Valuing Long-Term Commodity Assets

Eduardo S. Schwartz

Eduardo S. Schwartz is a Professor of Finance at the Anderson Graduate School of Management, University of California, Los Angeles.

I present a one-factor model for the stochastic behavior of commodity prices that retains most of the characteristics of a more complex two-factor stochastic convenience yield model in terms of its ability to price the term structures of futures prices and volatilities based on the pricing and volatility results of the two-factor model. When applied to the valuation of long-term commodity projects, it gives practically the same results as the more complex model. The inputs to the model are the current prices of all existing futures contracts (and their maturities) and the estimated parameters of the two-factor model. It requires the numerical solution corresponding to a simple, one-factor model. Existing computer programs can be modified to incorporate the essential elements of the new model.

■ The application of option concepts to value real assets has been an important growth area in the theory and the practice of finance. These methods have been especially successful in the valuation of natural resource investments, such as copper mines and oil deposits. The reason for this is the existence of well-developed futures markets for these commodities from which essential market information can be extracted.

The traditional approach to valuing investment projects is the net-present-value approach, which essentially involves discounting the expected net cash flows from the project at a discount rate that reflects the risks of those cash flows (the risk-adjusted discount rate). In this approach, the adjustment for risk is made to the discount rate. An alternative approach is to make the adjustment for risk to the cash flows and to discount the certainty-equivalent cash flows, instead of the expected cash flows, at the risk-free rate of interest. The certainty-equivalent cash flows are the certain amounts which would have the same value as the uncertain cash flows. The net-present-value approach is the one normally used in practice because it is felt that it is easier to estimate the risk-adjusted discount rate than the certainty-equivalent cash flows.[1] However, in the case of commodities, the certainty-equivalent cash flows are known since they can be obtained from forward (or futures[2]) prices. So, when valuing projects in which the main uncertainty is the commodity price and forward prices for the commodity exist, it is much easier to use the certainty-equivalent approach, since it avoids the need to compute a risk-adjusted discount rate. Once the adjustment for risk has been made to the cash flows, the relevant discount rate is the risk-free rate of interest.

The second advantage of working in this risk-neutral environment in which the relevant discount rate is the risk-free rate of interest is that it is also the environment of option pricing. The multiple operating options available in a typical project can then be naturally incorporated in the analysis. These options include the optimal time to exercise the option to invest in the project, the options to stop and restart production in response to price changes, and the option to abandon the project if prices are too low to justify maintaining the operations.

In the first attempt to value investment projects in natural resources using this new approach (see, for example, Brennan and Schwartz, 1985), the spot price of the commodity was assumed to follow geometric Brownian motion similar to the process assumed for stock prices in the option-pricing literature. This approach allowed for the extension of the option-

This paper was adapted from the Keynote Address, Financial Management Association International 1997 Annual Meeting, Honolulu, HI.

[1]The difficulties of accurately estimating risk-adjusted discount rates, however, have recently been emphasized by Fama and French (1997).

[2]In this article, I will not distinguish between forward and futures prices since the analysis will be carried out in an environment with constant interest rates.

pricing framework to value real assets. Futures prices were used to determine the average convenience yield, which plays the same role in the commodity spot price process as the dividend yield in the stock price process. In theory, the convenience yield is the flow of services that accrue to the holder of the spot commodity but not to the holder of a futures contract. In practice, the convenience yield is the adjustment needed in the drift rate of the spot price process to price existing futures contracts properly.

The options approach to valuation has major advantages.[3] First, it avoids the need to make assumptions about the trajectory of spot prices in the future since it uses the information contained in futures prices (through the convenience yield). Second, it does not require the estimation of a risk-adjusted discount rate, since it uses the risk-free rate of interest. And finally, it explicitly allows for managerial flexibility in the form of options in the valuation procedure. In certain situations, these options can represent a significant part of the value of the project.

The stochastic process assumed for the spot commodity price, however, has some drawbacks. First, since the convenience yield is assumed constant, the model is unable to capture changes in the term structure of futures prices (for example, from backwardation to contango or vice versa). In reality, the convenience yield experiences significant changes through time. Second, the model implies that the volatility of all futures returns is equal to the volatility of spot returns. The data show, however, that the volatility of futures returns decreases with the time to maturity of the futures contract. Third, geometric Brownian motion implies that the variance of the distribution of spot prices grows linearly with time, whereas supply and demand adjustments to changing prices would suggest some type of mean reversion in spot commodity prices.

In the last few years, several attempts to resolve the drawbacks of the basic model discussed above have been made.[4] In Schwartz (1997), I compare three models of the stochastic behavior of commodity prices in terms of their ability to price the term structure of futures prices and the term structure of futures return volatility. The first model is a one-factor model in which the log of the spot price of the commodity is assumed to follow a mean-reverting process. The second model assumes that the convenience yield is also stochastic and follows a mean-reverting process. In this model, the convenience yield plays the role of a stochastic dividend in the spot-price process. The third model extends the second by assuming also stochastic interest rates. For the two commercial commodities considered, copper and oil, the one-factor model does a poor job of explaining the characteristics of the data. The other two models, however, are able to capture many of the characteristics of the term structure of futures prices and volatilities. Since these two models are empirically very similar, in this paper, I will deal only with the two-factor model, which assumes constant interest rates.

A major difficulty in applying these ideas to the valuation of projects is that commodity futures contracts traded on exchanges have maximum maturities that are typically less than two years, whereas most of the projects we want to value have much longer maturities. It is therefore very important to assess the implications of the different models with respect to futures prices beyond the existing exchange-traded contracts. Fortunately for my study, Enron Corp. made available some proprietary oil forward curves with maturities up to ten years, which allowed me to assess the implications of the models with respect to long-term futures prices.

The two-factor, stochastic-convenience-yield model has many advantages with respect to simpler one-factor models. The valuation procedure, however, is substantially more difficult, especially in the presence of complex operating options, since it requires the solution of a second order partial differential equation with two state variables (and time). This disadvantage can be quite important from the point of view of the practical implementation of the approach.

In this paper, I develop a one-factor model that retains most of the characteristics of the more complex two-factor stochastic-convenience-yield model in terms of its ability to price the term structure of futures prices and volatilities. The model is based on the pricing and volatility results of the two-factor model, but, when applied to value long-term commodity projects, it only requires the numerical solution corresponding to a typical one-factor model. The inputs to the model are the current prices of all existing futures contracts (and their maturities) and the estimated parameters of the two-factor model. Existing computer programs can be easily modified to incorporate the essential elements of the new model.

Section I develops the simple one-factor model and obtains the main results of the paper. Section II shows how to implement the simple model and compares the results obtained using the simple model with those obtained using the substantially more complicated, two-factor stochastic-convenience-yield model to value long-term commodity assets. Section III compares the simple model and the two-factor model with respect to their optimal exercise criteria, and Section IV provides some concluding remarks.

[3]For recent applications to similar investment problems, see Lee (1997) and Smit (1997).

[4]See, for example, Brennan (1991), Gibson and Schwartz (1990, 1991), Ross (1995), Cortazar and Schwartz (1994), and Bessembinder, Coughenour, Seguin, and Smoller (1995).

I. The Model

In this section, I present the basic constant-convenience-yield model and the two-factor stochastic-convenience-yield model. Then, I show that simple modifications of the basic model can take into account many of the important features of the two-factor model, especially when applied to value longer-term commodity assets. For this reason, I call the model developed in this article the "long-term model."

Since my main concern here is the valuation of commodity-contingent claims, I will define all the stochastic processes under the equivalent martingale measure (i.e., risk neutrality). Also, since I assume that interest rates are constant, I do not distinguish between forward and futures prices.

A. The Basic Model

Assuming a constant convenience yield, c, interest rate, r, and volatility, σ, for the rate of return on the spot commodity price, S, the stochastic process for the spot price under the equivalent martingale measure is given by:

$$\frac{dS}{S} = (r-c)dt + \sigma dz \tag{1}$$

where dz is an increment to a standard Brownian motion process.

In this model, the futures price, F, with maturity, T, for a spot price, S, is given by:

$$F(S,T) = Se^{(r-c)T} \tag{2}$$

By applying Ito's Lemma to Equation (2), it can be shown that the volatility of futures returns (dF/F) is also given by σ.

In the basic model, the value of any contingent claim on the commodity, V(S,T), must satisfy the partial differential equation

$$0 = \frac{1}{2}\sigma^2 S^2 V_{SS} + (r-c)SV_S - V_T - rV \tag{3}$$

subject to the appropriate boundary conditions. If the contingent claim is a project, we also have to add the cash flows on the project, CF(T), to Equation (3).

B. The Two-Factor Model

In the two-factor model, the convenience yield, denoted by δ to distinguish it from the constant convenience yield in the basic model, is assumed to be stochastic and to follow a mean-reverting process. The joint stochastic process for the spot price and the convenience yield under the equivalent martingale measure is:

$$dS = (r - \delta)Sdt + \sigma_1 S dz_1 \tag{4}$$

$$d\delta = \kappa(\hat{\alpha} - \delta)dt + \sigma_2 dz_2 \tag{5}$$

$$dz_1 dz_2 = \rho dt \tag{6}$$

The magnitude of the speed of adjustment, $\kappa > 0$, measures the degree of mean reversion to the long-run mean convenience yield, α, and ρ is the correlation between the two processes.

Futures prices in this model, $F(S,\delta,T)$, are given by:

$$F(S,\delta,T) = S\exp\left[-\delta\frac{1-e^{-\kappa T}}{\kappa} + A(T)\right] \tag{7}$$

where

$$A(T) = \left(r - \hat{\alpha} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1\sigma_2\rho}{\kappa}\right)T + \frac{1}{4}\sigma_2^2\frac{1-e^{-2\kappa T}}{\kappa^3} + \left(\hat{\alpha}\kappa + \sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa}\right)\frac{1-e^{-\kappa T}}{\kappa^2} \tag{8}$$

and the risk-adjusted, long-run mean of the convenience-yield process is given by

$$\hat{\alpha} = \alpha - \frac{\lambda}{\kappa} \tag{9}$$

where λ is the market price of convenience yield risk, which is assumed to be constant.

By applying Ito's Lemma to Equation (7) it can be shown that the variance of the futures returns is independent of the value of the state variables and that it depends only on the time to maturity of the futures contract:

$$\sigma_F^2(T) = \sigma_1^2 + \sigma_2^2\frac{(1-e^{-\kappa T})^2}{\kappa^2} - 2\rho\sigma_1\sigma_2\frac{(1-e^{-\kappa T})}{\kappa} \tag{10}$$

Note that this variance converges to a fixed value as the maturity of the futures contract tends to infinity:

$$\sigma_F^2(\infty) = \sigma_1^2 + \frac{\sigma_2^2}{\kappa^2} - \frac{2\rho\sigma_1\sigma_2}{\kappa} \tag{11}$$

In the two-factor model, the value of any commodity contingent claim must satisfy the partial differential equation

$$0 = \frac{1}{2}\sigma_1^2 S^2 V_{SS} + \sigma_1\sigma_2\rho SV_{S\delta} + \frac{1}{2}\sigma_2^2 V_{\delta\delta} + (r-\delta)SV_S + \kappa(\hat{\alpha} - \delta)V_\delta - V_T - rV \tag{12}$$

subject to the appropriate boundary conditions. If the contingent claim is a project, we must add the cash flows to Equation (12).

C. The Long-Term Model

In Schwartz (1997), I show that the two-factor model has implications with respect to the term structure of futures prices and volatilities which are consistent with the data for the two commodities I consider (copper and oil). These two term structures determine the risk-neutral distribution of spot prices. Given stochastic processes (4)-(6), the risk-neutral distribution of spot prices is lognormal with mean equal to the forward price in Equation (7) and a variance (of the log price) which can be obtained by integrating the variance of the forward price in Equation (10), as I show later in Equations (21) and (22). My objective here is to develop a model which is simpler than the two-factor model, but which matches as closely as possible the term structure of futures prices and the term structure of futures volatilities implied by the two-factor model. As I show, I accomplish the first objective very accurately for long-term futures prices and the second objective exactly for all futures prices.

In the two-factor model, as the maturity of the futures contract increases, the rate of change in the futures price converges to a fixed rate that is independent of the initial value of the state variables:

$$\frac{1}{F}\frac{\partial F}{\partial T}\,(T \rightarrow \infty) = r - \alpha + \frac{\sigma_2^2}{2\kappa^2} - \frac{\rho\sigma_1\sigma_2}{\kappa} \tag{13}$$

As in Section II, for the oil and copper data available, close convergence can be obtained in approximately three years. Note that in the basic model in Equation (2) the rate of change in the futures price is constant and equal to

$$\frac{1}{F}\frac{\partial F}{\partial T} = r - c \tag{14}$$

So, if in the long-term model, the constant convenience yield is defined as

$$c = \alpha - \frac{\sigma_2^2}{2\kappa^2} + \frac{\rho\sigma_1\sigma_2}{\kappa} \tag{15}$$

it will have the same rate of change in futures prices as the two-factor model when Equation (13) is a good approximation.

My objective, however, is to match the futures prices. So, in addition to having the correct rate of change in futures prices, I must start from an appropriate spot price to give the futures prices in Equation (7) when the constant convenience yield in Equation (15) is applied. I define this price as the shadow spot price, Z, which is given by:

$$Z(S,\delta) = \lim_{T \rightarrow \infty} e^{(-(r-c)T)} F(S,\delta,T) \tag{16}$$

After simplifying, Equation (16) becomes:

$$Z(S,\delta) = S\, e^{\left(\frac{c-\delta}{\kappa} - \frac{\sigma_2^2}{4\kappa^2}\right)} \tag{17}$$

In other words, given the state variables of the two-factor model, S and δ, and the parameters of the model, I can obtain the shadow spot price, Z, from Equation (17). When this price is used as a single, state variable in a model with the constant convenience yield, c, from Equation (15), the model will give futures prices F(Z,T) that are very close to F(S,δ,T) when the time to maturity of the contract is greater than three years.

Next, define the stochastic process for the shadow spot price as:

$$\frac{dZ}{Z} = (r - c)dt + \sigma_F(t)dz \tag{18}$$

Note that now the volatility is a function of time and is defined as in Equation (10).

In the long-term model, the futures price, F, with maturity, T, for a shadow spot price, Z, is then:

$$F(Z,T) = Ze^{(r-c)T} \tag{19}$$

By applying Ito's Lemma to Equation (19), it can be shown that the volatility of futures returns is given by $\sigma_F(T)$. In this model, then, the volatilities of futures returns are exactly the same as those in the two-factor model, and futures prices are very close to those in the two-factor model for maturities greater than three years.

In the long-term model, the value of any claim on the commodity, V(Z,T), must satisfy the partial differential equation

$$0 = \frac{1}{2}\sigma_F^2(T)Z^2V_{ZZ} + (r-c)ZV_Z - V_T - rV \tag{20}$$

By using the two-factor model to redefine a single state variable, the shadow spot price of the commodity, with a volatility that depends on the maturity of the contract valued, and by also defining a constant long-term convenience yield, a one-factor model very similar to the basic model is obtained. The only important difference is that the volatility is time dependent. This is not a critical issue in the valuation of European claims since the accumulated volatility to the maturity of the claim

$$v(T) = \int_0^T \sigma_F^2(t)dt \tag{21}$$

has a closed-form solution

$$v(T) = \left(\sigma_1^2 + \frac{\sigma_2^2}{\kappa^2} - \frac{2\rho\sigma_1\sigma_2}{\kappa}\right)T + \frac{\sigma_2^2(1-e^{-2\kappa T})}{2\kappa^3} + 2\sigma_2\left(\sigma_1\rho - \frac{\sigma_2}{\kappa}\right)\frac{(1-e^{-\kappa T})}{\kappa^2} \tag{22}$$

The risk-neutral distribution of the shadow spot price is lognormal with mean equal to the forward price given in Equation (19) and a variance (of the log price) given in Equation (22). Note that this distribution has the same variance as the distribution of the spot price in the two-factor model. The mean of the distribution approximates the mean of the distribution in the two-factor model for longer maturities since Equation (19) converges to Equation (7).

For American claims, the fact that the volatility in Equation (20) is time dependent does not complicate the numerical solution of the differential equation.

D. Valuing European Options

Since the two-factor model and the long-term model imply very similar means and the same variances for the risk-neutral distributions of spot prices, both models will give very similar results when valuing long-term options. Consider the valuation of a European call option on the commodity with an exercise price of K. In the forward price space, the value of the call option according to both models can be expressed as:

$$C(\cdot, T) = e^{-rT}c(F(\cdot, T), T) \tag{23}$$

where

$$c(F,T) = FN(d) - KN(d - v(T)) \tag{24}$$

and where N(d) is the standard normal distribution function and

$$d = \frac{\ln\frac{F}{K}}{v(T)} + \frac{1}{2}v(T) \tag{25}$$

For each model, the call price and the futures price are functions of the appropriate state variables of the respective models: the spot price and the instantaneous-convenience yield in the two-factor model, and the shadow spot price in the long-term model. The variance is the same for both models and is given by Equation (22). Naturally, at the maturity of the option the relevant variable for determining the exercise strategy is the actual spot price of the commodity and not the shadow price.

II. Implementation

In this section, I will demonstrate how to implement the long-term model using the parameters estimated for the two-factor model in Schwartz (1997). Table 1 shows these parameters for oil and copper. The parameters for copper were estimated using publicly available futures prices for contracts for the period July 1988 to June 1995. All the contracts had maturities of less than two years. The parameters for oil were estimated using proprietary oil forwards curves which were made available by Enron for the period January 1993 to May 1996. For this data, the maximum maturity used was 9 years. Note that μ, the true total expected return on the spot commodity, does not enter into our calculations and is only reported in Table 1 for completeness.

Table 1. Parameter Values for Oil and Copper

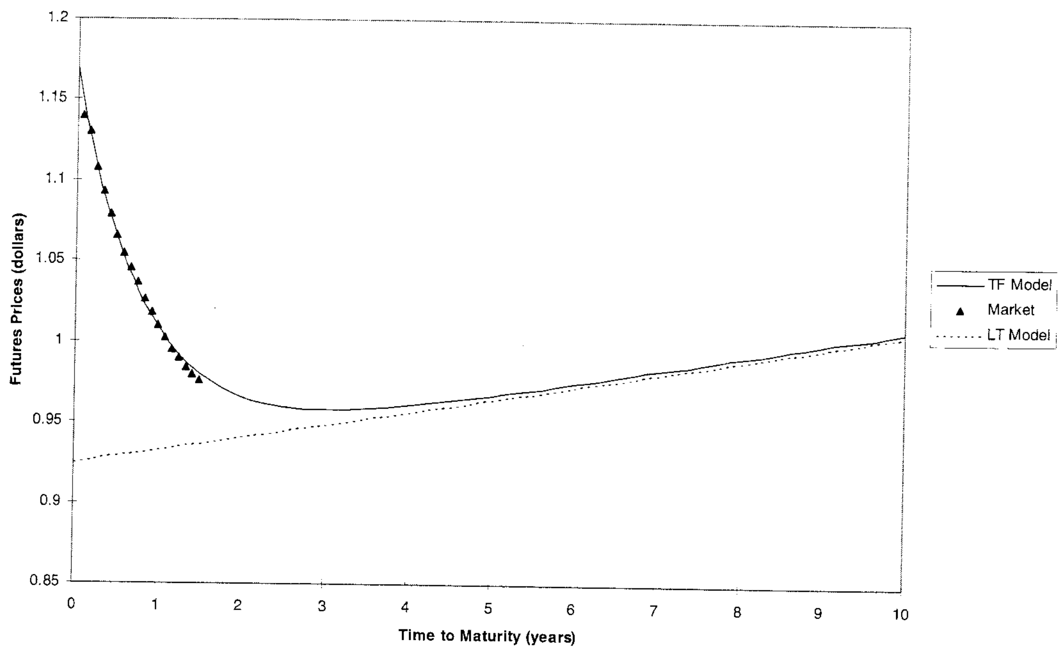
This table shows the parameters of the two-factor model estimated in Schwartz (1997) and used in the implementation of the long-term model. (Standard errors are in parenthesis.)

	Copper	Oil
Period	7/29/88 to 6/13/95	1/15/93 to 5/16/96
Contracts	F1, F2, F3, F5, F7, F9	Enron Data
NOBS	347	163
μ	0.326 (0.110)	0.082 (0.120)
κ	1.156 (0.041)	1.187 (0.026)
α	0.248 (0.098)	0.090 (0.086)
σ_1	0.274 (0.012)	0.212 (0.011)
σ_2	0.280 (0.017)	0.187 (0.012)
ρ	0.818 (0.020)	0.845 (0.024)
λ	0.256 (0.114)	0.093 (0.101)
r	0.06	0.05

Given the parameters of the two-factor model and all the futures prices with their respective maturities on any given day, I use the futures pricing Equation (7) in a double-grid search routine to estimate the state variables, S and δ, which minimize the squared deviations between model and market prices. With the estimated state variables S and δ, Equation (7) allows us to construct the term structure of futures prices implied by the two-factor model, and Equation (17) allows us to compute the shadow spot price, Z. Finally, Equation (19) allows us to construct the term structure of futures prices implied by the long-term model.

Figure 1 graphs the market prices of all the copper futures contracts reported in the *Wall Street Journal*

Figure 1. Market and Model Copper Futures Prices, 3/31/97



for 3/31/97. The figure also shows the term structure of copper futures prices implied by both the two-factor model (TF Model in Figure 1) and the long-term model (LT Model in Figure 1) for that day using the parameters for copper in Table 1. Note that the maximum maturity of existing copper futures contracts is less than two years. The estimated spot price is \$1.169, and the estimated instantaneous convenience yield is 0.305. The corresponding shadow spot price is \$0.9252, and the long-term convenience yield is 0.0516. The figure shows that the long-term model approximates the two factor model for maturities greater than three years.

Figure 2 displays the market prices for all oil futures contracts reported in the *Wall Street Journal* for 3/31/97 and the term structure of futures prices implied by the two models for the oil parameters in Table 1. Note that in this case there are prices reported for contracts up to almost seven years to maturity. The estimated spot price is \$20.79, and the estimated instantaneous convenience yield is 0.117. The corresponding value for the shadow spot price is \$19.16 and for the long-term convenience yield is 0.0275. Note once again that both models give very close prices for contracts with maturities greater than three years. The models, however, do not fit the oil futures prices so well, though some of these prices do not represent actual trades.

Figure 3 graphs the values of European copper call values obtained from the two-factor model and the long-term model for 3/31/97 for an exercise price of \$1.00. The figure shows that for maturities greater than three years, the two models give very similar results.

III. Optimal Exercise Criteria

I have shown that the simple long-term model has valuation implications very similar to the more complex two-factor model when valuing futures and options with maturities greater than three years. An issue that still remains to be discussed is how the models compare with respect to the optimal time to undertake a project, or more generally, the optimal exercise of American contingent claims. This is important since in the two-factor model the critical spot price above which it is optimal to invest (or exercise the American option) depends on the current instantaneous convenience yield, whereas in the one-factor, long-term model there is only one critical shadow spot price above which it is optimal to invest. In this section, we investigate this issue in the context of a simple imaginary copper mine.

Consider a simple copper mine that can produce one ounce of copper at the end of each year for ten years. Assume that production starts at the end of the fourth year since it takes three years to make the necessary investments in the project. Suppose that the present value of the initial investment required is K=\$2 and that the constant unit cost of production is C=\$0.40. Assume that once the initial investment is made, further investments will take place for three years and then production will commence and continue for the following ten years; that is, we neglect in this analysis

Figure 2. Market and Model Oil Futures Prices, 3/31/97

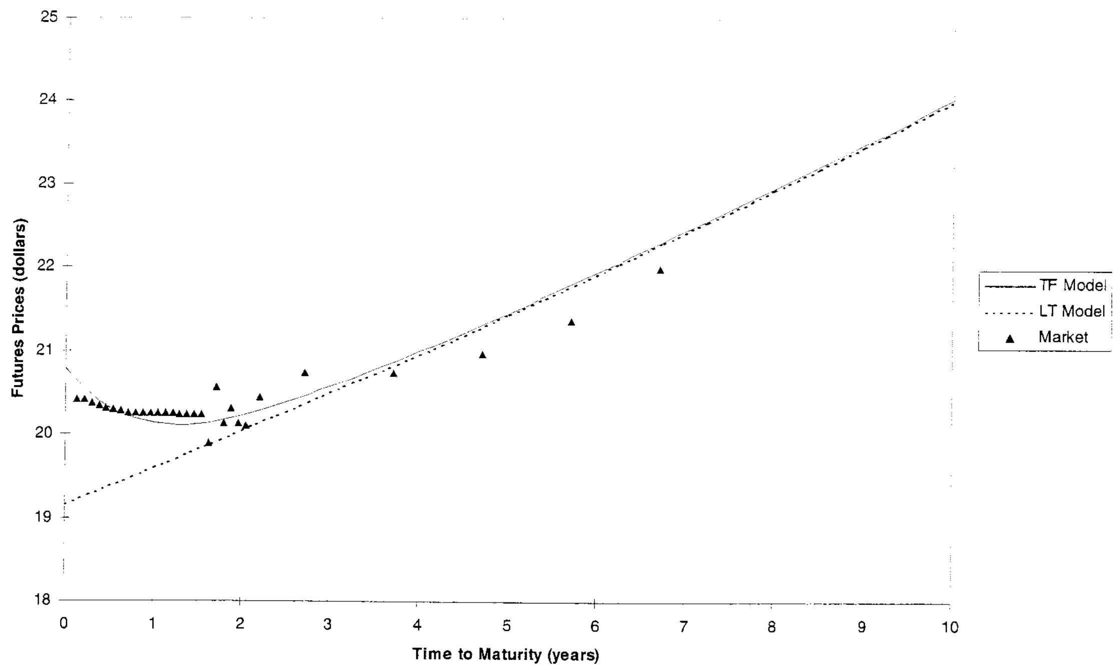
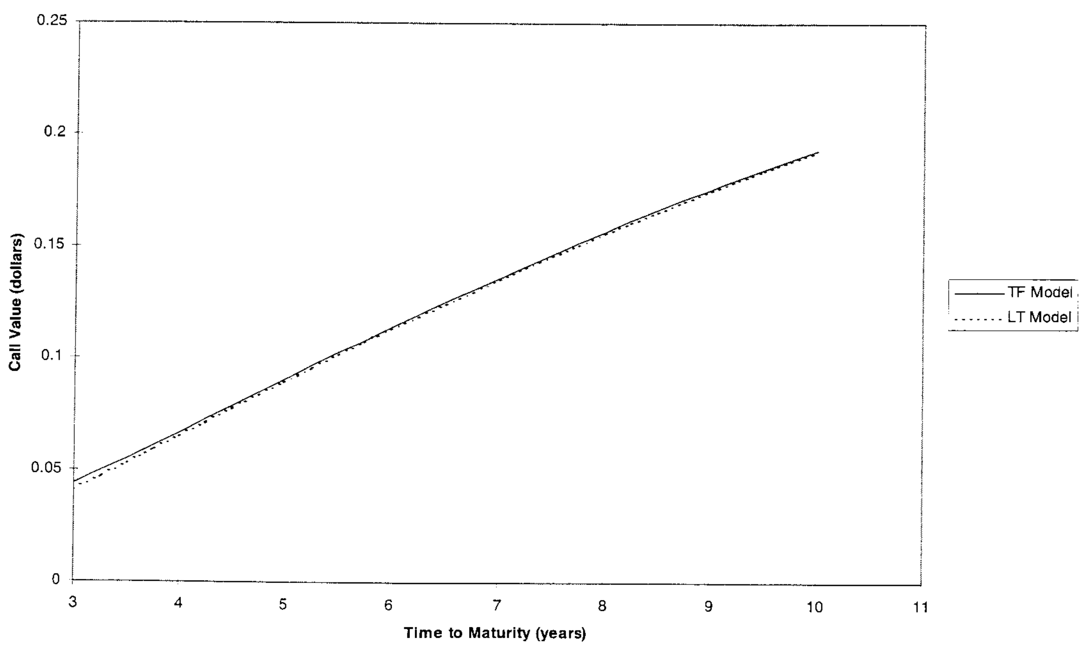


Figure 3. Long-Term Copper Call Values for K=$1.00, 3/31/97



the options to close and open the mine and the option to abandon it,[5] and concentrate entirely on the option to invest. The first step in the procedure consists of determining the net present value of the project once it has been decided to go ahead with the investment (this is the boundary condition of the second step). The second step consists of evaluating the option to invest. The net present value of the project once the commitment to invest has been made is:

[5]For a detailed discussion of these options, see Brennan and Schwartz (1985). Since the procedures to evaluate the mine are numerical, it would be easy to incorporate them in the analysis.

$$NPV(\cdot) = \sum_{T=4}^{13} e^{-rT} F(\cdot, T) - C \sum_{T=4}^{13} e^{-rT} - K \tag{26}$$

For each model, the futures prices and the net present value are functions of the appropriate state variables of the respective models: the spot price and the instantaneous-convenience yield in the two-factor model, and the shadow spot price in the long-term model. Note that the summation starts at T=4 since it is assumed that production starts in year four.

To value the option to invest and determine the price of copper above which it is optimal to invest in the project, we need to solve numerically the partial differential Equation (12) for the two-factor model and Equation (20) for the long-term model, both with boundary condition (26).[6]

In the two-factor model, the value of the mine is a function of both the spot price of copper and the instantaneous-convenience yield. The optimal spot price above which it is optimal to invest is a function of the instantaneous convenience yield. Figure 4 shows this optimal price as a function of the convenience yield.[7] Notice that for a convenience yield of zero the optimal price is $1.10, whereas for a convenience yield of 0.40 the optimal price is $1.52. Due to the strong mean reversion in the convenience yield and the high correlation between the convenience yield and the spot price, a high value for the convenience yield implies that the spot price will tend to come down, therefore requiring a higher trigger value for investment. The value of the mine on 3/31/97, when the spot price was $1.169 and the instantaneous convenience yield was 0.305, would have been $1.75, and the optimal trigger price for investment would have been $1.41. At the current copper spot price it would not have been optimal to start investing.

Figure 4 also shows the corresponding optimal shadow spot prices computed using Equation (17) and implied by the combinations of optimal spot prices and convenience yields obtained from the two-factor model. Notice that there is surprisingly little variation in these optimal shadow spot prices: every combination of spot price and convenience yield implies practically the same shadow spot price (values vary between $1.10 and $1.13). It is clear from the figure that only for very low levels of the convenience yield the optimal spot price is below the corresponding optimal shadow spot price.

When solving the partial differential equation for the one-factor long-term model, we obtain an optimal shadow spot price for investment of $1.12, which is indistinguishable from the one obtained from the two-factor model. Figure 5 graphs the value of the simple copper mine as a function of the shadow spot price. It also shows the net present value of the project, which is positive when the shadow spot price is greater that $0.69. The value of the investment option is larger than the net present value up to a price of $1.12 where both lines converge since at that point it is optimal to invest in the project. The value of the mine on 3/31/97, when the shadow spot price was $0.9252, would have been $1.75, which to two significant figures is the same one obtained using the two-factor model. Since the shadow spot price was below the trigger price for investment, it would not have been optimal to start investing.

The previous discussion reveals that when valuing commodity projects where cash flows start a few years into the future, a simple one-factor model can give practically the same results as the two-factor model. When using the long-term model, the current convenience yield together with the optimal shadow spot price obtained from the model can be used in Equation (17) to determine the actual spot price at which investment should proceed. If there are project cash flows in the first two years, the value of the project can be adjusted after the initial valuation has been completed to take into account the possibly large differences in futures prices implied by the two models in these years.

IV. Conclusion

In my presidential address to the American Finance Association, I showed that a two-factor model for the stochastic behavior of commodity prices fits quite well the term structure of futures prices and the term structure of futures return volatility for two key commercial commodities, copper and oil. The two-factor model, however, is relatively difficult to apply when valuing complex investment projects with multiple options, such as the investment option, the option to close the operation temporarily and then re-open it, and the abandonment option.

In this article, I develop a simple one-factor model that has practically the same implications as the two-factor model, when it is applied to value long-term commodity assets. It derives directly from the two-factor model: the single factor in the simple model is a function of the two factors in the more complex model. The constant convenience yield in the model is a function of the parameters of the two-factor model and the time-dependent volatility is the same as in the two-factor model. The inputs to the model are the prices of all current futures contracts and the estimated parameters for the two-factor model.

This long-term model can then be applied to value projects with complex options without sacrificing any

[6]In both cases, I assume that the investment option has a maturity of ten years.

[7]The discreteness in the figure is due to the discreteness in the numerical solution.

Figure 4. Simple Copper Mine Optimal Exercise Prices

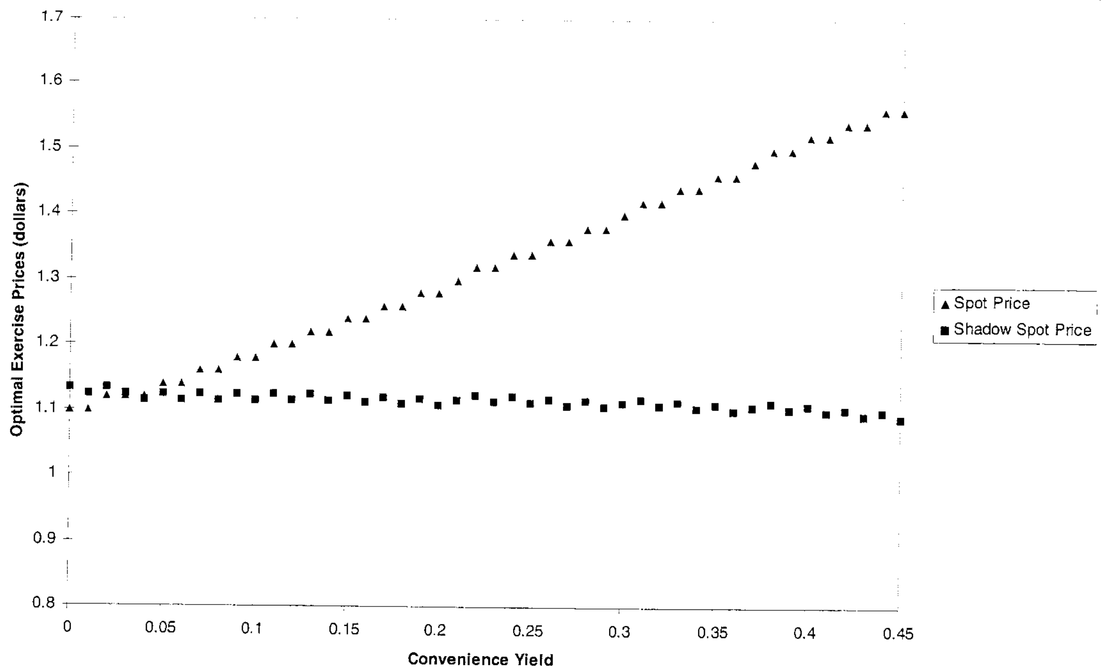
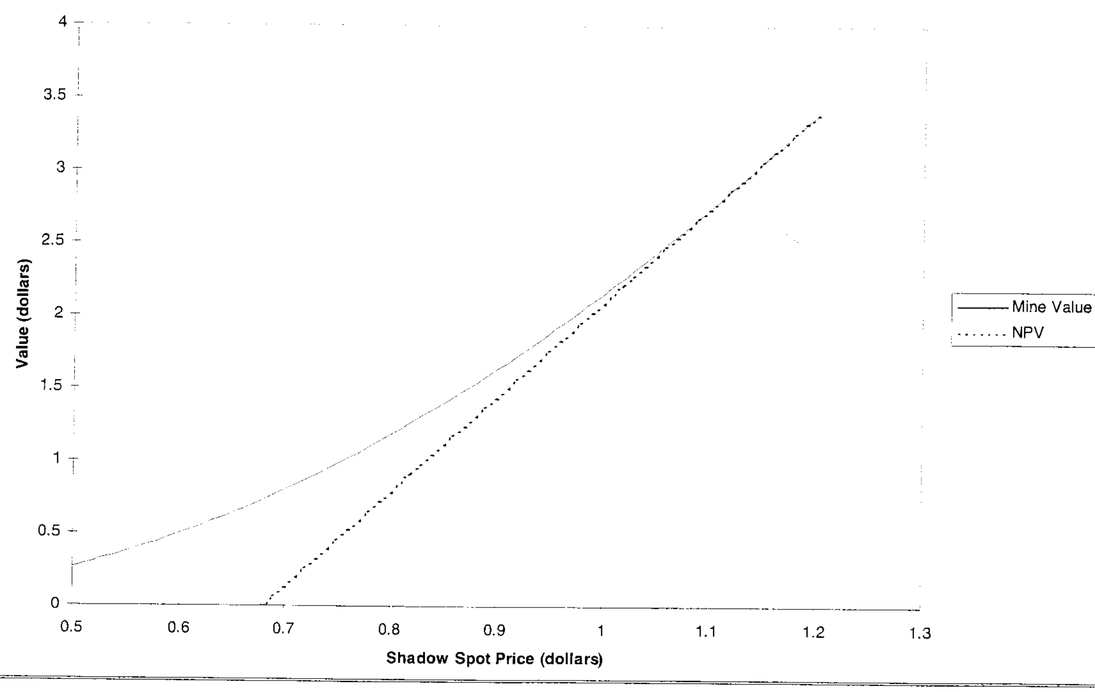


Figure 5. Simple Copper Mine Value



of the advantages of the two-factor model. Existing computer programs for one-factor models can be easily modified to take into account most of the properties of the two-factor model. ■

References

Bessembinder, H., J.F. Coughenour, P.J. Seguin, and M.M. Smoller, 1995, "Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure," *Journal of Finance* (March), 361-375.

Brennan, M.J., 1991, "The Price of Convenience and the Valuation of Commodity Contingent Claims," in D. Lund and B. Oksendal, Eds., *Stochastic Models and Option Values*, Elsevier, North Holland.

Brennan, M.J. and E.S. Schwartz, 1985, "Evaluating Natural Resource Investments," *Journal of Business* (April), 133-155.

Cortazar, G. and E.S. Schwartz, 1994, "The Evaluation of Commodity Contingent Claims," *Journal of Derivatives* (October), 27-39.

Fama, E.F. and K.R. French, 1997, "Industry Costs of Equity," *Journal of Financial Economics* (February), 153-193.

Gibson, R. and E.S. Schwartz, 1990, "Stochastic Convenience Yield and the Pricing of Oil Contingent Claims," *Journal of Finance* (July), 959-976.

Gibson, R., and E.S. Schwartz, 1991, "Valuation of Long-Term Oil-Linked Assets," in D. Lund and B. Oksendal, Eds., *Stochastic Models and Option Values,* Elsevier, North Holland.

Lee, M.H., 1997, "Valuing Finite-Maturity Investment-Timing Options," *Financial Management* (Summer), 58-66.

Ross, S.A., 1995, "Hedging Long-Run Commitments: Exercises in Incomplete Market Pricing," Yale University Working Paper.

Schwartz, E.S., 1997, "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging," *Journal of Finance* (July), 923-973.

Smit, H.T.J., 1997, "Investment Analysis of Offshore Concessions in The Netherlands," *Financial Management* (Summer), 5-17.