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The term structure of very short-term rates: New evidence for the expectations hypothesis☆

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Abstract

Empirical researchers have frequently rejected the expectations hypothesis. The expectations hypothesis, however, has seldom, if ever, been tested at the extreme short end of the term structure where maturities are measured in days or weeks. Using overnight, weekly, and monthly repo rates, I find that term rates are almost unbiased estimates of the average overnight rate. This evidence provides new support for the expectations hypothesis. © 2000 Elsevier Science S.A. All rights reserved.

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That this interest is truly an average is attested both by the comparative stability of the rate of interest realized on long time bonds as compared with the fluctuations of the rate of interest in the short time money market .... The investor who holds a bond a long time realizes an interest which is an "average" of the oscillating rates of those who speculate during the interim.

Irving Fisher (1896)

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1. Introduction

After more than a century, the expectations hypothesis remains the best-known and most intuitive theory of the term structure of interest rates. Because of its central role in term structure theory, the expectations hypothesis has also been one of the most intensively studied models in financial economics. Important recent examples of papers testing the expectations hypothesis include Roll (1970), Shiller et al. (1983), Fama (1984a), Fama and Bliss (1987), Stambaugh (1988), Froot (1989), Campbell and Shiller (1991), Campbell et al. (1996), Buser et al. (1996), Bekaert et al. (1997), Balduzzi et al. (1997), and Backus et al. (1998). Virtually all of these studies have rejected the expectations hypothesis.

However, the expectations hypothesis has seldom, if ever, been tested at the extreme short end of the term structure where maturities are measured in days or weeks. Testing whether the expectations hypothesis holds at the extreme short end is important because on the one hand, if it cannot explain how one-week rates are related to overnight rates, it likely cannot explain the rest of the term structure. On the other hand, finding that the expectations hypothesis holds at the extreme short end would add an entirely new dimension to the important issue of how interest rates are determined in financial markets.

In this paper, I test the expectations hypothesis using short-term repurchase (repo) rates ranging from one day to three months in maturity. The high frequency of this data is ideally suited to studying the term structure of very short-term rates. Using this unique data set, I test the implications of the expectations hypothesis at both an unconditional and conditional level, as both types of tests provide insights about the term structure.

The results of the unconditional tests of the expectations hypothesis are surprising. I find that term premia in weekly and monthly rates are small in economic terms and statistically insignificant. Thus, at the unconditional level, even the simplest version of the expectations hypothesis in which term premia are zero cannot be rejected. These results are in stark contrast with earlier research on Treasury bill markets, which finds evidence of large unconditional term premia in Treasury bill yields. The results support the widespread Wall Street view that much of the apparent term premium in Treasury bills results from other factors such as liquidity. Because repo rates represent the actual cost of capital for holding riskless securities, repo rates could be better measures of the short-term riskless term structure than Treasury bill rates.

I then conduct conditional tests of the expectations hypothesis. In light of an important paper by Bekaert et al. (1997) demonstrating that the highly persistent nature of interest rates severely affects the small-sample properties of many traditional tests of the expectations hypothesis, I pay particular attention to the small-sample distribution of the estimators. Specifically, I study the small-sample properties of my tests under the assumption that interest rates follow a VAR-GARCH process similar to that in Bekaert, Hodrick, and Marshall.

The results of the conditional tests of the expectations hypothesis also have important implications for term-structure theory. In particular, I find that longer-term rates are nearly unbiased forecasts of the average overnight rate during the term of the longer rate. This is true for all maturities from one week to three months. These results again provide support for the simplest version of the expectations hypothesis in which term premia are zero.

Taken together, the results from these tests suggest that the expectations hypothesis serves as an accurate description of the behavior of very short-term interest rates. The remainder of this paper is organized as follows. Section 2 discusses the expectations hypothesis. Section 3 describes the data. Section 4 reports the results of the unconditional tests of the expectations hypothesis. Section 5 reports the results of the conditional tests of the expectations hypothesis. Section 6 makes concluding remarks.

2. Expectations hypothesis

Many different versions of the expectations hypothesis are presented in the literature. Cox et al. (1981) show that a number of traditional forms of the expectations hypothesis are inconsistent with each other and argue that some versions imply the existence of arbitrage opportunities.[1] Campbell (1986) demonstrates, however, that differences between the various forms of the expectations hypothesis stem from small volatility or convexity effects that are typically of little empirical significance. In this paper, I consider only rates with maturities of three months or less. Following Campbell, it is easily shown that the differences in the various forms of the expectations hypothesis are virtually zero for rates this short.[2] Consequently, I do not differentiate among the various forms of the expectations hypothesis in this paper.

As suggested by the quotation from Fisher (1896), the expectations hypothesis can be viewed as requiring that the rate on a long-term riskless loan be equal to the expected average short-term rate from now until the maturity date of the longer-term loan. This can be expressed as

$$\mathrm{E}[R_{t+n} \mid \Omega_t] = Y_t(n) + a_n, \tag{1}$$

where R_{t+n} is the average short-term rate from time t to time $t+n$, Ω_t is the information set at time t, $Y_t(n)$ is the n-period term rate observed at time t, and

[1] McCulloch (1993) and Fisher and Gilles (1998), provide counterexamples showing that these forms of the expectations hypothesis do not necessarily imply the existence of arbitrage. Longstaff (2000) shows that these forms of the expectations hypothesis can hold generally without arbitrage when markets are not complete.

[2] Following the analysis in Cox et al. (1981), the difference between the various forms of the expectations hypothesis for the three-month repo rate can be shown to be on the order of one-tenth of a basis point, where return volatility is estimated using either historical returns or implied volatilities from short-term over-the-counter Treasury bond options.

a_n is a constant term premium that can differ across horizons n. Thus, $Y_t(n) + a_n$ is the conditional expected value of average short-term rate from time t to time $t + n$. When a_n is zero, this form of the expectations hypothesis is sometimes termed the pure expectations hypothesis. If a_n is constant but not zero, the yield $Y_t(n)$ moves in a one-to-one relation with the expected average short-term rate, provided that Eq. (1) holds.

3. The data

The expectations hypothesis is a theory of the term structure of interest rates on riskless loans. Traditionally, researchers have used the Treasury bill rate as the measure of the riskless rate in empirical studies. This approach has many advantages. For example, Treasury bills can clearly be viewed as defaultfree. In addition, the Treasury bill market is highly liquid and market quotations are reliable indications of where trades can be executed. Furthermore, Treasure bill rates for maturities ranging from three months to twelve months are readily available.

The objective of this paper, however, is to test the expectations hypothesis at the extreme short end of the term structure using data with the highest frequency possible. In particular, a time series for the overnight rate is required as the measure of the short-term riskless rate in the tests. Note that using rates with maturities of more than one day as a proxy for the short-term rate is not appropriate given that term rates could include term premia and introduce biases into tests of the expectations hypothesis. Because Treasury bills are auctioned weekly, however, obtaining a daily series of overnight rates from the Treasury market is not possible. A yield on a Treasury bill with one day to maturity is only available once per week. Alternative measures for the riskless term structure must be considered.

Even without this data limitation, Treasury bill rates may not provide the optimal measure of the riskless term structure. Extensive interviews with traders, brokers, dealers, and other Treasury market participants reveal a widespread view on Wall Street that Treasury bill rates are lower than the true riskless rate. The institutional demand for Treasury bills with their regulatory, tax, credit, and liquidity characteristics makes Treasury bills generically special.[3] Because this specialness persists through the life of the Treasury bill, the yield on Treasury bills converges to the equilibrium special rate, which is lower than the pure interest rate on a riskless loan. In effect, a Treasury bill is more valuable than the present value of its cash flows. Investors are willing to pay something extra for a Treasury bill because of its characteristics as a security. Duffee (1996a)

[3] For a discussion of specialness and special repo rates, see Duffie (1996b). For a discussion of the valuation effects of liquidity and marketability, see Longstaff (1995).

provides empirical evidence suggesting that Treasury bill yields display liquidity-related idiosyncratic variability.

In this paper, general collateral short-term repo rates are used as an alternative measure of the riskless term structure. Repo rates provide a realistic alternative in the context of this paper for a number of reasons. First, the repo rate is virtually a default-free rate by the nature of the repo contract. Specifically, when an investor borrows money in the repo market, the investor must provide collateral to the counterparty in the form of liquid securities.[4] Standard practice is to overcollateralize the repo loan to maintain full collateralization even with a large market movement.[5] In this paper, only rates on repo loans that are fully collateralized by Treasury securities are considered; general collateral government repo rates. Hence, the repo loan is fully collateralized by default-free collateral and the repo rate can essentially be viewed as the riskless rate.[6] Because the collateral is general instead of specific, posting specific bonds as collateral is not required. The borrower has the choice of which Treasury issues are posted as collateral. Second, because repo loans are pure financial contracts, not publicly traded securities, repo rates should not be affected by the various liquidity and other factors driving the specialness of Treasury bills. As a result, the repo rate may better reflect the pure cost of riskless borrowing and lending.[7] Third, the repo market is one of the most active fixed income financial markets in existence, as many large participants finance their inventories of securities via the repo market. Because of this, reliable repo rate quotations are readily available in the financial markets for maturities ranging from one day to three months. Finally, because the repo markets are the primary source of capital for financing inventories of Treasury securities, the repo rate for Treasury collateral is essentially the equilibrium cost of capital for investors holding generic Treasury securities, which can be viewed as an alternative definition of the riskless rate.[8]

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[4] For a discussion of the repo markets, see Stigum (1989, 1990).

[5] During periods of higher market volatility, repo dealers generally increase the level of collateralization required to ensure that repo loans remain overcollateralized.

[6] Given the possibility of fraud, settlement risk, or misuse of collateral, the repo rate cannot be riskless in the strictest sense. Technically, however, a similar argument could be applied to the Treasury bill market because institutional investors typically deal through government bond dealers when taking positions in the Treasury bill market.

[7] I am grateful to Mark Grinblatt for making this point. Also see Kamara (1994).

[8] A number of other papers have used the Fed Funds rate as a proxy for the short-term riskless rate, including Roberds et al. (1996) and Balduzzi et al. (1997). Using the Fed Funds rate could be problematic in the context of this paper for two reasons. First, Fed Funds rates are unsecured and, consequently, are not defaultfree. Thus, some of what appears to be a term premium in the Fed Funds rate could be term credit preima. Second, because of their role in the Federal Reserve banking system, Fed Funds can acquire a special nature similar to that of Treasury bills and on-the-run Treasury bonds. Fed Funds rates have sporadically been below fully secured rates in recent years.

The data for the study consists of daily observations of the closing overnight, one-week, two-week, three-week, one-month, two-month, and three-month general collateral government repo rates. The period covered by the study is May 21, 1991 through October 15, 1999.[9] The data are obtained from the Bloomberg system and the source of the data is Garban, a large and well-known Treasury securities broker. Repo rates are quoted on an actual/360 basis and the rate quotations in the Bloomberg system are given in increments of a basis point. Only days for which a complete set of rates for all maturities are available are included in the sample.[10] The total number of daily observations in the sample is 2,095.

Table 1 provides summary statistics for the levels and first differences in the short-term repo rates in the sample. The average term structure of repo rates is very flat during the sample period. The mean overnight rate is 4.7145, which is slightly higher than the mean one-week rate of 4.7070. The mean three-month rate is 4.7554, which is only slightly more than 4 basis points higher than the mean overnight rate. Fig. 1 graphs the overnight repo rate during the sample period. Fig. 2 graphs the spread between the three-month repo rate and the overnight rate. As shown, the term structure is often steeply upward or downward sloping. Although flat on average, the term structure of repo rates is rarely flat at a given point in time.

Table 1 also reports the mean repo rates for the different maturities by day of the week. The results show a number of regularities in the data. For example, the mean overnight rate on Monday is 4.7497, which is about 9 basis points higher than the mean overnight rate on Friday of 4.6602. This difference is highly significant. A similar pattern is observed for all of the other rates, that is the mean rate on Friday is always lower than the mean rate on Monday.

Also reported are the standard deviations of daily changes in the various rates. The overnight rate is much more variable than the other rates. The overall standard deviation of daily changes in the overnight rate is roughly 19 basis points per day, while the standard deviations for the other rates are typically on the order of 5 to 7 basis points per day. This pattern is consistent with Fisher's (1896) observation that short-term rates are more variable than longer-term rates. Note also the small differences in the volatility of changes in rates across days, presumably resulting from differences in the release of information. Finally, observe that all of the repo rates display a high level of persistence. However, the overnight repo rate is somewhat less persistent that the other repo rates. Daily changes in repo rates also display patterns of serial correlations.

[9] May 21, 1991 is the earliest date for which repo rates are available from Bloomberg.

[10] This criterion resulted in only 11 days being dropped from the sample.

Table 1
Summary statistics for repo rates and daily changes in repo rates. The data set consists of daily observations of the indicated term government general collateral repo rates during the period May 21, 1991 to October 15, 1999. The daily change in the repo rate for the indicated weekday is measured from the indicated day to the next business day. The term ρ_i denotes the ith order serial correlation coefficient. The total number of observations for each rate is 2,095.

	Repo rates							Daily changes in repo rates						
Statistic	Overnight	One-week	Two-week	Three-week	One-month	Two-month	Three-month	Overnight	One-week	Two-week	Three-week	One-month	Two-month	Three-month
Mean	4.7145	4.7070	4.7137	4.7183	4.7297	4.7414	4.7554	− 0.0003	− 0.0003	− 0.0003	− 0.0003	− 0.0003	− 0.0003	− 0.0003
Monday mean	4.7497	4.7201	4.7225	4.7257	4.7337	4.7425	4.7541	− 0.0419	− 0.0082	− 0.0050	− 0.0050	0.0009	0.0026	0.0049
Tuesday mean	4.7187	4.7125	4.7176	4.7206	4.7334	4.7456	4.7595	0.0062	− 0.0068	− 0.0031	− 0.0025	− 0.0051	− 0.0043	− 0.0057
Wednesday mean	4.7315	4.7134	4.7210	4.7254	4.7360	4.7491	4.7616	− 0.0118	− 0.0067	− 0.0075	− 0.0056	− 0.0041	− 0.0034	0.0017
Thursday mean	4.7154	4.7004	4.7095	4.7157	4.7276	4.7404	4.7575	− 0.0433	− 0.0042	− 0.0017	− 0.0015	0.0002	− 0.0022	− 0.0047
Friday mean	4.6602	4.6897	4.6986	4.7046	4.7180	4.7293	4.7439	0.0862	0.0241	0.0156	0.0129	0.0070	0.0063	0.0031
Standard deviation	0.9982	0.9806	0.9798	0.9782	0.9758	0.9768	0.9799	0.1926	0.0729	0.0570	0.0557	0.0515	0.0546	0.0602
Monday standard deviation	0.9822	0.9868	0.9847	0.9819	0.9786	0.9773	0.9793	0.1701	0.0680	0.0568	0.0625	0.0493	0.0454	0.0626
Tuesday standard deviation	0.9791	0.9797	0.9798	0.9803	0.9786	0.9800	0.9836	0.2010	0.0749	0.0534	0.0528	0.0536	0.0578	0.0588
Wednesday standard deviation	0.9921	0.9820	0.9820	0.9804	0.9773	0.9780	0.9803	0.2036	0.0664	0.0508	0.0454	0.0509	0.0510	0.0542
Thursday standard deviation	0.9921	0.9744	0.9745	0.9721	0.9721	0.9743	0.9789	0.1615	0.0509	0.0513	0.0540	0.0568	0.0627	0.0684
Friday standard deviation	1.0162	0.9849	0.9829	0.9807	0.9770	0.9786	0.9816	0.1917	0.0924	0.0680	0.0610	0.0452	0.0528	0.0556
ρ_1	0.9809	0.9970	0.9981	0.9981	0.9984	0.9980	0.9979	− 0.3063	− 0.0598	− 0.1700	− 0.2808	− 0.2651	− 0.3249	− 0.3678
ρ_2	0.9734	0.9943	0.9967	0.9972	0.9975	0.9974	0.9972	− 0.1009	− 0.0191	0.0629	0.0729	0.0271	0.0064	0.0896
ρ_3	0.9698	0.9918	0.9952	0.9960	0.9966	0.9967	0.9962	− 0.0443	− 0.0956	− 0.0276	− 0.0331	− 0.0003	− 0.0025	− 0.0492
ρ_4	0.9678	0.9898	0.9937	0.9950	0.9958	0.9960	0.9953	− 0.0204	− 0.1335	− 0.0114	0.0252	− 0.0003	0.0459	0.0299
ρ_5	0.9665	0.9885	0.9923	0.9939	0.9948	0.9951	0.9944	− 0.0146	− 0.0493	− 0.0247	− 0.0167	− 0.0003	− 0.0624	− 0.0339

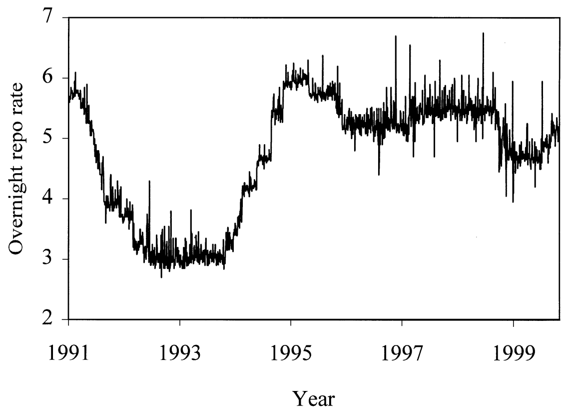

Fig. 1. Graph of daily observations of the overnight repo rate from May 21, 1991 to October 15, 1999.

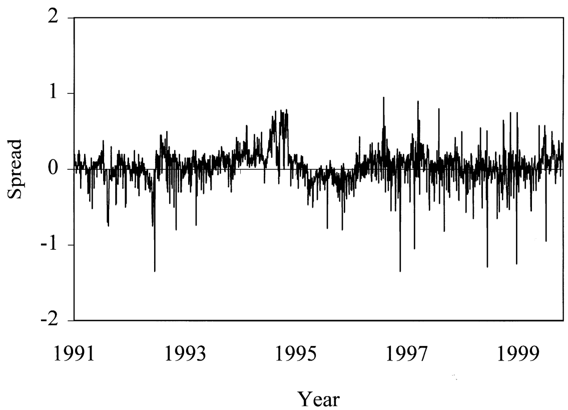

Fig. 2. Graph of daily observations of the spread between the three-month term repo rate and the overnight repo rate from May 21, 1991 to October 15, 1999.

4. Unconditional tests

Earlier research documents the existence of large term premia in short-term rates. For example, Fama (1984b) finds that the term premium in two-month Treasury bills relative to one-month Treasury bills is approximately 38.4 basis points. Similarly, the term premium in three-month Treasury bills relative to one-month Treasury bills is 68.4 basis points. These estimates of the term premium correspond closely with those reported by McCulloch (1987) and

Richardson et al. (1992). Term premia of this magnitude are clearly very large relative to the average level of interest rates. Even moderate time variation in these term premia could drive a wedge between longer-term rates and expected short-term rates.

As an unconditional test of the expectations hypothesis, I examine whether evidence exists of term premia in the term repo rates. Specifically, I compare the average overnight rate from time t to $t+n$, designated R_{t+n}, to the yield on a n-period term repo loan $Y_t(n)$. Under the null hypothesis that the expectations hypothesis holds, Eq. (1) implies that $\mathrm{E}[R_{t+n} - Y_t(n) \mid \Omega_t] = a_n$. Taking the expectation over all information sets gives the result that the unconditional mean of $Y_t(n) - R_{t+n}$ is a constant. Because only the first moment is used in this unconditional test, the results in Fuller (1976) imply that the sample mean of $Y_t(n) - R_{t+n}$ is unaffected by the effects of persistence in the interest rate process. Thus, while unconditional tests are usually less powerful than conditional tests, this approach has the advantage of being free from the small-sample persistence-induced problems identified by Bekaert et al. (1997).

The unconditional term premia estimates are reported in Table 2. Also reported are the t-statistics for the hypothesis that the term premia are zero, where standard errors are corrected for the overlap in the observations using the Hansen and Hodrick (1980) approach. As shown in the table, the unconditional term premia are monotonic in maturity, ranging from 0.56 basis points for the one-week repo rate to 3.19 basis points for the three-month repo rate. None of the term premia is significantly different from zero. Thus, the results from these unconditional tests are consistent with the pure form of the expectations hypothesis in which $a_n = 0$.

These unconditional term premia are much smaller than those reported by Fama (1984b), McCulloch (1987), and Richardson et al. (1992). Because the

Table 2

Summary statistics for the term premia in term repo rates. The term premium is computed as the difference between the term repo rate for the indicated maturity and the average overnight repo rate for the horizon of the term repo rate. The t-statistics reported are based on the Hansen and Hodrick (1980) covariance estimate where lag length equals the length of the overlap in observations. N denotes the number of observations.

Repo maturity	Average term repo rate	Average overnight repo rate	Term premium	t-statistic	N
One-week	4.7062	4.7006	0.0056	1.35	2,091
Two-week	4.7118	4.7028	0.0090	1.48	2,086
Three-week	4.7152	4.7038	0.0114	1.45	2,081
One-month	4.7248	4.7045	0.0203	1.91	2,074
Two-month	4.7313	4.7065	0.0248	1.41	2,052
Three-month	4.7417	4.7098	0.0319	1.15	2,031

period covered by these studies is earlier than that in this study, the issue is raised of whether the differences in results stem from sample period or the use of repo rates instead of Treasury bill rates. To address this, monthly data are collected from the Bloomberg system on constant-maturity one-month and three-month Treasury bill yields for the same period as the repo rate sample, May 1991 to October 1999. The unconditional term premium in the three-month Treasury bill rates is computed by taking the difference between the three-month Treasury bill rate and the average of the one-month Treasury bill rates for the current and the two subsequent months. Averaging these differences over the entire sample period results in an estimate of the unconditional term premium in three-month Treasury bills relative to one-month Treasury bills of 38.5 basis points. This unconditional term premium is highly significant. The Hansen and Hodrick t-statistic for this term premium is 9.53. The size of this term premium is clearly on the same order of magnitude as those documented by Fama (1984b), McCulloch (1987), and Richardson et al. (1992) and indicates that the difference between my results and the earlier literature stem entirely from the use of repo rates instead of Treasury bills. Thus, at an unconditional level, the pure expectations hypothesis holds for repo rates but not for Treasury bill rates. This is consistent with the common Wall Street view than the yields on short-term Treasury bills are lower than the pure riskless rate because of their liquidity or security-specific features. If the yields on short-term bills are below the pure riskless rate, then term premia measured relative to short-term Treasury bills will appear larger.

Note that the difference in the size of the estimated term premia between Treasury bills and repo rates cannot be attributed to the possibility that repo rates include a credit-spread component. If there was a credit spread, the high credit quality of repo loans would imply an upward sloping term structure of credit spreads.[11] Thus, the estimated term premia in repo rates would be even larger than those in Treasury bills if they resulted from credit spreads.

5. Conditional tests

In this section, I test the expectations hypothesis at a conditional level. Specifically, I estimate the regression

$$R_{t+n} - Y_t(n) = a_n + b_n Y_t(n) + \varepsilon_{t+n}. \tag{2}$$

Under the null hypothesis of Eq. (1), the conditional mean of the dependent variable in Eq. (2) is a constant and is reflected in the estimated intercept a_n.

[11] For empirical evidence on the shape of the term structure of credit spreads, see Sarig and Warga (1989) and Longstaff and Schwartz (1995).

Thus, variables in the information set Ω_t should not have explanatory power for the ex post value of $R_{t+n} - Y_t(n)$. Under the null hypothesis, the dependent variable in this regression is orthogonal to any variable in the information set Ω_t. If the expectations hypothesis holds, b_n should be indistinguishable from zero. $Y_t(n)$ is included as an explanatory variable because it represents the time-varying portion of the conditional mean of R_{t+n} under the null hypothesis. In addition, including $Y_t(n)$ as an instrument parallels the traditional specification of tests of the expectations hypothesis in the literature.[12]

The high persistence of interest rates raises a number of econometric issues. In an important paper, Bekaert et al. (1997) use simulation to explore the small-sample properties of a number of standard tests of the expectations hypothesis. They demonstrate convincingly that large biases can exist in many estimators used to test the expectations hypothesis and that their small-sample distribution can be significantly different from their asymptotic distribution. A key implication of their findings is that inferences about the expectations hypothesis should be based on a thorough analysis of the small-sample distribution of estimated parameters.

The results based on the asymptotic distribution of the estimated parameters are shown in Table 3, for each of the six term repo rates in the sample. As illustrated, the expectations hypothesis cannot be rejected for any of the six term repo rates based on the asymptotic t-statistics. The point estimates for b_n are all numerically very close to zero, ranging from 0.00361 to -0.02121. Multiplying these coefficients by the mean value of the term repo rates results in values on the order of only a few basis points, suggesting that any deviations from the expectations hypothesis are small in economic terms. In addition, even the intercept terms are never statistically significant. This is again consistent with the pure form of the expectations hypothesis in which term premia are zero.[13]

The properties of the small-sample distribution of the coefficients, are explored under the assumption that the stochastic process driving the term structure follows a VAR-GARCH model similar to that used by Bekaert et al. (1997). Specifically, I assume that the overnight repo rate and the spreads between the one-week and overnight repo rates, the one-month and overnight

[12] Often, tests of the expectations hypothesis use the spread between the term rate and the short-term rate as the explanatory variable in the regression. As shown by Bekaert et al. (1997), however, the small-sample properties of this type of specification are particularly poor. The Bekaert, Hodrick, and Marshall result is confirmed in my data set. The small-sample properties of the specification in Eq. (2) are much better than those in specifications where both $Y_t(n)$ and the overnight repo rate appear as explanatory variables.

[13] I also test the expectations hypothesis using the overnight rate as the explanatory variable instead of the term-repo rate. The results from this specification are very similar to those reported here.

Table 3
Conditional tests of the expectations hypothesis. The results reported are from the regression of the difference between the realized average overnight rate R_{t+n} for the indicated horizon n and the corresponding term repo rate $Y_t(n)$ regressed on the term repo rate $Y_t(n)$. The t-statistics reported are Hansen and Hodrick (1980) t-statistics for the regression coefficients, where the lag length equals the length of overlap in the observations.

$$R_{t+n} - Y_t(n) = a_n + b_n Y_t(n) + \varepsilon_{t+n}$$

Statistic	One-week	Two-week	Three-week	One-month	Two-month	Three-month
a_n	− 0.02424	− 0.02433	− 0.02846	− 0.03996	− 0.00971	0.04006
t-statistic	− 1.06	− 0.85	− 0.79	− 0.83	− 0.15	0.44
p-value	0.856	0.804	0.785	0.797	0.560	0.330
b_n	0.00361	0.00243	0.00233	0.00219	− 0.00721	− 0.02121
t-statistic	0.75	0.41	0.31	0.22	− 0.54	− 1.20
p-value	0.226	0.343	0.379	0.414	0.705	0.886
Number of observations	2,091	2,086	2,081	2,074	2,052	2,031

repo rates, and the three-month and overnight repo rates follow a fifth-order vector autoregressive process with heteroskedastic innovations. A fifth-order process instead of the second-order process of Bekaert, Hodrick, and Marshall is used because of the day-of-the-week regularities in short-term repo rates documented in Table 1. Following Section 5 of Bekaert, Hodrick, and Marshall, let $z_t = [r_t, S_t(1W), S_t(1M), S_t(3M)]'$ where r_t is the overnight rate and $S_t(1W)$, $S_t(1M)$, and $S_t(3M)$ denote the spreads between the indicated term repo rates and the overnight rate. The fifth-order VAR describing the four time series is

$$z_t = \mu + \sum_{i=1}^{5} c_i r_{t-i} + \sum_{i=1}^{5} d_i S_{t-i}(1W) + \sum_{i=1}^{5} k_i S_{t-i}(1M) + \sum_{i=1}^{5} l_i S_{t-i}(3M) + \varepsilon_t. \tag{3}$$

I model the innovation vector ε_t as a factor structure with the innovations of the overnight rate and the three-month term spread as the factors. Thus, $\varepsilon_t = F e_t$, where

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ f_{21} & 1 & 0 & f_{24} \\ f_{31} & 0 & 1 & f_{34} \\ f_{41} & 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

In this notation, the vector e_t represents the idiosyncratic innovations. Thus, $\mathrm{E}[e_t e_t' | \Omega_{t-1}] = H_t$, where H_t is a diagonal matrix. Consequently, the conditional covariance matrix of the innovations ε_t is given by FH_tF'. Each diagonal element in H_t is assumed to follow a GARCH(1, 1) process, augmented with the square root of the overnight rate as in Gray (1996) and Ang and Bekaert (1998, 1999),

$$h_{it} = \gamma_i \sqrt{r_{t-1}} + \alpha_i e_{it-1}^2 + \beta_i h_{it-1}, \quad i = 1, 2, 3, 4. \tag{5}$$

Proceeding as in Bekaert et al. (1997), the model is estimated by first estimating the $4 \times 21 = 84$ VAR parameters by least squares. I then correct the VAR parameters for small-sample bias in the following way. I estimate the unconditional covariance matrix of the innovations based on the least squares point estimates. I simulate a path of 2,195 realizations of z_t under the assumption that the innovations are normally distributed, discard the first 100 observations to avoid dependence on the starting values, and reestimate the fifth-order VAR. I repeat this process 200,000 times and bias-correct the original OLS estimates

Table 4
Bias-corrected VAR parameters for the VAR-GARCH model. The parameters reported below are obtained by bias-correcting the least squares estimates of the parameters of the VAR model below, where the vector z_t of time series modeled by the VAR consists of the overnight repo rate r_t, the spread $S_t(1W)$ between the one-week rate and the overnight rate, the spread $S_t(1M)$ between the one-month rate and the overnight rate, and the spread $S_t(3M)$ between the three-month rate and the overnight rate. The bias correction is done by estimating the correlation matrix of the residuals using the least squares point estimates of the parameters, simulating the evolution of the VAR process, reestimating the regression parameters using the simulated time series, repeating the experiment 200,000 times, and adjusting the original point estimates by the difference between the point estimates and the mean values from the simulation. Asymptotic standard error (s.e.) is given in parentheses. The sample period is May 21, 1991 to October 15, 1999, consisting of 2,095 daily observations.

$$z_t = \mu + \sum_{i=1}^{5} c_i r_{t-i} + \sum_{i=1}^{5} d_i S_{t-i}(1W) + \sum_{i=1}^{5} k_i S_{t-i}(1M) + \sum_{i=1}^{5} l_i S_{t-i}(3M) + \varepsilon_t$$

Parameter	r_t	(s.e.)	$S_t(1W)$	(s.e.)	$S_t(1M)$	(s.e.)	$S_t(3M)$	(s.e.)
μ	−0.00058	(0.016)	−0.00621	(0.015)	−0.00294	(0.016)	−0.00665	(0.017)
c_1	0.85546	(0.116)	0.06692	(0.104)	−0.13214	(0.109)	−0.18451	(0.113)
c_2	0.18057	(0.122)	−0.08122	(0.122)	0.01953	(0.124)	0.14073	(0.132)
c_3	−0.12560	(0.125)	0.08955	(0.120)	0.10949	(0.125)	0.06835	(0.133)
c_4	0.09217	(0.111)	−0.11538	(0.104)	0.02503	(0.111)	0.00680	(0.124)
c_5	−0.00216	(0.084)	0.03996	(0.081)	−0.02247	(0.084)	−0.03066	(0.094)
d_1	0.59957	(0.136)	0.14482	(0.164)	−0.57052	(0.144)	−0.56164	(0.141)
d_2	−0.07278	(0.133)	0.07915	(0.153)	0.06260	(0.142)	0.04959	(0.140)
d_3	0.06907	(0.124)	−0.16548	(0.125)	−0.13645	(0.124)	−0.13020	(0.128)
d_4	−0.01745	(0.110)	−0.05023	(0.113)	0.00332	(0.111)	0.04455	(0.114)
d_5	−0.03952	(0.059)	0.14951	(0.061)	0.04769	(0.058)	0.02350	(0.061)
k_1	−0.02126	(0.119)	0.19210	(0.130)	0.63504	(0.131)	0.05961	(0.139)
k_2	0.23585	(0.116)	−0.18889	(0.121)	−0.02918	(0.122)	−0.18176	(0.129)
k_3	−0.15482	(0.102)	0.20208	(0.099)	0.15125	(0.104)	0.08559	(0.110)
k_4	0.08600	(0.118)	0.00847	(0.110)	0.02044	(0.121)	−0.06638	(0.122)
k_5	0.06176	(0.092)	−0.11689	(0.094)	−0.10011	(0.097)	−0.11601	(0.100)
l_1	−0.03339	(0.108)	−0.00845	(0.079)	0.09330	(0.105)	0.60889	(0.120)
l_2	0.04636	(0.098)	0.00335	(0.076)	−0.04731	(0.094)	0.25110	(0.112)
l_3	−0.01844	(0.090)	0.02679	(0.075)	0.06806	(0.087)	0.09070	(0.095)
l_4	0.03066	(0.078)	−0.06722	(0.070)	−0.01176	(0.080)	0.01023	(0.084)
l_5	−0.04847	(0.069)	0.02597	(0.067)	0.03928	(0.071)	0.08236	(0.075)

of the VAR parameters by the difference between the OLS estimates and the mean of the OLS parameters from the 200,000 simulated experiments.[14] The bias-corrected VAR parameters are reported in Table 4.

[14] The bias adjustments are generally fairly small and are on the same order of magnitude or smaller than those reported in Table 4 of Bekaert et al. (1997).

Table 5
Volatility parameter estimates for the VAR-GARCH model. This table reports the volatility parameters estimated by quasi-maximum likelihood from the VAR-GARCH model. The sample period is May 21, 1991 to October 15, 1999, consisting of 2,095 daily observations. The time series modeled by the VAR are the overnight repo rate r_t, the spread $S_t(1W)$ between the one-week rate and the overnight rate, the spread $S_t(1M)$ between the one-month rate and the overnight rate, and the spread $S_t(3M)$ between the three-month rate and the overnight rate. The residuals from the bias-corrected fifth-order VAR estimated in Table 4 are assumed to follow a factor structure with the short-term rate and the three-month spread as factors as described in Eq. (4), where $f_{21}, f_{31}, f_{41}, f_{24}$, and f_{34} are the parameters governing the factor structure. The idiosyncratic innovations are assumed to follow the GARCH process shown below. Asymptotic standard errors are given below the parameter estimates.

$$h_{it} = \gamma_i \sqrt{r_{t-1}} + \alpha_i e_{it-1} + \beta_i h_{it-1}, \quad i = 1, 2, 3, 4.$$

Time series	γ_i	α_i	β_i	f_{i1}	f_{i4}
r_t	0.00449	0.27429	0.36611	1.00000	0.00000
	(0.00057)	(0.04131)	(0.06301)		
$S_t(1W)$	0.00087	0.06981	0.11053	− 0.86123	0.20708
	(0.00006)	(0.00730)	(0.03589)	(0.00845)	(0.02081)
$S_t(1M)$	0.00065	0.00770	0.01744	− 0.96090	0.47377
	(0.00008)	(0.00235)	(0.09690)	(0.00684)	(0.01569)
$S_t(3M)$	0.00117	0.02463	0.00621	0.00000	1.00000
	(0.00013)	(0.00544)	(0.08870)		

Using the bias-adjusted VAR parameters, I compute the residual vector ε_t and estimate the 17 parameters defining the factor GARCH process by quasi-maximum likelihood. The estimated parameters governing the volatility of the VAR-GARCH model are reported in Table 5. These parameters are generally similar to those estimated by Bekaert, Hodrick, and Marshall.

The small-sample distributions of the regression coefficients in Eq. (2) are examined in the following way. Under the null hypothesis that the expectations hypothesis holds, the term repo rate $Y_t(n)$ equals the expected average value of the overnight rate from time t to $t + n$. This expected average value is given by the standard technique of forecasting the VAR model and then taking averages as in Bekaert et al. (1997). Using this, I simulate paths of 2,095 observations of z_t and estimate the regression in Eq. (2). I repeat the process 5,000 times and then report in Table 6 the means and standard deviations of the simulated regression coefficients, along with the p-values for the coefficients estimated in Table 3 based on the percentiles of the simulated distribution.

As shown, the small-sample results also imply that the expectations hypothesis cannot be rejected. The p-values for the term repo rate are all greater than

Table 6
Simulation results based on the VAR-GARCH model for the conditional tests reported in Table 3. The simulation is conducted by generating values of the overnight repo rate using the four-factor VAR-GARCH model, and then solving for term repo rates under the assumption that the expectations hypothesis holds. The simulated differences between the realized average overnight rates and the corresponding term repo rate are regressed on the term repo rate as in Table 3. The table reports the means and standard deviations of the indicated parameters over 5,000 simulated sample paths of the overnight repo rate, where each path contains 2,095 daily observations. The small-sample p-values are for the regression coefficients reported in Table 3 based on the percentiles of the distribution obtained from the simulation.

$$R_{t+n} - Y_t(n) = a_n + b_n Y_t(n) + \varepsilon_{t+n}$$

Results	One-week	Two-week	Three-week	One-month	Two-month	Three-month
Mean a_n	0.00666	0.01525	0.02505	0.04132	− 0.00920	− 0.01368
Standard Deviation a_n	0.02472	0.03751	0.04871	0.06593	0.09187	0.13850
Small-sample p-value a_n	0.993	0.906	0.922	0.954	0.512	0.357
Mean b_n	− 0.00131	− 0.00296	− 0.00480	− 0.00782	0.00348	0.00513
Standard Deviation b_n	0.00422	0.00652	0.00859	0.01176	0.02051	0.03087
Small-sample p-value b_n	0.071	0.130	0.129	0.123	0.694	0.808

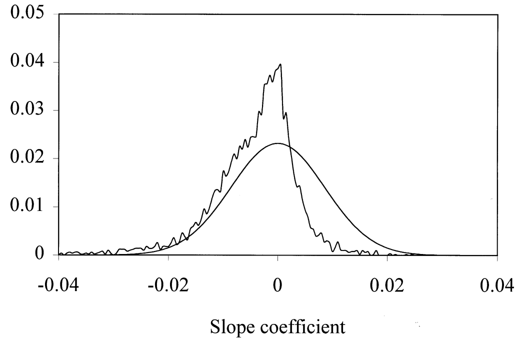

Fig. 3. Monte Carlo and asymptotic distribution (scaled histogram) of the slope coefficient in the regression of the difference between the simulated average overnight rate and the three-week term repo rate on the three-week term repo rate as described in Table 6. The smooth density is the asymptotic distribution of the slope coefficient under the null hypothesis that the expectations hypothesis holds and the data are generated by the VAR-GARCH model. The jagged distribution represents the histogram of slope coefficient estimates from 5,000 Monte Carlo replications under the null hypothesis.

0.050 and are typically greater than 0.100. Similarly, most of the intercepts are not significant based on their small-sample distributions. Again, this is consistent with the pure form of the expectations hypothesis. The only exception is the intercept for the one-week term repo rate, which is approximately 3.10 basis points below its small-sample mean, with a *p*-value of 0.993.

The results in Table 6 also confirm the Bekaert, Hodrick, and Marshall finding that the small-sample distribution of the parameters can differ from the asymptotic distribution. In particular, the *p*-values for the slope coefficient b_n are all somewhat lower based on the small-sample distribution than on the asymptotic distribution. To illustrate this, Fig. 3 graphs the asymptotic and small-sample distribution of the slope coefficient b_n for the three-week repo rate.

6. Conclusion

Having tested the expectations hypothesis at the extreme short end of the term structure using short-term repo rates, I cannot reject the expectations hypothesis at either the conditional or unconditional level. Except for the one-week term repo rate, I cannot reject the hypothesis that the term premia in repo rates are zero. This is consistent with the pure form of the expectations hypothesis in which longer-term rates equal the expected average short-term rate over the horizon of the longer-term rate.

These findings differ from earlier work on Treasury bill markets, which finds evidence of large time-varying term premia in the prices of Treasury bills. The difference in results is directly attributable to the use of repo rates instead of Treasury bill rates. A widely-held view on Wall Street that Treasury bill rates are poor measures of the riskless rate because Treasury bills are influenced by security-specific features such as their liquidity. While this paper cannot completely resolve this issue, finding that repo rates conform much more closely to the expectations hypothesis helps build a case that repo rates may be better measures of the riskless rate.

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