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The Valuation of Forestry Resources under Stochastic Prices and Inventories

Randall Morck, Eduardo Schwartz, and David Stangeland*

Abstract

A contingent claims approach to capital budgeting may be preferable to traditional methods where uncertainty and managers' strategic reactions to changing conditions are important. As an example of such a case, we solve the classical problem of the duration of an investment in forestry resources (i.e., when to cut down the trees) in the general case of stochastic output prices and stochastic natural growth rate and timber inventories. A contingent claims approach is used to value the forestry resources as a function of: (1) stochastic prices and inventories, and (2) an asymmetric, optimal production policy that incorporates the option to halt timber production temporarily.

I. Introduction

Classical capital budgeting techniques are based on the assumption that future cash flows follow a rigid pattern and can be accurately predicted far into the future. On the basis of such cash flow predictions, the project is either accepted or rejected. The uncertainty in the project, and management's strategic reactions to changing conditions are dealt with only superficially. At best, a risk-adjusted interest rate is employed and various deterministic scenarios are plotted out. In situations where uncertainty, and management's strategic reactions to it, are important, the classical technique is apt to lead to wrong decisions. The technology needed to deal with such problems more accurately is becoming available. The purpose of this paper is to present a stylized example of how it can be used.

Our example is a general solution to the classical "duration" problem of the optimal control of a long-term, renewable resource investment, such as forestry resources (i.e., when to chop down a tree).[1] If the prices and the natural growth rate of the inventory of forest timber are assumed to be deterministic, the optimal control is very simple to derive. But when both the price and inventory of timber are stochastic, the problem becomes both stochastic and asymmetric. The asym-

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[1] The model developed in this paper can be applied to any renewable resource management problem. For example, exactly the same equations describe an optimal management strategy for fisheries. The general methodology can be applied to many problems in diverse areas.

metry is due to the possible exercise of an option to halt timber production temporarily if prices are too low. The valuation of forestry resources must explicitly incorporate this stochastic, asymmetric optimal control framework.

We follow a contingent claims approach within the context of Merton's (1973) intertemporal capital asset pricing model. We calculate the value of a forestry lease as the value of an option to cut down the trees at the most advantageous time. The point of the contingent claims approach is that it ties the value of an asset to one or more variables whose stochastic properties can be easily estimated. It is not necessary to formulate predictions of prices and inventories far into the future. Instead, the stochastic properties of these variables are fed directly into the analysis. The only assumption about prices and inventories in the future is that they follow certain well-defined stochastic processes.

This simple and elegant framework allows for the incorporation of both the underlying stochastic price and inventory processes; and the stochastic, asymmetric nature of the optimization problem. Brennan and Schwartz (1985) use a contingent claims approach to develop a model for the valuation of a nonrenewable natural resource investment. Their model incorporates stochastic output prices in a stochastic, asymmetric optimal control framework. This paper develops a critical extension of the contingent claims approach to cover forestry resources, where *both* prices and inventories are stochastic. It generalizes the classical "duration" literature to cover renewable resource investments in which prices and inventories are stochastic.

Section II develops a valuation model of forestry resources, which involves a partial differential equation, production constraints, inventory constraints, and boundary conditions. Section III presents a specific example by valuing a hypothetical Canadian white pine forest. Section IV provides a conclusion.

II. A Valuation Model of Forestry Resources

The value of the timber in a particular forestry leasehold is assumed to be affected by two stochastic variables: (1) the price of timber, and (2) the inventory of timber in that leasehold. Define P as the price of timber, I as the inventory of timber, and t as the current time. Assume that these variables are governed by the stochastic differential equations

$$(1) \qquad \frac{d\tilde{P}}{P} = \mu_P(P,t)dt + \sigma_P(P,t)d\tilde{z}_P ,$$

$$(2) \qquad d\tilde{I} = \left[\mu_I(I,t) - q(P,I,t)\right]dt + \sigma_I(I,t)d\tilde{z}_I ,$$

where $\tilde{z}_p$ and $\tilde{z}_I$ are possibly correlated Wiener processes in $\mathscr{R}^2$ with correlation ρ, $\mu_P(P,t)$, and $\mu_I(I,t)$ are instantaneous drift coefficients, $\sigma_P(P,t)$ and $\sigma_I(I,t)$ are instantaneous diffusion coefficients, and $q(P,I,t)$ is the instantaneous quantity of timber produced, i.e., the cutting rate. The price dynamics in (1) are standard in the literature.[2] The expected rate of change in inventories, μ_I, in (2) is instan-

[2] A jump process also could be incorporated into the model. For simplicity, however, we consider only the standard Itô processes (1) and (2).

taneously decreased by the cutting rate q, which is precisely the stochastic optimal control.

Consider a logging company whose sole asset is a lease that gives it the right to harvest timber in a particular forest from t to a maturity date T. The company's after-tax cash flow from timber production on the leasehold is

$$f(t) = \left(1-\tau_C\right)\left[\left(1-\tau_R\right)Pq(P,I,t) - A(q,t)\right] - \lambda V(P,I,t), \tag{3}$$

where τ_c $\equiv$ corporate tax rate,[3]

τ_R $\equiv$ royalty rate on timber sales,

$A(q,t)$ $\equiv$ operating costs of logging quantity q,

λ $\equiv$ property tax rate, assumed proportional to the value of the logging company, and

$V(P,I,t)$ $\equiv$ value of the logging company.

The logging lease is a contingent claim on the underlying timber. For a given production policy, $q(P,I,t)$, the value of the logging company $V(P,I,t)$ can be determined. Applying Itô's lemma, the value of the leasehold will evolve according to the stochastic differential equation

$$\begin{aligned} d\tilde{V} &= V_P d\tilde{P} + V_I d\tilde{I} + V_t dt \\ &\quad + \frac{1}{2}V_{PP}\left(d\tilde{P}\right)^2 + \frac{1}{2}V_{II}\left(d\tilde{I}\right)^2 + V_{IP}\left(d\tilde{I}\right)\left(d\tilde{P}\right). \end{aligned} \tag{4}$$

Substituting (1) and (2) into (4), and rearranging terms, leads to

$$d\tilde{V} = \mu_V V dt + \sigma_{VP} V d\tilde{z}_P + \sigma_{VI} V d\tilde{z}_I, \tag{5}$$

$$\begin{aligned} \text{where } \mu_V = &\Big\{\mu_P P V_P + \left(\mu_I - q\right)V_I + V_t \\ &+ \frac{1}{2}\sigma_P^2 P V_{PP} + \frac{1}{2}\sigma_I^2 V_{II} + \rho\sigma_P\sigma_I P V_{IP}\Big\}V^{-1}, \end{aligned} \tag{6}$$

$$\sigma_{VP} = \sigma_P P V_P V^{-1}, \text{ and} \tag{7}$$

$$\sigma_{VI} = \sigma_I V_I V^{-1}. \tag{8}$$

If we assume that Merton's (1973) intertemporal asset pricing model holds, that

[3] Actual tax payments are discrete and, therefore, difficult to model in a continuous time framework. Note that in the above model corporate taxes are symmetric. There is a full offset if income is negative. Given that the solution procedure is numerical, the model can easily be adapted to allow either for a full offset or for no taxes if income goes negative. Brennan and Schwartz (1985) use the latter representation of taxes. A realistic treatment of the tax code would involve modeling tax loss carryforwards, and would, therefore, require the addition of a time-dependent state variable. In employing an offset provision, we are attempting to move closer to such an approach while keeping the analysis as straightforward as possible.

the risk-free rate r is constant, and that the utility function of the representative investor is logarithmic, then

$$V^{-1}f(t) + \mu_V - r = \sigma_{VW} , \tag{9}$$

where σ_{VW} is the covariance of the return on V with aggregate wealth, and $f(t)$ is as defined in Equation (3). Assume that aggregate wealth W follows the stochastic process

$$d\tilde{W} = \mu_W W dt + \sigma_W W d\tilde{z}_W . \tag{10}$$

Equations (5) and (10) imply

$$\begin{aligned} \sigma_{VW} &= \frac{\operatorname{Cov}\left[V^{-1}d\tilde{V}, W^{-1}d\tilde{W}\right]}{dt} \\ &= \rho_{WP}\sigma_W\sigma_{VP} + \rho_{WI}\sigma_W\sigma_{VI} , \end{aligned} \tag{11}$$

where ρ_{WP} and ρ_{WI} are the instantaneous correlations of the price and inventory of timber with aggregate wealth. Let

$$\begin{aligned} \phi_P &= \rho_{WP}\sigma_W\sigma_P , \\ \phi_I &= \rho_{WI}\sigma_W\sigma_I . \end{aligned} \tag{12}$$

Substituting (6), (7), (8), (11), and (12) into (9) yields[4]

$$\begin{aligned} &\left(1-\tau_C\right)\left(1-\tau_R\right)\left[Pq(P,I,t) - A(q,t)\right] - \lambda V - rV \\ &+\left(\mu_P - \phi_P\right)PV_p + \left(\mu_I - q(P,I,t) - \phi_I\right)V_I + V_t \\ &+\frac{1}{2}\sigma_P^2 P^2 V_{PP} + \frac{1}{2}\sigma_I^2 V_{II} + \rho\sigma_P\sigma_I PV_{PI} = 0 . \end{aligned} \tag{13}$$

The logging company is assumed to choose the optimum production policy, $q^*(P,I,t)$, so as to maximize the company's value. This leads to the stochastic Bellman equation

$$\begin{aligned} 0 = \max_{q\in[0,M]} \Big\{ &\left(1-\tau_C\right)\left[\left(1-\tau_R\right)Pq(P,I,t) - A(q,t)\right] \\ &- \lambda V - rV + \left(\mu_P - \phi_P\right)PV_P \\ &+\left(\mu_I - q(P,I,t) - \phi_I\right)V_I + V_t \\ &+\frac{1}{2}\sigma_P^2 P^2 V_{PP} + \frac{1}{2}\sigma_I^2 V_{II} + \rho\sigma_P\sigma_I PV_{PI} \Big\} , \end{aligned} \tag{14}$$

where M is the maximum rate of production that is technologically feasible.

---

[4] This equation can be derived in several ways. It is equivalent to Equation (31) in Cox, Ingersoll, and Ross (1985), and also can be derived using a contingent claims hedging procedure as in Brennan and Schwartz (1985). The latter derivation requires that an instrument be available for hedging inventory risk.

To find the optimum production policy, we set the partial derivative of (14) with respect to q to zero, and solve for q. The optimum production policy also must incorporate the constraints that: (1) production must be nonnegative, and (2) if production falls too low, the production site can be shut down and later reopened in order to avoid ongoing operating costs.[5] The optimum production policy is then substituted into Equation (14) to obtain the concentrated PDE to be solved.

A. Boundary Conditions and Constraints

The value of the logging company must satisfy the Bellman equation subject to the following boundary conditions:

1. On the terminal date T of the lease, the value of the company must fall to zero. That is,

$$V(P,I,T) = 0 . \tag{15}$$

2. If the price of timber drops to 0, then stochastic process (1) implies that the value of the company must also go to zero. That is,

$$V(0,I,s) = 0 \qquad s \in [t,T] . \tag{16}$$

3. If the price of timber becomes very large, then changes in the value of the firm due to changes in the price level will be proportional to the level of inventory held, that is, V_P will approach a linear function of I given by

$$\lim_{P \to \infty} \frac{\partial V(P,I,s)}{\partial P} = kI \qquad s \in [t,T] . \tag{17}$$

The proportionality factor k will depend on the cost function and other parameters and will be specified in more detail below.

4. If there is a natural maximum density of trees in the forest, $I_{\max}$, then at this point, the value of the company cannot be increased further due to increased inventory. This is mathematically modeled as a reflecting boundary condition

$$\left. \frac{\partial V(P,I,s)}{\partial I} \right|_{I = I_{\max}} = 0 \qquad s \in [t,T] . \tag{18}$$

5. If there are regulatory constraints that limit the minimum inventory in the forest, $I_{\min}$, this means that, if the inventory falls to this level, all lumbering must stop. This imposes the following constraint on the stochastic control

$$q(P,I,s) = 0, \quad \forall s \in [t,T] , \quad \text{if } I \leq I_{\min} . \tag{19}$$

The boundary conditions listed above are not meant to be realistic. They are chosen primarily for simplicity. In each specific case where this methodology is applied, the relevant constraints on minimum or maximum inventories and prices due to regulatory, financial, or technological restrictions would have to be incorporated. The complete valuation model for forest resources is the PDE in Equation (14), after concentrating with respect to the optimal production policy,

[5] See Brennan and Schwartz (1985) for how to incorporate optimal shutdown, reopening, and abandonment. See also Myers and Majd (1983).

boundary conditions such as Equations (15) through (19), and any additional constraints. Together, these equations uniquely determine the value of the company $V(P,I,t)$ and the optimal operating policy $q^*(P,I,t)$. We are unaware of any analytic solution to this model, however numerical methods can solve it.

III. An Example

A. Specification and Coefficient Estimates

The usefulness of this model can be demonstrated by solving a specific example. Below, we assume specific functional forms for the instantaneous drift coefficients, instantaneous diffusion coefficients, and cost function. As we mentioned above, the only assumption we need make about the future is that these functional forms and the parameters of the stochastic processes do not change over time.

To implement the model developed in the previous section, we make the following additional simplifying assumptions:

1. Stochastic changes in the inventory of timber in this leasehold are uncorrelated with changes in aggregate investor wealth. The factor risk premium, ϕ_I, is thus equal to zero.

2. There are futures markets for timber and the convenience yield is proportional to the spot price of timber, P.[6] Brennan and Schwartz (1985) show in a similar context that the risk-adjusted drift of the price process $\mu_P - \phi_P$ is equal to the riskless interest rate minus the proportional convenience yield $r-\kappa$.

3. The standard deviation of the return process in (1) is constant. The expected change and the standard deviation of changes in inventory in (2) are proportional to the level of the inventory.[7]

Under assumptions 1, 2, and 3 above, the risk-adjusted stochastic process corresponding to (1) and (2) can be written as

$$(1') \qquad \frac{d\tilde{P}}{P} = (r-\kappa)dt + \sigma_p d\tilde{z}_p ,$$

$$(2') \qquad d\tilde{I} = \left[\mu I - q(P,I,t)\right]dt + \sigma I d\tilde{z}_I .$$

4. The leasehold is assumed to be small relative to the timber industry so that the stochastic processes $\tilde{z}_P$ in (1′) and $\tilde{z}_I$ in (2′) are uncorrelated, i.e., ρ equals zero. Changes in this forest's inventory of trees do not affect the market

[6] The convenience yield is defined as the value of the flow of services that accrues to the owner of a physical commodity but not to the owner of a contract for future delivery of the same commodity. It can be thought of as valuing the advantage to be gained by exploiting temporary shortages and price fluctuations. See Kaldor (1939), Working (1948), Brennan (1958), and Telser (1958). The assumption that the convenience yield is proportional to the spot price is very strong. We employ it solely for simplicity, following the tradition of constant proportional volatility, etc. in the continuous time finance literature. In many cases, such assumptions may be highly problematic—see Brennan (1986). The issues involved here are developed in connection with long-term oil-linked assets by Gibson and Schwartz (1989). Dealing effectively with these issues is quite difficult and is beyond the scope of this paper.

[7] This is only a simplifying assumption. We would, in fact, expect a much more complex relation between μ and I.

price of timber. This "small firm" assumption could be relaxed at the expense of a more complicated numerical problem.[8]

5. The cost function is given by the quadratic equation

$$(20) \qquad A(q,t) = \begin{cases} a_0 + a_1 q + \frac{1}{2} a_2 q^2 & \text{if } f(t) > 0 \\ 0 & \text{if } f(t) \leqslant 0 \end{cases},$$

where a_0 is the fixed cost, a_1 is the variable cost, a_2 is the quadratic term reflecting increasing marginal cost, and where the cost function reflects an assumption that production can be costlessly shut down and reopened. The quadratic functional form is not critical, and is chosen solely for its algebraic simplicity.

Substituting these specific functional forms into Equation (14) produces

$$(21) \qquad \begin{aligned} 0 = & \max_{q \in [0,M]} \left(1-\tau_C\right)\left[\left(1-\tau_R\right)Pq(P,I,t) - A(q,t)\right] \\ & -\lambda V - rV + (r-\kappa)PV_P + [\mu I - q(P,I,t)]\,V_I + V_t \\ & + \frac{1}{2}\sigma_P^2 P^2 V_{PP} + \frac{1}{2}\sigma^2 I^2 V_{II} . \end{aligned}$$

To find the optimal production policy, set the partial derivative with respect to q equal to zero

$$(22) \qquad \left(1-\tau_C\right)\left[\left(1-\tau_R\right)P - a_1 - a_2 q^*(P,I,t)\right] - V_I = 0 .$$

Imposing the constraints that (1) production must be nonnegative and (2) production can be costlessly shutdown and reopened, Equation (22) can be solved for the optimum production policy

$$(23) \quad q^*(P,I,t) = \begin{cases} \max\left[0, \dfrac{-V_I}{\left(1-\tau_C\right)a_2} + \dfrac{\left(1-\tau_R\right)P - a_1}{a_2}\right. & \text{if } f(t) > 0 \\ 0 & \text{if } f(t) \leqslant 0 \end{cases}$$

[8] We abstract from the effect that the level of aggregate inventory has on the market price. To incorporate this effect would require an additional state variable, a stochastic convenience yield. We leave this issue as an interesting topic for future research.

Substituting Equation (23) into Equation (21) gives the concentrated partial differential equation to be solved

$$
\begin{aligned}
0 = {} & \frac{1}{2a_2(1-\tau_C)} V_I^2 + \left[\mu I - \frac{(1-\tau_R)P - a_1}{a_2} \right] V_I \\
& + \left[\frac{(1-\tau_C)}{2a_2} \left[(1-\tau_R)P - a_1 \right]^2 - (1-\tau_C)a_0 \right] \\
& - rV - \lambda V + (r-\kappa)PV_P + V_t + \frac{1}{2}\sigma^2 P^2 V_{PP} \\
& + \frac{1}{2}\sigma^2 I^2 V_{II} \, .
\end{aligned}
\tag{24}
$$

Note the quadratic expression in V_I. Equation (24) is a *nonlinear*, partial differential equation. This equation, along with the boundary conditions in Equations (15) through (19)[9] and the production contraints in Equation (23), is to be solved numerically.

As a specific application, consider a hypothetical white pine forest in Canada. Estimated coefficients for this application are given in Table 1. The analysis is carried out in real terms; the parameters of the cost functions are assumed constant throughout the lease period; and the historical standard deviation and trend, as well as the convenience yield of the white pine, are computed using deflated prices. We employ a real interest rate.

Using the coefficients in Table 1, the concentrated PDE in Equation (24) can be solved numerically subject to the boundary conditions in Equations (15) through (19), and the production constraints in Equation (23). The numerical technique used is a fourth order Runge-Kutta algorithm with a variable step size. This is implemented using a software package called FORSIM.[10]

B. Results

With the parameters given in Table 1, the value of the logging company with a lease of 10 years (40 quarters) is \$545,600. Note that this figure is derived without any specific predictions about future prices or inventories. The only assumption is that the stochastic processes that govern these variables are stationary. This figure takes into account the uncertainty in the underlying price and inventory processes, and the potential strategic reactions of the firm's management to changes in these variables.

[9] For the example considered, the term kI in the right-hand side of (17) is approximated by $(1-\tau_C)(1-\tau_R)(I-I_{\min})/(1+\lambda)$. The boundary condition (19) is replaced by $V(P, I_{\min}, t) = 0$, so that if the inventory reaches the regulatory minimum, the lease is terminated. These boundary conditions are highly stylized and are chosen primarily for simplicity.

[10] The Runge-Kutta method employs a weighted average of four successive incremental ratios to approximate the behavior of the function at each grid point. A variable step size is a device to increase the performance of the numerical procedure by locally refining the grid size where required for a better fit. The Runge-Kutta method is described in Braun (1975), p. 159. The package FORSIM is available from the Chalk River Nuclear Laboratories, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada.

TABLE 1

Estimated Coefficients for a Hypothetical White Pine Forest

White Pine Inventory:[a] 150,000 cubic meters	
Average Growth Rate (μ):[a]	1.7%/year
Standard Deviation (σ):[a]	9.3%/year
White Pine Price: $50/cubic meter	
Standard Deviation (σ_P):[b]	10.4%/year
Average Growth Rate (μ_P):[b]	0.4%
Convenience Yield (κ):[c]	4.6%/year
Risk-Free Interest Rate (r):[c]	5%/year
Logging Costs:[d]	
Fixed Costs (a_0):	$100,000/year
Variable Costs (a_1):[e]	$28/cubic meter
Quadratic Costs (a_2):	$2/cubic meter
Taxes:[f]	
Corporate Tax Rate (τ_C):	33%
Royalty Rate (τ_R):	5%
Property Tax Rate (λ):	2%

[a] Alberta Phase 3 Forest Data.

[b] Based on Canada's softwood lumber index, using the Canadian industrial price index series to adjust for inflation.

[c] Brennan and Schwartz (1985) argue that $\mu_P - \phi_P = r - \kappa$. The assumption made above is consistent with ϕ_P equal to zero and forms our base case. This is reasonable if timber price movements are largely idiosyncratic. See Mills (1988) and Binkley and Washburn (1988) for empirical evidence on this point. Results using other values of κ will be compared with the base case.

[d] Based on 150,000 cubic meters, the average inventory of Alberta forest companies.

[e] From the *Alberta Economic Timber Supply Analysis* (1980). Reforestation is required by law and its costs are included in these estimates. Logging of reforested areas on the leasehold is not permitted.

[f] All logging is done on Crown lands and is taxed accordingly.

These strategic reactions define an implicit optimal cutting rate. To explore an explicit optimal cutting policy, we must make assumptions about future price and inventory levels. If we assume that these variables grow at their historical trends, an optimal cutting policy through time can be derived. This is shown by the middle curve of Figure 1. The optimal cutting rates shown in this figure and in those that follow are based on two assumptions. First, that ex post the initial timber price (in the base case $50) grows at its expected trend, a real rate of 0.4 percent per year. Second, that ex post the inventory remaining after each quarter's logging also grows at its expected trend of 1.7 percent per year. The inventory is initially 150,000 cubic meters in the base case. Note that most of the allowable timber is cut during the first half of the lease period. The upper curve shows the optimal cutting rate for an inventory 50 percent larger (225,000 cubic meters), and the lower curve gives the same information for an inventory 50 percent smaller (75,000 cubic meters). The pattern is similar to the base case with the cutting rate and period of cutting being increased or decreased, respectively.[11]

[11] A comparison of the management strategy described in Figure 1 and in subsequent figures with the actual practices of the forestry industry would have been informative. Unfortunately, we were unable to obtain the necessary data.

Figure 2 illustrates the sensitivity of the cutting rate through time to different tree growth rates. The higher the growth rate, the lower is the cutting rate and the longer the forest lasts. Figure 3 shows that the convenience yield can have an important impact on the cutting rate. When the convenience yield is high (2.5 percent per quarter or 10 percent per year), there is an incentive to cut the forest fast since risk-adjusted prices are expected to decrease in the future. Alternatively, when the convenience yield is low, (0 percent, there is an incentive to delay cutting down the trees.

Figure 4 shows that a higher initial price is an incentive to cut the forest early. Figure 5 illustrates the sensitivity of the cutting rate to the standard deviation of price changes. A higher standard deviation implies a higher option value of the forest and a greater value to postponing the exercise of the option to harvest the trees.

Figures 6 through 10 illustrate the sensitivity of the initial value of the logging company ten years prior to the maturity of the lease for various initial prices, inventory levels, and parameters of the stochastic process. As expected, the value of the forest is an increasing function of the tree growth rates, inventory levels (Figure 6), and the price of trees (Figures 7 and 8). Figure 9 illustrates the value of the leasehold as a function of the inventory level for different convenience yields. Value is inversely related to convenience yield. Finally, Figure 10 shows the option value of the forest. Higher price volatilities result in higher forest values.

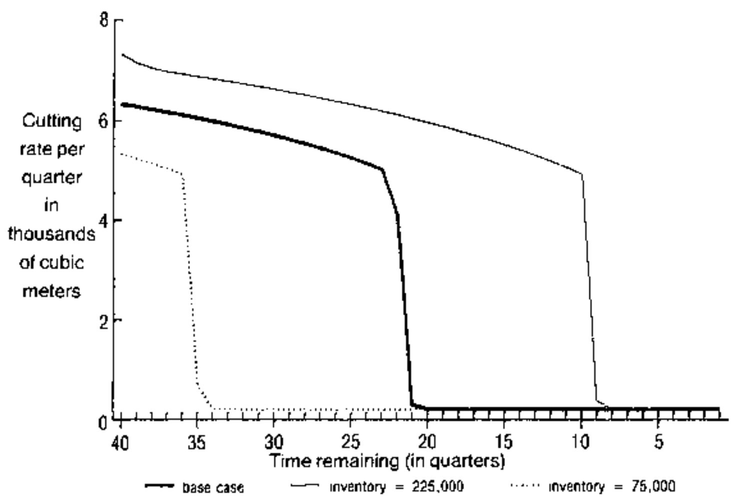


FIGURE 1

Cutting Rate vs. Time to End of Lease for Various Initial Inventory Levels

IV. Summary and Conclusions

A contingent claims approach to capital budgeting may be preferable to standard methods if uncertainty and management's strategic reactions to changing conditions are important. As an example of such a case, we value a forestry resource in an environment where prices and inventories change stochastically

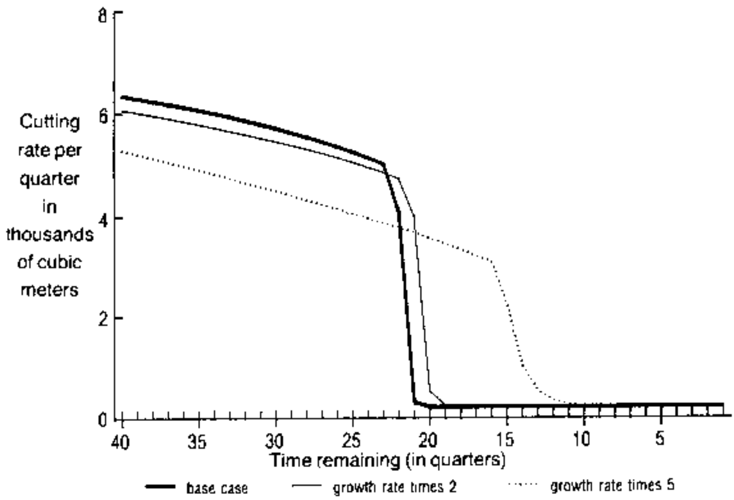


FIGURE 2

Cutting Rate vs. Time to End of Lease for Various Tree Growth Rates

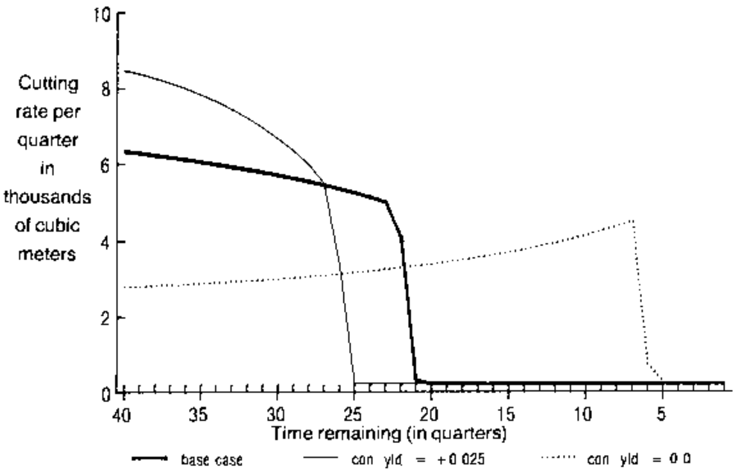


FIGURE 3

Cutting Rate vs. Time to End of Lease for Various Convenience Yields

through time. Managers can react to adverse conditions by reducing or shutting down logging operations. The setup is one in which the forest is leased for a fixed amount of time, but the framework can be easily extended to deal with different situations.

The model jointly determines the value of the forest and the implicit optimal cutting rate as a function of the stochastic properties of the price of timber and the firm's inventory of trees, as well as the time to maturity of the lease. The model is then applied to a hypothetical white pine forest in Canada.

The model developed gives rise to a nonlinear partial differential equation that must be solved by numerical methods. We believe that this is the first paper in the finance literature that solves this type of equation.

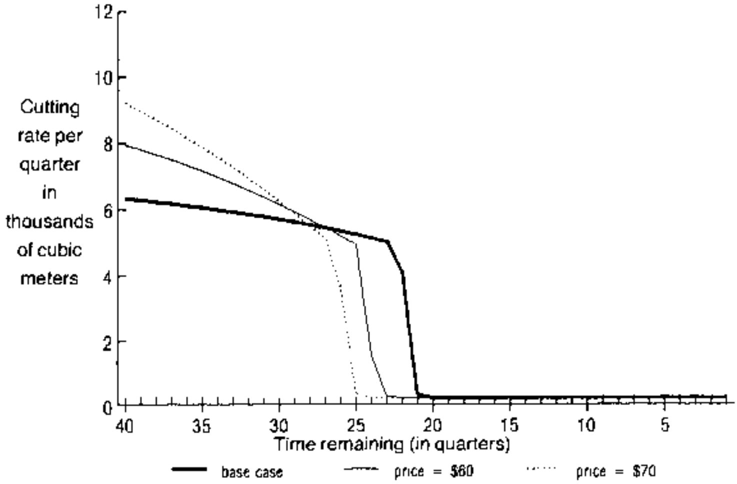


FIGURE 4

Cutting Rate vs. Time to End of Lease for Various Initial Price Levels

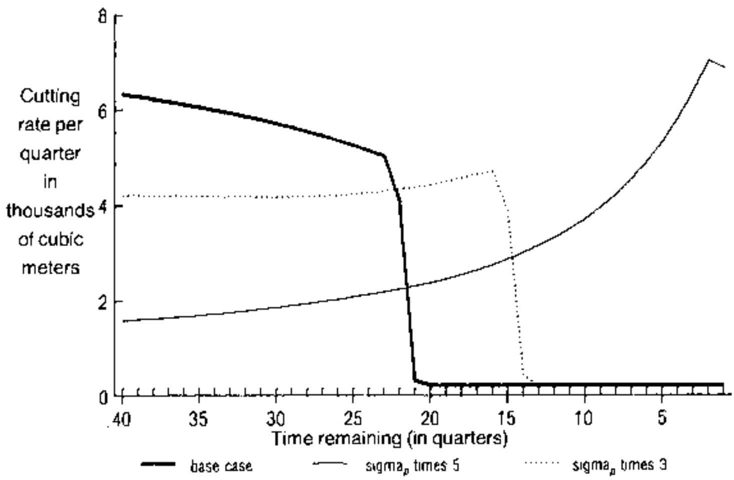
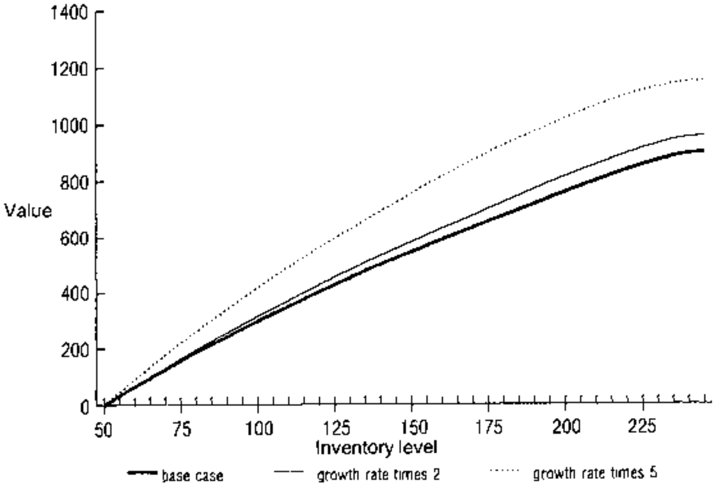


FIGURE 5

Cutting Rate vs. Time to End of Lease for Various Levels of σ_p

Obvious extensions of the analysis are to consider stochastic interest rates and convenience yields. This would, however, somewhat complicate the numerical solution procedures. It also would be of interest to extend the problem over a much longer time horizon in order to consider issues such as optimal reforestation policies.

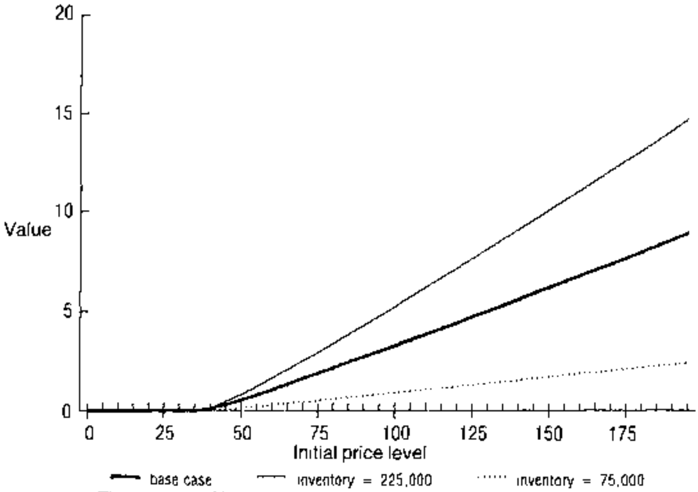
Indeed, in focusing on the optimal policy for a short-term private leaseholder, we abstract from a host of problems involving social vs. private optimization, which these methods might illuminate. The prospect of applying this methodology to general capital budgeting problems is promising, and there is great scope for future work.



* Time to end of lease = 40 quarters

FIGURE 6

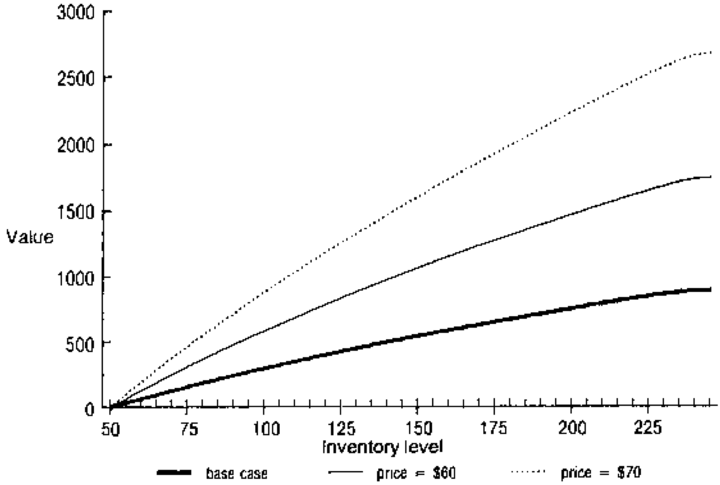
Value (in thousands) vs. Inventory for Various Tree Growth Rates*



* Time to end of lease = 40 quarters

FIGURE 7

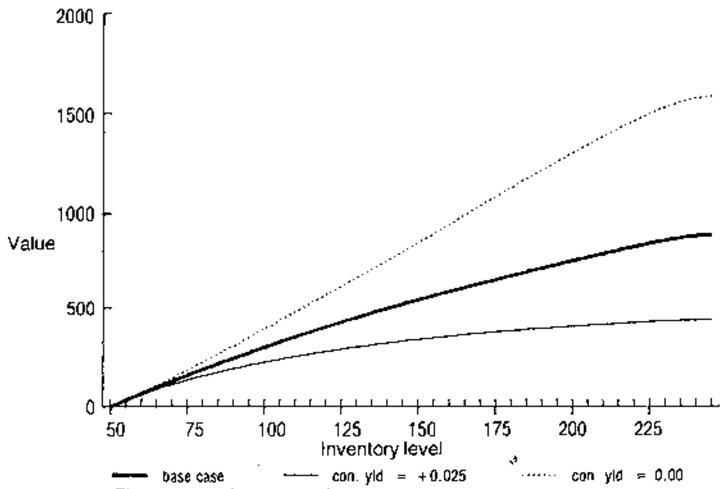
Value (in millions) vs. Price for Various Initial Inventory Levels*



* Time to end of lease = 40 quarters

FIGURE 8

Value (in thousands) vs. Inventory for Various Initial Price Levels*



* Time to end of lease = 40 quarters

FIGURE 9

Value (in thousands) vs. Inventory for Various Convenience Yields*

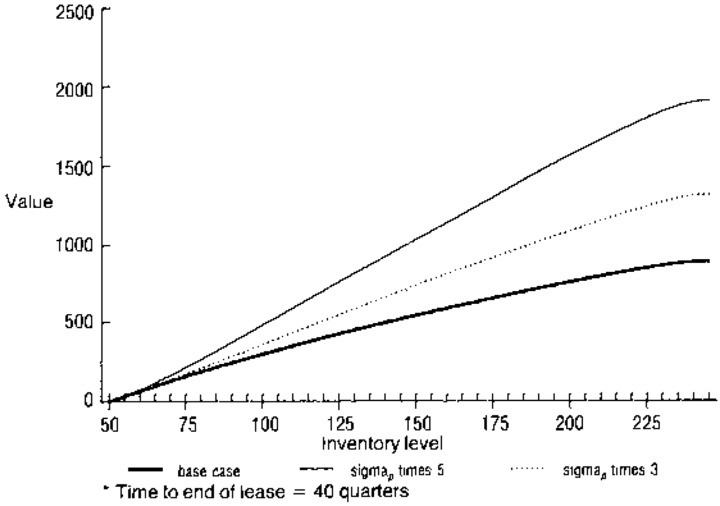


FIGURE 10

Value (in thousands) vs. Inventory for Various Levels of σ_p*

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