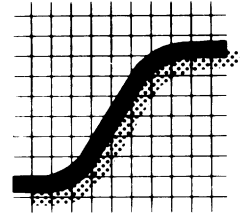# TECHNICAL
Note

SIMPLIFIED ESTIMATION PROCEDURES
FOR MCI MODELS*

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Structural transformations of the MCI model are presented which make the model easily estimated using dummy variables with widely available regression packages. The MCI model is empirically shown to provide better predictive power than several other models of similar form, but ones which do not produce logically consistent market share estimates.

(Multiplicative Competitive Interaction Models; Dummy Variable Regression; Logical Consistency)

1. Introduction

To encourage wider application of multiplicative competitive interaction (MCI) models, this note presents a method for further reducing the computa-

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tional burden associated with estimating the parameters of MCI models and producing logically consistent estimates of market shares (choice probabilities) from MCI procedures. We present a series of structural transformations of MCI models which allow parameters to be estimated by dummy variable multiple regression. The MCI model is then compared to three market share models with similar dummy variable form which do not produce logically consistent market share estimates.

In Nakanishi and Cooper (1974) we proposed a least squares approach for estimating parameters of a generalization of the Huff (1962) model. We call models of the following general type multiplicative competitive interaction (MCI) models:

$$\pi_{ij} = \prod_{h=1}^{H} X_{hij}^{\beta_h} \delta_{ij} \Big/ \sum_{j=1}^{m_i} \prod_{h=1}^{H} X_{hij}^{\beta_h} \delta_{ij} \tag{1}$$

where:

π_{ij} = the probability that a consumer in the ith choice situation (period and/or area) selects the jth object ($i = 1, 2, \ldots, I; j = 1, 2, \ldots, m_i$),

X_{hij} = the value of the hth variable for object j in choice situation i ($X_{hij} \geqslant 0, h = 1, 2, \ldots, H$),

β_h = the parameter for the sensitivity of π_{ij} with respect to variable h.

δ_{ij} = an independently and log-normally distributed, specification error.

Equation (1) produces logically consistent market share estimates (i.e. the estimates of market share are all nonnegative and sum to one over all choice alternatives in the market). Naert and Bultez (1973) argue that logical consistency should be required as a criterion for the appropriateness of market share models.

Model (1) may be transformed into a linear form in the parameters by applying the following transformation to π_{ij}.

$$\log(\pi_{ij}/\tilde{\pi}_{i.}) = \sum_{h=1}^{H} \beta_h \log(X_{hij}/\tilde{X}_{hi.}) + \log(\delta_{ij}/\tilde{\delta}_{i.}) \tag{2}$$

where $\tilde{\pi}_{i.}$, $\tilde{X}_{hi.}$ and $\tilde{\delta}_{i.}$ are the geometric means of π_{ij}, X_{hij} and δ_{ij} over j in choice situation i, respectively. The above transformation will be referred to as "log-centering" hereafter.

Further, we combined the specification error first suggested by a draft of Bultez and Naert (1975) with multinomial sampling error which Bultez and Naert do not consider. In Nakanishi and Cooper (1974), we provided generalized least squares estimation procedures for three cases: only specification error present, only sampling error present, and both types of error present.

While the least squares procedures proposed by Nakanishi and Cooper make MCI models much more available than the functional iteration or direct search approaches of Urban (1969), Kuehn, McGuire and Weiss (1966), Hlavac and Little (1966), and Haines, Simon and Alexis (1972), even less

cumbersome procedures are presented in the next section devoted to estimation of the MCI model by dummy variable regression.

2. Estimation by Dummy Variable Regression

Based on equation (2), we can estimate:

$$\log(p_{ij}/\tilde{p}_{i.}) = \sum_{h=1}^{H} \beta_h \log(X_{hij}/\tilde{X}_{hi.}) + \epsilon_{ij} \tag{3}$$

where:

p_{ij} = an estimate of π_{ij} ($p_{ij} > 0$),
$\tilde{p}_{i.}$ = the geometric mean of p_{ij} over j in situation i,
ϵ_{ij} = the stochastic disturbance term.

The stochastic disturbance term ϵ_{ij} is a function of specification errors, δ_{ij}, and multinomial sampling errors, η_{ij}, resulting from the disparity between sample proportions p_{ij} and population probabilities π_{ij}. Because of this combination of influences the ordinary least squares (OLS) estimates proposed below are not the minimum variance estimators. (The generalized least squares procedures of Nakanishi and Cooper (1974) produce asymptotically minimum variance estimators.)

To estimate market shares from (3) one can use the estimated regression coefficients $\hat{\beta}_h$ and note that

$$\hat{\pi}_{ij} = \prod_{h=1}^{H} X_{hij}^{\hat{\beta}_h} \bigg/ \sum_{j=1}^{m_i} \prod_{h=1}^{H} X_{hij}^{\hat{\beta}_h}. \tag{4}$$

Alternatively one can let the estimate of the dependent variable denoted by

$$\hat{y}_{ij} = \sum_{h=1}^{H} \hat{\beta}_h \log(X_{hij}/\tilde{X}_{hi.}). \tag{5}$$

It follows from (4) and (5) that

$$\hat{\pi}_{ij} = \exp(\hat{y}_{ij}) \bigg/ \sum_{j=1}^{m_i} \exp(\hat{y}_{ij}). \tag{6}$$

We call the transformation in (6) the "inverse log-centering" transformation.

Equation (3) is somewhat cumbersome since it does require the computation of geometric means for several variables. One also needs to suppress the

intercept, a feature available in SAS, SPSS and BMD, but not common to all packaged multiple regression programs. Equation (3) can be rewritten as

$$\log p_{ij} = \sum_{h=1}^{H} \beta_h \log X_{hij} + \log \tilde{p}_{i\cdot} - \sum_{h=1}^{H} \beta_h \log \tilde{X}_{hi\cdot} + \epsilon_{ij}. \tag{7}$$

If we let

$$\alpha_i = \log \tilde{p}_i - \sum_{h=1}^{H} \beta_h \log \tilde{X}_{hi}. \tag{8}$$

α_i does not change over j in a given choice situation, which suggests what is demonstrated in the appendix, that the ordinary least squares estimates from (3) are numerically equivalent to those obtained without log-centering from the following dummy variable regression model.

$$\log p_{ij} = \sum_{i'=1}^{I} \alpha_{i'} D_{i'} + \sum_{h=1}^{H} \beta_h \log X_{hij} + \epsilon_{ij} \tag{9}$$

where:
 $D_{i'}$ = a dummy variable which is equal to 1 if $i' = i$ and 0 otherwise,
 $\alpha_{i'}$ = the regression coefficient for $D_{i'}$.
Equation (9) suppresses the common intercept term. If it cannot be suppressed in a particular multiple regression package one may substitute

$$\log p_{ij} = \gamma_0 + \sum_{i=1}^{I-1} \gamma_i' D_i' + \sum_{h=1}^{H} \beta_h \log X_{hij} + \epsilon_{ij} \tag{10}$$

where:
 $\gamma_0 = \alpha_I'$,
 $\gamma_i' = \alpha_i' - \alpha_I'$.
To estimate choice probabilities from (10) one should use the inverse log-centering transformation, that is if we let $\hat{y}_{ij}^*$ be the estimate of the dependent variable in (10) then as per Equation (6) we obtain

$$\hat{\pi}_{ij} = \exp(\hat{y}_{ij}^*) \Big/ \sum_{j=1}^{m_i} \exp(\hat{y}_{ij}^*). \tag{11}$$

For the purpose of the regression analysis then one may use either (3), (9) or

(10) all of which will produce identical estimates $\{ \hat{\beta}_h \}$. When the total number of choice situations (periods or areas), I, is small, one should be indifferent between the three regression models on all criteria except those dealing with adaptability to computational software. When I is large, the prereduction of data by log-centering reduces computational expense by not having to estimate the parameters corresponding to dummy variables.

The dummy variable regression technique described above is also applicable to the so-called multinomial logit (MNL) models, a typical representation of which takes the following form:

$$\pi_{ij} = \exp\left(\sum_{h=1}^{H} \beta_h X_{hij} \right) \Big/ \sum_{j=1}^{m_i} \exp\left(\sum_{h=1}^{H} \beta_h X_{hij} \right). \tag{12}$$

One may view the MNL formulation as an MCI model in which all explanatory variables are exponentially transformed. A regression model analogous to (9) for the model in (12) is given by:

$$\log p_{ij} = \sum_{i'=1}^{I} \alpha_i' D_{i'} + \sum_{h=1}^{H} \beta_h X_{hij} + e_{ij}. \tag{13}$$

Although more complex estimation techniques for MNL models have been proposed elsewhere (cf. Manski and McFadden, 1981) equation (13) provides an easy estimation technique when ratio-scaled estimates of π_{ij}'s are available. Choice probabilities for MNL models may be estimated by the inverse log-centering transformation (11).

3. Comparison with Related Models

The dummy variables regression model (9) is directly comparable with other log-linear models. Model (9) clearly is a generalization of the log-linear regression model of the form

$$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log X_{hij} + \epsilon_{ij}. \tag{14}$$

Model (14) is a special case of model (9) in which the intercept term is assumed to be identical for each choice situation. This special case does not produce logically consistent market share estimates.

To illustrate the advantage of logically consistent models, regression models

(9) and (14) are fitted to data from Huff (1963),[1] along with two variations of (14), namely,

$$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log\left(X_{hij} \Big/ \sum_{j=1}^{m_i} X_{hij} \right) + \epsilon_{ij} \quad \text{and} \tag{15}$$

$$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log\left(X_{hij}/\overline{X}_{hi.} \right) + \epsilon_{ij}, \tag{16}$$

where $\overline{X}_{hi.}$ is the arithmetic mean of X_{hij} over j in choice situation i. The explanatory variables in (15) are in a share form and those in (16) are in a normalized form. Both have been used as market share models before (e.g., Lambin 1972, Weiss 1968, and Wildt 1974). None of those models satisfy the logical consistency requirement.

Table 1 shows the OLS estimates of the parameters of the four models. Model (9) gives a marginally better fit as indicated in the R^2-values. When the estimates of π_{ij}'s are computed from respective models (by inverse log-centering in the case of (9)), marked differences emerge. The first column of Table 2 shows the mean squared deviation between $\hat{\pi}_{ij}$ and p_{ij} of (analogous to the variance of estimation errors) for each model. The mean squared deviation for the MCI model is by far the smallest. Since (9) includes dummy variables, it has fewer degrees of freedom than other models, and its better fit is not strong evidence of its superiority.

A cross validation analogous to the one in Nakanishi, Cooper and Kassarjian (1974) may be performed here. The parameters of all models may be estimated from each pair of choice situations and used to "predict" the choice probabilities in each remaining choice situation. The average cross validity correlations between actual and predicted probabilities are all very high. But the average of the mean squared errors strongly favors the MCI model (9), which produces mean squared errors three to ten times smaller, as shown in Table 2.

The dummy variable regression (10) should make the estimation of multiplicative competitive interaction (MCI) models easy. It should be emphasized,

[1] The data taken from Huff (1963, pp. 453–454). The estimated values of choice probabilities are $p_{ij} = n_{ij}/n_i$, where n_i is the sample size in situation i and n_{ij} is the number of respondents who chose object j (a shopping center in this case). The variables are shopping center size in thousands of square feet (used as a surrogate for width of display space), and travel time. Finding a scalar to convert center size into width of display space is superfluous in model (1) since multiplying each explanatory variable by a constant, possibly unique to each choice situation, does not affect the estimates of β_h. Choice measure were collected on 14 shopping centers in three neighborhoods. Centers which received no share of a neighborhood were dropped from that choice situation. This resulted in $m_1 = 5$, $m_2 = 8$ and $m_3 = 12$ shopping centers.

TABLE 1

Parameter Estimates* for Log-Linear Models with Standard Errors (in Parentheses)

Model	Parameters						
(Equation)	α_0	α_1	α_2	α_3	β_1	β_2	R^2
(9) MCI	—	− 6.22	− 5.82	− 5.79	1.46	− 2.41	0.78
		(1.38)	(1.33)	(1.31)	(0.26)	(0.29)	
(14)	− 5.73	—	—	—	1.40	− 2.34	0.77
	(1.28)				(0.25)	(0.28)	
(15)	− 4.48	—	—	—	1.60	− 2.11	0.69
	(0.68)				(0.29)	(0.22)	
(16)	− 3.45	—	—	—	1.44	− 2.40	0.76
	(0.16)				(0.26)	(0.29)	

* All parameter estimates are significant at the 0.01 level.

however, that (10) is an ordinary least squares procedure and does not produce the best estimators in terms of minimum variance. Generalized least squares procedures (cf. Nakanishi and Cooper 1974) are needed to produce (asymptotically) minimum variance estimators.

By following an approach analogous to that indicated here, the dummy variable regression (10) can also be generalized to extended MCI models which allow parameters to vary over objects, j, or models in which the market share of one brand is affected by the marketing efforts of others. The methodological issues involved in these generalizations are the topic of separate development.

Appendix

Consider the least-squares estimates $\hat{\alpha}_i$ and $\hat{\beta}_h$ and (9):

$$\operatorname*{Min}_{\alpha_i, \beta_h} \sum_{i=1}^{I} \sum_{j=1}^{m_i} \left[\log p_{ij} - \sum_{i'=1}^{I} \alpha_{i'} D_{i'} - \sum_{h=1}^{H} \left(\beta_h \log X_{hij} \right) \right]^2 .$$

TABLE 2

Mean Squared Error Between Actual and Estimated Market Shares

Model	Mean	Average Cross Validation Results	
(Equation)	Squared Error[a]	Correlation of p and $\hat{p}$	Mean Squared Error
9 (MCI)[b]	0.005	0.988	0.005
14	0.017	0.987	0.016
15	0.039	0.962	0.050
16	0.022	0.988	0.032

[a] Degrees of freedom are 20 for the MCI and 23 for other models.

[b] Estimated market shared are computed through the inverse log centering transformation.

For any given β_h, the optimal α_i has the property:

$$\underset{\alpha_i}{\text{Min}} \sum_{j=1}^{m_i} \left[\log p_{ij} - \alpha_i - \left(\sum_{h=1}^{H} \beta_h \log X_{hij} \right) \right]^2.$$

Note that given $\{\beta_h\}$, the problem of finding the optimal α_i becomes separable. Thus

$$\alpha_i = \frac{1}{m_i} \sum_{j=1}^{m_i} \left(\log p_{ij} - \sum_{h=1}^{H} \beta_h \log X_{hij} \right)$$

$$= \frac{1}{m_i} \sum_{j=1}^{m_i} \left(\log p_{ij} - \sum_{h=1}^{H} \beta_h \frac{1}{m_i} \sum_{j=1}^{m_i} \log X_{hij} \right)$$

$$= \left(\log \tilde{p}_{ij} - \sum_{h} \beta_h \log \tilde{X}_{hi \cdot} \right)$$

By substituting the optimal value $\hat{\alpha}_i$ into (9) we get (3) thereby proving the result that the optimal $\hat{\beta}_h$ for (3) are the same as those for (9).

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