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Sovereign Debt: Optimal Contract, Underinvestment, and Forgiveness

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ABSTRACT

In this paper we develop a time consistent rational expectations model which analyzes the equilibrium loan contract between a borrowing country and a foreign bank. The loan contract specifies both the amount of the loan and the promised interest payments, and rationally reflects the investment decisions of the country and the possibilities of renegotiation and repudiation of the debt. An important feature of the model is that at the initial negotiation of the loan there is uncertainty about whether the country will renegotiate for partial forgiveness in the future, and whether it will eventually repudiate the debt, even having successfully renegotiated. Moreover, the probabilities of renegotiation and repudiation, and the amount of possible forgiveness are endogenously determined. In the model the repudiation decision is directly related to the underinvestment problem; the objective of the renegotiation is precisely to alleviate this problem. The model is used to analyze the effects of four variables on both the optimal contract and the country's welfare: the degree of penalties that a bank can impose on a defaulting country, the uncertainty of production, the productivity of investments and the riskless interest rate. The analysis has policy implications as well as testable predictions.

DURING THE LAST TEN years the issues related to less developed countries' (LDCs) debt have become increasingly more important in the world economy. The main reason for this is the significant increase in the amount of debt outstanding and the consequent higher yearly interest payments. By 1988, seventeen highly indebted countries had an aggregated debt of 510 billion dollars (see Rogoff (1990)). A second important reason is that starting in 1982 some of these countries suspended debt service payments, creating a serious crisis of confidence in the banking system of the lending countries.

As a consequence, borrowers and lenders have had to engage in negotiations including the rescheduling of the repayments and possibly partial forgiveness. A number of papers have suggested that forgiveness could conceivably make both lenders and borrowers better off because the country

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will be more likely to pursue the type of investments that lead to higher levels of output and therefore of debt repayment, if the banks are willing to absorb a smaller part of the country's output (see for example Sachs (1985, 1988), Kenen (1990), Portes (1987) and Krugman (1987b)). The "overhang" of external debt creates a tax on new investment, as the debtor country realizes that the creditor banks will be entitled to a share of any economic growth (see Sachs (1990)). This underinvestment problem was first identified at the firm level by Myers (1977), and gives rise to a situation that resembles the 'Laffer curve' from tax policy, in which case creditors can increase their expected repayments by forgiving part of the debt (see Eaton (1990)).

There is a second type of inefficiency that is peculiar to sovereign debt. Given that governments are usually the main debtors in the highly indebted countries, they will have incentives to keep a lower than optimal exchange rate since increasing the value of the foreign currency will automatically increase the budget deficit in local currency (see Dornbusch (1988)).

Another issue discussed in the literature is the possibility of indexing the debt repayments either to a variable endogenous to the country, such as output or exports of the country, or to an exogenous variable, such as a commodity price or interest rate. Lessard (1987) and Helpman (1987) argue that output indexation may be an effective risk sharing device since lenders are better able to diversify their portfolios than the borrowing country. Krugman (1987a), however, points out that obvious moral hazard considerations make indexation to an exogenous variable preferable. Froot, Scharfstein and Stein (1989) suggest that an endogenous variable may still be desirable when there is asymmetric information about some characteristic of the country, since indexation to an endogenous variable may help to solve the misrepresentation problem in a separating equilibrium.

The literature has also studied the repudiation decision of a sovereign borrower based on reputation arguments. The benefit of a continued access to the capital markets are compared against the cost of making the scheduled repayments. For papers on this topic see Eaton and Gersovitz (1981a, 1981b), Kletzer (1984), Grossman and Van Huyck (1988), Eaton, Gersovitz, and Stiglitz (1986), Kulatilaka and Marcus (1986), and Chowdhry (1991).

Giammarino and Nosal (1991) look at the rescheduling problem and examine its efficiency with respect to investment opportunities which emerge subsequent to a major default in existing debt.

As the above discussion indicates, the literature has mainly focused on the problems that arise when a country that has borrowed in a previous period is unable or unwilling to meet its debt obligations. An exception is a recent paper by Bulow and Rogoff (1989) who use Rubinstein's (1982) bargaining framework to develop a dynamic model in which renegotiation of the debt can take place continuously and where the amount that the banks can bargain out of the country in the future determines the maximum that the country is initially allowed to borrow. Eaton and Gersovitz (1981a, 1981b) look at the initial loan contract and take into account the possibility of repudiation, but do not consider the possibility of renegotiation.

In this paper we develop a time consistent rational expectations model which analyzes the equilibrium loan contract between a borrowing country and a foreign bank. The loan contract specifies both the amount of the loan and the promised interest payments, and rationally reflects the investment decisions of the country and the possibilities of renegotiation and repudiation of the debt. An important feature of the model is that at the initial negotiation of the loan there is uncertainty about whether the country will renegotiate for partial forgiveness in the future, and whether it will eventually repudiate the debt, even having successfully renegotiated. Moreover, the probabilities of renegotiation and repudiation, and the amount of possible forgiveness are endogenously determined. In the model the repudiation decision is directly related to the underinvestment problem; the objective of the renegotiation is precisely to alleviate this problem. The model is used to analyze the effects of four variables on both the optimal contract and the country's welfare: the degree of penalties that a bank can impose on a defaulting country, the uncertainty of production, the productivity of investments and the riskless interest rate. The analysis has policy implications as well as testable predictions.

Underinvestment is more likely to be a problem in the case of sovereign debt considered here than in the case of corporate debt originally studied by Myers (1977), as in the case of countries it is more difficult to enforce restrictive covenants on the debtors. For example, dividend restrictions at the firm level are easier to enforce than consumption constraints at the country level. While the underinvestment problem can be considered a good explanation for the existence of restrictive loan covenants in the case of firms, these covenants seem to go a long way in solving the problem.

The paper is organized as follows. In Section I we develop the model. In Section II we present numerical simulations and comparative statics with respect to key parameters of the model. Finally, in Section III we summarize the main results of the paper and suggest some empirical and policy implications of the model.

I. The Model

In this section we develop a three-period model which captures some of the essential elements of the negotiation and renegotiation process for LDCs debt. Figure 1 illustrates the sequence of events of the model. In Period 0 a country agrees to take a loan from a bank to be repaid in Period 2. The loan is used to finance an investment project, which requires an additional "maintenance" investment in Period 1. In Period 1 the outcome of the Period 0 investment project is realized and the country decides whether to renegotiate the debt or not. If the decision is positive the renegotiation process takes place and the country decides how much to reinvest. In Period 2 the outcome of the Period 1 investment is realized and the country repays the debt.

The model considers three forms of uncertainty. The first source of uncertainty relates to the productivity of the investments, the second to the costs of

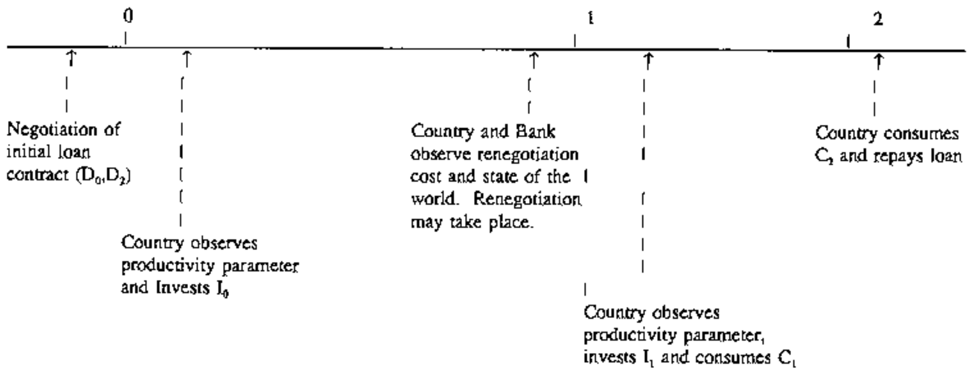

Figure 1. Chronology of the model.

renegotiation and the third to the "state of the world." Neither the country nor the bank know some productivity parameters when the initial negotiation in Period 0 and the subsequent renegotiation in Period 1 take place, but the country learns them before making the investments. The state of the world and the costs of renegotiation in Period 1 are unknown in Period 0 when the debt is taken and the first investment is made. The realizations of these random variables are learnt simultaneously by both the country and the bank in Period 1 before the renegotiation process takes place.

The production shocks that the country learns before making the investment decisions are meant to reflect the fact that after the negotiations, as time goes by, the country gains more information pertinent to the productivity of its investments, and will use this information in its best interest. Even though there is no asymmetric information at the time of the negotiations, both parties know that the country will have an informational advantage when making the investment decisions.

Renegotiation is costly to the country, but does not represent income for the bank. The probability distribution of the renegotiation cost is common knowledge in Period 0 and its realization is simultaneously observed by the bank and the country in Period 1 before the renegotiation process starts. The renegotiation cost summarize different types of costs that the country faces if it renegotiates, all having in common that they do not represent income to the bank: The verification costs of having a committee coming and checking that the country has been adversely affected by external circumstances and has real difficulties in honoring the initial contract, the transaction costs of the renegotiation itself, and the loss in reputation that affects its trade credit and its future access to the capital markets. This cost is important in our model, because without it the country would always renegotiate, making the promised debt payment irrelevant and the promised interest payment indeterminate. Renegotiation costs are random since this cost may depend upon the situation of other highly indebted countries and upon the fiscal and monetary policies followed by this country and by the others. The model also allows the bank to contribute to the renegotiation costs to facilitate the renegotiation process in the case that the country would not, by itself, renegotiate, and renegotiation is in the best interest of the bank.[1]

For simplicity we assume that there are only two possible states of the world in Period 1, denoted by a and b, each one occurring with probability 0.5. This is modeled by assuming that in Period 0 there are two alternative distributions for the productivity in Period 1. Both parties learn the state of the world, i.e., the relevant distribution, in Period 1 before the renegotiation process starts. The state uncertainty attempts to summarize the exogenous future conditions of the world economy that affect the ability of the country to pay its debt, and that determine the price of the commodity. The fact that this source of uncertainty is symmetric in the model would make it possible to design a contract contingent on the state of the world. Even though this type of contract is more efficient, in this paper we assume that contracts contingent on the state of the world are not possible, because even if the state of nature is observable by both parties it may not be possible to validate it in court, or, alternatively, because in reality there is a very large number of states of the world, making such a contingent contract complex and expensive. In essence, we focus on one way to alleviate the Myers underinvestment problem: the possibility of renegotiation of the debt and we abstract from contracts that would eliminate the problem in the first place.[2]

For tractability we will assume that both the country and the bank are risk neutral.[3] The country is assumed to make its borrowing, investment, and renegotiation decisions so as to maximize the expected value of a linear utility of consumption in Periods 1 and 2. To simplify the problem and without loss of generality, we have assumed that there is no consumption in Period 0. The country's utility function can then be written as:

$$U(C_1, C_2) = C_1 + \rho C_2 \tag{1}$$

where C_1 and C_2 are consumptions in Periods 1 and 2, respectively, and ρ is the time preference parameter.

We assume that the bank operates in a competitive environment so that in equilibrium the terms of the loan will be set such that the bank's net expected profits are zero.

Let I_0 and I_1 be the investments in Periods 0 and 1, and X_1 and X_2 the outputs of these investments in Periods 1 and 2, respectively. The outputs in Periods 1 and 2 are given by:

$$X_1 = g(I_0)\varepsilon_1 \tag{2}$$

$$X_2 = g(I_1)\varepsilon_2 \tag{3}$$

[1] The Brady plan, unveiled by the Secretary of the Treasury Nicholas Brady in March 1989, has endorsed the concept of voluntary debt reduction.

[2] Short-term debt may play a similar role in eliminating the underinvestment problem.

[3] By using a linear utility function we abstract from risk-sharing issues.

where $g(\cdot)$ is the production function satisfying $g(0) = 0$, $g' > 0$ and $g'' < 0$, and ε_1 and ε_2 represent stochastic shocks to the production process. When the negotiation of the debt occurs in Period 0, both the bank and the country know the probability distribution of ε_1. However, the country learns the realization of ε_1 before making the investment decision, also in Period 0. Similarly, only the probability distribution of ε_2 is known to both parties when the renegotiation decision is made, but the country observes the realization of ε_2 before making the investment decision in Period 1.[4]

Let D_0 be the amount of loan given in Period 0, and D_2 the amount contracted to be paid back in Period 2. The promised interest rate on the loan, R, is given by $(1 + R)^2 = D_2/D_0$. We assume for simplicity that there are no promised interest payments in Period 1. Let D represent the relevant repayment obligation, defined as the original face value of the debt D_2 if there is no renegotiation and the renegotiated face value of the debt, otherwise. Our model assumes that the bank can imposed penalties on the country in the event of default. These penalties include: (1) seizure of the country's overseas assets, (2) reduction of the country's gains from trade since the bank might seize payments to firms that attempt to export to the country and payments made by firms that attempt to import from it, and (3) restricted access to the capital markets in the future. These penalties are modeled as a fraction β of the country's total output. Therefore the actual payment of the debt in Period 2, P, is given by:

$$P = \min\{D, \beta X_2\} \tag{4}$$

where $0 < \beta < 1$.

We further assume that the bank is able to capture only a lower fraction α of the country's output. The lost product, $(\beta - \alpha)X_2$, represents the administrative and legal costs of default and confiscation. It also can be interpreted as a measure of the loss of reputation of the country in the event of default. Therefore, the receipts of the bank in Period 2 are given by:

$$R = \begin{cases} D & \text{if } P = D \\ \alpha X_2 & \text{if } P < D \end{cases} \tag{5}$$

where $0 < \alpha < \beta < 1$.

Bulow and Rogoff (1989b) argue that there are many uncertainties surrounding the actual damage that a lender can inflict on a sovereign country, and that this is a "gray area" in Western Law. Our assumption that the bank can confiscate a fraction of the country's output is more general than the assumption of Froot, Scharfstein, and Stein (1989) who have $\alpha = \beta = 1$, which is similar to Bulow and Rogoff (1989a).

We assume that in the event of renegotiation the country will pay the full renegotiation costs if the total benefits that it derives from renegotiation are

[4] Since there is no consumption in Period 0, the investment in Period 0 is not affected by the country's observation of the productivity parameter.

larger than the costs, and otherwise it will pay only its total benefits, with the bank paying the difference.[5]

Consumption in Periods 1 and 2 is then given by:

$$C_1 = X_1 - I_1 - c_r \tag{6}$$

$$C_2 = X_2 - P \tag{7}$$

where c_r represents the net cost of renegotiation for the country, i.e., zero if there is no renegotiation, and the actual renegotiation costs net of side payments from the bank, if any, in the event that renegotiation occurs. The actual renegotiation cost for the country is the minimum of the renegotiation costs and the total benefits that the country obtains from renegotiation.

To avoid unnecessary notation we assume away endowment flows to the country from other sources. Because of this, and since there is no consumption in Period 0, the initial investment in Period 0 is financed entirely by the loan (i.e., $I_0 = D_0$). Also to simplify the algebra we assume that there are no liquidity problems in Period 1, that is to say, we assume that the country will have enough resources in Period 1 so that the optimal investment will be an interior solution and consumption will be nonnegative. Either introducing endowments flows from other sources or by appropriately choosing the parameters of the problem we can make sure that this will be the case.

In what follows we will develop in detail a special case of the model. This is done to provide a better intuition and to avoid excessive notation. Consider a square root production function and uniform distributed stochastic shocks to production and renegotiation costs. That is:

$$X_1 = \sqrt{I_0}\,\varepsilon_1, \qquad X_2 = \sqrt{I_1}\,\varepsilon_2 \tag{8}$$

$$\varepsilon_1 \sim U(A, A + y_1), \qquad \varepsilon_2 \sim U(A, A + y_2) \tag{9}$$

where y_1 is known in Period 0, and y_2 can take two possible values in Period 1, y_{2a} and y_{2b}, with equal probability. These values of y_2 summarize the two possible states of the world in Period 1; the actual state of the world is learnt in Period 1; A represents a lower bound of the productivity of the investments. Let k represent the total renegotiation costs and K the maximum value that this random variable can take:

$$k \sim U(0, K) \tag{10}$$

Finally, we assume that ε_1, ε_2 and k are independent random variables.

The country is faced with three decisions. In sequential order they are: (1) the negotiation of the debt and investment in Period 0, (2) the possible renegotiation of the debt in Period 1, and (3) the investment decision in Period 1. In the spirit of dynamic programming we solve the problem backwards, starting with the productive decision in Period 1.

[5] Obviously, this is a particular specification. There are many other conceivable ways to divide the surplus from the renegotiation.

A. The Period 1 Investment Decision

We will analyze the investment decision in Period 1 in two steps. Initially we assume that the country will repay the debt D and we compute the optimal investment in this case. Then we assume that the country will repudiate the debt and we compute the optimal investment under these conditions. Finally, we compare the two solutions to find the global optimum.

To determine the optimal investment in Period 1 the country maximizes its utility. This is equivalent to maximize the net present value (NPV) of the Period 1 investment using as discount rate the time preference parameter:

$$I_1^* = \operatorname{argmax}[-I_1 + \rho(X_2 - P)] \tag{11}$$

From equation (4) and equation (8), the optimal investment of equation (11) for the no-default and default cases are respectively given by:

$$I_1^* = \operatorname{argmax}\left[-I_1 + \rho\left(\sqrt{I_1}\,\varepsilon_2 - D\right)\right] \tag{12}$$

$$I_1^* = \operatorname{argmax}\left[-I_1 + \rho(1-\beta)\sqrt{I_1}\,\varepsilon_2\right] \tag{13}$$

Figure 2A shows the NPV evaluated for the optimal investments (12) and (13) for different values of ε_2. The intersection of these two curves defines a threshold for ε_2, below which it is optimal for the country to default. This critical value, denoted by $\bar{\varepsilon}$, is given by:

$$\bar{\varepsilon} = \sqrt{\frac{4D}{\beta(2-\beta)\rho}} \tag{14}$$

and clearly depends on the relevant repayment obligation, D.

The first order conditions in the two cases considered above give the following global optimal investment policy:

$$I_1^* = \begin{cases} \left[\dfrac{\varepsilon_2\,\rho(1-\beta)}{2}\right]^2, & \text{if } \varepsilon_2 < \bar{\varepsilon} \\ \left[\dfrac{\varepsilon_2\,\rho}{2}\right]^2, & \text{otherwise} \end{cases} \tag{15}$$

The second order condition for a maximum is always satisfied due to the concavity of the production function. The output corresponding to this investment is:

$$X_2 = \begin{cases} \dfrac{\varepsilon_2^2\,\rho(1-\beta)}{2}, & \text{if } \varepsilon_2 < \bar{\varepsilon} \\ \dfrac{\varepsilon_2^2\,\rho}{2}, & \text{otherwise} \end{cases} \tag{16}$$

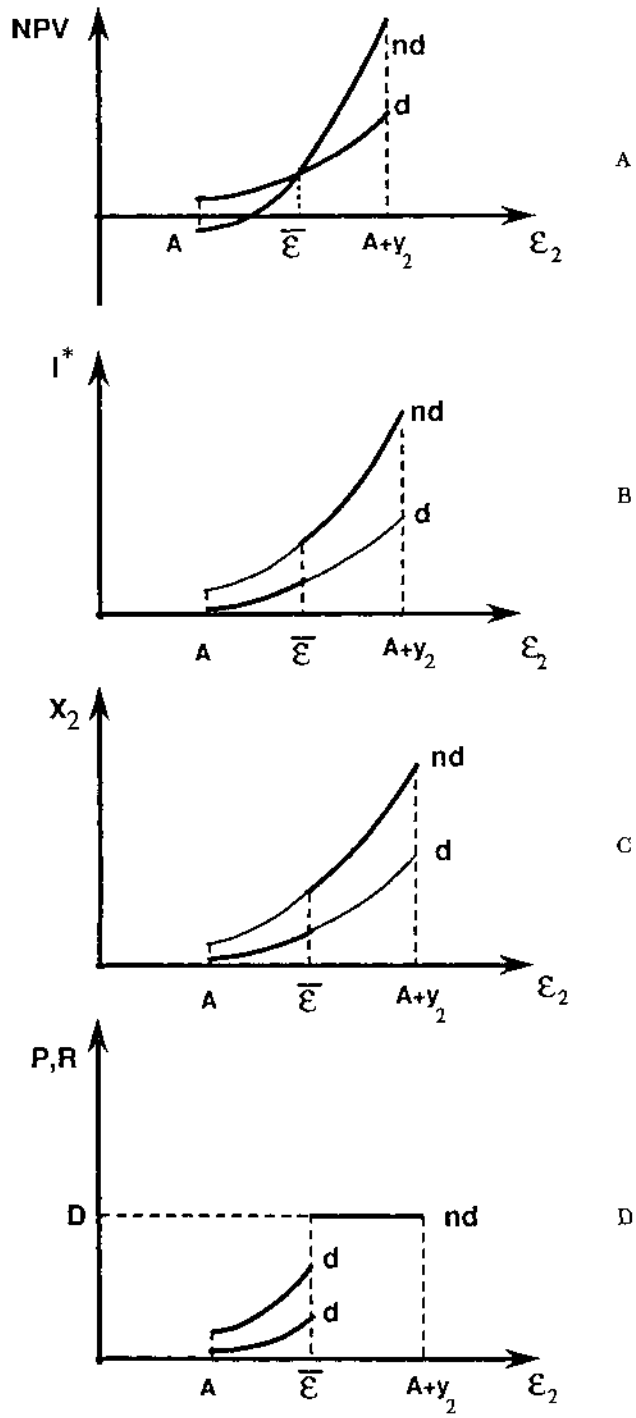

Figure 2. A, net present value. B, investment. C, production. D, payment, receipts.

The country's payment is then:

$$P = \begin{cases} \dfrac{\varepsilon_2^2 \rho\beta(1-\beta)}{2}, & \text{if } \varepsilon_2 < \bar{\varepsilon} \\ D, & \text{otherwise} \end{cases} \tag{17}$$

and the bank's receipt is:

$$R = \begin{cases} \dfrac{\varepsilon_2^2 \rho\alpha(1-\beta)}{2}, & \text{if } \varepsilon_2 < \bar{\varepsilon} \\ D, & \text{otherwise} \end{cases} \tag{18}$$

Note that the optimal investment, the Period 2 production, the country's payment and the bank's receipt, all functions of ε_2, are discontinuous at $\varepsilon_2 = \bar{\varepsilon}$, passing from the default solution to the no-default one. However, the *NPV* in Period 1 from which the optimal investment is derived is continuous, with the country being indifferent between the default and no-default solutions at $\varepsilon_2 = \bar{\varepsilon}$. These functions are depicted in Figure 2.

The underinvestment problem arises when it is optimal for the country to repudiate the debt (i.e., when $\varepsilon_2 < \bar{\varepsilon}$). In this case, since the country will lose a fraction β of its output, the incentives to invest decrease and the country produces less than the socially optimal amount. In the case of total expropriation ($\beta = 1$), if the country invests, it will always invest the socially optimal amount and the underinvestment problem will manifest itself in the possibility of the country not investing at all under certain circumstances.

B. The Renegotiation Process

The country has the option to initiate the renegotiation process by incurring in a cost k. If the country asks for renegotiation, the bank will set the optimal level of renegotiated debt, D^*, taking into account the trade-off between the probability of repayment and the amount repaid. Recall that at the time of renegotiation in Period 1 both the state of nature and the country's cost of renegotiation are known, but the country has not yet observed the realization of ε_2. Also, if the country finds it not worthwhile to renegotiate, but the total benefits from renegotiation of the country and of the bank combined are larger than the renegotiation cost k, then the bank will find in its interest to give a side payment to the country to facilitate the renegotiation process. In this circumstances the surplus of the country will be zero, since the bank will give just the amount that will leave the country indifferent between renegotiating and not. We say that in this second case renegotiation is initiated by the bank.

In order to study the renegotiation process we first compute the bank's expected receipts, and then we compute the optimal level of renegotiated debt, D^*, as the level of debt that maximizes the expected receipt of the bank. We use the results of the renegotiation to investigate in which cases renegotiation will take place and when it will be initiated by the bank and when by the country.

Taking the expected value of (18) over the distribution of the stochastic shock to production ε_2, it is possible to obtain the bank's expected receipts as a function of the face value of the debt. Since $\bar{\varepsilon}$ is positively related to D there are three cases to consider:

Case 1. *No-Default for Sure* ($\bar{\varepsilon} \leq A$)

Since A is the lower bound for ε_2, $\bar{\varepsilon} \leq A$ implies that always $\varepsilon_2 \geq \bar{\varepsilon}$. In this case the debt will be repaid with probability 1, because the amount of debt is small enough to always induce an efficient investment behavior. Thus,

$$ER(D) = D \tag{19a}$$

Case 2. *Uncertain Default* ($A < \bar{\varepsilon} < A + y_2$)

In this intermediate case there is some probability (less than 1) that the debt will be repudiated:

$$ER(D) = \frac{\rho\alpha(1-\beta)}{6y_2}[\bar{\varepsilon}^3 - A^3] + \frac{D}{y_2}[A + y_2 - \bar{\varepsilon}], \tag{19b}$$

Case 3. *Default for Sure* ($\bar{\varepsilon} \geq A + y_2$)

Because $A + y_2$ is the upper bound for ε_2, in this case ε_2 is always smaller than $\bar{\varepsilon}$ and the debt will be repudiated for sure. In this case the debt is large enough to always induce the inefficient investment behavior. Since the debt will always be repudiated the expected receipt of the bank is independent of the face value of the debt:

$$ER(D) = \frac{\rho\alpha(1-\beta)}{6y_2}\left[(A + y_2)^3 - A^3\right] \tag{19c}$$

Figure 3 illustrates the expected bank's receipts as a function of the face value of the debt. Maximizing this expected receipt function with respect to the debt level D we obtain the optimal renegotiated debt D^*:

$$D^* = \begin{cases} \left[\dfrac{A + y_2}{3\beta(2-\beta) - 2\alpha(1-\beta)}\right]^2 \beta^3\rho(2-\beta)^3, \\ \quad \text{if } y_2 > \dfrac{A[\beta(2-\beta) - 2\alpha(1-\beta)]}{2\beta(2-\beta)} \\ \left[\dfrac{\beta(2-\beta)\rho}{4}\right]A^2, \qquad \text{otherwise} \end{cases} \tag{20}$$

Note that there are two possible solutions for the renegotiated face value of the debt. The first one corresponds to risky debt, i.e., debt that may eventually be repudiated. The second is the risk free debt solution. If the global conditions of the economy summarized by y_2 are promising, the bank will prefer the risky solution.

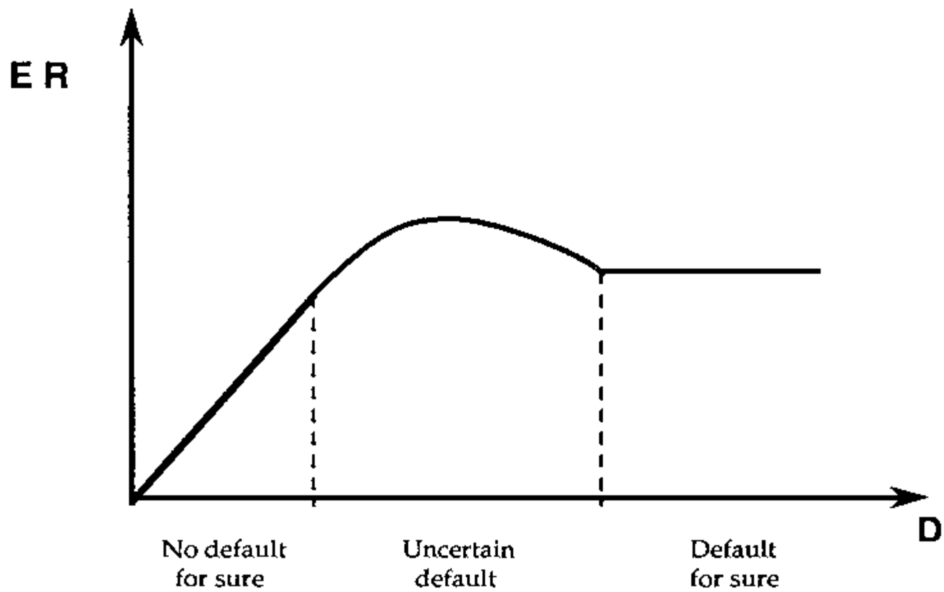

Figure 3. Bank's expected receipt.

Looking at Figure 3, the optimal renegotiated debt is risk free when the maximum of the expected receipt function is attained at the end of the no-default area; this occurs when the expected receipt function is decreasing in all the uncertain default area.

In order to find out whether the country will decide to initiate a renegotiation or not we first concentrate on the country's benefits of the renegotiation. To do this we compute its Period 1 expected utility for a given level of debt and without considering the renegotiation costs, $EV(D)$. We need to consider the same three cases of (19).

Case 1. No-Default for Sure ($\bar{\varepsilon} \leq A$)

$$EV(D) = X_1 + \frac{\rho^2}{12 y_2}\left[(A + y_2)^3 - A^3\right] - \rho D \tag{21a}$$

Case 2. Uncertain Default ($A < \bar{\varepsilon} < A + y_2$)

$$EV(D) = X_1 + \frac{[\rho(1-\beta)]^2}{12 y_2}[\bar{\varepsilon}^3 - A^3] + \frac{\rho^2}{12 y_2}\left[(A + y_2)^3 - \bar{\varepsilon}^3\right] - \frac{\rho D}{y_2}[A + y_2 - \bar{\varepsilon}] \tag{21b}$$

Case 3. Default for Sure ($\bar{\varepsilon} \geq A + y_2$)

$$EV(D) = X_1 + \frac{[\rho(1-\beta)]^2}{12 y_2}\left[(A + y_2)^3 - A^3\right] \tag{21c}$$

Figure 4 shows the country's Period 1 expected utility as a function of the level of debt and not including the renegotiation costs. Both the country and the bank anticipate the result of the renegotiation process, D^*, and in order to make the renegotiation decision they compare their respective situation with and without renegotiation. The country's expected gain in utility in Period 1 from renegotiation is:

$$\nu^c = EV(D^*) - EV(D_2) \tag{22}$$

and the bank's expected gain from renegotiation in Period 2, discounted to Period 1, is:

$$\nu^b = \frac{ER(D^*) - ER(D_2)}{1 + r} \tag{23}$$

where r is the risk free rate of interest.

As described before we model the renegotiation process in the following way: Renegotiation takes place if the benefits of the country and the bank combined are higher than the renegotiation cost k. If the country's benefits are larger than the cost k, then renegotiation is initiated by the country, which pays the full costs k. Otherwise, if the renegotiation costs are higher

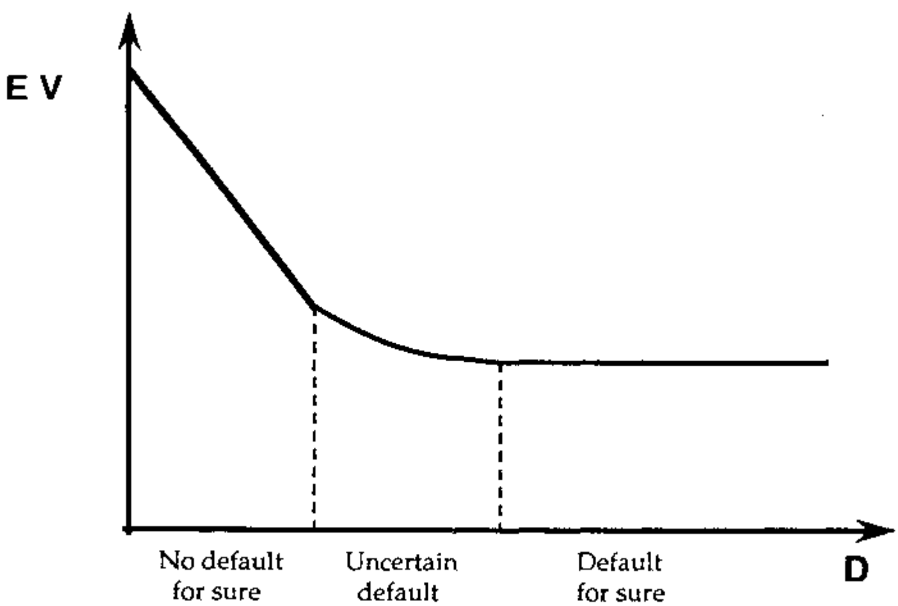

Figure 4. Country's expected utility.

than the country's benefits but less than the global benefits, then the bank initiates the renegotiation process by giving a side payment to the country that leaves it just indifferent between renegotiation or not renegotiation. The country rationally anticipates the outcome of the renegotiation process and decides whether it is optimal to renegotiate, taking into account the renegotiation cost and the amount to be forgiven, $D_2 - D^*$. In order to make this decision the country compares its utility with versus without renegotiation.

C. The Loan Negotiation

In Period 0 when the loan contract takes place, both the country and the bank face three sources of uncertainty: the state of the world, the cost of renegotiation, and the productivity parameters. They will assess the probabilities of every possible outcome and competition among banks will lead to the contract (D_0, D_2), which maximizes the expected utility of the country from the set of contracts that yield a zero net profit for the bank.

In order to compute the expected utility of the country in Period 0 we need to obtain the Period 0 expected output and renegotiation costs in Period 1 and the probabilities of renegotiation of the debt. The expected value of X_1 at time 0 is:

$$E[X_1] = \sqrt{D_0}\left[A + \frac{y_1}{2}\right] \tag{24}$$

The country's expected renegotiation cost at time 0 conditional on it initiating the renegotiation is:

$$R_c = \frac{m}{2}, \tag{25}$$

where

$$m = \min(K, \nu^c)$$

since the country will pay the renegotiation costs up to the total benefits that it derives from the renegotiation.

The probability that renegotiation will occur is equal to the probability that the total gains from renegotiation be less than the renegotiation costs:

$$\phi = \begin{cases} \dfrac{\nu^c + \nu^b}{K}, & \text{if } \nu^c + \nu^b \le K \\ 1, & \text{if } \nu^c + \nu^b > K \end{cases} \tag{26}$$

More precisely, the probability that the renegotiation be initiated by the country is:

$$\phi_c = \text{prob}(k < \nu^c) = \begin{cases} \dfrac{\nu^c}{K}, & \text{if } \nu^c < K \\ 1, & \text{if } \nu^c > K \end{cases} \tag{27}$$

And the probability that the renegotiation be initiated by the bank is:

$$\phi_b = \text{prob}(\nu^c < k < \nu^c + \nu^b) = \begin{cases} 0, & \text{if } \nu^c > K \\ \dfrac{K - \nu^c}{K}, & \text{if } \nu^c \leq K \leq \nu^c + \nu^b \\ \dfrac{\nu^b}{K}, & \text{if } K > \nu^c + \nu^b \end{cases} \tag{28}$$

Note that in all these probabilities it is assumed that the country's benefits are positive. This would not the case when the optimal renegotiated debt is higher than the current promised debt, but in this situation renegotiation does not make sense.

In (21) we computed the Period 1 expected utility for the country excluding the renegotiation cost and for any face value of the debt. Now we calculate the Period 0 expected utility including the renegotiation cost and incorporating the optimal renegotiation decision. If the country does not renegotiate, it obtains the utility associated with the promised loan repayment D_2. If it does renegotiate, the country obtains a positive surplus only if it initiates the renegotiation, since in the other case the bank will give the country a payment that makes it indifferent between renegotiation or not renegotiation of the debt. Then the Period 0 expected utility of the country for a given state of the world and for a given loan contract is:

$$E[U(D_0, D_2)/y_2] = E_0[EV(D_2)/y_2] + \phi_c[\nu^c - R_c] \tag{29}$$

where R_c is defined in (25). The expected utility of the country in Period 0 for the contract $(D_0,\ D_2)$ is equal to the expected value in Period 0 of the expected utility of the same contract in Period 1 plus the expected surplus from changing the terms of the contract. Equation (29) holds for each of the two possible states of the world y_{2a} and y_{2b}. By the law of iterative expectations the country's unconditional expected utility for the contract $(D_0,\ D_2)$ is given by:

$$E\{U(\cdot)\} = E\{E[U(\cdot)/y_2]\} = 0.5\{E[U(\cdot)/y_{2a}] + E[U(\cdot)/y_{2b}]\} \tag{30}$$

In order to compute the Period 0 expected profits of the bank we need to calculate the expected side payments to the country in order to facilitate renegotiations. The bank's expected renegotiation cost conditional on it initiating the renegotiation is:

$$R_b = \frac{M - \nu^c}{2}, \tag{31}$$

where

$$M = \min(K, \nu^c + \nu^b)$$

since the bank will start paying for the renegotiation cost only after the country has paid its total benefits from renegotiation, and the bank will pay only up to the total benefits that it derives from the renegotiation.

The Period 0 expected benefits of the bank as a function of the loan contract and the state of nature are:

$$E[\pi(D_0, D_2)/y_2] = -D_0 - \frac{\phi_b R_b}{(1+r)} + \frac{(1-\phi)ER(D_2) + \phi ER(D^*)}{(1+r)^2} \quad (32)$$

where the first term is the initial loan, the second term is the expected Period 1 renegotiation expenses of the bank discounted at the risk free rate, and the last term corresponds to the expected Period 2 receipt taking into account the possibility of renegotiations and discounted at the risk free rate. Equation (32) holds for the two possible states of the world y_{2a} and y_{2b}. By the law of iterative expectations the bank's unconditional expected profit for the contract (D_0, D_2) is:

$$E\{\pi(\cdot)\} = E\{E[\pi(\cdot)/y_2]\} = 0.5\{E[\pi(\cdot)/y_{2a}] + E[\pi(\cdot)/y_{2b}]\} \quad (33)$$

Figure 5 illustrates the six possible outcomes in Period 2 for each state of nature, determined by the country's actions in Period 1: no renegotiation, renegotiation initiated by the country, and renegotiation initiated by the bank in the two different states of the world.

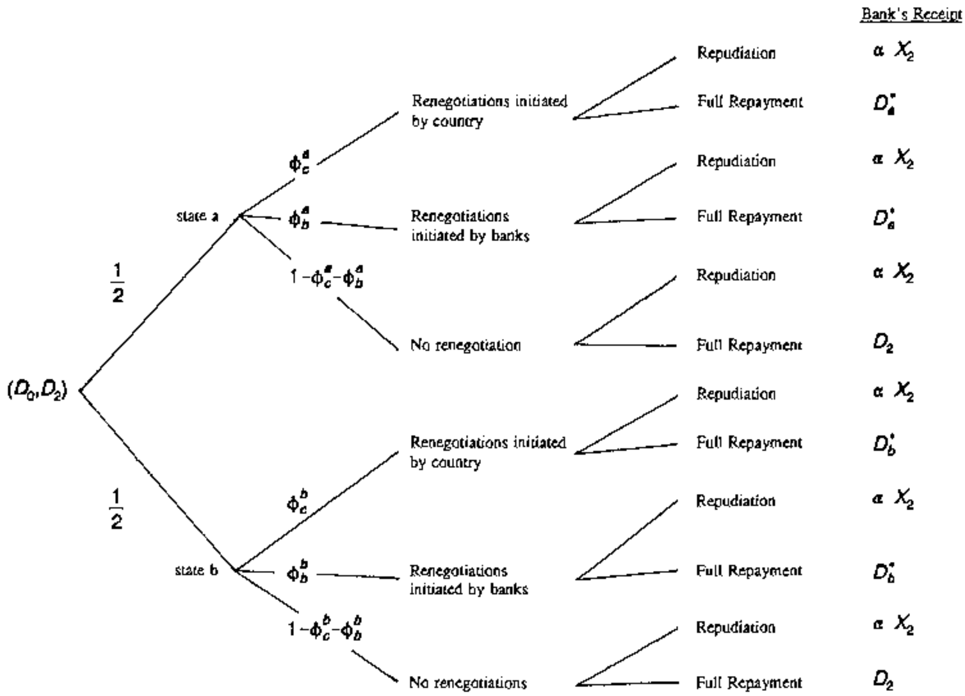

Figure 5. Outcomes and repayments.

Finally, the equilibrium contract is obtained by:

$$(D_0, D_2)^* = \text{argmax}\{EU(\cdot) \quad \text{s.t.} \quad E[\pi(\cdot)] = 0\} \tag{34}$$

The country will take this optimal loan contract if its expected utility with the loan is greater than without it. Since we have assumed no endowments this rationality constraint translates into the condition that the expected utility of the optimal contract must be greater than zero.

Even for the simple case that we have been considering, the maximization of the expected utility of the country subject to the zero expected profits for the bank in (30) gives rise to a nonlinear system of equations in D_0 and D_2 which does not have a closed form solution and must be solved numerically.

II. Comparative Statics

In this section we illustrate the nature of the equilibrium by numerically solving for the optimal contract for a basic example defined by the set of parameter values presented in Table I. We also investigate the effects on the optimal contract of perturbing the key parameters of the model.

The lower part of Table I reports the optimal contract for the basic example, the equilibrium risky interest rate on the loan, and the country's expected utility for this contract. In this example the difference between the risky interest rate and the riskless rate is 10.4% indicating a significant probability of default since by assumption the bank's expected profits are zero.

In the comparative statics analysis that follows we will focus on the optimal contract and on the country's expected utility for this optimal contract. A debt contract is defined by two variables: the amount of the loan and the contracted risky interest rate. These variables reflect the real and financial implications of the possibility of renegotiation and repudiation of

Table I

Parameter Values of Basic Example

Minimum production in Periods 1 and 2 (A)	1.5
Maximum production in Period 1 ($A + y_1$)	3.5
Maximum production in Period 2 in state a ($A + y_{2a}$)	3.0
Maximum production in Period 2 in state b ($A + y_{2b}$)	4.0
Maximum renegotiation costs (K)	0.01
Riskless interest rate (r)	0.05
Country's penalty rate (β)	0.55
Dead weight loss rate in case of default ($\beta - \alpha$)	0.05
Time preference (ρ)	0.95
Optimal contract	
Optimal loan (D_0^*)	0.604
Promised payment (D_2^*)	0.804
Risky interest rate	0.154
Country's expected utility	2.739

sovereign debt contracts. Since the bank has zero expected profits, the welfare implications of the analysis are captured by the country's expected utility. In other words, any increase in the country's expected utility is Pareto improving from a social point of view.

The comparative statics will be done with respect to four main parameters: the degree of penalties that can be imposed on a defaulting country, the production uncertainty of the investment projects, the productivity of the investments and the risk free rate of interest.

In our model the degree of penalty that a bank can impose on a defaulting country is given by the fraction β of Period 2 output that the country loses if it does not pay its debt. Figure 6 shows the effect on the optimal contract of changing the penalty rate β between 10% and 95% of output keeping all other parameters as in Table I. As shown in the figure, when the penalty increases from 10% to 30% the risky interest rate drops by more than 20% and the amount of the loan more than doubles reflecting a dramatic decrease in the riskiness of the loan. As penalties increase above 30% the optimal amount borrowed increases at a decreasing rate reaching a maximum when penalty rates are of the order of 60%. However, the risky interest rate decreases consistently reaching a minimum of 7% when the penalty rate is 95%.

Both the drop in the interest rate and the increase in the amount of the loan explain the sharp increase in the country's expected utility when penal-

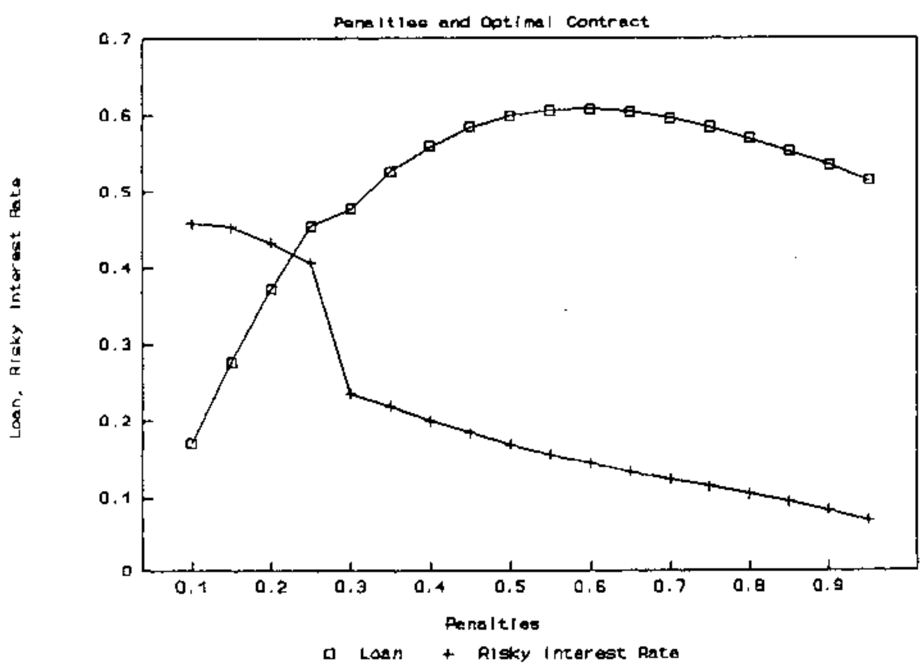

Figure 6.

ties increase from 10% to 30% observed in Figure 7. The most striking aspect of this figure, however, is that there is an interior level of penalties that maximizes the expected utility of the country. That is, there is a rate of penalty that maximizes social welfare. Ex-ante the country is better of with higher penalties since they give it a better access to the international credit market. Ex-post, however, higher penalties may produce larger inefficiencies in the investment decisions of the country lowering its expected utility. This trade-off is responsible for the concavity of the expected utility curve. To the extent that the level of penalties can be influenced by international regulation, our analysis has important policy implications.

We measure the production uncertainty of the investments by the spread between the maximum production attainable in the two possible state of the world in Period 1. We investigate the effect on the optimal contract of increasing the spread between these two variables from 0.2 to 2.0 while keeping constant their mean. As can be seen in Figure 8, the risky interest rate remains stable whereas the optimal amount of the loan deceases moderately as the spread increases from 0.2 to 1.6. When the spread increases above 1.6 the optimal contract jumps to a riskier equilibrium where both the risky interest rate and the amount borrowed increase significantly. An interesting result is that the country's expected utility of the contract is monotonically increasing with the uncertainty of the investments, reflecting the option features imbedded in the investment projects. It is well known

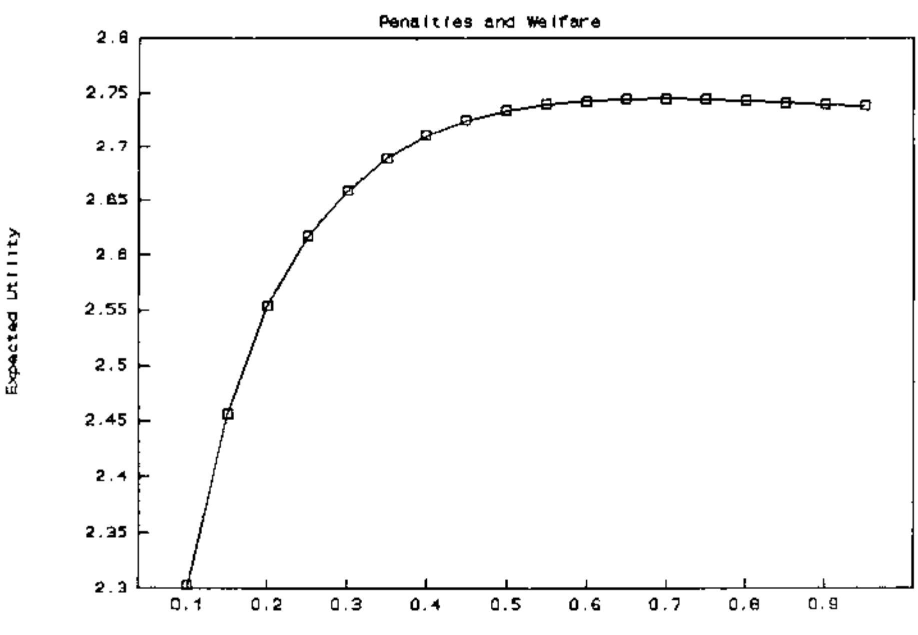

Figure 7.

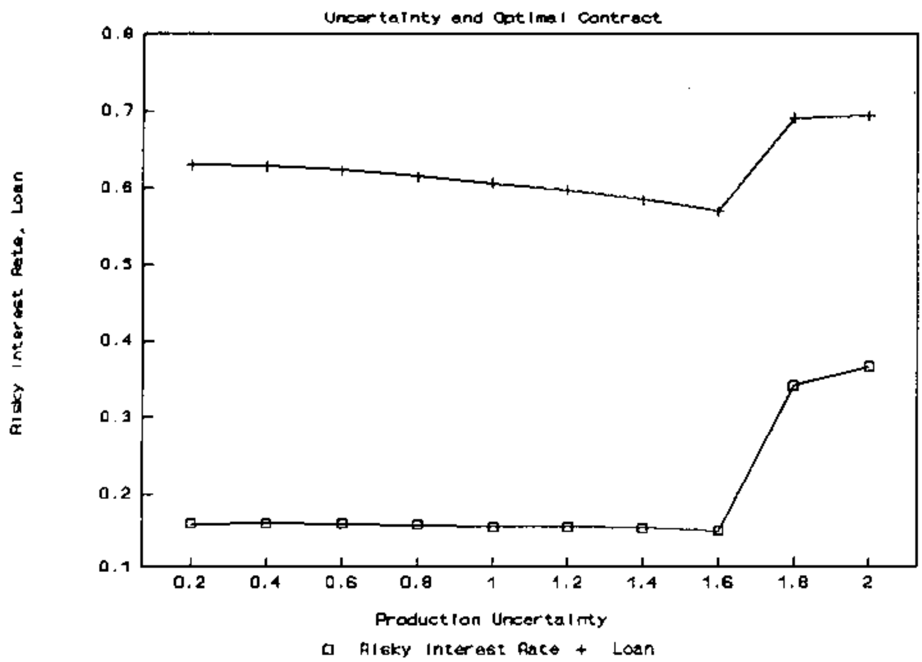

Figure 8.

that the value of an option increases with the volatility of the underlying asset; this result is preserved even in this situation where the bank is charging an interest which reflects the risk of the project.

The productivity of the investments is measured by the minimum production attainable. Changing this variable modifies the expected production without changing production volatility. Not surprisingly, increasing this variable results in higher expected utility and larger loans contracted at lower interest rates. Optimal contracts for different levels of productivity are shown in Figure 9.

The effects of changing the riskless interest rate on the optimal contract are shown in Figure 10. As can be expected increases in this variable result in lower expected utility for the country and smaller loans contracted at higher risky interest rates.

III. Summary and Conclusions

In this paper we have developed a framework to study sovereign debt starting from the initial negotiation process. Most of the literature up to date has concentrated in the renegotiation stage once the country has fallen into financial distress. Our main contribution has been to model the possibility of renegotiation and of repudiation of the debt and how it is taken into account in the initial loan contract.

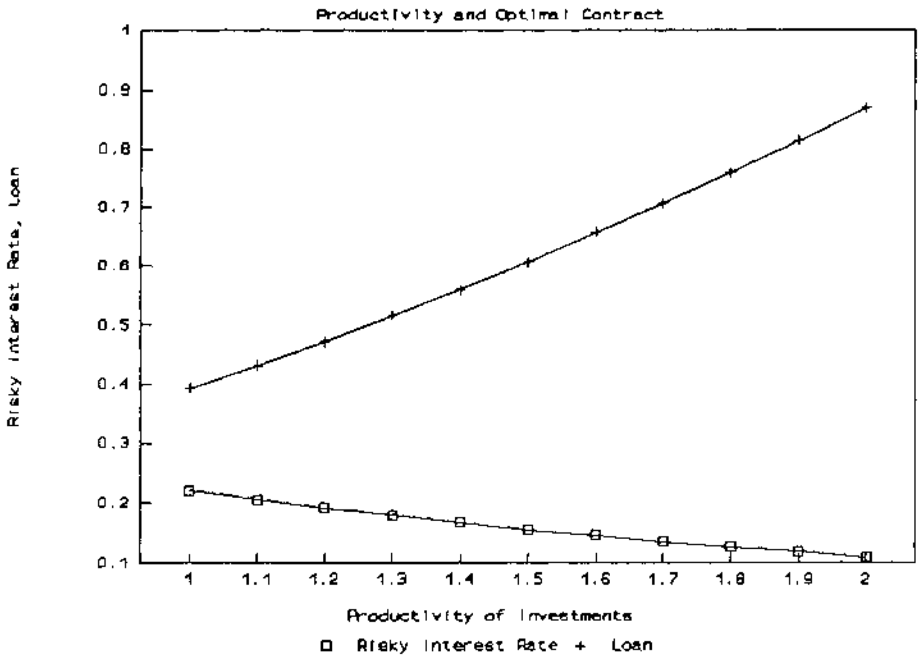

Figure 9.

The model allows us to determine the optimal amount of debt to borrow, based on the production possibilities of the country, its time preferences and the riskless interest rate prevailing in the world economy. In addition, we are able to determine a meaningful promised interest rate even with the possibility of renegotiation. To our knowledge this result has been absent from the literature.

The factors that determine the penalties that the country experiences in the event of default (summarized by the parameter β) can be broadly classified in two groups. The first one has to do with the characteristics of the country or its exports. The second one relates to the international regulatory environment. From this last perspective, considering β as a decision variable, our model can be used to derive policy implications for international regulation.[6] An implication of our model is that the higher the penalty that can be imposed on the country in the event of default, the lower will be the probability of default, but the higher will be the underinvestment problem in the event of default. Thus, comparative statics shows the existence of an optimal penalty from a welfare point of view, which in our model translates

[6] In 1976 the United States passed the Foreign Sovereign Immunities Act and in 1978 Great Britain passed the similar State Immunity Act. By clarifying and strengthening the righs of the creditor banks these laws made it easier for countries to borrow in the industrialized countries' financial markets.

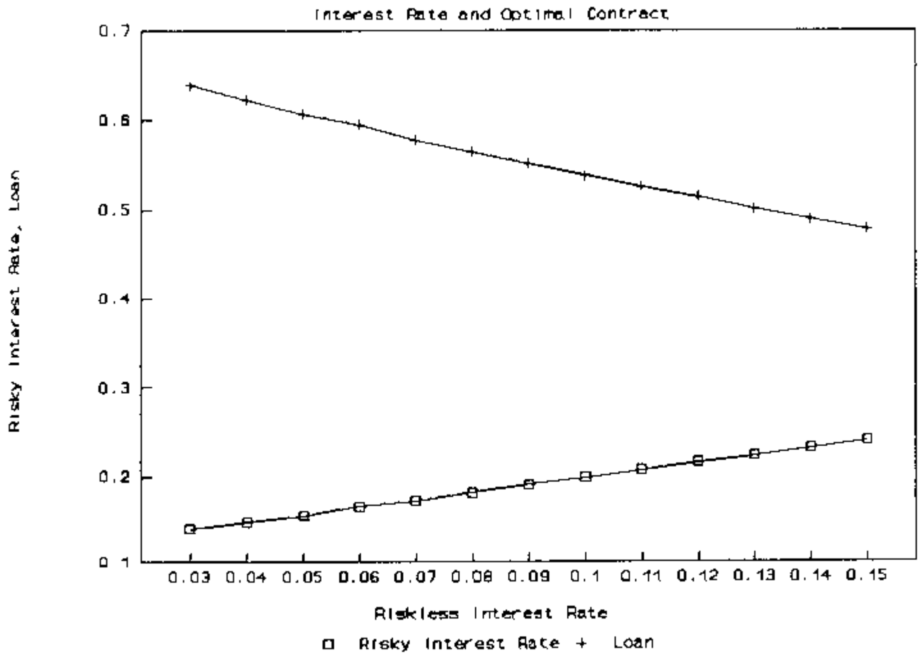

Figure 10.

into finding the value of β that maximizes the country's utility of the equilibrium contract.

Another interesting result of the model is that production uncertainty may benefit the country since it increases the value of the options features imbedded in its investment projects. This can occur in an equilibrium where banks take into account the option elements both in the amount they lend and in the interest rate they charge.

We have presented the discussion in the context of the less developed countries which have already experienced financial distress. We believe, though, that the arguments are equally relevant for considering possible new loans to the countries of Eastern Europe.

By modelling the problem as a one bank-one country contract, we abstract from issues of coordination and free rider problems that are present in the LDC debt crisis. The problem is that even if it were beneficial for the lending banks as a whole to forgive part of the debt, each bank will optimally try to free ride in the forgiveness of the others. This can lead to a less than socially optimal amount of forgiveness. A related problem that we do not consider is that of interdependence of renegotiations: when a bank renegotiates with any particular country, it is really weighing the effects of any concessions, not just with regards to that country, but also with regards to all the other countries that might press for similar terms.

Signalling mechanisms are absent from our model since there is no asymmetric information at the time of negotiations. This contrasts with Froot, Scharfstein and Stein (1989), where output indexation is used as a device for a separating equilibrium in which the countries reveal their types.

Many of the features of our model, and most models of LDCs debt, also apply to local debt. There seems to be, however, some differences in the mechanisms to deal with default at the local firm level, i.e., Chapter 11, and we do not observe to the same degree some of these issues, such as forgiveness, raised in this context. The absence of bankruptcy law in the international context means that debtor countries are unable to obtain a fresh start by having unpaid obligations discharged by a court. The fresh start aspect of bankruptcy law establishes property rights to investments opportunities that emerge. In contrast, sovereign borrowing can only be discharged through repayment, repudiation or forgiveness (see Giammarino and Nosal (1991)). This makes particularly relevant the study of underinvestment in the sovereign debt context.

The model developed in this paper has some possible empirical implications. For example consider the parameter β which represents the fraction of the total output that the country loses in the event of repudiation of the debt. It is reasonable to interpret this parameter as a summary of all the factors that determine the ability of the bank to impose costs on the country if it decides not to pay the debt: the international trade of the country (measured, say, by the proportion of exportable in the total output), the dependence of the country on international lending, etc. Because β varies across countries and for a given country across time it could be possible to design both cross sectional and time series tests of the effect of this factor on the equilibrium contract. This is an interesting topic for further research.

Another line of empirical work comes from the possibility of parameterizing the production technology by assuming, for example, that is of the Cobb-Douglas type with parameter γ:

$$X_2 = (I_1)^{\gamma} \varepsilon_2$$

With this specification it may be possible to test the implications on the optimal contract of different production functions, by estimating this parameter in a cross section across countries.

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