard Roll

In portfolio performance evaluation, one compares the return obtained on a managed portfolio to the return expected on an unmanaged portfolio having the same risk. The benchmark is the expected return on the unmanaged portfolio. It should accurately reflect the risk associated with the managed portfolio during the evaluation period. However, since it is always difficult to measure the risk associated with a managed portfolio, there is always potential for error in the benchmark. The purpose of this paper is to analyze benchmark error, and I do so in the context of the current widespread practice of using the capital asset pricing model (CAPM) to measure risk. As we shall see, performance evaluations based on the CAPM are prone to systematic errors of various kinds.

Error in performance measurement can be ascribed to two sources. The first is random variation: The actual return is in part a function of unforeseeable events that cause parameter mis-estimation, events that tend to cancel each other's effects over repeated measurements. A second source of error is in the *ex ante* CAPM benchmark, an error that cannot be eliminated by repeated evaluations. Thus, *ex ante* benchmark errors are much more important than errors due to random causes; they make particular managers appear to "outperform" expectations when they fortuitously choose portfolios with negative errors in the benchmark, while managers unfortunate enough to choose portfolios with positive benchmark error will appear to do relatively poorly. We must remember that *true portfolio management ability is not accurately indicated if the measured performance reflects the benchmark's own error*. Thus, the elimination of benchmark error is an extremely important practical problem for the evaluator.

BENCHMARK ERRORS OF THE CAPITAL ASSET PRICING MODEL (CAPM)

I have chosen to use the simplest version of the CAPM in this analysis of benchmark error. This version involves a linear relationship between the evaluator's expected return on a given asset or portfolio and the beta coefficient (which is supposed to measure the "systematic" risk of the asset[1]). This securities market line (SML), depicted in Figure 1, has an intercept equal to the risk-free rate of interest (E_F) and a slope equal to the difference between the evaluator's expected return (E_m) on the market index and the risk-free rate. I shall assume that a given market index has been selected for these evaluation procedures and that a nominal risk-free asset is available. My analysis assumes not that the simple CAPM is correct but only that it is used as a benchmark for performance evaluation.[2]

We can readily see from Figure 1 that an inaccurate assessment of risk will cause true performance to differ from measured performance. The measured performance is $\hat{\alpha}$, the vertical distance between the securities market line and the actual return (R_p) of the evaluated portfolio at the mis-assessed risk level ($\widehat{Risk_p}$). In this particular example, the performance is positive, because the observed return R_p lies above the securities market line at the assessed level of risk. The assessed risk level implies that the portfolio was expected to return $\hat{E}_p$. The actual performance (α) is negative, however, because the true risk associated

* This is the first part of an analysis of performance evaluation that will be continued in our issue of Winter 1981.

1. Footnotes appear at the end of the article.

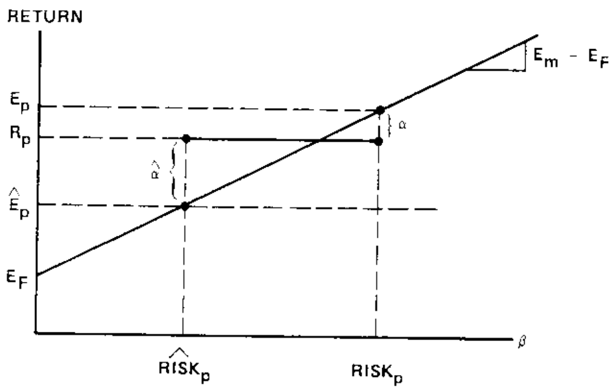


Figure 1. Mis-assessment of Performance Caused by Error in the Ex-ante Risk Measure. N.B.: the Risk Error is not Caused by Statistical Variation.

with this portfolio is larger than the measured risk. Thus, the true expected return of this portfolio is E_p, which lies above the observed return R_p.

In Figure 1 I illustrate benchmark error for the case in which one accurately assesses the position of the securities market line but inaccurately assesses the appropriate risk level for the portfolio. This error is *not* a statistical estimation error in the beta. It is possible to assess inaccurately the risk of a portfolio even if one knows the true expected returns and there is no statistical estimation problem at all.

How can this happen? It will occur if the market index is not on the evaluator's *ex ante* mean/variance efficient frontier; i.e., when the index is not an "optimized" portfolio. Unlike common estimation errors in statistics, one cannot eliminate this error in beta by using larger sample sizes. It will remain no matter how large the sample is. It is not an estimation error in the beta of the asset as measured against the market index *in use.* Instead, it is the difference between the measured beta and that beta which should have been calculated using an optimized index.

Figure 2 illustrates the situation in which the true and measured performances differ because the

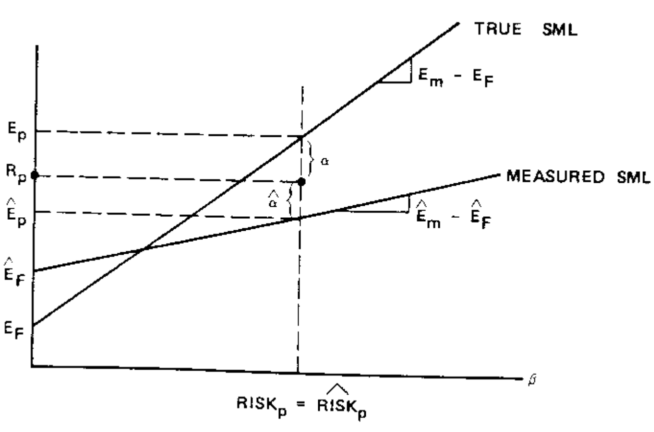


Figure 2. Mis-assessment of Performance Caused by Error in the Securities Market Line. This error is not due to statistical variation.

security market line's position is incorrect. The error in position is the result of two problems, neither of which is related to statistical variation: First, a non-optimized market index has been employed, an index whose expected return $\hat{E}_m$ differs from that (E_m) of the optimized index appropriate for the true risk-free asset. Second, the true risk-free asset has a return (E_F) that is different from the return on the nominal "riskless" asset used to measure $\hat{E}_F$. The net result is measured performance $\hat{\alpha}$ that differs from true performance α. As illustrated in Figure 2, the measured $\hat{\alpha}$ is positive, while the true performance α is negative. (R_p is the observed return.)

Figure 3 illustrates all possible non-statistical evaluation errors. It also introduces a number, π, that

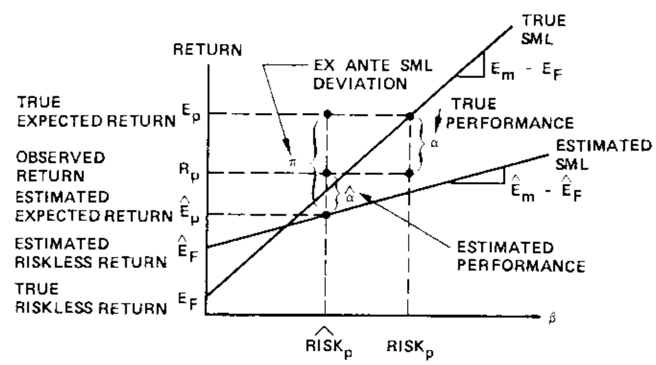


Figure 3. Mis-assessment of Performance Caused by Non-Statistical Errors in the Evaluation Benchmark, the Securities Market Line (SML).

measures the extent of these errors. In Figure 3, both the market index and the risk-free asset have been chosen incorrectly and in such a manner that the true risk of the portfolio is larger than the measured risk. Consequently, although the estimated performance is positive, the true performance is negative.

Before analyzing the ex-ante performance error that captures the essence of this problem, I must emphasize that true performance is an *ex post* quantity equal to the difference between the observed return and the *true* expected return. Of course, true performance is subject to statistical variation from one sample period to another. Clearly, the difference between an observed return and a true expected return consists of both random variation and true ability in portfolio management. On the other hand, if over time we repeatedly measure performance, we should find that the random variability tends to average out, leaving only true ability reflected in the average of such performance measurements. Notice, however, that repeated evaluations will not eliminate error in estimating the expected return, since the error will be present in the difference between the true performance and the estimated performance in *every one* of the evaluation periods.

The average performance evaluation error that remains as the number of evaluations grows large is equal to π, the deviation of the true expected return from the inaccurately assessed securities market line. This *ex ante* performance error is set forth algebraically in equation (1) and is derived formally in the footnote:[3]

$$\left.\begin{array}{l}\text{EX POST}\\ \text{PERFORMANCE}\\ \text{EVALUATION}\\ \text{ERROR IN}\\ \text{PERIOD t}\end{array}\right\} \hat{\alpha}_t - \alpha_t = \pi \left\{\begin{array}{l}\textit{EX ANTE}\\ \text{SML}\\ \text{DEVIATION}\end{array}\right. \quad (1)$$

The simple relationship in equation (1) makes it clear that the causes of deviation from the *ex ante* estimated securities market line are very important. If the evaluator could estimate such deviations independently, he could correct the traditional CAPM performance evaluations and thereby derive a more accurate assessment of true management ability.

THE CAUSES OF *EX ANTE* DEVIATIONS FROM THE SECURITY MARKET LINE

The entire error between true performance and estimated performance is due to deviation of the portfolio's position from the assessed securities market line. We shall now investigate why such deviations occur. Although it might seem that they could be caused by errors in assessing any of three components of the securities market line (the riskless rate of interest, the beta coefficient, or the expected return on the market index), we shall see that there is only one cause: failure to choose the proper optimized portfolio as the market index.

Figure 4 illustrates why the optimality or non-optimality of the chosen index is the critical ingredient for whether or not there are deviations from the securities market line. In the left panel of Figure 4, the il-

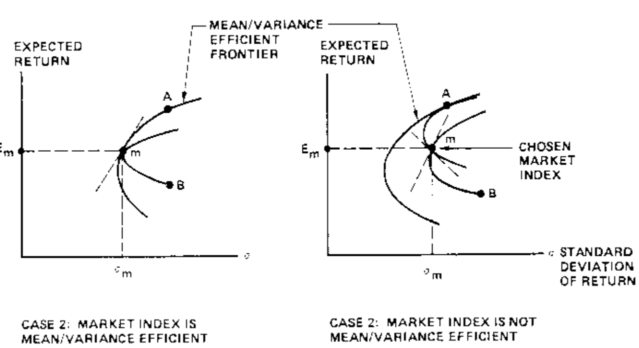


Figure 4. The Slope of Portfolio Loci (Dotted Lines) when the Market Index is Optimal (left) and Non-optimal (right).

lustrated index (m) is an optimized portfolio and is therefore located on the *ex ante* mean/variance efficient frontier. Imagine forming a hybrid portfolio from an arbitrary asset and the market index. For example, if asset B were located as shown in the diagram, it could be combined in varying proportions with the market index to trace a locus of portfolios depicted by the curve mB. Of course, points on the curve between m and B indicate some positive amounts invested in both m and B, whereas points outside this range indicate a short position either in m or in B. A similar locus can be created with m and any other asset; for example, with A. In this illustration, A is also an optimal portfolio.

The key principle in the diagram is: At the point where m is located, for any asset that is combined with m, the slopes of all such loci are equal to each other and to the slope of the efficient frontier at point m. This slope is indicated by the dotted line in the left hand panel.

In the right panel, I illustrate the second possibility. The chosen market index is not an optimized portfolio and therefore lies strictly inside the efficient frontier. Now, imagine combining assets with this index to generate a hybrid portfolio. The curve connecting m and B is again the locus of portfolios that can be generated by combining asset B with the market index. Similarly, the curve combining m and A indicates the portfolios that can be generated by combining the market index with A. Since m is strictly inside the efficient frontier, it is clear that the slope of these loci need not be equal at point m, unlike the case in which m is optimal. In fact, as the right panel in Figure 4 shows, the slope of the portfolio locus connecting m and A is negative at m, whereas the slope of the portfolio locus for m and B is positive.

It is possible to prove that there *must* be disagreements among these slopes when m is within the efficient frontier. Indeed, one correct definition of an optimized portfolio is that all slopes connecting any asset with the optimized portfolio are equal at the point where the optimized portfolio is located.

We can easily prove that in the general case the dotted lines in Figure 4 have slopes given by equation (2).[4]

$$\gamma_j = \frac{E_j - E_m}{\sigma_m(\beta_j - 1)} \quad (2)$$

If E_j is the return expected on an arbitrary asset j and if β_j is its (true) beta computed against m, then E_m is the *true* expected return on the market index, and σ_m is the index's true standard deviation. Equation (2) gives the loci slope when the true market portfolio is not "optimized" as well as when it is "optimized" and located on the efficient frontier.

BRINGING IN THE CAPITAL MARKET LINE

We are now ready to introduce the final link in the chain connecting the position of the measured market index and the error in performance measurement. This final link is the estimated capital market line, the line between E_f and m in Figure 5. Since the

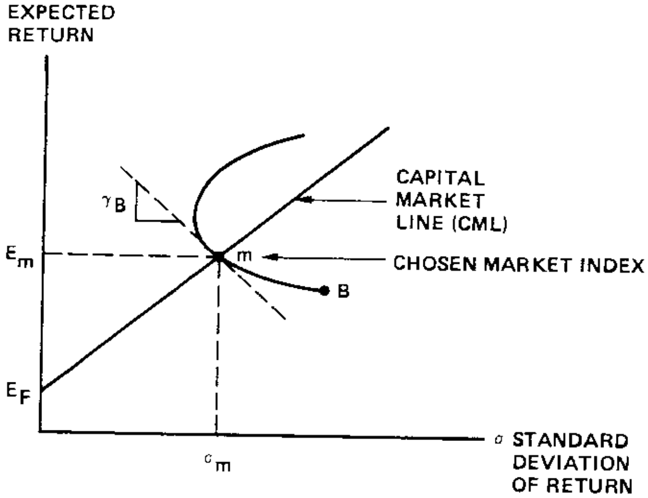


Figure 5. The Capital Market Line (CML) When the Market Index (m) Is not optimized.

market index is not (necessarily) an optimal portfolio, the locus of portfolios formed from any asset, say B, and the market index may pass *through* the capital market line at m as shown in Figure 5. The estimated capital market line has a slope equal to

$$\frac{E_m - E_F}{\sigma_m}.$$

For the case illustrated in Figure 5, notice that the expected true return on asset B is less than the (true) expected return E_m on the market index. Since the slope of the locus of portfolios formed by combining m with B is negative at m, we infer from equation (2) that β_B must be greater than one. Comparing the slope of the locus of portfolios with the capital market line, we must have the following inequality:

$$\gamma_B = \frac{E_B - E_m}{\sigma_m(\beta_B - 1)} < \frac{E_m - E_F}{\sigma_m}. \tag{3}$$

The standard deviation σ_m of the market index return is positive, and since $\beta_B > 1$, inequality (3) reduces to the following expression:

$$E_B < E_F + \beta_B(E_m - E_F). \tag{4}$$

This is equivalent to the *ex ante* deviation from the securities market line being negative for asset B; i.e., it is equivalent to

$$\pi_B < 0. \tag{5}$$

Thus, for any portfolio under evaluation, the *ex ante* deviation π from the securities market line depends upon three considerations. First, the index used in the evaluation must not be an optimized portfolio. Given this condition, the relationship between the position of the index and that of the portfolio under evaluation causes an *ex ante* securities market line deviation whose sign is dependent on two factors: (1) whether the measured beta is greater or less than unity and (2) whether the slope of the portfolio locus γ is greater or less than the capital market line's slope.

The general relationship looks like a sharp-edged saddle and is depicted in Figure 6. Performance is judged to be better than it really is when β is less than one and γ is less than the slope of the capital market line or when β is greater than 1 and γ is greater than the slope of the capital market line. Performance

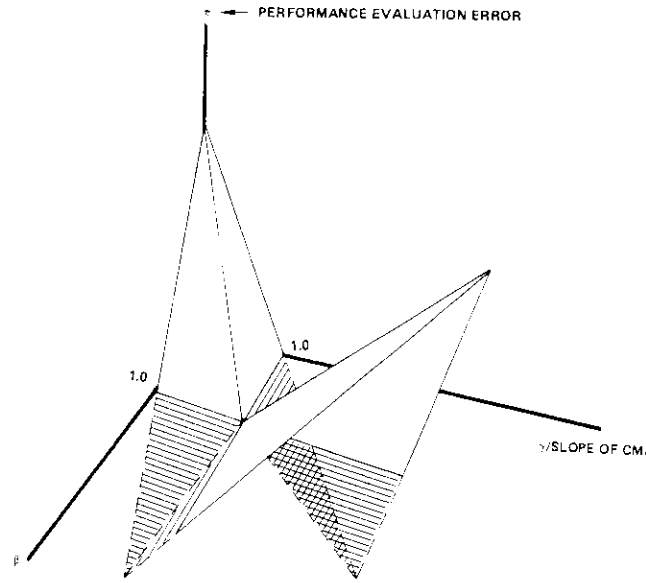


Figure 6. Non-Statistical Performance Evaluation Error as a Function of Beta (β) and Gamma(γ).

is better than it is assessed to be in the other two quadrants, when β is greater than 1 and γ is less than the slope of the capital market line or vice versa.

Equation (2) shows that γ depends on the difference between the true expected return of the portfolio under evaluation and the true expected return of the market index being employed. This permits a finer categorization of performance evaluation errors. Table 1 lists the six possibilities that produce positive or negative evaluation errors, and these possibilities are illustrated in the six panels of Figure 7. The letter corresponding to each of the six portfolios in Figure 7 is circled in the six panels of Table 1. Figure 7

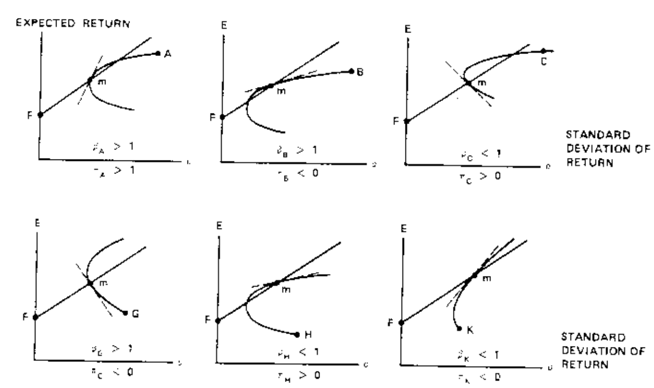
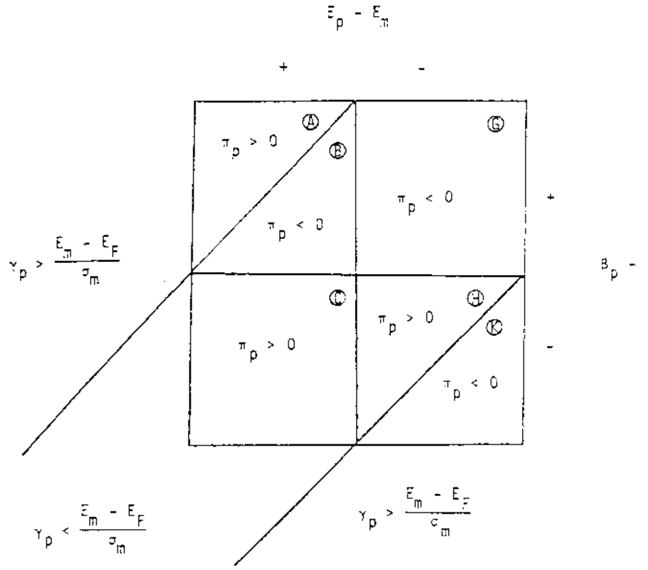


Figure 7. The Six Possible Configurations of Ex ante Deviations (π) from the Securities Market Line for Various Levels of Systematic Risk (β) and Expected Returns ($E_i - E_m$). The Dotted Line has a Slope Equal to $\gamma_i = (E_i - E_m)/[\sigma_m(\beta_i - 1)]$

TABLE 1

The Relationships Among Performance Evaluation Error π_p, Expected Return E_p, beta (β_p), and Portfolio Locus Slope (γ_p). Compare Figure 7. This tableau assumes $E_m > E_F$.



and Table 1 apply *only* in the "usual" case — when the CML has a positive slope or, equivalently, when $E_m > E_F$.

There are three cases in which performance may be evaluated as better than it actually is, and these are illustrated by portfolios A, C, and H. Of these three cases, two are associated with portfolios whose expected returns are larger than the market index's expected return. (These are portfolios A and C). Portfolios C and H have betas less than one, and portfolio A has a beta greater than 1. Thus, if the portfolio manager wants to appear to have more ability than he does have, he will choose a portfolio like A, C, or H. Given that the market index is not an optimized portfolio, any of these three cases will consistently produce "superior" results. Of course, this appearance is completely illusory, is due to benchmark error, and is not an indication of the portfolio manager's true ability.

Conversely, true ability will be offset by negative performance evaluation error if a manager is unfortunate enough to have chosen a portfolio such as B, G, or K. Of these three cases, two — G and K — are associated with portfolios whose expected returns are less than the market index. Portfolios B and G have betas greater than 1, and K has a beta less than 1.

Two of the six portfolios illustrated in Figure 7 — G and H — are dominated by the market index in the sense that the index has both a higher expected return and a lower variance of return. Such portfolios would be dominated by index funds that were successful in mimicking the market index. Nevertheless, though dominated by an index fund, the performance evaluation benchmark would be negatively biased in the case of G and positively biased in the case of H. It is hard to imagine how any portfolio dominated by an index fund can be considered to be successfully managed. Yet, in the case of portfolio H, even if the manager had no ability whatsoever, he would be consistently judged to have superior ability, since the deviation π_H from the securities market line is positive.

On the other hand, the mirror image case is not possible: Provided that the market portfolio has a larger expected return than the risk-free interest rate (i.e., that the capital market line has a positive slope), no portfolio that dominates the market index can have a negative benchmark error. Thus, it is not possible for a managed portfolio that dominates an index fund to have consistently negative performance evaluations in the absence of ability.

ALTERATIONS IN THE CHOICE OF MARKET INDEX AND THE BENCHMARK ERROR

Frequently, the management performance evaluator is concerned with whether he would obtain a vastly different evaluation if he chose one market index rather than another. For example, would there be a major difference in the ranking of managed portfolios if one used, say, the Standard and Poor's 500 Index rather than the New York Stock Exchange Index? The purpose of this section is to show that a change in the market index need not produce a markedly different set of evaluations. But we will also show that this fact does not mitigate the basic benchmark error problem. Different indices can produce the same or similar benchmark errors. Agreement among evaluators who use different indices, therefore, does not imply that the evaluations are correct.

Suppose, for example, that a given portfolio has been evaluated with a particular market index m and that the evaluation contains a benchmark error, π_p. The relationship between the expected return on the portfolio, the beta, the risk-free asset, and the expected return of the market index is given by

$$E_p = \pi_p + E_F + \beta_p(E_m - E_F). \qquad (6)$$

How does the benchmark error π_p change as a function of the choice of index? Let us consider, for example, an alternative index, say m', which need not fall on the securities market line produced by the original index. The expected return on the new index satisfies

$$E_{m'} = \pi_{m'} + E_F + \beta_{m'}(E_m - E_F) \qquad (7)$$

where $\pi_{m'}$ is the benchmark error for the new index evaluated against the old one. By combining (6) and (7) we can eliminate the original market index and obtain an equation based on the new index that looks very much like a securities market line. The original benchmark error for portfolio p (the portfolio being evaluated) and the benchmark error for the new index are combined into the hybrid error in brackets:

$$E_p = [\pi_p - \pi_{m'}/\beta_{m'}] + E_F + (\beta_p/\beta_{m'})(E_{m'} - E_F). \quad (8)$$

The only difference between equation (8) and a securities market line (plus p's deviation) is that the beta for p computed with index m′ may not be exactly equal to $\beta_p/\beta_{m'}$. It is quite easy to prove, however, that when the new and old indices are perfectly correlated, the new beta for portfolio p will be exactly equal to $\beta_p/\beta_{m'}$. In general, the new benchmark error will be $\pi_p - \pi_{m'}/\beta_{m'}$, plus some increment that depends on the indices' correlation. For high levels of positive correlation between the two indices, the new beta should be close to the ratio $\beta_p/\beta_{m'}$. Thus, the new benchmark error will be close to the old one plus the constant $-\pi_{m'}/\beta_{m'}$.

In Figure 8 we illustrate graphically how a change in the index affects benchmark error. The original securities market line with the original index m is the upper line, and the portfolio p being evaluated has a negative benchmark error with respect to that line. In this example, we assume the new index lies *on* the original securities market line at m′, so that $\pi_{m'} = 0$.

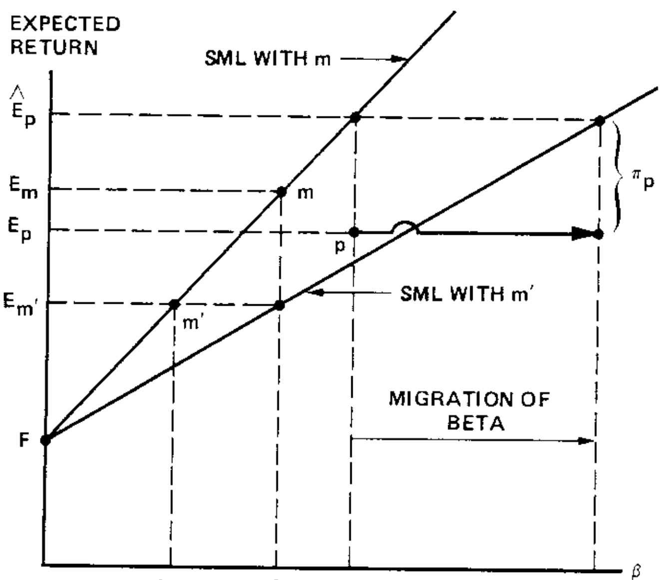


Figure 8. Beta "Migration" and Constancy of Benchmark Error (π) with Change in Market Index (m to m′).

When one uses the new rather than the old index, the evaluated portfolio's position will change; its beta will migrate to the right, as the arrow shows. If the old and new indices are perfectly correlated, the evaluated portfolio will maintain the distance under the new securities market line that it had under the old line. The benchmark error is constant because the beta has migrated from β_p to $\beta_p/\beta_{m'}$ with the index change.

Most of the commonly-used stock market indices, such as the Dow Jones Industrial Average, the New York Stock Exchange Index, and the Standard and Poor's 500, are very highly correlated. Although they are not perfectly correlated, the correlations are sufficiently high that the benchmark errors need not be significantly altered by using one index rather than another. However, as illustrated in Figure 8, this does *not* imply that there is no benchmark error. The benchmark errors are close to each other under alternative index choices, but the errors still exist and must be corrected if the evaluator is to obtain an accurate assessment of the manager's ability.

When a new index is associated with a non-zero benchmark error using the original securities market line (i.e., when $\pi_{m'} \neq 0$), the value of each new benchmark error will be different. Since the change in error, $-\pi_{m'}/\beta_{m'}$, is constant across evaluated portfolios, however, the *rankings* of estimated ability can remain unchanged. Thus, if with one index manager A has an algebraically larger benchmark error than manager B, he can also have a larger error with another index.

If there happen to be *no* differences in the evaluations produced by two indices, significant benchmark errors can still be present. The agreement in evaluations across indices does not guarantee that management ability has been properly assessed. On the other hand, if the old and new indices are not perfectly correlated, it is *possible* that substantial differences will occur in the benchmark errors produced by different indices. (See Roll [1978]).

To understand how a change in the market index can lead first to an alteration in benchmark error and then to a reversal of estimated management ability, consider the situation depicted in Figure 9. We

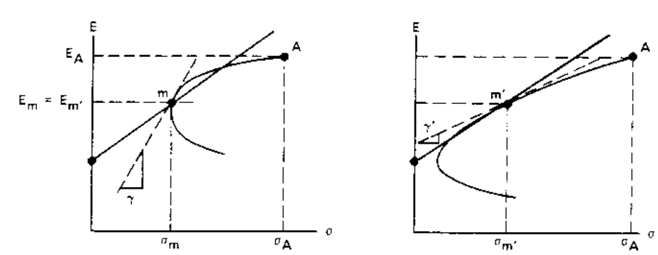


Figure 9. How the Benchmark Error of a Given Evaluated Portfolio (A) can Change Sign when the Index is Changed (from m to m′).

evaluate a given portfolio, labeled A in both panels of the figure, against the CAPM benchmark. We use a given market index, m, in the left panel and a different index, m[1], in the right. To illustrate the nature of the evaluation process, we assume both indices have the same expected return and standard deviation of return. However, they are not perfectly correlated.

In this case, the evaluated portfolio has a β greater than unity with both indices, *but* the degree of correlation is larger between portfolio A and index m′ (on the right) than between A and index m (on the left). This difference in correlation results in the left-hand locus being more broadly curved, and, as a result, the γ for portfolio A, the dotted slope of the locus at m(m′), is larger (smaller) than the capital market line's slope for m(m′). As we have already seen, if the expected return on the evaluated portfolio exceeds the market index's expected return, if the β exceeds unity, and if γ exceeds the slope of the capital market line, the benchmark error will be positive. (See the top left panel in Figure 7.) Under the same circumstances for expected returns and β, if γ is less than the capital market line's slope, the benchmark error will be negative. (See the top center panel of Figure 7.)

The upshot? A change in the market index one uses in performance evaluation can result in a reversal of the benchmark error. In the absence of ability, this will cause a previously well-considered manager to fall into disfavor. The direction of change in esteem will be the same even if ability is present — the over-estimated manager will become under-estimated.

Figure 9 gives a special case because the means and variances of the two indices are identical, something that one cannot expect for most changes in index. For example, the NYSE index has a considerably lower variance than the AMEX index does. Such differences in mean or variance would serve to increase the possibility of changes in benchmark error, given the degree of correlation between the indices.

HOW TO DETECT AND CORRECT BENCHMARK ERROR

To detect and correct an error in the CAPM benchmark, the portfolio management evaluator must obtain an independent estimate of the error's two components, β and γ. (See Figure 6.) This is tantamount to obtaining independent estimates of the evaluated portfolio's expected return.[5]

One fairly straightforward method for obtaining such estimates is to apply the classification scheme of Table 1 (or Figure 7) to the individual securities approved for purchase by the portfolio manager. During some validation period (different from the period of management evaluation), each approved security would provide a sample estimate of β and γ from observed rates of return. These γ and β estimates must be calculated with the market index that will be used in performance evaluation.

One could proceed to form a qualitative judgment about the benchmark error for a given evaluated manager by noting whether he selected securities falling more heavily into the $\pi > 0$ cells of Table 1; i.e., whether he selected securities that had characteristics like those of portfolios A, C, or H in Figure 7. Selecting such securities would be evidence that he was attempting to "game" the evaluation by choosing securities with positive benchmark errors.

It is possible to be more precise and quantitative by estimating the *ex ante* SML deviation π_j for each approved security j. Then a quantitative benchmark error π_p would be simply an investment-weighted average of the π_j's constituting the portfolio. Unfortunately, some knotty statistical problems are associated with this procedure. During any validation period used to estimate the vector of approved π_j's, cross-sectional *de*pendence will be present. Furthermore, since a vector is to be predicted, less familiar methods such as Stein-type estimators[6] should be employed. Although the expense of developing a satisfactory procedure may be substantial, the benefits will continue because the same mechanism can be employed to correct management evaluation in every period. For this reason, we should soon see such sophisticated correction methods put into practice.

An easy way to infer the existence of benchmark errors with the simple CAPM is to notice the ability of other variables to predict expected returns. Recently, such variables as dividend yield, price/earnings ratio, and firm size have been found to be useful return predictors,[7] and some have already received practical application.[8]

The very fact that such variables are useful implies the existence of ex ante deviations from the simple securities market line. Take the case of dividend yield: Although its importance is sometimes attributed to a tax differential between capital gains and ordinary income,[9] dividend yield is a surrogate, albeit a very imperfect one, for nominal expected return. Even with *no* effective tax differential,[10] "dividend tilt" could improve performance simply because high dividend yields are associated with positive benchmark errors (which also are related positively to nominal expected returns). To the extent that dividend yields are positively related to *beta* risk-adjusted nominal expected returns, they must be explaining benchmark errors in the simple no-tax CAPM. This implies that the tilting would become worthless if the market index currently in use were replaced by an "optimized" index.

SUMMARY

As a benchmark for evaluating portfolio management ability, the capital asset pricing model (CAPM) is subject to persistent error. This error is not due to statistical variation or estimation, and it will not average out over repeated manager evaluations. CAPM benchmark error is present whenever the market index is not "optimized"; i.e., whenever the index is not an ex ante mean/variance efficient portfolio. Whether a particular managed portfolio has a positive or negative benchmark error depends upon a complex set of factors, including the portfolio's expected return, beta, and variance of return. One can systematically categorize and correct benchmark errors by using additional sources of information concerning expected return.

It is possible for different market indices to produce different benchmark errors for the same managed portfolio. On the other hand, this need not happen. Agreement across indices in management evaluation implies neither the absence of benchmark error nor the validity of the evaluations.

The effectiveness of variables like dividend yield in explaining risk-adjusted returns is evidence of the presence of benchmark error.

[1] The "beta" for security j is

$$\beta_j \equiv \rho_{jm} \frac{\sigma_j}{\sigma_m}$$

where σ_j and σ_m are the standard deviations of returns on security j and the market index, respectively, and ρ_{jm} is the correlation coefficient between these returns.

[2] Sometimes researchers use more complicated versions of the CAPM. For example, if no risk-free asset exists, one can replace it with the expected return on a "zero-beta" portfolio. Because it is unclear which version of the asset pricing model is correct, such refinements introduce additional sources of error. However, since each version of the CAPM is prone to similar kinds of error, in this analysis I use the simplest capital asset pricing model for ease of exposition.

[3] For a given evaluation period t, the true and estimated performances are given by, respectively,

$$\alpha_t = R_{pt} - E_p$$
$$\hat{\alpha}_t = R_{pt} - \hat{E}_p.$$

So, the performance evaluation error is $\hat{\alpha}_t - \alpha_t = E_p - \hat{E}_p$.
Given the capital asset pricing model and the true risk of the portfolio, the true expected return is

$$E_p = E_F + \beta_p (E_m - E_F).$$

However, the *estimated* expected return is

$$\hat{E}_p = \hat{E}_F + \hat{\beta}_p (\hat{E}_m - \hat{E}_F),$$

where each component ($\hat{E}_F$, $\hat{E}_m$, and $\hat{\beta}_p$) of the estimated securities market line could be inaccurately assessed. If one does inaccurately assess these, the true expected return will deviate from the *estimated* SML on an *ex ante* basis, so that

$$E_p = \pi + \hat{E}_F + \hat{\beta}_p (\hat{E}_m - \hat{E}_F),$$

with $\pi \neq 0$. Thus, the performance evaluation error is given by

$$\hat{\alpha}_t - \alpha_t = E_p - \hat{E}_p = \pi.$$

[4] Let δ be the proportion invested in portfolio j (or in individual asset j) and $1 - \delta$ be invested in the market index m. This hybrid portfolio p then satisfies $E_p = \delta E_j + (1 - \delta)E_m$ and $\sigma^2_p = \delta^2\sigma^2_j + (1 - \delta)^2\sigma^2_m + 2\delta(1 - \delta)\sigma_{jm}$. It is easy to see that p's mean and variance change with δ. At $\delta = 0$ (100% invested in the market portfolio), the rate of trade-off between mean and standard deviation of p is given by $(\partial E_p/\partial\delta)/(\partial\sigma_p/\partial\delta)|\delta = 0 = (E_j - E_m)/\sigma_m(\beta_j - 1)$.

[5] Cornell [1980] argues forcefully for a portfolio management evaluation method based solely on independent estimates of expected return and not based on the CAPM or on any asset pricing model.

[6] See Efron and Morris [1975].

[7] See Litzenberger and Ramaswamy [1979], Basu [1977], Banz [1979], and Reinganum [1978].

[8] See W. F. Sharpe's excellent description [1978] of the Wells Fargo yield "tilt" portfolio management process.

[9] Litzenberger and Ramaswamy [1979].

[10] Miller and Scholes [1978] argue that there is no effective difference between the taxation rates of dividends and capital gains, since the former can be converted into the latter by appropriate financial planning.

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