

# Valuing the Implicit Guarantee of the Federal National Mortgage Association

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## *Abstract*

The paper analyzes the guarantee of the Federal National Mortgage Association (FNMA). Rather than try to price the guarantee, we used time-series estimates of its value from Kane and Foster to infer the behavior of FNMA in exploiting the guarantee. The results are consistent with a model that predicts that FNMA does not take as much risk as it might. Rather, it trades off risk and return, but it does increase risk and exploit the guarantee when it gets in trouble (as it did in 1981).

## **1. Introduction**

This paper is a preliminary attempt at analyzing the pricing of the implicit guarantee of debt issued by the Federal National Mortgage Association (FNMA). We do this by applying a contingent claims model, viewing the guarantee as a put option, giving holders of FNMA debt the right to sell their debt to the guarantor in the event of bankruptcy. We also model equity in the firm as a call on the firm's assets at a price equal to the value of the (guaranteed) liabilities. Because both the debt and the assets are risky, we use an extension of the standard Black-Scholes (1973) model, developed by Margrabe (1978), to analyze the options of exchanging one risky asset for another.

Our model begins with a variant of approaches used first by Merton (1977) and later by Marcus and Shaked (1984) and Ronn and Verma (1986) to price deposit insurance, which is a similar sort of guarantee. These models assume that banks are audited every  $T$  years, and at time  $T$  the institutions are shut down if they have negative net worth. If they have positive net worth the guarantee is repriced. Hence the guarantees are equivalent to options with term  $T$ .

This is our point of departure, but it needs to be supplemented if it is to be used to take a serious look at most government guarantees. The reason is that most guaranteed institutions are not regularly audited, and we do not know much about

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conditions under which the institutions would go bankrupt. Thus a crucial "boundary condition" necessary to solve the option-pricing model is unknown. It is because of this fundamental problem that we invert the process and solve for the implied length of the term as the guarantee.

Following Kane and Foster (1986) (henceforth K-F), the guarantee is modelled as an implicit asset on FNMA's balance sheet. The value of FNMA's stock should reflect the value of the guarantee. K-F estimate the items on FNMA's balance sheet, which in turn allows them to back into FNMA's guarantee from the balance sheet condition that assets (including the guarantee) equal liabilities. We take the value of the guarantee as given by the K-F calculations and assume that the guarantee and the stock can be modelled as options of term  $T$ , but where  $T$  and the volatility of FNMA's assets are unknown. We then find values of  $T$  and volatility that best predict the calculated guarantee.

While our results are clearly preliminary (we look at end-of-year data from 1978 through 1985: only 8 observations), they are consistent with two propositions: (1) the market does not seem to react as if the guarantee is fully exploited; i.e., the implied term is generally only 1 or 2 years; (2) however, when FNMA got into serious trouble in the early 1980s, the market did expect it to take increased advantage of the guarantee, and the implied term of the guarantee increased, as did the implied volatility of FNMA's assets. We discuss why that might be expected.

## 2. FNMA

FNMA is a "quasi-private" corporation which is chartered to make mortgage loans. It was founded in the 1930s as a federal agency, but it was "privatized" in 1968. Privatization took it off the federal budget and made it much like a private corporation. It has issued stock, which trades like other stocks, and it behaves in many ways like any private corporation. It is, however, also connected with the government. It is regulated by the Department of Housing and Urban Development (HUD), its Chief Executive Officer is a presidential appointment, and it has a wide range of implicit and explicit connections with the government (see K-F for further discussion). It is fair to say, however, that for the most part government control has been minimal and that, within its charter limits, FNMA has generally operated like a private corporation. However, because of its government connection, FNMA is perceived<sup>1</sup> as having a federal guarantee. As a result, it can borrow at close to risk-free rates even though it takes on a lot of risk. It is the value of this perceived guarantee with which we are concerned.

FNMA does two things: it manages a portfolio consisting primarily of mortgages; and it guarantees pools of mortgages packaged as mortgage-backed securities (MBS). With respect to its portfolio, which is the main source of risk-taking (some \$100 billion at the end of 1985), FNMA operates like a large savings and loan, and it has experienced the same problem as savings and loans: both interest-rate risk and credit risk. Like savings and loans, FNMA borrows at close

to risk-free rates, because of its guarantee; but unlike them it does not have explicit insurance, and it has not been charged a premium. While it is regulated by HUD, it does not have a formal audit process comparable to that of the savings and loans.

We assume that FNMA does in fact have a guarantee. This follows from what analysts seem to think (see K-F for a discussion), and from FNMA's borrowing rates. In general these have been quite close to Treasury rates, despite a highly leveraged and risky portfolio. On the dates on which we make our estimates (end-of-year, 1978–1985), the spread between FNMA and Treasury two-year debt (approximately the average duration of FNMA debt) was generally 20 basis points or less. It was largest at the end of 1981, at which time it was as high as about 40 basis points.

### 3. The model

Our point of departure is Kane and Foster (1986), who begin by viewing the guarantee as an implicit asset on FNMA's balance sheet. If we let  $G$  be the value of the guarantee,  $A$  the value of FNMA's other assets (mainly mortgages),  $L$  the value of its debt, and  $E$  the value of its equity, its balance sheet is given by:

$$A + G = L + E \quad (1)$$

Since  $E$  and  $L$  are traded, we need only estimate  $A$  in order to back into an estimate of  $G$ . K-F use data on FNMA's assets to estimate the market value of  $A$ , which consists mostly of mortgages (plus an estimate of the present value of profits from such other activities as mortgage-backed securities). They then produce a time series of estimates of  $G$  from 1978 through 1985 (reproduced in our Table 1).

This strikingly simple approach has the major advantage of being independent of any particular pricing model, thus avoiding the very serious problem of modelling FNMA bankruptcy. The approach is practicable because of the extreme simplicity of FNMA's balance sheet, which consists overwhelmingly of priceable mortgages, unlike, e.g., a commercial bank whose assets are heterogenous and generally not traded frequently. But the K-F approach has disadvantages as well:

1. It assumes that all of the residual on FNMA's balance sheet is accounted for by  $G$ . There may be other implicit assets, however, such as charter value or monopoly power, or "goodwill." There may also be implicit liabilities due to the extent that it is regulated.
2. There may be errors in pricing FNMA's mortgages or other items in FNMA's assets (e.g., the value of its MBS business).

While these might be significant problems with other firms, we are inclined to

Table 1. Net worth and value of guarantee for FNMA\* (Billions of dollars)

End of Year	Value of Assets	Value of Liabilities	Net Worth	Value of Stock	Value of Guarantee
1978	39.6	40.0	-0.4	.9	1.3
1979	42.6	46.7	-4.1	.9	5.0
1980	45.3	52.7	-7.4	.7	8.1
1981	45.9	56.6	-10.8	.5	11.3
1982	67.0	73.7	-6.7	1.6	8.3
1983	72.3	77.2	-4.9	1.5	6.4
1984	84.5	88.0	-3.4	1.0	4.4
1985	103.3	102.0	1.3	1.9	0.6

\*From Kane and Foster (1986), various tables.

agree with K-F (again because of the simplicity of FNMA's business) that they are not major sources of error. What is more important to us is that while the K-F approach probably does give good insight into the value of the guarantee, it does not tell us about the structure of the guarantee. That is, the analysis tells us at what price the stock market is implicitly valuing the guarantee, but it does not tell us about the parameters of the guarantee in the minds of the traders; nor does it say anything about what traders are revealing concerning their assumptions about FNMA's behavior. Such aspects cannot be disclosed without a different model.

Our model is an elaboration of Merton's (1977) application of the Black-Scholes (1973) option-pricing model to deposit insurance. The driving force of the Merton model is the auditing process. Merton assumes that the institution is audited every  $T$  years. At that time it is shut down if it has negative economic net worth, and the insurer pays off deposit claims. If its net worth is positive, the insurance is repriced, which means either that the insurance premium is changed or the firm changes its balance sheet to make the value of the insurance equal to the current price. Hence,  $T$  might be thought of as a repricing or planning interval. After  $T$  years the firm is forced to change its policy or pay a different premium.

The insurance thus has the characteristics of a put on the firm's assets, with exercise price equal to the value of the institution's liabilities and term equal to the time between audits,  $T$ . At the same time, the equity in the firm can be characterized as a call on the firm's assets at a price equal to  $L$ . To model this we use a model developed in Margrabe (1978) to price an option to exchange two risky assets. This generalization of Black-Scholes is necessary because both FNMA's assets and liabilities are risky (e.g., due to interest-rate risk).

As discussed above the major problem with applying the model to FNMA is that it is not audited, at least not in the sense of banks. Indeed, it is not at all clear how or if FNMA would be shut down. We could use  $T$  as a sort of metaphor for the political process and its reaction to FNMA's position. A more plausible interpretation of the "term" of the guarantee is that it represents a planning period. That is, every  $T$  years FNMA adjusts its portfolio in order to control its risks, e.g., by ad-

ding equity or changing the structure of its portfolio (to take on more or less risk) in order to correct for increases or decreases in its net worth; and the  $T$ , which we infer below, is the time between changes. Whether it does this of its own accord or because it is induced to by regulators is something that our model cannot distinguish.

We take the size of  $T$ , then, to be a measure of the degree to which the guarantee is exploited. We interpret a large  $T$  as meaning that FNMA chooses to or is allowed to continue risky strategies for some time without adjusting its portfolio. If  $T$  is small, we assume that FNMA is expected to adjust quickly to correct changes in its situation or, perhaps, that regulators are expected to keep a tight rein. Because the value of the guarantee increases with  $T$ , a small  $T$  implies that FNMA is not expected to exploit its guarantee fully. Why it might choose to do this depends on what it is expected to maximize.

Before discussing what FNMA might optimize, we put forth the option-pricing model, given  $T$  and given a "strategy," which is characterized by stochastic processes for its assets and liabilities over the period  $T$ . We assume that FNMA's assets and liabilities are given by the following stochastic processes:

$$\frac{dA}{A} = m_A dt + \sigma_A dz_A \quad (2)$$

$$\frac{dL}{L} = m_L dt + \sigma_L dz_L \quad (3)$$

where the  $\sigma$ 's are constant but the  $m$ 's can be stochastic,<sup>2</sup> the  $dz$ 's are Wiener processes and  $\rho$  is the correlation between  $dz_L$  and  $dz_A$ . The equations imply, for instance, that if the  $m$ 's are constant, future values (after a period of length  $T$ ) of  $A$  and  $L$  are lognormally distributed with mean equal to  $A_0 \exp(m_A T)$  and variance equal to  $A_0^2 \exp(2m_A T) [\exp(\sigma_A^2 T) - 1]$  for  $A$  and similar expressions for  $L$ .

The guarantee is a  $T$ -year option to sell (put) the assets,  $A$ , to the insurer, presumably the Treasury, at price equal to  $L$ , the market value of the liabilities. We can think of this as an option to exchange FNMA debt for comparable Treasury securities. The equity,  $E$ , in the firm is taken to be an option to buy the firm's assets at time  $T$  at a price equal to  $L$ , i.e., a call option on the firm's assets. Put-Call parity leads to the balance sheet condition.

Both  $A$  and  $L$  are risky.  $A$ , which, again, is essentially mortgages, involves both interest-rate and credit risk.  $L$  does not involve credit risk, but, because duration of FNMA's liabilities has been around two years, it involves interest-rate risk (3). The major risk in the portfolio is, of course, that the assets are longer in duration than the liabilities. However, our model does not require specifying the details of this. In particular the model allows us to solve for  $\sigma_A$ , eliminating the need to specify the source of risk.

Assuming that there are no dividend payouts on the stock, that the bonds pay all interest at maturity, and that the above processes hold for  $A$  and  $L$ , the standard

arbitrage arguments which are used in Margrabe (1978) lead to a closed-form solution for the value of the firm's stock, given by

$$E = A \cdot N(d) - L \cdot N(d - \sigma\sqrt{T}) \quad (4)$$

where

$$d = \frac{\ln A/L}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \quad (5)$$

$$\sigma^2 = \sigma_A^2 - 2\sigma_A\sigma_L\rho + \sigma_L^2 \quad (6)$$

and  $N$  is the cumulative normal density function.

From this the value of  $G$  can be deduced from (1) if we know  $A$  and  $L$ . The reader will note that if  $\sigma_L = 0$  and  $L = Ke^{-rt}$ , where  $r$  is the  $T$ -period interest rate and  $K$  the exercise price, (4) coincides with the usual Black-Scholes formula.

Following arguments similar to those in Merton (1973), it can be shown that the instantaneous volatility of equity,  $\sigma_E$ , is given by

$$\begin{aligned} \sigma_E^2 = & \left( \frac{A \cdot N(d)\sigma_A}{E} \right)^2 + \left( \frac{L \cdot N(d - \sigma\sqrt{T})\sigma_L}{E} \right)^2 \\ & - 2 \frac{A \cdot L \cdot N(d) \cdot N(d - \sigma\sqrt{T})}{E^2} \sigma_A\sigma_L\rho. \end{aligned} \quad (7)$$

#### 4. FNMA behavior

We take FNMA's portfolio and the stochastic processes governing the values of the items in it as given during the period of the option. These can however change over time, and, indeed, we should expect optimizing behavior to lead to changes. In particular, the simple nature of FNMA's business, holding mortgages, and the fact that it pays nothing for its guarantee suggest a simple strategy: FNMA should maximize its value by maximizing  $G$  (for a given size of  $L$ ). Merton (1977) shows that in a model like ours the partial derivative of  $G$  with respect to  $\sigma$  is always positive. This suggests the desirability of taking on as much risk as possible.

However, there are at least three reasons for limiting risk-taking:<sup>3</sup>

1. A part of  $G$  may be FNMA's "charter value," or some measure of monopoly power. As shown by Pennacchi (1985), in a model that allows for monopoly power, risk-taking can lower the charter value, leading to a limited amount of risk-taking. That is, in a broader model the partial derivative of  $G$  with respect to  $\sigma$  may eventually be negative.

2. While the stockholders of FNMA may benefit from a risky, value-maximizing strategy, management, which has its human capital tied into the firm, is more like a debt-holder and may not want to increase  $\sigma$ . Hence there may be a principal-agent issue limiting FNMA risk-taking.
3. Anticipated future sanctions by FNMA's regulators and the "shadow prices" representing the price of their reaction can also limit risk-taking.

We therefore suspect that the appropriate underlying model of FNMA behavior is one of choosing its asset-liability mix, and in turn the processes for  $A$  and  $L$ , by trading off the benefits of a higher value of  $G$  from risk-taking with the costs of the risk-taking.

Consider the second argument above, the principal-agent issue. Suppose managers maximize a utility function that depends on the value of  $E$  (because of bonuses paid in stock or stock options), and on the probability of bankruptcy at the end of  $T$  (because managing a bankrupt firm lowers a manager's reputation and hence the value of his human capital). Assume, as is the case with FNMA, that  $L$  is fixed in the short run by regulation.<sup>4</sup> Then  $E$  is simply an increasing function of  $G$ , and the only parameters left to vary are  $\sigma$ , which depends on the riskiness of  $A$  and  $L$ , and leverage,  $A/L$ .

However, if the institution has sufficient net worth, choosing high levels of  $\sigma$  or  $A/L$  also increases the probability that the institution will go bankrupt or that there will be a reorganization and new management. Hence we have the tradeoff described above.

Suppose, however, that given past policy (and as a result of bad luck, we find the institution beginning its plan with negative economic net worth. A low-risk strategy will simply ensure that the institution will go under. At this point managers' and owners' interests tend to coincide, taking on more risk increases both  $G$  and the value of the manager's human capital. Risk-taking should increase, although it may still be limited by regulators' sanctions. Hence, in a period like 1981 when interest rates increased dramatically and net worth fell precipitously, we should expect to see risk-taking increase. This should be manifest in: (1) a larger value of  $T$ , i.e., a feeling by stockholders that FNMA will exploit its guarantee for a longer period; and (2) a higher level of  $\sigma_A$ , reflecting a belief that FNMA will choose riskier assets.

We have then two sorts of things to look at: (1) Is  $T$  large or small? If it is small, it means that the FNMA is not perceived in the market as fully exploiting its guarantee, or it as not being allowed to. (2) How do  $T$  and asset volatility change over time? In particular, when its net worth was quite negative around 1981, was FNMA expected to take on more risk? We examine these questions by using the model in section 3 to estimate  $T$  and  $\sigma_A$  over time.

## 5. Results

We have to solve three equations, (1), (4), and (7), for  $G$ ,  $A$ , and  $\sigma_A$ , given  $E$ ,  $L$ ,  $\sigma_E$ ,  $\sigma_L$ , and  $\rho$ . We have, from K-F, data on  $E$  and  $L$ , and we can estimate the annual

volatility (monthly standard deviation) of FNMA stock from the CRSP equity file, and  $\sigma_L$  from the CRSP government bond file. Because K-F estimate a duration of FNMA liabilities of about two years, we use the volatility of 2-year Treasury bonds. For both equity and debt our standard deviation at the end of each year is the standard deviation of monthly percentage price changes over the previous 12 months. That these measures are subject to measurement error presents a problem in solving the equations. For positive values of  $\sigma_A$ , and  $\sigma_L$  there will always be a solution for  $E$ ,  $G$ , and  $\sigma_E$ . It will not be the case that when we invert the system to solve for  $\sigma_A$  there will always be positive real solutions for arbitrary positive values of  $\sigma_E$ ,  $\sigma_L$ ,  $\rho$ ,  $E$ , and  $L$ . In particular, measurement error in  $\sigma_E$  and  $\sigma_L$  can (and apparently sometimes does) lead to no solution.

To solve the three equations we need to know  $\rho$ . Because the major risk of both  $A$  and  $L$  is due to interest-rate changes, we should expect  $\rho$  to be high; but because they have different durations and  $A$  (mortgages) is subject to credit and prepayment risks, they should be imperfectly correlated. We ran solutions with  $\rho$  ranging from .2 to 1.0 at intervals of .2. Occasionally the  $\rho = 1$  assumption produced different results; but the results were largely insensitive to values of  $\rho$  in the .4 to .8 range. Because  $\rho = 1$  seems unlikely we ignore those results and focus on those from  $\rho = .8$ . We solved the system for  $G$ ,  $A$ , and  $\sigma_A$  for  $T$  varying from 1 to 5 years and for a range of  $\rho$ 's. The value of  $T$  that comes closest to giving the K-F estimate of  $G$  (and hence  $A$ , which is tied to  $G$  by the balance sheet identity) is our estimate of the market's implied value of  $T$ ; and the corresponding  $\sigma_A$  is our estimate of the market's estimate of FNMA's volatility.

Table 2 provides results for  $\rho = .8$ . Comparison of the estimated value of  $G$  with the K-F estimates indicates that  $T$  was never as long as three years and was generally one or two years. Again, none of the results is affected by varying  $\rho$  from .2 to .8. Note that the values of  $G$  vary considerably with changes in  $T$ , so that small errors in the K-F estimates of  $G$  will not significantly affect our results.

As we just discussed, there are situations, particularly 1979/1980 and for some simulations in 1981, in which there are no solutions. In the 1979, 1980 and 1981 cases this is because  $\sigma_L$  is large and  $\sigma_E$  small, which requires that  $\sigma_A$  be small to make (7) determine  $\sigma_E$ . In the nonsolution cases there is no positive real  $\sigma_A$  that can do this. Similarly in some of the cases where  $T$  was large, the guarantee was so valuable that  $A$  had to be negative (because  $A + G$  have to add up to  $L + E$ ), and there was no solution.

We assumed this to be due to measurement error, and we reran the simulations using an average value of  $\sigma_E$  for the entire period.<sup>5</sup> Table 3 provides the same sort of results, but with  $\sigma_E$  always set at 1.22, the average for the entire period. In that case we had convergence more frequently and had results for 1979 and 1980. Again note that  $T$  was generally one or two years. Hence the model is consistent with traders believing that FNMA did not (or was not allowed to) exploit its guarantee very fully.

However, in both tables  $T$  and the  $\sigma_A$  associated with that  $T$  both rose to a peak (of about three years) in 1981, when  $G$  was greatest and net worth smallest (about



Table 2. Value of guarantee ( $\rho = .8$ ) with variable stock volatility

1978			1984		
$\sigma_L = .029, \sigma_E = .980, K-F \text{ value of } G = 1.3$ (Estimates)			$\sigma_L = .097, \sigma_E = 1.5820, K-F \text{ value of } G = 4.4$ (Estimates)		
<i>T</i>	<i>G</i>	$\sigma_A$	<i>T</i>	<i>G</i>	$\sigma_A$
1	.21	.05	1	3.83	.10
2	1.27	.07	2	22.26	.23
3	3.75	.10	3	56.71	.46
4	8.14	.14	4	78.09	.76
5	14.26	.19	5	no solution ...	

  

1979			1985		
$\sigma_L = .113, \sigma_E = .660, K-F \text{ value of } G = 5.0$ (No solution)			$\sigma_L = .071, \sigma_E = 1.3818, K-F \text{ value of } G = 0.6$ (Estimates)		
<i>T</i>	<i>G</i>	$\sigma_A$	<i>T</i>	<i>G</i>	$\sigma_A$
			1	2.81	.10
			2	16.86	.18
			3	46.31	.32
			4	75.33	.52
			5	90.65	.73

  

1980			1981		
$\sigma_L = .300, \sigma_E = 1.20, K-F \text{ value of } G = 8.1$ (No solution)			$\sigma_L = .173, \sigma_E = 1.1985, K-F \text{ value of } G = 11.3$ (Estimates)		
<i>T</i>	<i>G</i>	$\sigma_A$	<i>T</i>	<i>G</i>	$\sigma_A$
1	no solution ...		1	no solution ...	
2	no solution ...		2	12.03	.17
3	12.03	.17	3	18.88	.23
4	18.88	.23	4	28.05	.31
5	28.05	.31			

  

1982		
$\sigma_L = .112, \sigma_E = 1.4631, K-F \text{ value of } G = 8.3$ (Estimates)		
<i>T</i>	<i>G</i>	$\sigma_A$
1	3.29	.13
2	17.28	.26
3	41.49	.45
4	59.89	.68
5	67.89	.90

  

1983		
$\sigma_L = .075, \sigma_E = 1.4293, K-F \text{ value of } G = 6.4$ (Estimates)		
<i>T</i>	<i>G</i>	$\sigma_A$
1	2.64	.11
2	15.95	.21
3	41.16	.39
4	61.67	.62
5	70.80	.84

Table 3. Value of guarantee with stock volatility fixed at average level (1.2218)

1978			1983		
$\sigma_L = .029$ , K-F value of $G = 1.3$ (Estimates)			$\sigma_L = .075$ , K-F estimate of $G = 6.4$ (Estimates)		
$T$	$G$	$\sigma_A$	$T$	$G$	$\sigma_A$
1	.66	.07	1	1.35	.07
2	4.21	.12	2	6.98	.13
3	12.30	.21	3	19.79	.21
4	22.82	.35	4	38.27	.32
5	30.97	.51	5	55.11	.46

  

1979			1984		
$\sigma_L = .113$ , K-F value of $G = 5.0$ (Estimates)			$\sigma_L = .097$ , K-F estimate of $G = 4.4$ (Estimates)		
$T$	$G$	$\sigma_A$	$T$	$G$	$\sigma_A$
1	no solution		1	no solution	
2	4.56	.15	2	6.52	.10
3	11.60	.23	3	16.59	.16
4	21.75	.33	4	33.83	.24
5	31.62	.45	5	54.02	.35

  

1980			1985		
$\sigma_L = .300$ , K-F value of $G = 8.1$ (Estimates)			$\sigma_L = .071$ , K-F estimate of $G = 0.6$ (Estimates)		
$T$	$G$	$\sigma_A$	$T$	$G$	$\sigma_A$
1	no solution		1	1.69	.07
2	no solution		2	8.78	.13
3	18.13	.29	3	25.27	.20
4	24.27	.37	4	49.64	.31
5	30.91	.44	5	72.33	.44

  

1981		
$\sigma_L = .173$ , K-F value of $G = 11.3$ (Estimates)		
$T$	$G$	$\sigma_A$
1	no solution	
2	no solution	
3	12.52	.18
4	20.13	.25
5	29.98	.33

  

1982		
$\sigma_L = .112$ , K-F value of $G = 8.3$ (Estimates)		
$T$	$G$	$\sigma_A$
1	no solution	
2	7.40	.16
3	18.93	.24
4	35.21	.34
5	50.64	.46

minus \$10 billion at the end of 1981). When FNMA was in the best shape and  $G$  smallest (1978 and 1985),  $T$  was only about one year and  $\sigma_A$  was at its lowest level (as low as .07 as compared with a peak of around .2 in 1980 and 1981).

The model is thus consistent with FNMA trading off value and risk and not exploiting the guarantee fully. When FNMA was in trouble it was expected to take on more risk and exploit the guarantee more fully. That  $T$  probably never exceeded three years suggests that the market did not expect anywhere near the exploitation of the guarantee that could have taken place.

### Comments

We only have 8 observations of FNMA's portfolio, and we have a deliberately simple model of that portfolio. Hence we can only point to our results as provisional. Because FNMA's business is restricted to mortgages, which can at least approximately be priced, and because everything else in its portfolio has a market price, we could use the model combined with the Kane-Foster (1986) data to generate some insights into what the market thinks about FNMA's behavior. Apparently it perceives FNMA either as not exploiting its guarantee very fully, or else as having its risk-taking controlled by regulators. The results do suggest that both the volatility of FNMA's assets and the degree of exploitation of the guarantee increased when it got into trouble. This is consistent with the discussion of FNMA trading off value and stability, perhaps because of the nature of management's own optimizing.

Finally, we note that there is much to be done in the way of both modelling government guarantees and of the optimization of the beneficiaries of the guarantees. Our model is deliberately simple; it yields a closed-form solution, which makes solution of the three equations needed to obtain  $A$ ,  $G$ , and  $\sigma_A$  feasible. But the model needs to be expanded. In particular, we assume that  $T$  is fixed and that FNMA is expected to correct itself at the end of  $T$ ; but, as we discussed above, it has the option to take on more risk if it gets into trouble.

In our calculations it is assumed that traders know the strategy for the next  $T$  years; e.g., at the end of 1981 they expect that FNMA will pursue a particular strategy in light of their previous history and luck. But we do not assume that at the end of 1978 traders took account of the fact that FNMA could change its strategy in 1981 if it got into trouble. Rather we assumed that traders thought that risk-taking would eventually (at  $T$ ) be controlled. What this probably means is that guarantees are inherently more risky than is apparent in our model. Indeed, an important aspect of decision making, on which further effort should be spent, is the option that a guaranteed institution has to change its strategy and exploit future options as new information becomes available. This suggests extending our model along lines in Geske (1979), who analyzes compound options.

## Notes

1. *Perceived* is the key word. FNMA does not have a legal guarantee. However, there is a widespread belief that the government would, and perhaps could be made to, bail out FNMA debtholders in the event of bankruptcy. See Kane and Foster for a discussion.

2. Because  $A$  and  $L$  are traded assets, their expected changes,  $m_A$  and  $m_L$ , drop out. This is typical in option-pricing models and quite useful, because they are likely to be difficult functions to estimate. We do have to know  $\sigma_A$  and  $\sigma_L$ , and in our formulation they do have to be constant for there to be the closed-form solution (4).

3. See Kane and Foster (1986) for a discussion.

4. FNMA has capital requirements which limit its liabilities. We are assuming that it cannot issue new stock in the short run. In the medium run an institution can take on risk by growing. Indeed, FNMA did grow more rapidly (see Table 1) after its net worth became quite negative in 1981.

5. We did not set  $\sigma_L$  at a constant level, because changing interest-rate volatility was an important aspect of the period.

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