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Bivariate time-series analysis of the relationship between advertising and sales

DOMINIQUE M. HANSSENS

University of California, Los Angeles, U.S.A.

This paper focuses on empirical model building of the sale–advertising relationship at the aggregate response level. Arguments are presented in favour of a combined Box–Jenkins econometric approach to model this relationship. The approach is illustrated and the resulting model is compared to competing models with respect to descriptive and forecasting performance.

I. INTRODUCTION

The successful introduction of the method of Box and Jenkins for univariate time-series model building has raised an important controversy about the value of integrated autoregressive-moving average models (*ARIMA*) versus econometric models. In the context of forecasting, *ARIMA* models sometimes outperform econometric equations, as reported, e.g. by Nelson (1972). This is a surprising finding, in light of the fact that *ARIMA* models use the information of only one time series for forecasting. However, it is known by now that *ARIMA* and econometric models are related to each other. This relationship removes much of the original controversy, but, more importantly, it creates some unique opportunities for empirical causal model building and testing on longitudinal data. In Trivedi's (1975) words, " . . . time series and structural models need not be thought of as competing approaches."

The purpose of this paper is to use the integration of structural models and time-series analysis to investigate empirically the relationship between product sales and advertising expenditures. In the literature this problem has been approached primarily with econometrics. For example, Clarke (1976) lists no less than 69 econometric sales–advertising studies performed between 1962 and 1975. Although this approach has certainly been successful, one has to be aware of its limitations and dangers. With respect to parameter estimation, the frequently occurring problems such as multicollinearity, heteroskedasticity and autocorrelation are well known and many technical solutions for them have been proposed. With respect to model specification, the critical factor is the availability of a tested theory to generate a model, e.g. price theory in economics. In the case of advertising, about the only element of theory on which there is general agreement is that it has some

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positive effect on sales, which is of rather low magnitude and possibly distributed over time, i.e. a carryover effect. In addition, there is evidence of a feedback effect, i.e. past sales may partially determine the level of future advertising expenditures (e.g. Bass, 1969). In conclusion, a substantial portion of sales–advertising model building is left to empirical research, i.e. to the search for statistical regularities in the data. While this inductive approach to research has its powerful advocates, e.g. Ehrenberg and Simon, caution is required with longitudinal data. As early as 1926 the dangers of correlational techniques on time-series data have been pointed out (Yule, 1926), yet in practice they are often neglected.

This brief description serves as a background for the present research which proposes to use multiple time-series analysis in conjunction with econometric model building for the study of the sales–advertising relationship. Specifically, the issues of *causality detection* (in grangian sense) and *lag specification* from data analysis are at stake. The opportunity for this research comes from recent advances in theoretical time-series analysis and focuses around the well known method of Box and Jenkins. The reader who is not familiar with this methodology is referred to Box and Jenkins (1976) and Haugh and Box (1977).

II. DATA

A multiple time-series model for advertising and sales will be developed for the Lydia Pinkham vegetable compound data. This product was introduced in 1873 as a remedy against menopausal malaise and menstrual pain and has been in the market ever since. The history of the company gained strong publicity on several occasions because of controversies around the product's components and a large court case which made an excellent company data base public. This history is described in full in Palda (1964), who also lists annual data on dollar sales and advertising for 1907 through 1960 and monthly data for January 1954 through June 1960.

This data base was selected for several reasons. First, the product is a frequently purchased, low-cost consumer nondurable, which is a category of substantial interest to empirical researchers in marketing and economics. Secondly, advertising was almost the exclusive marketing instrument used by the company; price changes were small and rare, whereas the distribution, mainly through drug wholesalers remained fairly constant. Furthermore, there were no direct competitors for this product, so that the market under study can be labelled as a closed sales–advertising system. Thirdly, the data series are sufficiently long to perform a meaningful statistical analysis and reserve a portion of the data for forecasting.

Since the Lydia Pinkham data are publicly available and of excellent quality they have been used by several researchers for various purposes. For example, Palda (1964) introduced the first empirical evidence of the advertising carryover effect on sales using this data. Clarke and McCann (1977) and Houston and Weiss (1975) argued about the role of serial correlation in the error term. Helmer and Johansson (1977) provided an application of Box–Jenkins transfer function analysis, Caines, Sethi and Brotherton (1977) presented a new system identification algorithm and Melrose (1969) did an empirical study on optimizing advertising policy.

In spite of these and other literature sources this study can not benefit too much from earlier findings. The reason is that virtually all previously published Lydia Pinkham models are based on the annual data series. In an important review of the field, Clarke (1976) found that such models are seriously affected by temporal aggregation bias. For example, sales response models to advertising based on yearly data suggest advertising carryover effects of several years, whereas models on monthly or quarterly data indicate that the effects last only a few months. Clarke presents strong arguments for using the shorter interval data, specially in the case of frequently purchased products. In this paper only the monthly data are considered.

III. EMPIRICAL ANALYSIS

Univariate models

The first part of the empirical analysis is the fitting of univariate time-series models to the two sets of observations. Since numerous examples of this process exist in the literature, this step is not discussed in great detail. It was found that the series exhibit a strong seasonal pattern, so the twelfth-order seasonal differences (∇^{12}) are taken first. A Prothero–Wallis (1976) test on the transformed series confirmed that they are stationary. The *ARIMA* models are

$$\nabla^{12}A_t = (1 - 0.477\,B^{12})\alpha_{A_t},\ RSS = 2\ 199\ 000,\ \chi^2(23) = 10.890,$$

$$\nabla^{12}S_t = -44.98 + (1 - 0.257\,B^{12} - 0.621\,B^{15})\alpha_{S_t},\ RSS = 1\ 083\ 800,\ \chi^2(21) = 9.407 \tag{1}$$

where RSS is the residual sum of squares and χ^2 (k) is the Box–Pierce Chi-square statistic for white noise.

Causality Detection

The first important conclusion from the *ARIMA* model building is that the autoregressive operators of the two variables are the same. Following Quenouille (1957), this implies that it is possible that a two-way causal structure between these series exist, i.e. advertising and sales can be both endogenous variables.

To test whether, in fact, there is empirical evidence of a two-way structure, the estimated *ARIMA* residuals α_A and α_S are cross-correlated at various lags. These cross-correlations are shown in Table 1; for comparison purposes we list the values for the original, non-whitened data. The patterns on the whitened data differ substantially from these on the raw data, which are very irregular and would suggest strong interrelationships between the two series at various lags. However, as Granger and Newbold (1974) and others have pointed out, many of the correlations on the original data are likely to be spurious, so that model specification should be performed on the white-noise series only.

First, the hypothesis of series' independence is tested using Haugh's M statistic:

$$M = N \sum_{i=-k}^{k} r^2 \alpha_A \alpha_S (i).$$

Haugh (1976) has shown that, under the null hypothesis of independence, the value M is

Table 1 *Cross-correlograms*

Lag[a]	Original Data	Whitened Model
−12	0.46	0.24
−12	0.57	0.08
−10	0.25	−0.08
−9	−0.09	0.01
−8	−0.31	−0.13
−7	−0.08	0.06
−6	0.40	0.01
−5	0.20	−0.09
−4	0.03	−0.02
−3	−0.28	0.22
−2	−0.42	0.18
−1	−0.12	−0.07
0	0.40	0.25
1	0.62	0.29
2	0.32	0.35
3	−0.11	0.04
4	−0.31	−0.24
5	0.02	−0.26
6	0.37	−0.17
7	0.31	0.19
8	−0.03	−0.10
9	−0.33	−0.15
10	−0.46	0.11
11	−0.11	0.14
12	0.23	−0.12

[a]Negative lags: $\alpha_S \rightarrow \alpha_A$. Positive lags: $\alpha_A \rightarrow \alpha_S$.

approximately Chi-square distributed with $(2k + 1)$ degrees of freedom. Using $k = 12$ the test result is

M	d.f	*Verdict* ($\alpha = 0.05$)
39.539	25	*reject* H_0

so that the possibility of series independence need not be considered further.

Next, the cross-correlograms are used to determine possible shapes of the dynamic shock model (Haugh and Box, 1977) relating α_A and α_S. Bearing in mind that individual coefficients in the neighbourhood of 0.25 $(2/\sqrt{N})$ or more in absolute value can be considered significant, the conclusions are: – the only possible feedback effect is at lag twelve, i.e.

$$\alpha_{A_t} = \omega'_{12} B^{12} \alpha_{S_t} + u'_{1t},$$

where u'_1 is the added noise term. Advertising appears to have a carryover effect on sales, at least in the first few periods. This effect can be represented explicitly, e.g.

$$\alpha_{S_t} = (\omega_0' + \omega_1' B + \omega_2' B^2)\alpha_{A_t} + u_{2t}',$$

or implicitly through a Koyck formulation, which assumes geometric decay of the advertising impact on sales, i.e.

$$\alpha_{S_t} = \frac{\omega_0'}{(1 - \delta_1' B)} \alpha_{A_t} + u_{2t}'.$$

For parameter estimation we make use of the fact that a dynamic shock model is a special case of Wall's Rational Lag Distributed Structural Form (*RSF*) for which a full information maximum likelihood (*FIML*) estimation method was developed (Wall, 1976). What makes this procedure more attractive than, say, ordinary least squares is the fact that it allows to estimate the added noise parameters and that several equations can be parameterized simultaneously.

Experiments with various dynamic shock model specifications yielded an interesting result: the contemporaneous effect of α_A on α_S is not significant. Since a contemporaneous effect in the other direction is ruled out on logical grounds, a model with lagged effects is specified. The strongest advertizing impact occurs at period one, after which there is a monotone decay. The fully parameterized dynamic shock model with standard errors of parameters between brackets is

$$\alpha_{A_t} = \underset{(0.239)}{0.396} B^{12} \alpha_{S_t} + e_{1t}, \tag{2a}$$

$$\sigma^2_{e_1} = 36\,570,$$

$$\alpha_{S_t} = \frac{0.271\ (0.111)\, B}{1 - \underset{(0.190)}{0.492} B} \alpha_{A_t} + e_{2t}, \tag{2b}$$

$$\sigma^2_{e_2} = 21\,010.$$

Overall, the relationships suggested by the cross-correlograms are confirmed. With respect to the error terms e_1 and e_2, they are not significantly correlated. Strictly speaking one expects an $AR(1)$ process on e_2 (see, e.g. Haugh and Box, 1977), but since its coefficient was not significant it was deleted from the analysis.[1]

[1] The equations for sales, including an $AR(1)$ noise process were

$$\alpha_{S_t} = \frac{0.244\ (0.121) B}{1 - \underset{(0.208)}{0.529} B} \alpha_{S_t} + \frac{1}{1 - \underset{(0.181)}{0.163} B} e_{2t}$$

$$\sigma^2_{e_2} = 21\,020.$$

Structural Model Building

The last step in model development consists of transforming the dynamic shock model into a structural form. This transformation is done quite easily in theory, since it amounts only to substituting the *ARIMA* Equations 1a and b in Equation 2

$$\alpha_{A_t} = \omega'_{12} B^{12} \alpha_{S_t} + e_{1t}$$

$$\alpha_{S_t} = \frac{\omega'_1 B}{1 - \delta'_1 B} \alpha_{A_t} + e_{2t}$$

becomes

$$\frac{1 - B^{12}}{1 - 0.45 B^{12}} A_t = k' + \omega'_{12} B^{12} \frac{B^{12}}{1 - 0.26 B^{12} - 0.62 B^{15}} S_t + e_{1t},$$

$$\frac{\cdot - B^{12}}{- 0.26 B^{12} - 0.62 B^{15}} S_t = \frac{\omega'_1 B}{1 - \delta'_1 B} \frac{1 - B^{12}}{- 0.45 B^{12}} A_t + e_{2t}, \tag{3}$$

or

$$\nabla^{12} A_t = k + \frac{\omega'_{12} B^{12} (1 - 0.45 B^{12})}{1 - 0.26 B^{12} - 0.62 B^{15}} \nabla^{12} s_t + (1 - 0.45 B^{12}) e_{1t},$$

$$\nabla^{12} S_t = \frac{\omega'_1 B}{1 - \delta'_1 B} \frac{1 - 0.26 B^{12} - 0.62 B^{15}}{1 - 0.45 B^{12}} \nabla^{12} A_t + (1 - 0.26 B^{12} - 0.62 B^{15}) e_{2t}. \tag{4}$$

This theoretically derived structural form is quite complex unless a few simplifying assumptions are made. In particular the high-order autoregressive denominator processes implied by the model are unrealistic for economic data and expensive to parameterize. Therefore a structural form on the seasonally differenced data ($\nabla^{12} z_t$) is specified and the assumption is made that the *MA* processes of advertising and sales cancel each other out.[2] The structural model then becomes:

$$\nabla^{12} A_t = k_1 + \omega_{12} B^{12} \nabla^{12} S_t + \psi_A (B) e_{1t},$$

$$\nabla^{12} S_t = k_2 + \frac{\omega_1 B}{- \delta_1 B} \nabla^{12} A_t + \psi_S (B) e_{2t}$$

where ψ_A and ψ_S are the added noise factors.

The estimation of this *RSF* model consists of two steps. First, the transfer function parts are parameterized, using *FIML*. Then, the residuals are examined, in order to find the added noise processes, *MA*(12) for ψ_A and *MA*(1, 2, 3) for ψ_S. The resulting model is

$$\nabla^{12} A_t = 0.416 B^{12} \nabla^{12} S_t + (1 - 0.568 B^{12}) e_{1t}, \tag{5a}$$

$$\sigma^2_{e_1} = 34\,481.$$

[2] This assumption implies primarily that the *MA*(15) term is ignored, which may result in a slightly more complex added noise level.

$$\nabla^{12} S_t = \frac{0.374\ (0.126)\ B}{1 - 0.546\ B} \nabla^{12} A_t + (1 + 0.082\,B - 0.068\,B^2 + 0.372\,B^3) e_{2t}, \qquad (5b)$$
$$\qquad (0.188) \qquad\qquad\qquad (0.177) \quad (0.184) \quad (0.193)$$

$$\sigma^2_{e_2} = 25\ 600.$$

IV. FORECASTING PERFORMANCE

The development of a structural model has requested a substantial research effort and the question is raised immediately whether this effort has been worthwhile. The answer can only be furnished by performing a model evaluation. Specifically, we would like to know whether Equation 5 is a superior forecasting tool and whether it provides better insight into market structure than competing approaches. The second evaluation is done verbally in the conclusions section. The forecasting evaluation is the subject of this section, using the eighteen holdout observations mentioned earlier.

Several sources in the literature (e.g. Nelson, 1972) have demonstrated the superiority of so-called naive *ARIMA* models over econometric models for forecasting, so it is logical to compare the forecasting performance of the *ARIMA* Equation 1 to our model. In addition, two econometric models developed on the same monthly data are selected for the test (see Palda, 1964, pp. 37–8). Since Palda used all 78 observations for model development and since his data were deseasonalized, the equations are re-estimated using *OLS* first

$$S_t = 1112.831 + 0.188A_t + 170A_{t-1} + 0.041A_{t-2} + u_t(R^2 = 0.368) \qquad (6)$$
$$(0.63) \quad (0.070) \quad (0.062)$$

$$S_t = 616.763 + 0.435S_{t-1} + 0.206A_t + u_t(R^2 = 0.377) \qquad (7)$$
$$(0.107) \quad (0.046)$$

Since there are no published advertising models on the monthly data the *ARIMA* Equation 1a will be used to generate forecasts of the explanatory variables in Equations 6 and 7. The results of this comparison are shown in Table 2.

Table 2 *Comparison of forecasting performance*[a]

	Advertising	*Sales*
ARIMA, Equation 1	678 863	357 772
RSF Equation 5	489 985	354 577
ARIMA Equation 1 on Advertising, *PALDA* Equation 6 on Sales	678 863	870 023
ARIMA Equation 1 on Advertising, *PALDA* Equation 7 on Sales	678 863	885 713

[a]Table entries are sums of squared forecast errors for eighteen holdout observations.

All models perform satisfactorily in predicting future patterns of sales and advertising. However, there are differences in the quality of this performance which can be measured by overall forecast errors. Equation 5 performs best on both series, reducing the forecast error for advertising by 28 percent and the error in sales by 1 to 60 percent. One should not make too strong inferences about a forecasting sample of only eighteen observations, but at least the conclusion can be made that Equation 5 performs consistently better than its competitors, sometimes substantially better.

Table 2 also shows that the 'naive' *ARIMA* models provide good forecasts. In fact, in a few experiments on one-step ahead forecasting, the univariate models occasionally out-performed all the causal models. These findings are not in conflict with reported results in the literature and, using Pierce and Haugh's terminology (1977), indicate the importance of examining the 'intrastructure' of time series as thoroughly as the 'interstructure.'

V. CONCLUSIONS

In this paper a structural model for monthly Lydia Pinkham advertising and sales was developed, combining the time-series properties and the causal relationships of the two series. This model has proven superior forecasting performance against several univariate and multivariate alternatives. What remains to be done is to evaluate Equation 5 as a description of the relationship between sales and advertising. The discussion centres around three contributions: the modeling of a feedback relationship, the specification of carryover effects of advertising and the distinction between time-series effects and exogenous effects on sales.

The *RSF* Equation 5 is the first one to acknowledge the feedback relationship between sales and advertising in the monthly Lydia Pinkham series. This relationship was found after the systematic elements in sales and advertising were removed, consistent with the causality detection framework of Granger–Newbold and Haugh. From an econometrician's standpoint it implies that a simultaneous-equation model is needed, a point which has been discussed and illustrated at length by Bass and Parsons (1969). In addition to the discovery of feed-back, the combined time-series econometric model also assesses the relative importance of the feedback function, which can be derived from the following summary of structural error variances

Model	Error Variance
Mean	202 833
$\nabla^{12}A_t$	48 620
Feedback Function	40 770
Feedback plus Noise model	34 481

A substantial portion of total variability in advertising is due to systematic, in this case seasonal, variation. As a result, it can be said that the sales feedback on advertising is important only to the extent that it explains deviations from the expected seasonal patterns in advertising. Especially in the absence of inside information about managerial decision making, the combined feedback–time series model for advertising provides a reliable picture of the aggregate outcome of the advertising budgeting process in this organization.

As a second contribution, the *RSF* model reveals a clear, empirically determined shape of the advertising carryover effects. The main difference with other published results is the absence of a contemporaneous effect, since Equation 5 suggests that the first advertising effect occurs after a one-period lag. This result is plausible because the Lydia Pinkham sales figures are recorded when the goods are shipped, whereas advertising expenditures are made when the advertisements actually appear. In itself, this finding may not be too important, but it does illustrate the risk of modeling spurious relationships among variables when one fails to take into account their time-series properties first. Finally, the magnitude of the carryover effect, which is measured by the output lag coefficient 0.518, suggests that the 90 percent duration interval of advertising is 4.5 months. This finding is in agreement with Clarke's conclusion that 'the published econometric literature indicates that 90 percent of the cumulative effect of advertising on sales of mature, frequently purchased, low-priced products occurs within 3 to 9 months of the advertisement' (Clarke, 1976).

Last but not least, this modeling approach allows one to compare the importance of the 'intrastructure' and the 'interstructure' for monthly sales. Again, consider the following list of structural error variances

Model	Error Variance
Mean	36 856
$\nabla^{12}S_t$	30 433
Advertising Effects	27 610
Advertising plus noise model	25 600

The situation is quite different from the advertising case. First, it appears that neither past sales nor the advertising history are very successful in reducing the variance in the series. In the literature on sales and advertising, several sources report R-squares of 0.70 and more, which leads one to observe that (1) R-square is not a good measure of association on longitudinal data (see Pierce, 1977) and (2) that one should not expect very good fit with a limited information set such as the one used in this study. Secondly, one can conclude that the exogenous influences and the added noise are about equally important in explaining deviations from the seasonal pattern in sales ($\nabla^{12}S_t$). The results also indicate that, while the advertising effects are significant, they do not have a very important influence upon month-to-month variations in sales.

Although this study of combined time-series and economic model building is limited because only two series were considered, it is hoped that it generates insight, not only into the empirical research opportunities offered by this relatively new methodology, but also into the value of the information captured in the history of an economic time series before inferences are made about causal relationships.

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