Valuing Stripped Mortgage-backed Securities

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I. Introduction

An important recent development in financial markets has been the impressive growth in the market for mortgage-backed securities. This growth has been accompanied by an increasing interest in the valuation of these securities. The critical feature that makes the pricing of mortgage-backed securities more difficult than other fixed income securities is the mortgagor's prepayment decision. It is clear that the speed of prepayment will affect the value of a particular underlying mortgage pool, but what is not so obvious are the factors that affect prepayments and the magnitude of their effects. Given their importance, practitioners have expended considerable resources to organize prepayment data and develop models to value mortgage-backed securities.

The purpose of this paper is to provide a framework to value "stripped" mortgage-backed securities (Roll, 1988) which represent unequal proportions of the cashflows from an existing pool of mortgages. The valuation of these securities allows us to highlight the significance of prepayments to the pricing of mortgage-backed securities. We do not impose an optimal, value-minimizing call condition to value stripped mortgage-backed securities. Rather, we recognize that the value of a mortgage-backed security must reflect the fact that at each point in time there exists a probability of prepaying, this conditional probability depending upon the prevailing state of the economy.

To value stripped and other mortgage-backed securities, we integrate an empirical prepayment function into the two-factor model of Brennan and Schwartz (1982, 1985) for valuing default-free interest-dependent claims; this model assumes that the term structure of

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default-free interest rates can be summarized by two state variables: the instantaneous riskless rate of interest and the default-free consol yield. We follow Schwartz and Torous (1989) and employ a proportional hazards model to estimate the influence of various covariates or explanatory variables on mortgagors' prepayment decisions. By adding state variables underlying the posited prepayment function, we integrate the estimated prepayment function into the valuation framework of Brennan and Schwartz, resulting in a complete model for valuing mortgage-backed securities. Monte Carlo simulation methods are used to solve the resultant second-order partial differential equation subject to the boundary conditions which characterize stripped mortgage-backed securities as well as the terminal condition that the underlying mortgage be fully amortized at its maturity.

Note that the analysis of this paper is partial equilibrium in nature. That is, we price mortgage-backed securities as a function of the assumed underlying state variables. Without comparing the resultant model prices to corresponding market prices, the model can say nothing about the economics of stripping mortgage-backed securities.

The plan of this paper is as follows. Section II introduces stripped mortgage-backed securities. We review our mortgage-backed security valuation framework in Section III; Section IV illustrates the application of our model to the valuation of stripped mortgage-backed securities. Section V presents a summary and our conclusions.

II. Stripped Mortgage-backed Securities

Stripped mortgage-backed securities represent unequal proportions of the cashflows from an existing pool of mortgages. Each class of these securities backed by a particular pool of mortgages differs in both its interest and principal payments.

For example, consider a two-class stripped mortgage-backed security backed by a 11% mortgage-backed security.[1] Assume mortgage rates are approximately 11% so that the underlying mortgage-backed security is priced at approximately par. We assume that the principal payments are equally divided amongst each class. However, the first class receives seven-elevenths of the interest payments, while the second class receives four-elevenths. As a result, we have a stripped mortgage-backed security with a 14% coupon which sells at a premium, and a stripped mortgage-backed security with a 8% coupon which sells at a discount.

Consider the deep discount 8% stripped mortgage-backed security. Since it is backed by a 11% mortgage-backed security, all other things being equal, this security has a faster prepayment speed than a

[1] This example is based on Roll (1988).

traditional 8% mortgage-backed security with otherwise identical features. Therefore, the discount 8% stripped mortgage-backed security is more valuable than the discount traditional 8% mortgage-backed security because of the benefits of rapid prepayments to discount security holders.

By contrast, the premium 14% stripped mortgage-backed security backed by a 11% mortgage-backed security, all other things being equal, has a slower prepayment speed than a traditional 14% mortgage-backed security. As a result, the premium 14% stripped mortgage-backed security is more valuable than the traditional 14% mortgage-backed security because of the benefits of slower prepayments to premium security holders. Furthermore, if mortgage rates rise, the underlying 11% mortgage backed security will experience fewer prepayments and the premium stripped mortgage-backed security may increase in value. In other words, the premium stripped mortgage-backed security can experience reversed interest rate sensitivity. However, if mortgage rates rise significantly, the traditional 14% mortgage-backed security is more valuable than the stripped 14% mortgage-backed security again because of the benefits of rapid prepayments to discount security holders.

III. Stripped Mortgage-backed Security Valuation Model

This section outlines briefly our stripped mortgage-backed security valuation model. We discuss the model's underlying assumptions and its empirical implementation. For further details, see Schwartz and Torous (1989).

We make the following assumptions to develop a model to value stripped mortgage-backed securities.

A1. Following Brennan and Schwartz (1982), we assume that all information about the term structure of default-free interest rates can be summarized by two state variables: the instantaneous risk-free rate of interest, r, and the yield on a default-free consol l.

A2. Dynamics of r and l are assumed to be described by:

$$\begin{aligned} \mathrm{d}r &= (a_1 + b_1(l - r))\mathrm{d}t + \sigma_1 r \mathrm{d}z_1 \\ \mathrm{d}l &= (a_2 + b_2 l + c_2 r)\mathrm{d}t + \sigma_2 l \mathrm{d}z_2, \end{aligned}$$

where z_1 and z_2 are standardized Wiener processes. Increments to z_1 and z_2 are assumed to be instantaneously correlated:

$$\mathrm{d}z_1 \mathrm{d}z_2 = \rho \mathrm{d}t,$$

where ρ denotes the instantaneous correlation coefficient.

A3. Mortgages are assumed to be prepaid at the instantaneous rate of prepayment:

$$\pi = \pi(l, x, y, t; c).$$

We posit that π depends upon the prevailing refinancing rate, proxied by l, relative to the mortgage's contract rate, c. Also, π depends upon lagged refinancing costs, summarized by the state variable $x(t)$ where

$$x(t) = \alpha \int_{\infty}^{0} \exp(-\alpha s) l(t - s) ds, \ \alpha > 0,$$

an exponential average of past refinancing rates. The state variable $y(t)$ gives the fraction of a pool of mortgages currently outstanding relative to their principal which would prevail in the absence of prepayments but reflecting amortization and captures any heterogeneity in mortgagors. Time, t, affects the instantaneous rate of prepayment by determining both the age of the mortgage and the season of the year.

Given these assumptions, the value of the stripped mortgage-backed security can be expressed as

$$B = B(r, l, x, y, t).$$

Standard arbitrage arguments result in the following second-order partial differential equation which the value of the mortgage-backed security must satisfy

$$1/2r^2\sigma_1^2 B_{rr} + rl\rho\sigma_1\sigma_2 B_{rl} + 1/2l^2\sigma_2^2 B_{ll} + (a_1 + b_1(l - r) - \lambda_1\sigma_1 r)B_r$$
$$+ l(\sigma_2^2 + l - r)B_l + \alpha(l - x) + B_x + B_t - (r + \pi)B + \pi P(t) + A = 0,$$

where λ_1 is the market price of short-term interest rate risk, while A and $P(t)$ are the total payout rate and principal outstanding at time t, respectively, of the stripped mortgage-backed security. Since the underlying mortgage is assumed to be fully amortizing, the following terminal condition must be satisfied

$$B(r, l, x, 0, T) = 0.$$

We do not impose an optimal, value-minimizing call condition to value the stripped mortgage-backed security. Rather, the value of the mortgage-backed security reflects the fact that at each point in time there exists a probability of prepaying, this probability depending upon the

Table 1. Maximum Likelihood Estimates of Interest Rate Process Parameters

a_1	b_1	a_2	b_2	c_2	σ_1	σ_2	ρ
−0.0800 (0.0359)	0.0382 (0.0174)	−0.0033 (0.0063)	−0.0007 (0.0019)	0.0008 (0.0016)	0.0262	0.0173	0.3732

Standard errors are given in parentheses.

current state of the economy as summarized by the model's state variables.

We numerically solve this second-order partial differential equation to implement our valuation procedures. To do so requires that we specify the partial differential equation's coefficients which depend upon the parameters of the interest rate processes, the assumed prepayment function, and the market price of short-term interest rate risk.

Table 1 provides maximum likelihood estimates of the parameters of the interest rate processes. The instantaneous risk-free interest rate is approximated by the annualized 1-month CD rate, while the consol yield is approximated by the annualized running coupon yield on long-term U.S. Treasury bonds.[2] The sample period is December 29, 1982 through April 1, 1987. The empirical results are consistent with the short rate reverting to the long rate, while the long rate itself follows a random walk. Short rates were more volatile than long rates over our sample period and unanticipated proportional changes in short and long rates were positively correlated.

A prepayment function gives the probability of a mortgagor prepaying a mortgage during a particular period, conditional on the mortgage not having been prepaid prior to that period. We model the prepayment function by a proportional hazards model

$$\pi = \pi_0(t; \gamma, p)\exp(\underline{\beta v}),$$

where the baseline hazard function is given by the log-logistic hazard function

$$\pi_0(t; \gamma, p) = (\gamma p(\gamma t)^{p-1})(1 + (\gamma t)^p)^{-1}.$$

The baseline hazard function measures the conditional probability of prepayment as a function of the age of the mortgage only. According to the log-logistic specification, for $p > 1$ this conditional prepayment

[2] The running coupon yield is the coupon rate on a newly issued U.S. Treasury bond if the bond is then issued; otherwise it is the yield on the most recently issued U.S. Treasury bond.

probability increases with increasing mortgage age, reaches a maximum at $t^* = (p - 1)^{1/p}/\gamma$, and decreases thereafter with increasing mortgage age.

However, the conditional prepayment probability does not depend solely upon a mortgage's age. Our prepayment function takes into account the fact that various explanatory variables, $\underline{v}$, influence a mortgagor's prepayment decision with the regression coefficients $\underline{\beta}$ measuring the effects of the explanatory variables.

To empirically implement our prepayment function, we specify the following explanatory variables:

$$v_1(t) = c - l(t - s),\ s \geq 0.$$

This explanatory variable permits us to investigate the effects of refinancing rates, contemporaneous or lagged, upon a mortgagor's prepayment decision. If $v_1(t) > 0$ there exists an economic incentive to prepay, this incentive being larger the larger $v_1(t)$ implying that $\beta_1 > 0$.

$$v_2(t) = (c - l(t - s))^3,\ s \geq 0.$$

We allow the possibility that prepayments may further accelerate when refinancing rates are sufficiently lower than the mortgage's contract rate because of transaction costs. Since for $c > l(t - s)$ there is an economic incentive to prepay, we expect that $\beta_2 > 0$.

$$v_3(t) = ln\ (AO_t/AO_t^*)$$

where AO_t is the dollar amount of a pool of mortgages outstanding at t while AO_t^* is the pool's principal which would prevail at t in the absence of prepayments but reflecting amortization. The smaller the relative proportion of a pool of mortgages currently outstanding, the more likely mortgagors less prone to prepay remain, and as such we expect $\beta_3 > 0$.

$$v_4(t) = \begin{cases} +1 & \text{if } t = \text{May–August.} \\ 0 & \text{if } t = \text{September–April.} \end{cases}$$

More residential real estate transactions occur in the spring and summer, and therefore we expect greater prepayment activity in the spring and summer implying that $\beta_4 > 0$.

Table 2 provides maximum likelihood parameter estimates of our prepayment function given annualized monthly conditional prepayment rates over the period from January 1978 to November 1987 for a number of GNMA 30-year Single-Family Pools. Our empirical results are consistent with the conditional prepayment probability initially increasing with increasing mortgage age, this probability being maximized at approximately 6 years, and decreasing thereafter with increasing mort-

Table 2. Maximum Likelihood Estimates of Prepayment Function

γ	0.01496(0.00110)
p	2.31217(0.13919)
β_1	0.38089(0.06440)
β_2	0.00333(0.00134)
β_3	3.57673(0.34504)
β_4	0.26570(0.32870)
t^*	6.265 years

Jackknifed standard deviation estimates are shown in parentheses.

gage age. Note that all the posited explanatory variables influence prepayment decisions in the expected direction and, except for seasonality, are statistically significant.

We specify the market price of short-term interest rate risk in light of interest rate conditions prevailing at the end of November 1987. In particular, we determine iteratively that value of λ_1 such that a 30-year default-free nonprepayable fully amortizing mortgage with a 11% contract rate is priced at par for $r = l = 11\%$. The resultant estimate of the market price of short-term interest rate risk is $\hat{\lambda}_1 = -0.01$.

IV. Valuation Results

To illustrate our valuation procedures, we price 8% and 14% stripped mortgage-backed securities based upon a 11% mortgage-backed security. The prices of these stripped securities therefore reflect the prepayment behavior of a pool of 11.5% mortgages. Recall that the 8% stripped security receives half of the pool's principal payments and four-elevenths of its interest payments, while the 14% stripped security receives seven-elevenths of the interest payments in addition to half of the pool's principal payments. For comparison purposes, we also consider traditional 8% and 14% mortgage-backed securities. That is, securities backed by pools of 8.5% and 14.5% mortgages, respectively.

Given the estimated prepayment function, estimated parameters of the interest rate processes, and the specified market price of short-term interest rate risk, we employ Monte Carlo solution techniques to solve the second-order partial differential equation which characterizes mortgage-backed security prices. To begin with, we assume that all the underlying mortgage pools differ only in their contract rates. In particular, all pools are assumed to have been originated 5 years ago with 90% of their relative principal currently outstanding. Throughout we fix $r(t) = 11\%$ and let $l(t)$ vary from 5% to 17% in order to investigate the effects of prepayments on mortgage-backed security prices.

Table 3 presents simulated 8% mortgage-backed security prices. Note that, whenever the traditional security is selling at a premium, its

Table 3. Simulated 8% Mortgage-backed Security Prices

r	l	Traditional	Stripped
0.11	0.05	117.8	104.3
0.11	0.06	112.7	105.7
0.11	0.07	106.3	102.8
0.11	0.08	100.4	99.0
0.11	0.09	94.0	94.2
0.11	0.10	87.4	88.5
0.11	0.11	81.5	83.2
0.11	0.12	75.5	77.5
0.11	0.13	69.9	71.9
0.11	0.14	65.5	67.1
0.11	0.15	61.6	62.7
0.11	0.16	57.1	57.5
0.11	0.17	53.3	53.1

These prices are based on a pool of 11.5% mortgages originated 5 years ago with 90% of their relative principal currently outstanding.

value exceeds the value of the stripped mortgage-backed security. The traditional security is backed by a slower prepaying mortgage pool and slower prepayments are valuable to premium security holders. By contrast, when the stripped mortgage-backed security is selling at a discount, its value tends to exceed the value of the traditional mortgage-backed security. The stripped security is backed by a faster prepaying mortgage pool and faster prepayments are valuable to discount security holders.

Table 4 presents simulated 14% mortgage-backed security prices. Now the traditional security is backed by a faster prepaying mortgage

Table 4. Simulated 14% Mortgage-backed Security Prices

r	l	Traditional	Stripped
0.11	0.05	[a]	128.8
0.11	0.06	[a]	130.4
0.11	0.07	[a]	129.4
0.11	0.08	110.6	127.5
0.11	0.09	113.5	124.2
0.11	0.10	112.5	119.4
0.11	0.11	110.1	114.7
0.11	0.12	106.8	109.5
0.11	0.13	102.5	103.8
0.11	0.14	98.5	99.0
0.11	0.15	94.4	94.4
0.11	0.16	89.0	88.6
0.11	0.17	83.8	83.3

These prices are based on a pool of 11.5% mortgages originated 5 years ago with 90% of their relative principal currently outstanding.
[a] Underlying mortgages prepaid at these refinancing rates.

pool, whereas the stripped security is backed by a slower prepaying mortgage pool. As a result, we see that premium stripped mortgage-backed securities tend to be worth more than premium traditional mortgage-backed securities, whereas discount traditional mortgage-backed securities tend to be worth more than discount stripped mortgage-backed securities. Interestingly, both the premium 14% traditional and stripped mortgage-backed securities exhibit reversed interest rate sensitivity. Intuitively, interest rate increases dampen prepayment behavior. Given that interest rates are sufficiently low, the resultant increase in premium mortgage-backed security prices more than offsets the decline in prices due to the decrease in the present value of future mortgage payments.

The preceding simulation analyses assume that the underlying mortgage pools are identical in all respects save for their contract rates. However, the extent to which these mortgage pools have different contract rates reflects the fact that they were originated at different points in time and, as a result, are of different ages and are characterized by different relative proportions previously prepaid. These differences imply statistically significant differences in the pools' prepayment behavior which, in turn, imply differences in mortgage-backed security prices.

Table 5 presents simulated 14% mortgage-backed security prices when the underlying mortgage pools are no longer assumed to differ only in their contract rates. In particular, the 14.5% mortgage pool backing the traditional security is assumed to have been originated 10

Table 5. Simulated 14% Mortgage-backed Security Prices

r	l	Traditional	Stripped
0.11	0.05	[a]	128.8
0.11	0.06	121.9	130.4
0.11	0.07	126.6	129.4
0.11	0.08	127.5	127.7
0.11	0.09	125.3	127.5
0.11	0.10	121.4	119.4
0.11	0.11	117.7	114.7
0.11	0.12	111.1	109.5
0.11	0.13	105.5	103.8
0.11	0.14	100.3	99.0
0.11	0.15	94.2	94.4
0.11	0.16	89.1	88.6
0.11	0.17	84.9	83.3

The stripped mortgage-backed security prices are based on a pool of 11.5% mortgages originated 5 years ago with 90% of their relative principal currently outstanding; the traditional mortgage-backed security prices are based on a pool of 14.5% mortgages originated 10 years ago with 50% of their relative principal currently outstanding.

[a] Underlying mortgages prepaid at these refinancing rates.

years ago with 50% of the relative principal of the pool currently outstanding. Recall that the 11.5% mortgage pool backing the stripped security is assumed to have been originated 5 years ago, with 90% of the relative principal of the pool currently outstanding. For sufficiently low refinancing rates, the 14% stripped mortgage-backed security is still selling for more than the 14% traditional mortgage-backed security because of the greater interest rate sensitivity of 14.5% mortgage pool's prepayments. However, for higher refinancing rates, the premium traditional 14% mortgage backed security now sells for more than the stripped 14% mortgage-backed security because of the slower prepayment speed of the 14.5% mortgage pool. For discount 14% mortgage-backed securities, prepayment is less likely and the slower prepayment speed of the 14.5% pool tends to be offset by its shorter term to maturity.

These simulation results confirm the critical role of prepayments to the valuation of mortgage backed securities. All the factors underlying a mortgagor's prepayment decision must be explicitly taken into account when valuing mortgage backed securities. Otherwise, systematic mispricing results.

V. Summary and Conclusions

This paper has put forward a valuation framework for stripped mortgage-backed securities. The value of a stripped mortgage-backed security reflects the fact that at each point in time there exists a probability of prepaying, this conditional probability depending upon the prevailing state of the economy. A prepayment function links a mortgagor's conditional probability of prepaying to the prevailing state of the economy. By integrating an empirical prepayment function into our valuation framework, we provide a complete model for valuing stripped and other mortgage-backed securities.

By investigating the interest rate sensitivity of hypothetical stripped mortgage-backed securities, we highlight the important role that prepayment behavior plays in the valuation of these securities. When market prices of mortgage-backed securities become more available, future research should investigate the empirical accuracy of our valuation procedures, thereby allowing greater insights into the economics of stripping mortgage-backed securities.

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