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The author's principal objective is to present a framework for market analysis which specifically models primary demand, competitive reaction, and feedback effects of the market variables. The approach is an extension of earlier work by Clarke and by Lambin, Naert, and Bultez on the relationship among the elasticities of the marketing variables. The author develops this framework and formulates an approach for empirical applications based on principles of time series analysis. In particular, Granger's well-known causality definition is used in conjunction with Box-Jenkins analysis to find the nonzero elements in the marketing model. These principles are applied empirically to the case of a city pair of the U.S. domestic air travel market, where three major airlines compete on the basis of flight scheduling and advertising. The analysis reveals that flight scheduling has a market-expansive or a competitive effect, depending on the competitor, and that advertising does not have a significant impact on performance. In addition, several patterns of competitive reactions are found. The author offers observations on the theoretical and empirical aspects of this approach to marketing model building.

Market Response, Competitive Behavior, and Time Series Analysis

In an increasingly complex and risky business environment, the development of quantitative models of markets is a difficult but rewarding task. In the past decade such models have been of strong interest among academicians, as shown by the large number of articles published in this field. Also, models of markets are gaining popularity in industry, where they are used for forecasting as well as evaluation of market plans (e.g., Stryker 1978; *The Wall Street Journal* 1977).

The structural relationships that are part of a market mechanism can be categorized into sales response effects, competitive reactions, and feedback effects. Though the importance of these types of market relationships has certainly been recognized, very few empirical studies have included all of them simultaneously. For example, numerous studies have analyzed the marketing mix effects on sales of single products, isolated from the market in which the products operate. The main reason for such an approach is probably the lack of good data, because most market research is done in the profit sector where data are typically scarce and/or proprietary. Unfortunately, failure to include the relationships among certain variables may result in severe model misspecification and, ultimately, unreliable research findings.

The main purpose of this article is to propose a systematic modeling of the various relationships that characterize markets. To recognize the *dynamic nature* of market variables, interest is focused on *longitudinal* data sets, which are the most common and the most useful, because marketing planning is dynamic. The first part of the article is marketing theoretical. After a brief review of the literature, the theoretical models proposed by Clarke (1973), Lambin, Naert, and Bultez (1975), and Schultz and Wittink (1976) are examined. The findings of those researchers can be integrated and extended into a full-scale dynamic model of a market, i.e., a model which incorporates all the poten-

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tial structural relationships previously described. The complexity of such a model necessitates the description of an empirical approach for using the model. This is done by application of recent developments in univariate and multiple *time series analysis* in conjunction with econometrics. Specifically, the issues of discovering relationships and ruling out spurious associations are discussed.

The second part of the article is an empirical illustration of the theoretical ideas, based on a submarket of the U.S. domestic air travel industry. The model considers each of the three competitors in this market separately and includes the two most important marketing instruments for the period studied, number of flights and advertising expenditures.

PRIOR RESEARCH

Some important contributions in the evolution of modeling sales response, feedback, and competitive behavior are reviewed hereafter. Parsons and Schultz (1976) and Naert and Leeflang (1978) provide a more thorough discussion of the literature.

Sales Response Elements

Marketing model builders have devoted most of their efforts toward developing functions of sales response to the marketing mix. The earlier applications typically considered only one product and one marketing variable at a time, for example Palda's (1964) regression models on Lydia Pinkham sales and advertising, which stimulated a subsequent research stream on the dynamic or carryover effects of advertising (e.g., Bass and Clarke 1972; Houston and Weiss 1975). At about the same time, more studies appeared which considered two or more marketing mix variables, for example Lambin's (1970) study on small electric appliances and Little's (1975) work on a packaged food product. Also, some researchers added competitive marketing efforts as explanatory variables in response functions, for example Sexton (1970) and Urban (1969) on frequently purchased branded goods. Finally, there was a definite trend toward analyzing markets rather than single product sales, as exemplified by the more complex models by Beckwith (1972) and McCann (1974), also on frequently purchased branded goods.

One issue in sales response modeling which has not received sufficient attention is the distinction between market-expansive (i.e., primary demand) effects and competitive (i.e., secondary demand) effects of the marketing variables. As Parsons and Schultz (1976) point out for the case of advertising, most studies really have not been designed to test for the presence of primary demand effects. Yet the question is very important because in the absence of market expansion the marketing efforts of the competitors may cancel each other out. For example, a study by Metwally (1978) indicates ". . . that advertising in a number of Australian industries is self-cancelling and escalating." Fortunately, two theoretical developments are very useful for the study of expansive versus competitive effects: (1) a simple mathematical equality, due to Clarke (1973), which states that the elasticity of a marketing instrument on sales (n_S) equals the primary demand elasticity (n_{PD}) plus the market share elasticity (n_m), and (2) a set of theoretical conditions for the existence of primary demand, primary sales, competitive, and mixed effects of advertising derived by Schultz and Wittink (1976). These contributions are used in the model development hereafter.

Feedback Elements

The possible presence of a feedback relationship between sales and the marketing mix variables has not been investigated thoroughly, in spite of an early warning by Quandt (1964). Bass and Parsons (1969) used predictive testing and simultaneous equations to include the effects of past sales on future advertising budgets in a model of the cigarette industry. Similar efforts were made by Schultz (1971) and Wildt (1974), but are encompassed under the more general case of endogenous marketing decision variables. In terms of model estimation, the modeling of feedback is necessary only if there is a true simultaneous relationship. Because most marketing models are built on data for relatively short time intervals such as months or quarters they are more likely to be recursive in sales and, say, advertising. In such cases failure to model feedback should not affect the reliability of the market response coefficients.

Competitive Behavior Elements

The explicit modeling of competitive behavior is also fairly rare in market model building, primarily because data on competitive marketing expenditures are very difficult to obtain. Some *empirical* examples are Lambin's (1970) study of a consumer durable, which includes reaction functions for advertising, and Schultz' (1971) model for air travel which considers flight share and advertising share equations. Perhaps the most complete empirical study is Wildt's (1974) which models competition on the basis of advertising, price, promotion, and new products. These and other studies confirm that competitive reactions are usually very strong. A *theoretical* contribution by Lambin, Naert, and Bultez (1975) also deserves close attention. The objective of these authors was to generalize the Dorfman-Steiner theorem to cases of oligopoly with competitive reactions and market expansion, for which they derived the following fundamental relationship.

$$E'_{q,u} = (I, R) \cdot (E_{QT} + E_{m_i}) \quad (1)$$

where:

$E_{q,u}$ = vector of total sales elasticities of firm *i*,
E_{QT} = vector of primary demand elasticities of the mar-

keting variables of firm i and its competition,

$\mathbf{E}_{m_i}$ = vector of market share elasticities of the marketing variables of firm i and its competition,

$\mathbf{I}$ = identity matrix, and

$\mathbf{R}$ = matrix of reaction elasticities (the effects of firm i's decisions on competitive decisions).

This formulation is an extension of Clarke's finding ($n_S = n_{PD} + n_{ms}$) to the case of competitive reactions. Lambin, Naert, and Bultez define "simple competitive reactions" as those which use the same marketing instruments (diagonal elements of R) and "multiple competitive reactions" as those which use different instruments (off-diagonal elements of R). They illustrate several special cases by constraining elements of R and $\mathbf{E}_{QT}$ to be zero and also provide an extensive empirical example of multiple competitive reactions for a consumer durable in a nonexpansive market. Equation 1, which is called the LNB model, is essential to the model development hereafter.

Comment

The author proposes that models of markets should combine sales response, feedback, and competitive reaction effects. Specifically, a model should make the distinction between primary demand and secondary demand effects and between sales response and feedback effects of the marketing mix variables. In addition, competitive activity may change the sales response effects drastically; for example, even though sales response to advertising may be positive, the real effect could be zero because of competitive advertising reactions. A model should be able to detect such situations.

AN EXTENSION OF THE LNB MODEL

Though equation 1 captures all of the relationships of interest, it has a few restrictions with respect to the reaction matrix R which may limit its use. First, the LNB model does not allow for cases of joint marketing decision making, i.e., the possibility that levels of one marketing instrument affect or are affected by levels of other marketing instruments within the (same) firm. The marketing literature cites many instances of such "intrafirm effects," such as the negative relationship between advertising and personal selling or the positive relationship between price and personal selling (Heskett 1976). As a result, numerous researchers have faced problems in estimating sales response coefficients because of multicollinearity among the marketing mix variables. One extension therefore is the inclusion of intrafirm reaction elasticities in the matrix R.

The second restriction of the LNB model is perhaps even more important. The LNB model treats a firm's competition as a whole, so the market is defined as the firm plus the other firms. If the total number of competitors is relatively small, more information and insight into market structures could be gained from considering each competitor separately, provided that the necessary data are available. Such an approach would allow for the study of segmentation strategies, or the development of "brand competition maps," as done by Clarke (1973). Though the treatment of individual competitors is not always necessary, the author extends the LNB model to include individual competitors' reaction elasticities.

The notation used in describing the extended R matrix follows.

x_{ij} = the level of the marketing mix variable i of competitor j, where $i = 1, M$ and $j = 1, J$,

e_{xy} = the elasticity of y with respect to x, i.e., $\frac{dy}{dx} \cdot \frac{x}{y}$,

PD = primary demand or total industry sales,

m_j = market share of competitor j,

S_j = sales of competitor j $\left(\sum_{j=1}^{J} S_j = PD\right)$.

Then:

$\mathbf{n}_S$ is a ($JM \times 1$) vector of sales elasticities of each firm's marketing variables,

$\mathbf{n}_{PD}$ is the ($JM \times 1$) vector of primary demand elasticities of the mix variables,

$\mathbf{n}_{m_j}$ is a ($JM \times 1$) vector of competitor j's market share elasticities of j's marketing variables and all competitors' marketing expenditures (i.e., the cross-elasticities).

The matrix of reaction elasticities R is of dimension ($JM \times JM$):

$$\mathbf{R} = \{e_{x_{ij}x_{kl}}\},$$

where:

$i,k = 1, M$ (mix variables),

$j,l = 1, J$ (competitors) and, by definition,

$e_{x_{ij}x_{kl}} = 1$ for all $i = k, j = l$.

As an illustration, consider a market where three competitors ($J = 3$) compete with each other on the basis of advertising (A) and distribution (D), so $M = 2$. The matrix R is shown in Figure 1.

The four partitions shown can be interpreted as follows. The main diagonal blocks are the simple competitive effects, which are by definition interfirm effects. The off-diagonal blocks are the multiple competitive effects. Within these blocks, the elasticities on the main diagonals are intrafirm reactions (shaded areas); those off-diagonal are interfirm effects. These reaction coefficients are to be interpreted as *dynamic*; otherwise, one could not necessarily distinguish the directions of the effects.

In this framework, the LNB equation 1 is extended as follows.

$$\mathbf{n}_S = \mathbf{R} \cdot \left[\mathbf{n}_{PD} + \sum_{j=1}^{J} \mathbf{T}^{(j)} \cdot \mathbf{n}_{m_j}\right] \quad (2)$$

Figure 1

$$
\left[\begin{array}{ccc|ccc}
1 & e_{A_1A_2} & e_{A_1A_3} & e_{A_1D_1} & e_{A_1D_2} & e_{A_1D_3} \\
e_{A_2A_1} & 1 & e_{A_2A_3} & e_{A_2D_1} & e_{A_2D_2} & e_{A_2D_3} \\
e_{A_3A_1} & e_{A_3A_2} & 1 & e_{A_3D_1} & e_{A_3D_2} & e_{A_3D_3} \\
\hline
e_{D_1A_1} & e_{D_1A_2} & e_{D_1A_3} & 1 & e_{D_1D_2} & e_{D_1D_3} \\
e_{D_2A_1} & e_{D_2A_2} & e_{D_2A_3} & e_{D_2D_1} & 1 & e_{D_2D_3} \\
e_{D_3A_1} & e_{D_3A_2} & e_{D_3A_3} & e_{D_3D_1} & e_{D_3D_2} & 1
\end{array}\right]
$$

where $\mathbf{T}^{(j)}$ is a dummy-variable matrix of dimension $(JM \times JM)$ for competitor j, defined as $\mathbf{T}^{(j)} = (T^{(j)}_{kn})$, where

$$T^{(j)}_{kn} = 1 \Leftrightarrow k = j, j + J, \ldots, j + (M - 1)J$$

$$(\forall n);\ 0 \text{ elsewhere.}$$

In the preceding example with three competitors and two marketing instruments the matrices $\mathbf{T}^{(j)}$ would be:

$$
\mathbf{T}^{(1)} = \begin{bmatrix} 111111 \\ 000000 \\ 000000 \\ 111111 \\ 000000 \\ 000000 \end{bmatrix}, \quad
\mathbf{T}^{(2)} = \begin{bmatrix} 000000 \\ 111111 \\ 000000 \\ 000000 \\ 111111 \\ 000000 \end{bmatrix},
$$

$$
\mathbf{T}^{(3)} = \begin{bmatrix} 000000 \\ 000000 \\ 111111 \\ 000000 \\ 000000 \\ 111111 \end{bmatrix}
$$

These dummy-variable matrices are needed to postmultiply the vector of market share elasticities with the appropriate elements of the reaction matrix **R**.

Several special cases can be derived from equation 2 by imposing restrictions on the elements of **R** and/or by constraining the vector $\mathbf{n}_{pn}$ to be zero. For example:

—no primary demand effects, i.e., stable industry: $\mathbf{n}_{pn} = 0$;

—no competitive reactions: $\mathbf{R} = \mathbf{I}$, an identity matrix of dimension JM;

—no intrafirm effects: $e_{x_{lj}x_{kl}} = 0\ \forall j = l$;

—only simple competitive reactions in the market: $e_{x_{lj}x_{kl}} = 0,\ \forall l \neq k$.

USING THE EXTENDED LNB MODEL

The theoretical model (2) describes a relationship among market parameters, whereas most models represent a structure of the market variables. The advantage of equation 2 is that the relationship is mathematical and hence not subject to statistical error. The disadvantage is that parameters are unobservables, so the model is necessarily an abstract description of a market. If the model is to be usable, one must determine which of the various response and reaction parameters are zero and which ones are not. Also, estimates of the magnitudes of the nonzero parameters must be found.

These questions are crucial as they will determine how complex equation 2 will be in any given empirical application. The first problem is one of *model specification*, which is different from (although not totally unrelated to) *parameter estimation*. The sources of information that can be used to specify equation 2 are *prior knowledge* of the market from management, principles of *theory* (marketing or economic theory), and *statistical procedures*. However, the amounts of information one can expect from these three sources are generally not the same. Table 1 is a summary of expected information from prior knowledge, theory, and statistical procedures (i.e., data analysis).

In this article the focus is on using theory from marketing and economics and statistical procedures to determine the nonzero elasticities in model 2. However, it should be emphasized that the use of prior knowledge, for example through conversations with managers, is at least equally important for this task. This is especially true for the intrafirm reaction effects, which can logically be explained best by the decision makers themselves. A vast literature is available on subjective estimation of marketing parameters, e.g., Lambin's SIMAREX and Little's BRANDAID (Lambin 1972; Little 1975).

Marketing theory in this field has been developed chiefly from empirical generalizations. Most of the studies on the marketing mix variables reveal a positive elasticity on market share, which is usually below unity (decreasing returns to scale). There are marked differences among the mix variables, price traditionally being the strongest followed by distribution and product, whereas advertising is almost always the weakest variable (elasticity typically less than 0.5). The evidence on primary demand effects is less conclusive; for example, Parsons and Schultz' table shows that advertising occasionally has a market-expansive effect, occasionally not (1976, p. 224–5). The cross-share

Table 1
EXPECTED INFORMATION FOR THE SPECIFICATION OF THE EXTENDED LNB MODEL

Source of information	Response parameters[a]			Reaction parameters	
	Primary demand	Market share	Cross-market share	Intrafirm	Interfirm
Prior knowledge from management	Usually assumed positive, but order of magnitude seldom known		Depends on industry knowledge	Perfect or near-perfect information	Depends on industry knowledge
Marketing and economic theory	Zero or positive	Mostly positive: decreasing returns	Negative or zero	Evidence of strong effects	Microeconomics; nonzero in oligopoly, zero elsewhere
Statistical procedures	Application of one-sided or two-sided hypothesis testing against the null of zero effect				

[a]Some of the statements have to be qualified for the case of *price*. In most cases one could reverse the sign of the effects.

elasticities would be negative if the marketing variable had a truly competitive effect.

Theoretical insights into the nature of the reaction parameters are not as readily available. The frequent multicollinearity among the marketing mix variables in sales response suggests that intrafirm reaction effects may be substantial. For the interfirm effects a distinction must be made between pure or monopolistic competition and oligopoly. In the first case there are too many firms in the market for any significant reaction effects to exist, so **R** = **I**. In the second case, reaction elasticities are zero if the firm is a follower (Cournot model) and nonzero if the firm is a leader (Stackelberg model) or if the market is collusive. However, microeconomic theory does not specify or predict which of the cases will hold in any given market.

The overall conclusion from this brief review of theory is that there are several expected relationships, but empirical evidence is needed to arrive at conclusions for a specific market. In addition, empirical analysis is necessary in most cases to estimate the nonzero parameters in the model. An excellent starting point is the work on measuring industry advertising effects by Schultz and Wittink (1976).

Schultz and Wittink derive a set of simple, logically consistent conditions for the presence of primary demand, primary sales, competitive, and mixed effects of advertising. For example, in a market with two competitors with respective sales S_1 and S_2, advertising A_1 and A_2, and market shares m_1 and m_2, advertising of firm 1 is said to have primary demand effects if and only if

$$\frac{dS_1}{dA_1} > 0, \frac{dS_2}{dA_1} > 0, \frac{dm_1}{dA_1} = \frac{dm_2}{dA_1} = 0.$$

The same authors also describe a "discrimination model" which is more useful for our purposes because it is based directly on Clarke's equation $n_S = n_{PD} + n_m$. The conditions for the simple cases are:

primary demand effect:

$$n_S > 0, n_m = 0$$

primary sales effect:

$$n_S > 0, n_m > 0, n_S > n_m, \frac{dS_2}{dA_1} = 0$$

competitive effect:

$$n_S > 0, n_m > 0, n_S = n_m, \frac{dS}{dA_1} = 0.$$

To develop statistical procedures from these conditions, one needs to estimate these elasticities and/or derivatives. Schultz and Hanssens (1977) used the general linear model in which the partial derivatives are the least-squares estimates of the marketing mix coefficients. Also, Lambin, Naert, and Bultez (1975) used a regression model to illustrate the LNB equation and derive optimal levels of the marketing instruments, but their example was a nonexpansive market case. These strictly econometric solutions, however, have two important disadvantages.

1. The Schultz-Wittink conditions are static whereas the relationships among market variables are typically dynamic. To make the transition, one must specify several lagged values of the variables, which causes the loss of degrees of freedom and possible confusion when different lags give different results.
2. It is very important that the estimates of the partial derivatives be unbiased or at least consistent (unbiased in the limit). But from econometrics it is known that these properties can hold only if the structural model from which the estimates are drawn is the

"true" model. Hence the researcher faces a dilemma of having to test for structure while assuming that the form of the structure is already known, which it is not.

These issues, especially the second one, are not unique to market model building. They relate in general to the problem of *testing for causality in a dynamic environment* using longitudinal data, a question that has been studied in great detail and with some remarkable results by numerous time series analysts (e.g., Box and Jenkins 1976; Granger 1969; Haugh 1976; Pierce 1977; Zellner and Palm 1974). In the following section some of their contributions are explained in the context of this article. It is important to understand that the techniques are empirical and that their value increases as a function of the researcher's uncertainty about the system, i.e., the market, under study. Because information about competitors in a market is typically very scarce, this case justifies the investment in time and learning which time series analysis requires.

AN ANSWER FROM TIME SERIES ANALYSIS

The success of the Box and Jenkins method has stimulated research on "integrated autoregressive-moving average" (ARIMA) time series models over the past 10 years. The underlying philosophy of these univariate models comes from Yule (1926) who postulated that most time series can be regarded as realizations of stochastic processes. Specifically, a stationary series of equal-interval observations z_t is considered to be generated by an unobservable white-noise series (i.e., a serially uncorrelated series) a_t, filtered by a linear ARMA process $\Psi(B)$, where B is the lag operator:

$$a_t \rightarrow \text{ARMA FILTER} \rightarrow z_t.$$

$$\Psi(B)$$

The objective of the time series analyst is to discover the filter $\Psi(B)$ so that the original white-noise series a_t can be estimated from the data. Box and Jenkins postulate that $\Psi(B)$ can be decomposed in an autoregressive part $\phi(B)$ and a moving average part $\theta(B)$, such that

$$\phi(B)z_t = \theta(B)a_t.$$

Numerous books and articles have been written on this subject, particularly on the applications of the method for forecasting. In marketing, the method of Box and Jenkins has been applied by Geurts and Ibrahim (1975), Helmer and Johansson (1977), and Moriarty and Adams (1979). The reader who is unfamiliar with these techniques is referred to Box and Jenkins (1976) or Nelson (1973) for a complete exposure.

The method of Box and Jenkins has gained its reputation largely from its outstanding forecasting performance in comparison with other univariate methods such as Holt-Winters or exponential smoothing, and even multivariate methods such as econometrics. But in recent years the method has been used for another, more ambitious purpose: the development and testing of structural models using the white-noise ARIMA residuals. In essence, this research has focused on the use of ARIMA residuals for testing the independence versus the causal relationship between two or more variables in a system. The key contribution of ARIMA modeling to structural model building is that the distinction can be made between the "intrastructure" (within series) and the "interstructure" (between series) of longitudinal data.

Theoretical statisticians have long known that inferences from longitudinal data analysis can be questionable and even misleading because of univariate time-series properties of the variables such as trend, seasonality, etc. For example, in 1926 Yule published an article on the subject under the meaningful title, "Why Do We Sometimes Get Nonsense Correlations Between Time Series?". The most common problem is spurious correlation, as in the many econometric studies on sales and advertising using annual data which suggest erroneously that advertising has multiple year-long carryover effects (Clarke 1976). The spurious correlation problem in econometrics is addressed most concisely by Granger and Newbold (1974). One of their striking examples is the following. Suppose x and y are perfectly independent of one another; however, they are both strongly first-order autocorrelated with a coefficient of 0.9. It can be shown that, for sample size 20, $E(R^2)$ in the regression $Y_t = bX_t + u_t$ is 0.47, which is highly significant! What happens is that the autocorrelation structure (intrastructure or within structure) of x and y is confused with the (presumed) interstructure (between structure) in this system and the unaware researcher could make the wrong conclusion that the series are related to each other.

Methods for detecting and removing intrastructure problems are well known, in particular the Durbin-Watson test for an AR(1) process on the regression residuals. However, these remedies all assume that the structural model is known prior to data analysis, i.e., they are cures for an estimation problem, not a specification problem. The unique feature of ARIMA modeling is that one can remove intrastructure *prior* to structural model building, which is very important in modeling situations such as the one presented here.

Although the methodological literature contains several suggestions for combined ARIMA/structural model building, there is general agreement on the principle to be used. This principle is known as Granger or Wiener-Granger causality, i.e., a variable X is said to cause another variable Y, with respect to a given information set containing X and Y, if future Y-values can be predicted better using past values of X and Y than using the past value of Y alone (see Granger 1969). In other words, the Grangean test on the significance of $b(B)$ in the regression

$$Y_t = a + b(B)\, X_t + u_t$$

is that the mean square forecast error (MSFE) of $Y|$ past Y, X is smaller than the MSFE of $Y|$past Y.

The direct implication of the Wiener-Granger concept for empirical work is that the contribution of X in explaining Y should be assessed using the *innovations* of the two series, which are usually measured by the ARIMA residuals. This means that the series are filtered to remove nonstationarity (e.g., trend) and systematic behavior (e.g., first-order autoregression), after which one investigates whether or not random shocks in one series are related to random shocks in the other series. In this context the pioneering work by Haugh (1976) and Pierce (1977) has resulted in the development of parametric tests that are easy to use. A critical evaluation of this approach is given in Appendix A.

Haugh (1976) studied the statistical properties of cross-correlation functions and found that, for ARIMA residuals, the cross-correlations of two independent series are normally distributed in the limit. On the basis of this finding, he developed a simple chi square test for the independence of two series x and y. First, the ARIMA models on x and y are estimated so that the white-noise residual series $a_{x,t}$ and $a_{y,t} (t = 1, N)$ are obtained. These series are cross-correlated at various lags k and for each k the sample cross-correlation $r_{a_x a_y}(k)$ is computed:

$$r_{a_x a_y}(k) = \frac{\sum_{k=1}^{N-k} (a_{x,t} - \bar{a}_x)(a_{y,t+k} - \bar{a}_y)}{s_{a_x} \cdot s_{a_y}},$$

where $\bar{a}_x$ and $\bar{a}_y$ are the sample means and s_{a_x} and s_{a_y} are the sample standard deviations of a_x and a_y.

The test statistic S_M is derived as:

$$S_M = N \sum_{k=-M}^{M} r^2_{a_x a_y}(k)$$

where M is the maximum lag value chosen by the researcher. This value S_M is approximately chi square distributed with $(2M + 1)$ degrees of freedom *if* x and y are independent. The null hypothesis of series' independence is accepted at level α if $S_M < \chi^2_{\alpha,2M+1}$. In the other case, the conclusion is that the series x and y are causally related to each other.

Haugh's work was extended by Pierce (1977) to include tests for the *direction* of causality:

$$x \text{ causes } y \text{ if } S_1 = N \sum_{k=1}^{M} r^2_{a_x a_y}(k) > \chi^2_{\alpha,M},$$

$$y \text{ causes } x \text{ if } S_2 = N \sum_{k=-1}^{-M} r^2_{a_x a_y}(k) > \chi^2_{\alpha,M},$$

and "instantaneous" causality is tested for by including $k = 0$.

Pierce applied these tests to the relationship between money and interest rates and found that it is not as strong as is traditionally believed by economists. A few other applications in finance and economics have consistently shown that it is necessary to take into account the series' time series properties before inferences are made about the causal structure in a system (e.g., Cramer and Miller 1976; Granger and Newbold 1977).

The advantage of Pierce's tests is ease of use and interpretation. One disadvantage is that the choice of the maximum lag (M) for the chi square value is subjective. Also, in small sample applications it is often difficult to reject the null hypothesis. For example one may find one or two significant cross-correlations, but, depending on the choice of M, the string of cross-correlations used to compute S_1 or S_2 may be nonsignificant as a whole. Therefore it is advisable not to use the chi square in isolation, but in conjunction with an inspection of individual cross-correlations of two series at various lags. These points are illustrated in the empirical part of the article.

In this article time series causality test is used only for the specification of zero and nonzero elements in the extended LNB model. However, for the sake of completeness, it should be pointed out that the shape of the cross-correlation function can also be used to determine the carryover effects of the marketing mix variables and, in general, to help specify the dynamic-causal structure of a system. These models are sometimes called transfer functions and have been used in marketing by Helmer and Johansson (1977) and in economics by Zellner and Palm (1974).

In summary, the author proposes that principles of multiple time series analysis be used on the extended LNB model as follows.

1. Develop univariate ARIMA models for primary demand, market shares, and the various marketing mix variables and save the white-noise ARIMA residuals.
2. Test the null hypothesis of independent series by cross-correlating the ARIMA residuals with each other for the cases of interest, i.e., primary demand effects, market share and cross-share effects, intra-firm and interfirm reactions. Use Haugh's chi square test and inspection of individual cross-correlations to reach verdicts.
3. Specify the zero and nonzero elements in model 2. The form of the resulting model will determine

the method of parameter estimation to be used, for example OLS on single response equations or 2SLS on a group of equations.

These steps are fully illustrated and discussed in a model of competition for a city pair in the U.S. domestic air travel market.

EMPIRICAL ANALYSIS

Data

The data set consists of quarterly observations on the marketing variables in an important city pair in the American domestic air travel market. The study period is the first quarter of 1965 through the third quarter of 1974.

The airline route is served by three major carriers, with a combined market share of more than 95%. The few smaller carriers that make up the rest of the market are not included in the analysis. Under Civil Aeronautics Board regulations, competition between 1965 and 1974 was limited to advertising expenditures in the cities and changes in the number of nonstop flights per week, subject to CAB approval. Two marketing variables are not considered in this study: air fares, which are equal for all airlines, and the qualitative variable "service."

The quarterly data show substantial variability. First, at least the first half of the period was characterized by overall market expansion, as all the competitors had an upward trend in passenger sales. This fact provides an opportunity for examining possible primary demand effects of the marketing mix variables.

The levels of the marketing variables "flights" and "advertising" also change substantially, although the patterns are not the same. An abrupt change in trend of the number of flights occurred in the last few quarters, corresponding to the beginning of the oil crisis in the United States.

The market shares of the three competitors did not remain stable, either. On average, airline 1 held about 42% of the market, followed by airline 3 with 32% and airline 2 with 22%. One complication is that two airline strikes occurred during the period, one in quarter 17 and another in quarter 36. Visual data inspection suggests that these strikes did not affect primary demand, but they logically had very strong share effects: in quarter 17 airline 1's market share dropped to 32% and in period 36 airline 2's share plunged to about 14%. The strikes are included in the modeling process where appropriate.

The notation can be kept simple by using the following symbols: *PD* (primary demand), *P* (passengers), *F* (flights), *A* (advertising in \$1000), and *m* (market share). The competitors are denoted by 1, 2, 3; for example, $F1_t$ is airline 1's number of flights at time t, $m3_t$ is the market share of 3 at time t.

Prewhitening the Data

The development of univariate ARIMA models proceeds through several steps. As the Box-Jenkins method is explained and illustrated at length in the literature, details are omitted here.

First, stationary data series must be obtained prior to Box-Jenkins analysis, i.e., the series' means and variances must be independent of time. The data plots and the autocorrelation functions reveal some trend (e.g., in flight levels), seasonality (e.g., in the number of passengers), and heteroskedasticity (e.g., in the advertising series). Consequently, first-order differences are taken for *F*1, *F*2, *F*3 and *m*1, *m*2, *m*3, and seasonal fourth-order differences are taken for *PD*, *P*1, *P*2, *P*3. In addition, the entire analysis is done in the natural logarithms of the data, which removes heteroskedasticity. Working in logarithms also offers the advantage that the subsequent multivariate data analysis will take into account possible nonlinearities in the response functions.

The ARIMA models for number of passengers and market shares are complicated by the potential effects of the strikes in quarters 17 and 36. Because a strike is a strictly exogenous, discrete event, its effect can be estimated by dummy variable analysis. In time-series context, this approach is called *intervention analysis* and was first introduced and applied to Los Angeles air quality data (Box and Tiao 1975). An intervention model is a special case of a transfer function model, where the explanatory variable is either a step or a pulse dummy variable. The general model is:

$$y_t = \frac{\omega(B)}{\delta(B)} S_t + \frac{\theta(B)}{\phi(B)} a_t$$

where:

S_t = 1 when t is an intervention period (i.e., a strike period), 0 otherwise, and
ω and δ are the intervention parameter polynomials.

The univariate Box-Jenkins and intervention models for the 13 series are shown in Table 2. Within each category of marketing variables, the autoregressive processes are the same and the moving-average parts are occasionally different. This similarity in AR processes is a first indication to the researcher that several variables may be related to each other, because it is known that, in theory, the AR-orders of all endogenous variables in a structural model are the same (e.g., see Zellner and Palm 1974).

The process of fitting univariate time series models did cause the loss of a few observations: the residual white-noise series for passenger demand has 35 observations and those of market shares and flights are reduced to 38. These series are too short for separation of the data into an estimation set and a holdout sample. However, the total absence of serial correlation makes

Table 2
UNIVARIATE BOX-JENKINS MODELS

$$(1 - B^4)PD_t = (1 + .404B + .417B^2 + .486B^3)a_{PD,t}$$

$$(1 - B^4)P1_t = (1 + .564B + .839B^2)a_{P1,t} - .300S1_t + .043S2_t$$

$$(1 - B^4)P2_t = (1 + .726B)(1 - .479B^4)a_{P2,t} + .245S1_t - .483S2_t$$

$$(1 - B^4)P3_t = (1 + .803B + .625B^2 + .842B^3)a_{P3,t} + .125S1_t + .349S2_t$$

$$(1 - B)m1_t = (1 - .830B^2)a_{m1,t} + (-.302 + .272B)S1_t$$

$$(1 - B)m2_t = (1 - .989B^2)a_{m2,t} + .171S1_t + (-.517 + .540B)S2_t$$

$$(1 - B)m3_t = (1 - .644B)a_{m3,t} + .020S1_t + .111S2_t$$

$$(1 - B)F1_t = (1 - .318B^4)a_{F1,t}$$

$$(1 - B)F2_t = a_{F2,t}$$

$$(1 - B)F3_t = (1 - .590B + .450B^3)a_{F3,t}$$

$$A1_t - 6.8138 = (1 - .563B^3)a_{A1,t}$$

$$A2_t - 6.7970 = (1 + .341B^4)a_{A2,t}$$

$$A3_t - 6.7088 = (1 + .524B^4)a_{A3,t}$$

multivariate inferences robust and reliable, as is illustrated in the following sections.

Testing for Primary Demand Effects of the Marketing Mix Variables

Because the total market size did not remain constant during the study period, the first step toward specifying equation 2 is to investigate whether or not market expansion is related to manipulation of the competitors' marketing mix variables. Formally, the hypotheses are:

H_1: x_{ij} does not affect PD (i = 1, 2 and j = 1, 3).
H_2: x_{ij} has an effect on PD.

From a marketing intuitive standpoint, H_2 could be stated more precisely as "x_{ij} has a positive effect on PD." However, Haugh's chi square test investigates only the presence versus absence of a causal relationship between two series. Inferences about the sign of an effect can be made from inspection of the signs of large cross-correlation values, but those will be subject to confirmation by structural model building.

The value of M, the largest lag, is set at four, which corresponds to an one-year maximum time span in the market response functions. Because the null hypotheses are unidirectional, the cross-correlations to be included are

$$r_{a_{x_{ij}}, a_{PD}}(k) \qquad k = 0, 1, 2, 3, 4$$

and Haugh's test statistics for each variable and each competitor are

$$M_{ij} = N \sum_{k=0}^{4} r^2_{a_{x_{ij}}, a_{PD}}(k).$$

Under H_1, the M_{ij} are approximately chi square distributed with five degrees of freedom. The critical chi square table values are 9.24 (α = 0.10) and 11.1 (α = 0.05). The results of the time-series tests are shown in Table 3.

The conclusions of the primary demand tests are intriguing. First, the advertising dollars did not generate any previously untapped markets or market segments, which means that any potential advertising effects in this market are competitive. Second, the primary demand effects of flights existed, but they were not symmetric; the two large competitors in this market were able to expand the market size by offering more flights. In contrast, the smaller competitor did not influence industry demand. The implication of these findings is that the primary demand elasticity vector in equation 2 will consist of four zero and two nonzero elements.

As a methodological comment, it should be noted that the lags k = 1,..., 4 in this cross-correlation analysis on white-noise series are not necessarily the "true" lags in a dynamic relationship between two variables. In particular, if the ARIMA processes on two causally related series are sufficiently different, it is likely that the lag structure is different from the one suggested by cross-correlating the white-noise series. As an illustration, it is not necessarily true that the primary demand effect of $F3$ has a one-year lag.

Table 3
TIME SERIES TESTS

	Primary Demand					
	F1	*F2*	*F3*	*A1*	*A2*	*A3*
lag $k=0$	−.19	.15	.04	−.06	.02	.13
1	.25	.09	.07	−.02	−.10	.12
2	.16	−.19	.26	−.32	−.07	.00
3	−.31	.09	.04	−.04	−.24	.16
4	.38	.18	.47	−.01	.10	.09
M	12.81[a]	3.72	10.29[b]	3.76	2.89	2.36

	Market Share					
	m1		*m2*		*m3*	
	F1	*A1*	*F2*	*A2*	*F3*	*A3*
lag $k=0$	.06	−.08	.43	.11	.33	−.14
1	.02	−.20	−.18	.24	.29	.09
2	.09	.32	.21	.07	.09	−.19
3	.18	.16	.11	.07	−.11	.13
4	−.09	−.04	.18	−.04	.04	.01
M	2.02	6.49	11.65[a]	3.10	8.11	3.01

	Cross-Market Share											
	m1				*m2*				*m3*			
	F2	*F3*	*A2*	*A3*	*F1*	*F3*	*A1*	*A3*	*F1*	*F2*	*A1*	*A2*
lag $k=0$	−.26	−.13	−.05	.05	−.20	.01	.23	−.15	.27	−.12	−.41	−.21
1	−.10	.00	−.18	−.18	.11	−.08	.06	.01	−.19	.28	−.01	−.11
2	.23	.05	−.19	.14	.01	.16	−.27	−.06	.09	−.06	.05	.09
3	−.00	.06	.21	−.19	.09	.04	−.23	−.09	.01	−.29	.04	−.25
4	−.34	−.18	−.35	−.28	.06	−.15	.09	.18	−.08	−.01	−.49	−.07
M	9.38[b]	2.13	8.94	6.42	2.35	2.14	7.24	2.50	4.67	6.85	15.18[a]	4.98

	Intrafirm Reactions					
Decision variable:	*F1*	*A1*	*F2*	*A2*	*F3*	*A3*
Affected by:	*A1*	*F1*	*A2*	*F2*	*A3*	*F3*
lag $k=0$	−.05	−.05	.06	.06	.16	.16
1	−.19	−.13	.40	−.24	−.09	−.02
2	−.14	−.12	−.33	.08	.22	.14
3	.16	.06	−.24	.13	−.21	−.05
4	−.32	.07	−.01	.18	.14	.17
M	7.00	1.64	12.56[a]	4.40	5.59	2.97

	Simple Competitive Reactions											
Decision variable:	*F1*	*F2*	*A1*	*A1*	*F2*	*F2*	*A2*	*A2*	*F3*	*F3*	*A3*	*A3*
Affected by:	*F2*	*F3*	*A2*	*A3*	*F1*	*F3*	*A1*	*A3*	*F1*	*F2*	*A1*	*A2*
lag $k=0$	.27	.48	.36	.34	.27	.31	.36	.44	.48	.31	.34	.44
1	−.08	.06	.10	−.10	.25	.14	.00	.17	.08	−.13	−.01	.26
2	.08	−.14	−.17	−.19	−.10	.02	−.10	.01	.25	−.11	−.07	.01
3	.13	.01	.00	.05	.04	.06	−.26	−.21	−.10	−.00	−.30	−.15
4	−.07	.14	.43	.19	−.07	.03	.11	.03	−.10	−.13	.23	.32
M	4.15	10.39[b]	13.46[a]	7.50	5.77	4.42	8.37	10.11[b]	11.75[a]	5.20	9.82[b]	14.82[a]

Table 3

Continued

Multiple Competitive Reactions

Decision variable:	F1		A1		F2		A2		F3		A3	
Affected by:	A2	A3	F2	F3	A1	A3	F1	F3	A1	A2	F1	F2
lag k=0	.04	.10	.10	−.23	.10	.16	.04	.18	−.23	.18	.10	.16
1	−.02	−.12	−.29	−.29	.11	−.12	−.20	−.28	−.21	−.06	−.07	−.15
2	−.18	−.07	−.13	.10	−.36	.02	.16	.00	−.03	− 19	−.02	−.29
3	−.13	.03	29	−.02	−.22	−.10	−.13	−.10	−.22	−.22	−.10	.16
4	.04	.13	07	.09	.17	.17	−.17	.09	−.12	.20	.14	.12
M	2.04	1.70	7 74	5.82	8.63	3.12	4.33	4.92	6.07	6.12	1.64	6.59

[a]Significant at the .05 level.
[b]Significant at the .10 level.

The procedure for testing the market share and competitive reaction hypotheses is similar to that used in the primary demand case. Only the test results are reported hereafter. The values of the statistics are shown in Table 3.

Market Share Effects

The null hypothesis of series' independence is rejected in only one case: the number of flights of airline 2 has a strong effect on its market share. In combination with the previous findings, a tentative conclusion for flights as a marketing instrument is that all three airlines could increase passenger sales by manipulating flight service, but for different reasons: the two major airlines drew upon new customers (or more frequent purchases by current customers), whereas the smaller airline caused brand switching by current users. These conclusions are reexamined and complemented by further insight in cross-share flight elasticities and competitive reactions.

There is no evidence that advertising has a direct impact on market share. Thus, one can tentatively conclude that it did not have an impact on passenger sales, in the absence of primary demand effects.

The results for the cross-share effects indicate that the direct market share effect of firm 2's flights occurs at the expense of firm 1, because the null hypothesis is rejected for this case. This finding is another piece of evidence of the lack of symmetry in this market. In addition, airline 3's market share was subject to advertising competition from the market leader, airline 1, although one expects this effect to be weak because the other market shares did not seem to be influenced by 1's advertising.[1]

Competitive Reaction Effects

The elements of the matrix **R** indicate the effect (elasticity) of changes in one marketing mix variable on another instrument within or outside the firm. In total there are 30 possible reactions, i.e., six intrafirm, 12 simple competitive, and 12 multiple competitive effects. *A priori* strong reactions are expected to exist in the advertising decision making, because this variable is not regulated. However, because the data plots on flights show some common patterns, significant reaction effects for this variable can also be expected.

The first category of reaction effects are *intrafirm*, i.e., reflecting joint marketing decision making. Here, the null hypothesis is rejected in one case (see Table 3): advertising decisions in firm 2 precede number of flights, which indicates that this airline prepares future changes in flight scheduling by advertising messages, a managerially plausible policy. All other flight and advertising decisions appear to be made independently of one another.

The second category of reaction effects are called *simple* competitive, i.e., using the same marketing instrument to react to competitive pressures on the market. The null hypothesis of no effect is rejected in several cases. First, there is strong evidence that the numbers of flights of the two leading competitors are related to each other. Although Haugh's chi square test is significant in both directions, it is not immediately obvious that there is two-way competitive reaction between these variables because only the zero-lag cross-correlation is significant. Strictly speaking the only valid conclusion is that these series covary in some way. It is tentatively concluded that there is a simultaneous relationship between $F1$ and $F3$, subject to confirmation by an appropriate structural model.

As expected, the advertising series are strongly related to each other, specifically $A1$ and $A2$, $A1$ and $A3$, and $A2$ and $A3$. This result (possibly a two-way

[1]The case of $A2$ and $m1$ is marginal: the overall chi square value is close to the 10% threshold level and one spike is barely significant. The relationship was included in the subsequent structural model, but the coefficient was not significant. Therefore, the null hypothesis was not rejected.

reaction, similar to the case of $F1$ and $F3$) indicates that a system of simultaneous equations will be needed to determine the elasticities of these simple competitive effects. Overall, the findings confirm the classical principles of oligopolistic decision making.

The last set of parameters for the extended LNB model are the *multiple competitive reaction* elasticities, i.e., the effects of changes in number of flights of one firm on another firm's advertising and vice versa. The conclusions are straightforward: the null hypothesis of zero effect cannot be rejected in any of the 15 cases. This result considerably simplifies the complexity of the matrix **R** and reduces the number of parameters to be estimated.

In addition to the competitive reaction effects, the levels of the marketing mix variables could also be influenced by past sales or market share, i.e., the feedback effect. These possibilities were examined for each of the marketing decision variables by using Haugh's test. No significant feedback effects were found.

Parameterizing the Extended LNB Model

The series of Haugh tests indicate which variables have an impact on market response and competitive activity, but they do not provide an accurate picture of the lag structure in these relationships. Methodologically the most complete way to determine this lag structure is by the following steps (see Haugh and Box 1977).

1. Specify dynamic shock models on the ARIMA residuals, i.e., structural models on the white-noise series, based on the shape of the cross-correlation functions.
2. Substitute the original variables for the ARIMA residuals and work out the lag polynomials.
3. Parameterize the derived structural models on the original variables.

This stepwise approach is time-consuming and usually complicated because the substitution of the ARIMA residuals by the original data creates long polynomials. In the end, several simplifying assumptions are typically made to ensure that the ultimate model can be estimated and is realistic for interpretation (see Hanssens 1980). For that reason, and also because the time series in this application are short, this methodological exercise is not undertaken. Instead, the loglinear models reflect a simple, marketing-plausible dynamic structure such as instantaneous effects of number of flights on passengers and short lags in the response to advertising. In addition, the time series patterns in the residuals are modeled when they are not white noise.

The model for *primary demand* includes $F1$ and $F3$ and seasonal dummy variables. An overall indicator of business activity (GNP) and the one-way air fare in coach class ($FARE$) are added as explanatory variables. The fact that the two flight variables are highly contemporaneously correlated causes an estimation problem. Two solutions are proposed: lagging one of the flight variables by one quarter or taking their sum as a variable (F), which assumes that their primary demand elasticities are equal. The results follow.[2]

(3a) $$PD_t = \underset{(1.520)}{e^{6.222}}\ \underset{(.084)}{F1_{t-1}^{.207}}\ \underset{(.067)}{F3_t^{.114}}\ \underset{(.276)}{GNP_t^{.659}}\ \underset{(.146)}{FARE_t^{-.378}}$$

$$e^{\underset{(.029)}{.204D2} + \underset{(.028)}{.157D3} + \underset{(.029)}{.144D4}}$$

$$(R^2 = .788,\ \mathrm{DW} = 1.413)$$

(3b) $$PD_t = \underset{(1.530)}{e^{5.694}}\ \underset{(.054)}{F_t^{.258}}\ \underset{(.293)}{GNP_t^{.697}}\ \underset{(.152)}{FARE_t^{-.293}}$$

$$e^{\underset{(.031)}{.196D2} + \underset{(.031)}{.151D3} + \underset{(.032)}{.144D4}}$$

$$(R^2 = .735,\ \mathrm{DW} = 1.682)$$

Because the estimate of the elasticity of F (.258) is more than one standard deviation away from the estimate of the elasticity of $F3$ (.114), equation 3a is selected.

The estimation of *market share elasticities* is simplified by the fact that nonmarketing exogenous variables can be omitted (as these factors usually are assumed to affect all competitors alike). Although the loglinear market share model does not guarantee the satisfaction of the range and sum constraints on the dependent variables, it is used here for the purpose of parsimony and because the three airlines do not make up the entire market for the city pair. Because the OLS residuals were AR(1), the SAS procedure AUTOREG[3] was used to estimate the parameters:

(4a) $$m1_t = \underset{(.133)}{e^{-.553}}\ \underset{(.037)}{F2_t^{-.072}}\ \underset{(.047)}{e^{-.301 S1_t}}\ \underset{(.047)}{S1_t^{+.079}}\ S2_t\ \underset{(.153)}{e^{(1-.302B)u_{1t}}}$$

$$(R^2 = .586)$$

(4b) $$m2_t = \underset{(.284)}{e^{-.2655}}\ \underset{(.079)}{F2_t^{.329}}\ \underset{(.064)}{e^{.195 S1_t}}\ \underset{(.064)}{S1_t^{-.688}}\ S2_t\ \underset{(.121)}{e^{(1-.658B)u_{2t}}}$$

$$(R^2 = .797)$$

(4c) $$m3_t = e^{-.795}\ A1_t^{-.048}\ e^{.125 S1_t}\ S1_t^{+.260}\ S2_t\ e^{(1-.565B)u_{3t}}$$

---

[2]The standard errors of the least-squares estimates are given in parentheses. All coefficients are significant at $\alpha = 0.10$ or better unless otherwise indicated.

[3]AUTOREG is a nonlinear estimation procedure developed by Gallant and Goebel (1975). The method first estimates OLS regression coefficients. Next, the autoregressive parameters of the residuals are estimated. Finally, the data are transformed by a linear combination of the AR parameters and the model is reestimated.

$$(.165)\ (.024)\ (.054)\ (.053)\qquad (.132)$$

$$(R^2 = .553)$$

Last, the parameters of the *reaction functions* for flights and advertising must be estimated. For the case of number of flights, recall that only a contemporaneous cross-correlation between $F1$ and $F3$ was found to be significant. This is one case where Granger's causality definition and, consequently, Haugh's test does not yield conclusive evidence. The proposed solution is to hypothesize that the effects between $F1$ and $F3$ are bidirectional, i.e., a truly simultaneous relationship exists. The hypothesis is tested by parameterizing the model

$$F1_t = f(F3_t, X1)$$

$$F3_t = f(F1_t, X2),$$

where $X1$ and $X2$ are exogenous variables necessary to make the model identified. In the absence of seasonal, multiple competitive reaction, or feedback effects, lagged flight values were selected as predetermined variables. The model was estimated by two-stage least squares because there was no significant correlation among the two equations' residuals.

$$F1_t = e^{-.078*}\quad F3_t^{.343}\quad F1_{t-1}^{.687}$$
$$(.317)\quad (.130)\quad (.090)$$

$$F3_t = e^{.764}\quad F1_t^{.323*}\quad F3_{t-1}^{.779}$$
$$(.417)\quad (.205)\quad (.210)$$

where an asterisk denotes nonsignificant coefficients.

This model, as well as several versions of it using different time lags, indicates that the competitive reaction goes in only one direction: flights of airline 1 are influenced by changes in flights of airline 3. In addition, the lagged flight coefficients are highly significant, which indicates that flight decision making is rather conservative and definitely not as volatile as the advertising expenditures.

Second, a model for the interdependencies in advertising decision making is developed. The procedure is very similar to that used in the case of flights with one exception: the model for advertising is necessarily recursive because competitors have no knowledge about each others' advertising expenditures until several quarters after the fact. As a result, the parameters can be estimated by ordinary least squares. The equation for $A1$ also includes an AR(3) parameter on the residuals. The final model follows.

$$A1_t = e^{4.336}\quad A2_{t-4}^{.369}\quad e^{(1+.350B^3)u_t}\qquad (R^2 = .179)$$
$$(.928)\quad (.137)\quad (.154)$$

$$A2_t = e^{5.189}\quad A3_{t-4}^{.247*}\qquad (R^2 = .039,\ DW = 1.535)$$
$$(1.436)\quad (.215)$$

$$A3_t = e^{2.931}\quad A1_{t-4}^{.191*}\quad A2_{t-4}^{.369}\ (R^2 = .303,\ DW = 2.215)$$
$$(1.081)\quad (.160)\quad (.140)$$

Although all the coefficients have the expected sign, only the effect of $A2$ on the other advertising budgets is statistically confirmed.[4] It shows that the aggressive airline 2, even though it is the smallest competitor, is the advertising leader in this market.

Last, the intrafirm relationship between advertising and number of flights for airline 2 is parameterized. The coordination between flight levels and advertising dollars in this firm appears to occur primarily with a one-quarter lag.

$$F2_t = e^{2.430}\quad A2_{t-1}^{.165}\quad e^{(1-.801B)u_t}\qquad (R^2 = .313)$$
$$(.294)\quad (.041)\quad (.097)$$

All the nonzero coefficients in equation 2 have now been estimated and the extended LNB model is fully specified.

Results

The fully parameterized equation 2 contains many zero elements and none of the reaction matrix parameters are multiplied with nonzero cross-share elasticities or primary demand elasticities. As a result, the ultimate sales elasticities are found directly by adding the direct share and primary demand elasticities.

$$e_{F1,P1} = .207 + 0 = .207$$
$$e_{A1,P1} = 0 + 0 = 0$$
$$e_{F2,P2} = 0 + .329 = .329$$
$$e_{A2,P2} = 0 + 0 = 0$$
$$e_{F3,P3} = .114 + 0 = .114$$
$$e_{A3,P3} = 0 + 0 = 0$$

These findings indicate, as noted before, that number of flights affects passenger levels positively, although with decreasing returns to scale. The reasons for this positive effect are market expansion in the case of the two major carriers and competition in the case of the smaller airline. There is no evidence, however, that advertising has a direct impact on market share or total industry demand. Consequently, it has no influence on number of passengers.

In addition to these conclusions about sales response, the model gives interesting insights into the nature of competition in this market. The complete matrix of reaction elasticities is shown in Figure 2.

[4]This means that the two-way reaction in advertising between 2 and 3 is rejected in favor of a one-way effect from 2 to 3, which is consistent with the time series test. However, the nonsignificant $A1$ coefficient in the equation for $A3$ is in conflict with the Haugh chi square test, probably because both $A1$ and $A3$ respond to $A2$.

Figure 2

	F1	A1	F2	A2	F3	A3
F1	1	0	0	0	0	0
A1	0	1	0	0	0	0
F2	0	0	1	0	0	0
A2	0	.37	.17	1	0	.37
F3	.34	0	0	0	1	0
A3	0	0	0	0	0	1

Although there are only four nonzero elasticities, the matrix indicates some intense competition in marketing decision making. The two major airlines compete primarily on the basis of flight scheduling. In addition, they respond to changes in airline 2's advertising efforts. Because advertising does not have a significant impact on passenger sales, this sequence of reactions could lead to escalation of advertising budgets, as in some of the markets analyzed by Metwally (1978). In this case, however, that is not necessarily true because the advertising leader coordinates its promotional efforts with changes in flight scheduling. As the number of flights of this airline is a highly competitive instrument, it is managerially meaningful for the two major carriers to defend their market positions. However, they would be more successful in doing so by using flight scheduling more than advertising. This conclusion is substantiated by observing that airline 2's market share gradually increased during the study period at the expense of the larger competitors.

The findings of this study can be compared with those of Schultz (1971), although the city pairs and time periods are different. Schultz' conclusion that flight scheduling is the most effective marketing instrument is confirmed; however, the present study explains the reasons for this high effectiveness by making the distinction between primary demand and competitive effects. A similar comparison can be made for advertising which is also found to be a weak marketing variable in Schultz' study. Finally, the present study examines competitive behavior at the individual firm level, whereas Schultz estimates "flight share" and "advertising share" equations which are, by definition, at a more aggregate level; the only implied competitive behavior in his study is a positive lagged market share effect on flight share. Unfortunately, direct parameter comparisons cannot be made between the two studies, except for the price elasticities on primary demand: about unity in Schultz' work versus less than .4 in this study. This difference probably can be explained by the fact that the route in this study is much shorter (and less expensive) than the route in Schultz' city pair. Overall the two reports are not in conflict, but the present analysis gives more information on market and competitive behavior.

CONCLUSIONS

The purpose of this article is twofold: to present a framework of market analysis which specifically models primary demand, competitive reaction, and feedback effects, and to introduce principles of time-series analysis in conjunction with standard econometric model building for using this framework.

To achieve the first objective, mathematical models by Clarke and by Lambin, Naert, and Bultez are extended. Although no claim is made that the resulting model is necessarily the "best" (i.e., produces the best fit with the data), it makes several useful marketing theoretical contributions. Most important is the fact that the model makes a sharp distinction between market-expansive and competitive effects of the marketing mix variables. In addition, it is shown how competitive reactions can have a major impact on the effectiveness of the marketing variables. For example, even though a marketing variable may positively affect market share, if it does not have primary demand effects, its impact on sales can be self-canceling, depending on the nature of competitive reactions. Because the model is built at the level of the individual firm, these and other conclusions can be made at a more disaggregate level than was done in prior research.

The second objective in this study is justified by the fact that there are few theoretical premises to rely on in analyzing response and competitive behavior in real markets. As a result, empirical analysis must be used to discover how specific markets work and why market shares have evolved to their present levels. Because longitudinal data are needed for this type of assignment, the author argues that principles of time-series analysis can be used to gain insight into the structure of the market and its competitive decision making. The need for time series analysis is inversely related to the amount of prior knowledge about the market. For example, the researcher who knows from management how advertising budgets are set has little need for these techniques for the specification of an advertising decision function. However, in most cases there is little or no prior information because competitive data are not easily obtained.

The two objectives are fully illustrated in the development of a model for a city pair in the domestic air travel market. Even with a limited data base the systematic analysis proposed in the first section gives a clear, nontrivial description of the market and the outcomes of competitive decision making. The analysis uses several research components, such as the Schultz-Wittink conditions for primary demand versus competitive effects, univariate Box-Jenkins analysis, multiple time-series modeling, and econometrics. As a result, the research is relatively time-consuming and requires training in various methodological techniques. At this time the contributions of the approach are primarily academic, but it is hoped that, with rapid advances in time-series analysis and the availability of more and better market data, further research in this area will be stimulated to a point where the theories and methods give a full understanding of the mechanisms of markets.

REFERENCES

Bass, Frank M. and Darral G. Clarke (1972), "Testing Distributed Lag Models of Advertising Effect," *Journal of Marketing Research*, 9 (August), 298-308.

——— and Leonard J. Parsons (1969), "Simultaneous-Equation Regression Analysis of Sales and Advertising," *Applied Economics*, 1 (May), 822-31.

Beckwith, Neil E. (1972), "Multivariate Analysis of Sales Response of Competing Brands to Advertising," *Journal of Marketing Research*, 9 (May), 168-76.

Box, G. E. P. and Gwilym M. Jenkins (1976), *Time Series Analysis, Forecasting and Control*, revised edition. San Francisco: Holden-Day, Inc.

——— and G. C. Tiao (1975), "Intervention Analysis with Applications to Economic and Environmental Problems," *Journal of the American Statistical Association*, 70 (March), 70-9.

Brunner, K. and A. Meltzer, eds. (1979), *Carnegie-Rochester Conference Series on Public Policy*, Volume 10. Amsterdam: North-Holland Publishing Co.

Clarke, Darral G. (1973), "Sales-Advertising Cross-Elasticities and Advertising Competition," *Journal of Marketing Research*, 10 (August), 250-61.

——— (1976), "Econometric Measurement of the Duration of Advertising Effect on Sales," *Journal of Marketing Research*, 13 (November), 345-57.

Cramer, Robert H. and Robert B. Miller (1976), "Dynamic Modeling of Multivariate Times Series for Use in Bank Analysis," *Journal of Money, Credit and Banking* (February), 85-96.

Gallant, A. R. and J. J. Goebel (1975), "Nonlinear Regression with Autoregressive Errors," Paper No. 986, Institute of Statistics, North Carolina State University.

Geurts, Michael D. and I. B. Ibrahim (1975), "Comparing the Box-Jenkins Approach with the Exponentially Smoothed Forecasting Model with Application to Hawaii Tourists," *Journal of Marketing Research*, 12 (March), 182-8.

Granger, C. W. J. (1969), "Investigating Causal Relations by Econometric Models and Cross-spectral Methods," *Econometrica*, 37, 424-38.

——— and P. Newbold (1974), "Spurious Regressions in Econometrics," *Journal of Econometrics*, 2, 111-20.

——— and ——— (1977), "Identification of Two-Way Causal Systems," in *Frontiers in Quantitative Economics*, Vol. III A, M. D. Intrilligator, ed. Amsterdam: North-Holland Publishing Co.

Hanssens, Dominique M. (1980), "Bivariate Time-Series Analysis of the Relationship Between Advertising and Sales," *Applied Economics*, 12 (September).

Haugh, Larry D. (1976), "Checking the Independence of Two Covariance-Stationary Time Series: A Univariate Residual Cross-Correlation Approach," *Journal of the American Statistical Association*, 71, 378-85.

——— and G. E. P. Box (1977), "Identification of Dynamic Regression (Distributed Lag) Models Connecting Two Time Series," *Journal of the American Statistical Association*, 72, 121-9.

Helmer, Richard M. and Johny K. Johansson (1977), "An Exposition of the Box-Jenkins Transfer Function Analysis with an Application to the Advertising-Sales Relationship," *Journal of Marketing Research*, 14 (May), 227-39.

Heskett, James L. (1976), *Marketing*. New York: Macmillan Publishing Co., Inc.

Houston, Franklin S. and Doyle L. Weiss (1975), "Cumulative Advertising Effects: The Role of Serial Correlation," *Decision Sciences*, 6 (July), 471-81.

Lambin, Jean-Jacques (1970), "Advertising and Competitive Behavior: A Case Study," *Applied Economics*, 2 (January), 231-51.

——— (1972), "A Computer On-Line Marketing Mix Model," *Journal of Marketing Research*, 9 (May), 119-26.

———, Philippe A. Naert, and Alain V. Bultez (1975), "Optimal Marketing Behavior in Oligopoly," *European Economic Review*, 6, 105-28.

Little, John D. C. (1975), "BRANDAID: A Marketing Mix Model, Part 1: Structure," *Operations Research*, 23 (July-August), 628-55.

McCann, John M. (1974), "Market Segment Response to the Marketing Decision Variables," *Journal of Marketing Research*, 11 (November), 399-412.

Metwally, M. M. (1978), "Escalation Tendencies of Advertising," *Oxford Bulletin of Statistics*, 253-63.

Moriarty, Mark and Arthur Adams (1979), "Issues in Sales Territory Modeling and Forecasting Using Box-Jenkins Analysis," *Journal of Marketing Research*, 16 (May), 221-32.

Naert, Philippe A. and Peter Leeflang (1978), *Building Implementable Marketing Models*. Leiden/Boston: Martinus Nijhoff Social Sciences Division.

Nelson, C. R. (1973), *Applied Time Series Analysis for Managerial Forecasting*. San Francisco: Holden-Day.

Palda, Kristian S. (1964), *The Measurement of Cumulative Advertising Effects*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

Palm, Franz (1977), "On Univariate Time Series Methods and Simultaneous Equation Econometric Models," *Journal of Econometrics*, 5, 379-88.

Parsons, Leonard J. and Randall L. Schultz (1976), *Marketing Models and Econometric Research*. New York: North-Holland Publishing Co.

Pierce, David A. (1977), "Relationships—and the Lack Thereof—Between Economic Time Series, with Special Reference to Money and Interest Rates," *Journal of the American Statistical Association*, 72 (March), 11-22.

Quandt, Richard E. (1964), "Estimating the Effectiveness of Advertising: Some Pitfalls in Econometric Methods," *Journal of Marketing Research*, 1 (May), 51-60.

Quenouille, M. H. (1957), *The Analysis of Multiple Time-Series*. New York: Hafner Publishing Co.

Schultz, Randall S. (1971), "Market Measurement and Planning With a Simultaneous-Equation Model," *Journal*

of Marketing Research, 8 (May), 153-164.
——— and Dominique M. Hanssens (1977), "Logical Implications of Competitive Behavior: An Approach to Model Specification," paper presented at Joint National ORSA/TIMS Meeting, Atlanta.
——— and Dick R. Wittink (1976), "The Measurement of Industry Advertising Effects," *Journal of Marketing Research*, 13 (February), 71-5.
Sexton, Donald E. (1970), "Estimating Marketing Policy Effects on Sales of a Frequently Purchased Product," *Journal of Marketing Research*, 7 (August), 338-47.
Stryker, Charles W. (1978), "Effective Allocation of Sales Resources," *Interfaces*, 8 (August), 1-11.
The Wall Street Journal (1977), "Econometrics Gains Many New Followers, But the Accuracy of Forecasts is Unproven," August 2, 38.
Urban, Glen L. (1969), "A Mathematical Modeling Approach to Product Line Decisions," *Journal of Marketing Research*, 6 (February), 40-7.
Wildt, Albert R. (1974), "Multifirm Analysis of Competitive Decision Variables," *Journal of Marketing Research*, 11 (February), 50-62.
Yule, G. U. (1926), "Why Do We Sometimes Get Nonsense Correlations Between Time Series?" *Journal of the Royal Statistical Society A*, 89, 1-64.
Zellner, Arnold and Franz Palm (1974), "Time Series Analysis and Simultaneous Equation Econometric Models," *Journal of Econometrics*, 2, 17-54.

APPENDIX A
ARMA AND DYNAMIC STRUCTURAL MODELS

The application of time series tests on structural models has caused some controversy and not all of the relevant issues have been completely resolved to date (see, for example, Brunner and Meltzer 1979). The most important question is probably whether or not the filters for removal of intrastructure could "accidentally" affect an existing interstructure as well, i.e., "the baby would be thrown out with the bathwater" and the researcher would face problems of "spurious independence." First, this question is answered from time series theory.

Let us assume that X affects Y in the following dynamic system.

$$y_t = \beta(B)\,x_t + \theta(B)\,\phi^{-1}(B)\,e_t \qquad \text{(A1)}$$

where x_t and y_t are stationary time series of X and Y, $\beta(B)$ = response parameter set, which could be rational, and $\phi(B)$, $\theta(B)$ = AR and MA components of the noise e_t which is assumed indendent of x_t.

Now, x_t follows the ARMA process $\phi_x(B)\,x_t = \theta_x(B)\,a_{x_t}$, so the dynamic structural form (*A*1) can be written in final equation form as:

$$y_t = \beta(B)\,\theta_x(B)\,\phi_x^{-1}(B)\,a_{x_t} + \theta(B)\,\phi^{-1}(B)\,e_t$$

or

$$\phi_x(B)\,\phi(B)\,y_t = \beta(B)\,\theta_x(B)\,\phi(B)\,a_{x_t} + \theta(B)\,\phi_x(B)\,e_t. \qquad \text{(A2)}$$

The final equation (A2) is in ARMA form: the left side is an AR process, whereas the right side represents the sum of two MA processes which is also MA (see Palm 1977). The finding that the final equations of a linear dynamic structural model are in ARMA form, first shown by Quenouille (1957), has three important implications.

1. it shows that ARMA models are *not* naïve, mechanical forecasting models but that they are intrinsically related to econometric models.
2. The final equations impose restrictions on the AR process of endogenous variables, a point which has been illustrated at length by Zellner and Palm (1974).
3. If X explains Y, then its ARMA residual a_x will also explain a_y: let y follow the ARMA process $\phi_y(B)\,y_t = \theta_y(B)\,a_{y_t}$, then its substitution in equation A2 yields:

$$\theta_y(B)\,a_{y_t} = \beta(B)\,\theta_x(B)\,\phi(B)\,a_{x_t} + \theta(B)\,\phi_x(B)\,e_t$$

or

$$a_{y_t} = \beta(B)\,\theta_x(B)\,\theta_y^{-1}(B)\,\phi(B)\,a_{x_t} + \theta(B)\,\phi_x(B)\,\theta_y^{-1}(B)\,e_t.$$

The question of spurious independence also needs to be examined from a practical, empirical standpoint. Here the crucial element is whether or not the assumption that time series variables can be represented as stochastic processes holds for a given application. In classical econometrics, such an assumption is usually not made, at least not for the exogenous variables in a system. If the assumption is false, spurious independence could occur. For example, if a deterministic trend in X "causes" a deterministic trend in Y, the time series of X and Y would be nonstationary and—*ceteris paribus*—ARIMA modeling would remove the trend and possibly yield uncorrelated white-noise residuals. But in this event, future values of Y could be predicted just as well from past Y alone as from past Y and X, so in the Wiener-Granger framework X would not cause Y. A judicious use of this powerful method is recommended. In particular, theoretical insights should be used in the selection of the *information set;* then, if Yule's philosophy of time series is appropriate, the methods discussed can be applied safely. In the end the researcher will avoid the two extremes of "theory without measurement" and "measurement without theory."