Competitive Pricing by a Price Leader

Abhik Roy • Dominique M. Hanssens • Jagmohan S. Raju
The A. Gary Anderson Graduate School of Management, University of California, Riverside, California 92521
The John E. Anderson Graduate School of Management, University of California, Los Angeles, California 90024
The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104

We examine the problem of pricing in a market where one brand acts as a price leader. We develop a procedure to estimate a leader's price rule, which is optimal given a sales target objective, and allows for the inclusion of demand forecasts. We illustrate our estimation procedure by calibrating this optimal price rule for both the leader and the follower using data on past sales and prices from the mid-size sedan segment of the U.S. automobile market. Our results suggest that a leader-follower system (Stackelberg) seems more consistent with the pricing behavior in this market, than a mutually independent pricing rule (Nash). We also find that our optimal price rule explains this market data better than other pricing schemes that do not account for optimizing behavior on the part of the leader and the follower.
(*Marketing—Competitive Strategy, Pricing; Games—Noncooperative, Sequential*)

1. Introduction

Price leadership has long been recognized as an important and frequently occurring phenomenon. U.S. steel producers maintained parallel price changes before foreign competition left them with excess capacity (Nagle 1987). General Motors acted as a price leader for many years and its prices were followed by Chrysler and American Motors (Carpenter, Cooper, Hanssens, and Midgley 1988). In some market segments, the dependency between Detroit manufacturers still prevails, though imports now play a much bigger role. Well documented cases of price leaders enforcing cooperation through price cuts are Shell Oil in California, and Standard Oil in the Ohio gasoline market (Nagle 1987). Other markets where price leadership patterns have been found include air travel, turbogenerators, personal computers, and some consumer packaged goods such as cigarettes and breakfast cereal (Scherer 1980).

How should a price leader act in such cases? We present an approach that may be used to arrive at the leader's price rule in a market where the leader knows that its competitors are likely to change their prices based on the actions of the leader. As compared to other pricing models presented in previous research, the distinctive features of our approach and its potential contributions are as follows.

(1) It includes forecasts of future demand in the leader's pricing rule.

(2) While much of the prior research provides good qualitative insights about how a firm should price its offerings over time, we actually develop an empirical procedure that may be used to calibrate the price rule.

(3) The ability to calibrate the price rule allows us to examine whether a particular set of data are more consistent with the leader-follower pricing, or the alternative where the brands are assumed to set their prices independently. For instance, the Wall Street Journal (July 28, 1992), recently reported that Chrysler's pricing strategy appears to follow in the footsteps of rival Ford. Our statistical tests help confirm such conjecture formally. We can also examine whether our pricing rule, which assumes optimizing behavior on the part of the competing brands, does a better job of explaining the data than other schemes which do not account for optimizing behavior.

Related previous research includes the monopoly pricing models examined in Robinson and Lakhani (1975), Dolan and Jeuland (1981), and Kalish (1983).

0025-1909/94/4007/0809$01.25

The three main pricing strategies that emerge from this body of research are skimming, penetration, and penetration followed by skimming. Among the papers that take competitive effects into account, Dockner and Jorgensen (1985), Erickson (1983), and Thompson and Teng (1984) examine pricing in a diffusion context. Eliashberg and Jeuland (1984) investigate dynamic pricing strategy when another firm enters the market in the second period. Rao and Bass (1985) derive industry price paths in the presence of saturation and cost learning effects. Narasimhan (1989) examines pricing strategy when consumers can form expectations about future price changes. Bensoussan, Bultez and Naert (1978) derive a market leader's optimal pricing strategy anticipating the follower's reactions. None of this research addresses the price leadership problem empirically.

We study the leader's problem of pricing over time in a market with the following characteristics.

(1) Since future demand is uncertain, the competing brands (both the leader and the follower) conduct market research to obtain sales forecasts. Continuing market research programs are common in the consumer packaged goods industry, service industry, and also in the automobile industry where brands regularly develop forecasts of future demand (Urban, Hauser, and Roberts 1990).

(2) We assume that the competing brands have information on past performance. More specifically, we assume that both brands use past sales and price data as inputs to set prices and sales targets for the next period. Own prices and sales can be obtained from internal records. Competitor's prices and sales in markets with a few brands are readily obtained from trade publications or other sources providing market data.

(3) Each brand sets a sales target for itself. While setting prices, the leader (follower) knows its own target, which implies an expected level of sales for the follower (leader).

We will use the term "optimal price rule" to describe a price rule that is optimal in the sense of best meeting the objective of minimizing deviations from pre-set target sales. We derive the optimal price rule and a procedure that can be used to calibrate it. We present tests that help us identify whether market data are more consistent with leader-follower pricing as opposed to the alternative where the competing brands are assumed to set prices independently. Our tests also examine whether market data are consistent with pricing rules, on the part of the competing brands, that assume optimizing behavior as we have defined it.

The rest of the paper is organized as follows. The leader's pricing problem is formulated in detail in §2. In §3 we present the optimal price rule for this problem. In §4, we describe a method to estimate the optimal price rule. In §5, we use this method to estimate the price rule, for both the leader and the follower, using data from the mid-size sedan segment of the U.S. automobile industry. Our analysis in § 6 formally examines whether a leader-follower price rule explains pricing behavior better than a rule that assumes that the competing brands act in a Nash manner. Section 7 summarizes our key results, and discusses limitations that may be overcome in future research.

2. Model Description

We assume that the market consists of two brands, labeled as Brand 1 and Brand 2. Although we focus on the pricing problem of the leader (Brand 1), our analysis also provides the optimal price rule for the follower (Brand 2). We assume that the pricing decisions are made in discrete time periods. For example, forecasts of future demand are obtained every month and the prices are based, in part, on these forecasts. We label each period by the subscript t, where $t = 1, 2, 3, \ldots, T$.

2.1. Demand Equations

Define q_t^i, to be the sales of Brand i in period t, and p_t^i to be the price of Brand i in period t. We assume that the sales in period t depend on the sales in period $(t-1)$, prices in period t, and other exogenous factors that are not known completely at the beginning of period t. More specifically,

$$q_t^1 = a_{11}q_{t-1}^1 + a_{12}q_{t-1}^2 - b_{11}p_t^1 + b_{12}p_t^2 + u_t^1, \quad (1$$

$$q_t^2 = a_{21}q_{t-1}^1 + a_{22}q_{t-1}^2 - b_{22}p_t^2 + b_{21}p_t^1 + u_t^2. \quad (2)$$

2.1.1. Key Features of (1) and (2). We focus our attention on (1); Equation (2) has similar characteristics. b_{11} reflects the effect of own price on demand in period

t, and its sign is consistent with the basic intuition that an increase in own price p_t^1 reduces demand in period t. b_{12} captures the effect of the competitor's price. An increase in the competitor's price p_t^2 raises the demand of Brand 1 in period t. Equation (1) also assumes that sales in period $(t-1)$ affect sales in period t, implying that there is a carryover effect from the past. This carryover effect may be due to inertia in the distribution system, or due to the fact that consumer preferences do not change instantaneously. Note that our model assumes that the entire carryover effect is captured by the sales in the immediate past period.[1]

Finally, the demand in period t also depends on exogenous factors, such as changes in consumer tastes or repositioning via design or advertising changes, that determine u_t^1. u_t^1 is not known at the beginning of period t, i.e., when the prices are being set. However, both firms conduct market research. Firm 1 obtains a forecast of u_t^1 given by f_t^{11}. A measure of 1's forecast accuracy is the covariance between f_t^{11} and u_t^1, which we label as γ_{1t}. Furthermore we define σ_{1t}^2, the variance of u_t^1, as a measure of the uncertainty in sales. Note that marginal costs are considered constant and we set them equal to zero.

A demand model linear in price of two firms has been derived from individual consumers' utility functions in Dixit (1979). Similarly an aggregate demand function of the form we posit in (1) and (2) can be derived from individuals' utility functions that are quadratic in quantities consumed of both brands, and where the coefficients reflecting preference weights for each brand are dependent on the quantity consumed in the previous time period.[2] The effect of prior purchase behavior on brand preferences is well documented in the marketing literature (see Kahn, Kalwani, and Morrison 1986).

[1] We want the model to capture industry trends as well as any preference trends that are specific to Brand 1. Consequently, we can express $q_t^1 = \alpha q_{t-1}^1 + \beta(q_{t-1}^1 + q_{t-1}^2) - b_{11}p_t^1 + b_{12}p_t^2 + u_t^1 = a_{11}q_{t-1}^1 + a_{12}q_{t-1}^2 - b_{11}p_t^1 + b_{12}p_t^2 + u_t^1$.

[2] Consider the following stylized example. Assume that consumer utility $u_t = \alpha_0 + [\alpha_1 q_{t-1}^1]q_t^1 + [\alpha_2 q_{t-1}^2]q_t^2 - 1/2[\beta_1 q_t^{1^2} + 2\gamma q_t^1 q_t^2 + \beta_2 q_t^{1^2}]$. If the consumer maximizes u_t subject to the income constraint $p_t^1 q_t^1 + p_t^2 q_t^2 = I$, we can show that, $q_t^1 = a_{11}q_{t-1}^1 + a_{12}q_{t-1}^2 + b_{11}p_t^1 + b_{12}p_t^2$, where $a_{11} = \alpha_1\beta_2/\beta_1\beta_2 - \gamma^2$, $a_{12} = -\gamma\alpha_2/\beta_1\beta_2 - \gamma^2$, $b_{11} = -1/\beta_1\beta_2 - \gamma^2$, and $b_{12} = -\gamma/\beta_1\beta_2 - \gamma^2$. The expression for q_t^2 is similar.

2.2. The Assumed Objectives of the Competing Brands

We assume that the leader's objective is to keep unit sales in each period as close as possible to a preset target. The follower is assumed to have a similar objective. We focus on Brand 1, since the process is identical for Brand 2. Brand 1's objective is to set price p_t^1 at the beginning of each period so that the realized sales are as close as possible to a preset target q^{11*}. Brand 1 also anticipates and plans for a certain level of Brand 2's sales q^{12*}. q^{11*} is the target Brand 1 wants to meet, while q^{12*} is that part of industry demand that Brand 1 expects 2 will meet. In other words, the objective is very similar to achieving a pre-set market share of expected industry sales.[3]

As an illustration, suppose Brand 1 sets a sales target of 600 units for itself, out of anticipated sales of 1000. This represents a 60% market share. Therefore it expects Brand 2 to achieve 40% share, or sales equal to 400 units. It is quite possible that Brand 2 actually sets a target of 500 for itself, and expects Brand 1 to sell the remaining 500 units. A strength of our approach is that it works even in a situation where the expectations do not match.

How realistic is this scenario? Lanzillotti (1958) studied the actual pricing objectives of large U.S. corporations, and found that staying within a minimum-maximum range of target market share (unit or dollar share) was a frequently stated objective in pricing decisions. In the same study General Electric claimed a policy of not exceeding 50% of any given market, while Johns-Manville did not want more than 20% of any market. Webster (1981) found that many marketing managers who have a key role in setting prices, have market share maintenance as their primary objective. Fruhan (1972) found similar results, and related this to the evaluation of marketing executives. As a more recent example from the automobile industry, Nissan executives stated that, in pricing their new Altima model, the goal was to achieve a level of 100,000 cars as quickly as possible (*Business Week*, April 20, 1992).[4]

[3] This interpretation makes it similar to the objective in Saghafi (1987), where he derives the pricing strategy in a competitive environment for a firm that has a market share stability goal.

[4] Meeting pre-set sales targets may be an important objective in many

Most marketers assume profit maximization in developing pricing theories. However, as Saghafi (1988) writes ". . . it has been well established that although profits are important, profit maximization is not the only or even the most widely used objective of firms in setting their prices."

Overall, the choice of the objective function depends on the context. As our focus in this paper is to understand and explain market behavior, the appropriate objective is the one that the firms actually use in their decision making process. Based on prior research referred to above, it appears that often times firms do set their prices to achieve a pre-set sales target. Our approach is useful in these cases.

From a modeling perspective, a major advantage of using this objective function is that a closed form solution to the optimization problem is obtainable. A numerical solution is possible if we assume brand level profit maximization as the objective, but it would not be possible to incorporate it in an estimation procedure of the type we develop in §4. Consequently, we would not be able to compare the various competing models as we do in §6.

2.3. Details of the Price Setting Process

A priori, Brand 1 does not know the demand errors u_t^1 and u_t^2 in (1) and (2). Instead, it uses f_t^{11} and f_t^{12} as forecasts for u_t^1 and u_t^2. The vector of own target and expectation about other's sales is labeled the own "target sales vector." The target sales vectors of Brands 1 and 2 are $\mathbf{q}^{1*} = (q^{11*}, q^{12*})$ and $\mathbf{q}^{2*} = (q^{21*}, q^{22*})$, respectively. If expectations and targets coincide, then $q^{11*} = q^{21*}$ and $q^{12*} = q^{22*}$. In our empirical illustration, described in §5, we consider the case where the target vectors of both brands coincide.

More specifically, we assume that each brand sets prices so as to minimize the discounted sum of squared deviations from the target vectors $\mathbf{q}^{1*}$ and $\mathbf{q}^{2*}$ over the T periods. We assume that the two brands have the same discount rate, ρ. Mathematically, for Brand 1, the optimization objective is stated in (3) as a *tracking control* problem (e.g., Shmueli and Tapiero 1980).

$$\min_{p_1, p_2, \cdots p_T} \sum_{t=1}^{T} \rho^{t-1}[(q_t^1 - q^{11*})^2 + (q_t^2 - q^{12*})^2], \quad (3)$$

subject to the constraints (1) and (2). Brand 2 has a similar objective.

3. Optimal Price Rule

The optimization problem is formally solved in Technical Appendix A1, where we show that the optimal price rule given in (4) has a simple linear form.[5] We note here that the optimal price rule for Brand 2 also has the same functional form.

$$p_t^{1*} = g^1 + \mathbf{G}^1\mathbf{q}_{t-1} + \mathbf{S}^1(\mathbf{f}_t^1 - \overline{\mathbf{f}^1}), \quad (4)$$

g^1 = (scalar) intercept of Brand 1's price rule.

$\mathbf{G}^1 = [G^{11}, G^{12}]$ is a vector whose first element reflects the weight given Brand 1's own lagged sales and the second element is the weight attached to Brand 2's lagged sales.

$\mathbf{S}^1 = [S^{11}, S^{12}]$ is a vector whose first element reflects the weight to the deviation of the forecast from the mean forecast prior to period t for Brand 1 and second element is the corresponding weight for the forecasts of the other brand.

$\mathbf{q}_{t-1} = [q_{t-1}^1, q_{t-1}^2]'$ is the vector of lagged sales.

$\mathbf{f}_t^1 = [f_t^{11}, f_t^{12}]'$ is the vector of forecasts made by Brand 1.

$\overline{\mathbf{f}^1} = [\overline{f_{t-1}^{11}}, \overline{f_{t-1}^{12}}]'$ is the average of forecasts made by 1 up to $t - 1$.

(5), (6), and (7) provide the formal expressions for g^1, $\mathbf{G}^1$, and $\mathbf{S}^1$. Once again, the detailed proofs are available in Appendix A1.

$$g^1 = (\mathbf{B}_1'\mathbf{B}_1)^{-1}\mathbf{B}_1'(\mathbf{q}^{1*} - \mathbf{B}_2 g^2) \quad (5)$$

$$\mathbf{G}^1 = -(\mathbf{B}_1'\mathbf{B}_1)^{-1}\mathbf{B}_1'(\mathbf{A} + \mathbf{B}_2\mathbf{G}^2) \quad (6)$$

$$\mathbf{S}^1 = -\mathbf{G}^1(\mathbf{A} + \mathbf{G}^{1\prime}\mathbf{B}_1' + \mathbf{G}^{2\prime}\mathbf{B}_2')\Sigma\Gamma_1^{-1} \quad (7)$$

$\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$ appearing in (5), (6), and (7) above contain the various parameters of the demand equations (1) and (2). More specifically, $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $\mathbf{B}_1 = (b_{11},$

industries. Exceeding sales targets may lead to unplanned, costly investments in plant and machinery, while under achievement may result in worker layoffs and a lower employee morale that can prove costly.

[5] Technical Appendices A1–A3 can be obtained by writing to Abhik Roy, 3168 Library South, Graduate School of Management, University of California, Riverside, CA 92521. Bitnet AB8HIKROY@UCRVMS. The solutions are based on the results of Roy (1990), and Bagchi & Basar (1981).

$b_{21})'$, $\mathbf{B}_2 = (b_{12}, b_{22})'$. Σ is the sales covariance matrix, and Γ_1 is the covariance matrix between forecasts made by Brand 1 and actual sales.

Linear Form of the Optimal Price Rule: (3) is a quadratic loss function, and the demand equations (1) and (2) are linear. Consequently, we have a linear-quadratic (LQ) control problem. Consequently, it is not very surprising that the optimal solution in (4) is linear. Note that each brand solves its own control problem, and in each case the optimal price rule is linear. The functional form of the price rule does not depend on whether the mode of competition is Stackelberg or Nash. However, the parameter magnitudes differ, depending on whether the competition is Nash or Stackelberg.

For the sake of completeness, the optimal price rule for Brand 2 is:

$$p_t^{2*} = g^2 + \mathbf{G}^2\mathbf{q}_{t-1} + \mathbf{S}^2(\mathbf{f}_t^2 - \overline{\mathbf{f}^2}).$$

The follower's optimization constraints are (5)–(7) with superscripts 1 and 2 interchanged.

4. Estimation Method for the Price Rule

We outline a procedure which may be used to estimate the optimal price rule for Stackelberg competition. Later, we explain how the estimation procedure can be suitably modified to a scenario where the brands act as Nash competitors. The data required for the empirical estimation are time series of sales and prices for both the leader and the follower, and a measure for the demand forecasts made by both firms in previous decision periods.

The optimal price rule can be estimated using maximum likelihood methods, since we can specify the likelihood function and also the complete set of first-order conditions (see Appendix). Chow (1975) describes the estimation of rational expectations models using maximum likelihood. Gasmi, Laffont, and Vuong (1990) point out that if the Stackelberg mode of competition is an accurate representation of market behavior, a maximum likelihood estimation procedure will yield consistent estimates.

The results of Breusch (1987), among others, show that in the limit, iterative generalized least-squares procedures yield global maximum likelihood estimates. This suggests that a full information method such as three-stage least-squares (3SLS) or seemingly unrelated regression (SUR) can be used in place of full information maximum likelihood (FIML). The availability of package programs for methods such as the 3SLS, make them viable alternatives to FIML, for which a program must be specially written.[6] Consequently, we present an estimation procedure that is based on iterative three stage least squares. We describe our approach in some detail below.

4.1. Constrained Iterative Three Stage Least Squares

We outline the procedure for the Stackelberg case first. The estimation steps follow the solution pattern for a Stackelberg game. Our solution is obtained by first estimating the follower's price rule (i.e., Equation (4) as applied to Brand 2), subject to the leader's price, and the follower's optimization conditions. The follower's estimated rule (a reaction function) is then used to determine the leader's optimal price rule, and the procedure is repeated till convergence. The following is a list of steps in estimation of optimal price rules for the leader and the follower.

(1) We start with the estimation of the follower's rule. OLS estimates of $\mathbf{G}^1$, $\mathbf{S}^1$ and g^1 serve as starting values. The simultaneous equations in our system are the price equation of the follower (Equation (4) for Brand 2), and the two sales equations, (1) and (2). These are estimated subject to the optimizing conditions (5)–(7). We outline the steps taken to incorporate (5)–(7).

(a) The expression for $\mathbf{S}^2$, the weight assigned to lagged sales by the follower, is stated below (see Technical Appendix A1 for details).

$$\mathbf{S}^2 = -\mathbf{G}^2(\mathbf{A} + \mathbf{G}^{1\prime}\mathbf{B}_1' + \mathbf{G}^{2\prime}\mathbf{B}_2')\Sigma\Gamma_2^{-1}. \qquad (8)$$

Note that $\mathbf{G}^2$ appears on the right hand side of (8) and it is not necessary to include Equation (6) for $\mathbf{G}^2$ separately. In the price rule derivation (see Appendix A1), (6) must be satisfied before we can use (8) to solve for $\mathbf{S}^2$. Equation (8) represents the relation between optimal

[6] Additionally our estimation has to be carried out subject to constraints on the parameters. We can employ constrained, iterative methods such as IT3SLS or ITSUR, using one of the many simultaneous equation packages available for the purpose (SYSLIN or SYSNLIN in SAS).

slope parameters of the price rule, S^2 and G^2. It is more specific than (6), and imposing (8) ensures that both optimality conditions are met.

(b) Equation (8) involves a quadratic term in G^2. With linear equations it is advisable to use linear constraints. Taylor series expansion around an OLS estimate of this parameter is used to linearize this constraint equation. Higher-order terms in the expansion are disregarded.

(c) Again referring to (8), based on the previous sales, prices, and forecasts, we know the elements of the sales variance/covariance matrix Σ as well as the covariance matrix Γ_2. We treat these as known and fixed, although at the end of estimation we could use (8) to check the value of Σ. If different from the start value we could iterate the entire procedure till convergence.

(d) For the constraint on the intercept term in the follower's price rule, we refer to Equation (5), with superscripts changed for Brand 2.

$$g^2 = (\mathbf{B}_2'\mathbf{B}_2)^{-1}\mathbf{B}_2'(\mathbf{q}^{2*} - \mathbf{B}_1 g^1) \qquad (9)$$

(e) Since Equation (8) is a 2×1 matrix equation, and is equivalent to two scalar equations, (8) and (9) together constitute a system of three constraints. The constraints operate across the three simultaneous equations of our model.

(2) The coefficients free to be estimated are sales equation parameters $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, the follower's target sales $\mathbf{q}^{2*}$, and the follower's price rule parameters G^2, S^2 and g^2.

(3) The estimates of G^2, S^2 and g^2 represent the follower's estimated reaction function. These values are substituted in the leader's optimizing conditions when we repeat the estimation for Brand 1. The three simultaneous equations in the leader's model are (4), price equation of Brand 1, and (1) and (2), the two sales equations. We estimate the leader's target sales $\mathbf{q}^{1*}$, and price rule parameters G^1, S^1 and g^1. We compare these with the start values. If they are different we take the more recent estimates and go back to step 1 for the follower.

(4) This back-and-forth estimation between the leader's rule and the follower's rule is continued until there is insignificant change from the previous iteration. The program usually converges rapidly if the assumptions about competition are met.

Note that in each iteration we estimate the price rule of a firm *along with the two sales equations*. We re-estimate the sales equation coefficients $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, along with the price rule, until they stabilize and we have a best fit demand function and the corresponding price rule. The optimal price rule of a firm depends on these sales function coefficients, and if we did not allow any iterations other than the first we would have a price rule that might be suboptimal and inconsistent with the best fit sales equations.

Specifically, we first estimate the follower's price rule along with the sales equations (1) and (2). Next, after substituting the follower's estimated price rule parameters in optimizing conditions (8) and (9), we estimate the leader's price rule (Equation 4), and simultaneously estimate the sales equations (1) and (2). In the second iteration we re-estimate the follower's price rule, using the updated estimates of the sales equation coefficients in the optimizing conditions (8) and (9). We continue the process till it converges.

Put differently, if we did not iterate we would not be using all the information in the data before arriving at a final set of price rules.

Estimation Procedure for the Nash Competitive Mode: A non-cooperative Nash solution can be obtained by simultaneous estimation of both price equations (Equation (4) for both brands together), with the sales equations, subject to *two* sets of optimal conditions imposed together (one set consisting of Equations (8) and (9) as shown, and the second set comprising of (8) and (9) but with superscripts 2 and 1 interchanged). We have four equations in the model, and six optimizing conditions. There is only one step in the estimation of the Nash model.

5. Empirical Study: Thunderbird vs. New Yorker

As an illustration of our methodology, we estimate the optimal price rule on data from the mid-size sedan segment of the domestic automobile market, using Ford Thunderbird as the leader, and Chrysler New Yorker as the follower. The parallels between this market segment and our analytical framework are discussed in more detail next.

U.S. Automobile Industry: It has long been hypothesized that, among the major U.S. automobile manufacturers, there existed an implicit understanding that prices for the coming year's car models would be decided by either General Motors or Ford, and that the peripheral players such as Chrysler and American Motors would base the prices of their new models on the prices indicated by the leaders. This view of leader-follower patterns in this industry is reflected in Scherer (1980), Carpenter, Cooper, Hanssens, and Midgley (1988) and Nagle (1987). While we examine the price interdependency between two car models we recognize that leader-follower behavior may apply between entire lines of automobiles.

Mid-Size Sedans: Although there are many cars, foreign and domestic, that can be categorized in the mid-size segment, only two auto models, namely, **Ford Thunderbird** and **Chrysler New Yorker,** have been consistently a part of this segment, going back as far as 1960. Thus, this segment allows us to use the duopoly framework developed in §§2–4. Both models were originally in the high-price family sedan segment with list prices above $3000 in the early 1960's. The Thunderbird underwent a change to a smaller, lower price car following the oil crisis of the early 70's, and is more recently positioned as a performance sedan. The Chrysler New Yorker became more of a luxury model after the restructuring of the Chrysler company in the early 80's. However, both cars are still in the same segment—medium-size sedan—as defined in a *Consumer Reports* issue on 1989 cars. They have been classified in the same category by *Ward's Automotive Handbook* over the entire period of our analysis.

Data and Measures: Estimation of our optimal price rule requires measures of the following variables:

(1) time series of prices of both brands,
(2) time series of unit sales of both brands,
(3) forecasts of unit sales of both brands.

Data on unit sales and prices are obtained from the annual new car registrations and list prices for each year's base model announced by manufacturers in September or October of the previous year (*Ward's Automotive Handbook* 1960–86). Yearly production figures are used as surrogates for demand forecasts. Considering the high manufacturing costs of automobiles, it is reasonable to use actual production as a substitute for demand forecasts. Demand forecasts are a means of determining how many units should be produced and reducing the risk of opportunity losses or inventory costs. Each brand obviously knows its own production plans. We assume that it has complete information about its competitor's planned production as well—a tolerable assumption in a close-knit industry with concentration of manufacturing units and suppliers in one area. For those seeking to apply our method, we note that, if any variable is substituted for forecasts in the model, one must ensure that it is correlated with forecasts, and it does not influence price in a way that is different from our model.

5.1. Leader/Follower Assumption: Preliminary Tests

Before proceeding with the estimation, we tested our assumption of a leader-follower relationship between Thunderbird and New Yorker using the concept of Granger Causality (Granger 1969) and the test developed by Pierce (1977), for determining causal flows based on cross correlations of residual terms from univariate time series models of each series (see Hanssens, Parsons, and Schultz 1990).

The test reveals an influence only in the direction of Thunderbird to New Yorker: the Pierce χ^2 value for the entire period of 27 years is significant at $p \leq 0.10$ and even at $p \leq 0.05$, when we exclude the first four years of data. In particular, there is a high correlation, at lags 3 and 4,[7] between the prewhitened price series of Thunderbird and that of the New Yorker. These negative correlations are consistent with our observation that the downsizing and price reductions in the Thunderbird motivated Chrysler to increase prices and reposition the New Yorker to fill the gap left in this segment by its rival's movement. Therefore statistical tests support our premise that the Thunderbird acted as a price leader for the duration of the study.

[7] We did not use contemporaneous correlations between the two price residuals because: i) they were found to be smaller than intertemporal correlations at lags 3 and 4, and ii) they do not allow us to distinguish the direction of causality. Same period correlations are less relevant because a Stackelberg leader need not necessarily move first during any period. It is enough for a brand to commit to the role of price leader for conditions to be met.

Additionally, we tested the assumption of linearity in sales response using Spitzer's (1982) variation of the Box-Cox test, and found it to be reasonable in the range of prices observed in our empirical study.

5.2. Accounting for Exogenous Variables

Throughout our analysis we used real prices adjusted for inflation. To account for potential autocorrelation in the data, we estimated a first-order autoregressive (AR(1)) model for both price series.

$$p_t = \theta_1 p_{t-1} + v_t$$

p_t and p_{t-1} are price at t and $t-1$ respectively; θ_1 is the first-order autocorrelation coefficient; v_t is the residual. The residuals of this filter represent the part of price that cannot be explained by itself. We applied our estimation method to these price residuals. Since an AR(1) filter was also sufficient to prewhiten the series for the Granger test above, a consistent feature of our study is that the residuals used to identify leadership were also used as inputs to the price rule estimation.

To examine whether other exogenous variables could explain variance in price or sales, we included a large number of variables related to the economy and the automotive industry, such as interest rates, gasoline prices, durable price indices, advertising expenditures, per capita incomes, and GNP. Single equation OLS regressions revealed that none of these variables is significant in any model of price or sales for either car.

If there *had* been a set of exogenous variables that helped explain variance in price, we would have dealt with them in the same manner as we did with autocorrelation, by regressing price on these variables, prior to estimating our model. The residuals from the first model would be the input to our estimation procedure. The final price rule would have included the additional variables.

5.3. Estimation Results for the Leader

Our estimation procedure is applied to a system of three equations—the equation for residual price (after AR(1) filter) of Thunderbird, and the sales equations for both brands. The final price is the sum of the price predicted by a univariate time series model and the optimal residual estimated by our method. To facilitate comparison with actual prices, we converted our estimated prices back to nominal levels.

Estimated Optimal Price Rule for the Ford Thunderbird: The estimated price rule for Ford Thunderbird is shown in Equation (10). Standard errors are in parentheses.

$$\begin{aligned} p_t^{1*} = {} & \underset{(23.30)}{1417} + \underset{(0.04)}{0.85} p_{t-1}^1 - \underset{(0.0007)}{0.0015} q_{t-1}^1 - \underset{(0.0008)}{0.0016} q_{t-1}^2 \\ & + \underset{(0.0045)}{0.0010} f_t^{11'} - \underset{(0.0009)}{0.0020} f_t^{12'} \end{aligned} \tag{10}$$

p_t^{1*} is the optimal price of Ford Thunderbird at time t; p_{t-1}^1 is the price of Thunderbird at time $t-1$; q_{t-1}^1 is volume sales of Thunderbird in time $t-1$; q_{t-1}^2 is volume sales of Chrysler New Yorker in time $t-1$; $f_t^{11'}$ is the deviation from the mean level of production of Thunderbird in time t; $f_t^{12'}$ is the deviation from mean production level of New Yorker (known to Ford) in time t. The intercept corresponds to g^1 in Equation (4). The additional term involving p_{t-1}^1 comes from the univariate AR(1) model estimated prior to our constrained estimation.

The estimated target sales for the Thunderbird, q^{11*}, turns out to be 298,000 units. This is the sales goal set for the Thunderbird, as inferred from the data. We note that this is quite close to the maximum sales level ever achieved by Thunderbird in a single year.

When we multiply the parameter estimates with the average values of the respective right-hand-side variables, we find that the contribution to Thunderbird price of own lagged sales, competitor's lagged sales, autoregressive price effects, and forecasts of own and competitor sales are of the same order of magnitude.

The negative signs of lagged sales coefficients suggest that high previous period sales of both brands are regarded as signals of lower demand in the current period and result in a lower price for the leader. The data suggest that an increase in new car sales in this segment is expected to be followed by a decline in the segment demand the following year. Furthermore, an increase in own sales forecast is a positive signal to raise price and a higher forecast for the competition alerts the leader to lower its price, or else it might lose sales to the New Yorker.

The autoregressive coefficient is found to be high, as is common in price series. All coefficients of (10) are significant at $p \leq 0.05$. If we estimate the AR(1) coefficient simultaneously with the other parameters it causes the standard errors of the other parameters to increase, although all remain significant at $p \leq 0.10$.[8]

Sales Equation for the Ford Thunderbird: The estimated sales equation for the leader brand is given in (11).

$$\begin{aligned} q_t^1 = &-4.0747 p_t^1 + 3.2838 p_t^2 \\ &\quad (1.123) \qquad (0.5741) \\ &+ 0.6162 q_{t-1}^1 - 0.2160 q_{t-1}^2 \\ &\quad (0.1482) \qquad (0.0572) \end{aligned} \tag{11}$$

p_t^1 and p_t^2 are the price of Thunderbird and New Yorker respectively. The other variables are the same as in Equation (10). All coefficients of (11) are significant at $p \leq 0.05$. Note that there is no intercept in this sales equation (see Equation (1)). The signs of the coefficients of the sales equation are as anticipated, and the effect of own price on sales is more than that of competitor's price, as we would expect. It is interesting to note that there is a positive influence of own lagged sales on current sales in (11), yet the impact of lagged sales on the price rule is negative in (10). We do not interpret (11) as a demand equation or infer elasticities from it because the prices are residuals from an autoregressive model. The overall fit statistics for the system of equations with constraints are high, and the R^2 value is 0.81 for the price equation (10).

5.4. Estimation Results for the Follower

As described in §4, our methodology involves estimation steps for the follower as well as the leader. As a result we obtain the optimal price rule and the sales equation for the follower too.

[8] If we constrained the leader to use the sales coefficients estimated by the follower and stopped the procedure after the first iteration the price rule of the leader (Thunderbird) would be:

$$\begin{aligned} p_t^{1*} = 1429 + 0.85 p_{t-1}^1 - 0.0016 q_{t-1}^1 - 0.0017 q_{t-1}^2 \\ + 0.0013 f_t^{11'} - 0.0024 f_t^{12'} \end{aligned}$$

In this case, the difference between the above rule and the calibrated rule represented by Equation 10 is minor. In general it is recommended that the iterative procedure be continued till convergence.

Estimated Optimal Price Rule for the Chrysler New Yorker: Equation (12) represents the estimated optimal price rule for the follower, Chrysler New Yorker.

$$\begin{aligned} p_t^{2*} = 1203 &+ 0.73 p_{t-1}^2 - 0.0342 q_{t-1}^1 - 0.1447 q_{t-1}^2 \\ (20.50)&(0.08) \qquad (0.0012) \qquad (0.0013) \\ &- 0.0014 f_t^{21'} + 0.0018 f_t^{22'} \\ &\quad (0.0059) \qquad (0.0007) \end{aligned} \tag{12}$$

p_t^{2*} is the optimal price of the New Yorker in time t; p_{t-1}^2 is the price of New Yorker in time $t - 1$; $f_t^{21'}$ is the deviation from the mean level of production of Thunderbird (known to Chrysler) in t; $f_t^{22'}$ is the deviation from mean production level of New Yorker in t. Other terms are the same as in (10) and (11). As compared to the leader's case, the follower's prices are less autocorrelated, but more dependent on own sales in the previous period. The follower also weighs forecasts more heavily than does the leader. The signs of the coefficients are consistent with the pattern found for the Thunderbird, and the intuition behind these signs is the same.

Estimated Sales Equation for the Chrysler New Yorker: Equation (13) gives the estimated sales equation for the New Yorker.

$$\begin{aligned} q_t^2 = 1.4565 p_t^1 &- 2.2638 p_t^2 - 0.0533 q_{t-1}^1 + 0.5747 q_{t-1}^2 \\ (0.4785) &\quad (0.3395) \qquad (0.0232) \qquad (0.2116) \end{aligned}$$

All coefficients in Equation (13) are significant at $p \leq 0.05$. As in the case of the Thunderbird, the signs of coefficients are as expected. The estimated target sales for the New Yorker, corresponding to the estimated parameters in (12) and (13), is 116,000 units. This is a little above the highest sales level achieved by this car during the period of analysis.

6. Model Comparisons

The two main objectives in this section are as follows:

(1) We want to examine whether the automobile data are more consistent with pricing behavior in a Stackelberg leader-follower market as opposed to a market where the competing brands set their prices independently (Nash).

(2) Our analytical procedure assumes that the competing brands set prices so as to keep their sales close to a preset target sales level. We want to examine if our model does better at explaining market data than other pricing schemes that do not assume such optimizing behavior on the part of the competing brands.

In the Appendix, we present likelihood functions for our optimal price model and some alternative models (including Nash). By substituting the parameter values obtained from our estimation procedure into the appropriate likelihood function, we can compute an estimate of the likelihood function and use it for comparing competing models. The tests are based on likelihood ratios.

6.1. Testing Stackelberg vs. Nash Behavior

Recently, the *Wall Street Journal* (July 28, 1992) reported that Chrysler's pricing strategy appears to follow in the footsteps of rival Ford. In this section, we present a statistical procedure for formally identifying leader-follower price behavior. This procedure can be used to formally confirm such types of conjectures.

More specifically, our comparisons in this section allow us to examine whether the pricing behavior in the Thunderbird–New Yorker market is more in line with the Stackelberg mode of competition than a Nash structure. To compare a Stackelberg system where Thunderbird acts as the price leader, with a Nash mode where Thunderbird and New Yorker set their respective prices independently, we use the specification tests between competing non-nested models described in Vuong (1989). These tests are similar to those used by Gasmi, Laffont, and Vuong (1988) in their empirical study of price and advertising decisions in the U.S. cola drink market.[9]

Test Details: To illustrate this test, let us consider the comparison of the Stackelberg vs. Nash models. The likelihood function under Stackelberg competition is given by (A.3) in the Appendix, and the function under Nash conditions is given by a Lagrangian extension of (A.4) which includes the optimizing equations (5)–(7). Using Vuong's (1989) notation, let us denote these two likelihood functions as $f(Y_t | Z_t, \theta_*)$ and $g(Y_t | Z_t, \gamma_*)$ respectively. Y_t are endogenous variables, Z_t a set of exogenous variables, θ_* and γ_* the two sets of (true) parameters under the two competing models. Under the null hypothesis H_0, $n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n)/\hat{\omega}_n$ converges in distribution to the standard normal $N(0, 1)$. Here the likelihood ratio $LR_n(\hat{\theta}_n, \hat{\gamma}_n)$ can be approximated as the difference in expected log likelihoods; $\hat{\omega}_n$ is an estimate of the variance of the likelihood ratio with respect to the true joint distribution of $(Y_t | Z_t)$; n is the number of observations, and $\hat{\theta}_n$, $\hat{\gamma}_n$ are maximum likelihood estimates of parameters in each model given a sample of size n.[10]

Results: Using Vuong's (1989) test, the value of the test statistic is found to be 1.821 which is significant at $p = 0.05$. *Hence, we can reject the null that the two models are equivalent, in favor of an alternative that the data are consistent with the Stackelberg model.*

6.2. Testing the Optimizing Behavior Assumption

We compare three alternative models. For ease of exposition we label each of these models using the following scheme. The letter S in the label indicates that the model uses the sales equations (1)–(2). The letter P indicates that it includes the optimal price rule (4). The letter O indicates that an assumption is being made about optimizing behavior on the part of the competing brands (Equations (5)–(7)). The letters NO indicate that no assumption is made about optimizing behavior.

1. Model SPO: This is our model accounting for optimal behavior. This model assumes that sales follow (1) and (2). Further, it assumes that the competing brands set their prices as per the price rule (4), and the coefficients of the price rule are determined by optimizing behavior (5)–(7). The likelihood function for this model is given in Equation (A.3) in the Appendix.

2. Model SPNO: This model assumes that sales follow (1) and (2), and the price rule used has the same specification as (4). But the parameters of the linear rule do not follow the optimizing conditions (5)–(7). The firm is assumed to want to obtain sales as close as possible to its target. It recognizes that the optimal price

[9] Their tests led Gasmi et al. (1988) to retain a model where the Coca Cola company acted as a Stackelberg leader until 1976 and (tacit) collusion in advertising prevailed thereafter.

[10] The test statistic is simply the difference in maximum log-likelihood values for the two models, suitably normalized. Further details of the normalization ($n^{1/2}\hat{\omega}_n$) can be found in Vuong (1989).

Table 1 Log-Likelihoods for Alternative Models

Model	Log-Likelihood
SPO	−30.2839
SPNO	−39.9674
SNO	−35.7936

rule is linear in state variables such as lagged sales and forecasts, but it does not know specifically which linear rule is optimal. The firm estimates a system of simultaneous equations using the likelihood function given in Equation (A.2) in the Appendix.

3. Model SNO: This model assumes that sales follow (1) and (2). However, the price rule in (4), as well as the optimizing behavior captured by (5)–(7) is ignored. The likelihood function for this model is given in Equation (A.1) in the Appendix.

The estimated log-likelihoods for these three models are summarized in Table 1.

Testing the Optimal Behavior Assumption: SPO, the model based on the joint distribution of errors from the price and sales equations and assuming optimizing behavior, has a much higher likelihood than either of the models that do not account for optimal behavior (SNO or SPNO).[11] The likelihood increases as we go from SPNO, which does not assume optimal behavior, to SPO which does. An econometric reason for the better fit of SPO is that imposition of optimal conditions accounts for a more complex error covariance structure than the non-optimal model. The likelihood function of SPO is based on the same joint normal distribution of independent errors as SPNO (see A.3 and A.2 in the Appendix), but the additional optimizing conditions in SPO implicitly recognize the across equation residual correlations. If this is closer to the true error covariance structure, then SPO should have a better fit.

As a formal test of the optimality of pricing decisions in our example, we use Vuong's (1989) test between overlapping models, in which one does not know if either of the competing models is correctly specified. This is a general test which involves a sequential procedure,[12] in which we first test whether $\hat{\omega}_n = 0$, where $\hat{\omega}_n$ is a sample estimate of the variance of the likelihood ratio as mentioned in §6.1. If we are able to reject the null hypothesis, or find that $\hat{\omega}_n \neq 0$, then we can proceed to test whether the difference in maximum log-likelihoods is significant using the same test statistic as in the Nash vs. Stackelberg comparison.

Comparing the models SPO and SPNO, we are first able to reject the null hypothesis that the variance of the likelihood ratio is equal to zero. We pass to the second step which is a test identical to the one outlined in §6.1. The value of the test statistic $n^{-1/2}LR_n(\hat{\theta}_n, \hat{\gamma}_n)/\hat{\omega}_n$ is 2.163, significant at $p \leq 0.05$. *We reject the null hypothesis of equivalence between SPO and SPNO in favor of an alternative hypothesis that our optimal model SPO provides better fit to the data.*

Testing the Effect of Including the Price Equation: We test SNO versus SPNO using Vuong's (1989) likelihood ratio tests between non-nested models, described in §6.1. A comparison of SNO and SPNO does not permit us to reject the null hypothesis of equivalence. The value of the test statistic is 1.581 and it is not significant at $p \leq 0.05$. Adding the price equation to a non-optimal sales model actually lowers the likelihood (compare A.1 and A.2 in Appendix), but there is no significant difference in this case.

6.2.1. Estimated Price Rules. It may be worthwhile at this stage to also compare the estimated price rules in each case to see if they differ qualitatively. The coefficients of the price rule obtained by estimating the model assuming optimal behavior (SPO) and the model (SPNO) without the optimizing assumption are summarized in Table 2. All price rule coefficients are significant at $p \leq 0.05$ for the optimal price rule SPO.[13]

[11] We are indebted to an anonymous reviewer for pointing out that simpler models, such as SNO, can also be estimated and tested using the same framework.

[12] See Vuong (1989) for details of this test, which is more complicated than the one described for non-nested models. The procedure also requires the c.d.f. of a weighted sums of chi-square distribution, values of which can be calculated using a program in Johnson and Kotz (1969).

[13] A common problem with constrained estimation is that the estimates are driven to satisfy exactly the constraint equations—a "settling at the boundary" problem. The non-zero standard errors and reasonable *t* statistics of our estimates suggest that this is not the case here. Another noteworthy point is that when estimation is done in two steps, with

Table 2 Estimated Price Rules

Price Rule Coefficients		SPNO	SPO
Intercept	g^1	162	1417
Lagged Sales	G^{11}	0.0158	−0.0015
	G^{12}	0.0022	−0.0016
Forecast	S^{11}	0.0007	0.0010
	S^{12}	−0.0012	−0.0020
Lagged Price		0.85	0.85

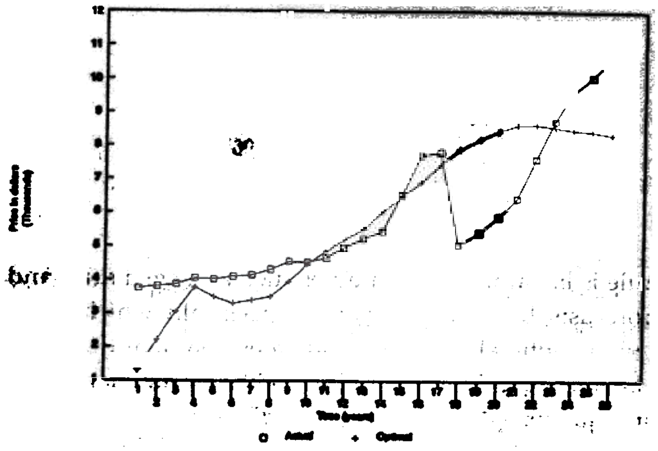
For the price rule without the optimal behavior assumption, the estimates of G^{11} and G^{12} are not significant, but the other coefficients of SPNO are significant at $p \leq 0.05$. There is a difference in the order of magnitude of the price intercepts between the two models. Further, the signs of the G coefficients are reversed. While SPNO places a high positive weight on own lagged sales in determining current price, SPO reduces the weight of past sales realizations and reverses the sign. The optimal pricing strategy also gives more weight to sales forecasts, as seen by comparing the S coefficients.

6.2.2. Price and Sales Paths. We graph the price path generated by our optimal rule and the price path generated by the model that does not assume optimal behavior. Estimated prices are transformed back to nominal levels for the sake of comparison with actual prices. The pricing strategy actually followed by Ford Thunderbird during this time period is also plotted in Figure 1. Note that the optimal price path is generally increasing over time and shows less fluctuations than the actual price path. Figure 2 shows the optimal sales path along with the actual sales path for the leader. The sales path resulting from the optimal pricing strategy is smoother than the actual sales. The linear form of the estimated sales equations is partly responsible for this smoothness.[14] Furthermore, as one would have anticipated, the optimal pricing strategy has a stabilizing influence on brand sales.

the simultaneous-equation model estimated after a univariate time series model has been fitted to the price series, the coefficients are of similar magnitude, but with lower standard errors, compared to coefficients from a simultaneous-equation model including a lagged price variable.

[14] The estimated sales equations (11) and (13) enable us to simulate the path sales would have taken if Ford Thunderbird management had followed our optimal pricing strategy.

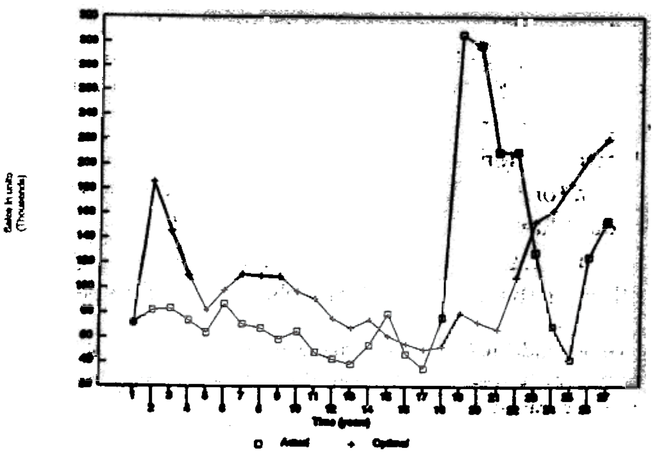
Figure 1 Price Paths for Ford Thunderbird



7. Limitations and Conclusions

We present a model and outline an estimation procedure for understanding pricing behavior in markets where one brand acts as the price leader. Our framework allows the competing brands to make forecasts about future demand and to incorporate these forecasts into the pricing rule. We develop a procedure to estimate the price rule, and illustrate it on data from the mid-size sedan segment of the U.S. automobile market. We present statistical tests that identify formally whether one brand acts as a price leader. Our statistical results

Figure 2 Sales Paths for Ford Thunderbird



confirm conjectures in the business press that Ford often acts as the price leader, with Chrysler as the follower. We note that auto manufacturers often announce price changes for entire product lines at one time. Therefore price leadership can exist for lines of automobiles and not just between car models as we have assumed in our illustration.

The model and the empirical procedure are subject to some limitations. First, the econometric estimation procedure assumes fixed coefficients in the demand equations, and the objective function (i.e., we assumed that the sales target remains unchanged). An alternative substitution method outlined in Technical Appendix A3, allows for time varying parameters.

Second, the specific objective function that assumes that firms set prices to achieve pre-set sales targets is representative of some real world situations. This assumption was also necessary to allow us to apply the estimation procedure. However, it limits our model to the type of scenarios described in § 2.2. The calibration of leader's price rules based on profit maximization is a logical next step.

Third, we have assumed the target sales as given, and estimated them from market data. An extension of our work would be to integrate the target setting process with the optimizing procedure we have presented.

Fourth, Stackelberg leadership is a specific form of price leadership. Other forms are prevalent, such as collusive price leadership, where each firm announces a price in advance and then both adjust. Our model could be extended to an n-brand market, possibly in the manner of a Stackelberg-Nash game (Choi, DeSarbo, and Harker 1990) where there is a single leader and multiple followers, and the followers behave in a Nash manner among themselves.

In conclusion, this paper presents and illustrates a methodology for estimating a pricing rule that accounts for leader-follower competition, utilizing sales forecasts that are routinely available in practice, but rarely linked directly to price setting.[15]

[15] The authors thank the Departmental Editor (Marketing), the Associate Editor, and three reviewers for their insightful comments.

A. Appendix

Estimation and Testing of a Stackelberg Model: A fundamental assumption in this approach to estimating price rule parameters, is that errors from each of the price and sales equations, are jointly distributed normal. In addition to the error terms u_i^t in the sales equations for brands $i = 1, 2$ in our system (Equations (1) and (2)), we assume there are errors v_s which represent the difference between actual price and optimal price at time t. To reiterate, the optimal price for the leader, which we label Brand 1, is given by:

$$p_t^{1*} = \mathbf{G}^1\mathbf{q}_{t-1} + \mathbf{S}^1(f_t^1 - \overline{f^1}) + \mathbf{g}^1$$

For various reasons the decision makers of both leader and follower brands may be implementing a suboptimal strategy.

A full information method can be used for estimation and likelihood ratio tests performed to compare models, given our knowledge of the maximum likelihood functions.

Likelihood Functions

In our empirical study we used full information methods, approximating maximum likelihood. Although the likelihood functions we present here are not the functions that are actually optimized in the simultaneous equation programs we used, we substitute our final parameter estimates to evaluate maximum log-likelihoods of each model we calibrate. We can do this because, in the limit, our estimation procedure yields global maximum likelihood estimates. Section 6 shows details of the model tests based on computed likelihoods.

(1) In the model we refer to as SNO in §6, we consider only errors from the sales equations and assume they are normally distributed. This is a sales equation model without optimal constraints.

$$L_{\widetilde{SNO}} = \text{const} - \frac{n}{2}\log|\Sigma| - \frac{1}{2}\mathrm{Tr}\cdot[\Sigma^{-1}(\mathbf{q}' - \mathbf{A}\mathbf{q}'_{-1} - \mathbf{B}_1\mathbf{p}^{1'} - \mathbf{B}_2\mathbf{p}^{2'}) \times (\mathbf{q} - \mathbf{q}_{-1}\mathbf{A}' - \mathbf{p}^1\mathbf{B}'_1 - \mathbf{p}^2\mathbf{B}'_2)] \tag{A.1}$$

$|\cdot|$ refers to the determinant of a matrix and Tr is the trace of a matrix. $\mathbf{q}$ is an $n \times 2$ matrix of current sales, $\mathbf{q}_{-1}$ a matrix of lagged sales, $\mathbf{p}^1$ an $n \times 1$ vector of leader's prices, $\mathbf{p}^2$ a corresponding vector of follower's prices; $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$ are coefficients of the demand model as defined in §3.

(2) An alternative model, also without optimal conditions, is referred to as SPNO in § 6 of the text. This considers the errors of the sales equations of both brands, and the price equation of a single brand, all assumed to be joint normally distributed. Let us consider the log-likelihood function for follower Brand 2, since the first step of estimation involves solving for the follower's price rule:

$$L_{\widetilde{SPNO}} = \text{const} - \frac{n}{2}\log|\Sigma| - \frac{n}{2}\log V_2$$

$$- \frac{1}{2}\mathrm{Tr}\cdot[\Sigma^{-1}(\mathbf{q}' - \mathbf{A}\mathbf{q}'_{-1} - \mathbf{B}_1\mathbf{p}^{1'} - \mathbf{B}_2\mathbf{p}^{2'})$$

$$\times (\mathbf{q} - \mathbf{q}_{-1}\mathbf{A}' - \mathbf{p}^1\mathbf{B}'_1 - \mathbf{p}^2\mathbf{B}'_2)]$$

$$- \frac{1}{2}[V_2^{-1}(\mathbf{p}^{2'} - \mathbf{G}^2\mathbf{q}'_{-1} - \mathbf{S}^2 f^{2'} - \mathbf{g}^2\mathbf{z}')$$

$$\times (\mathbf{p}^2 - \mathbf{q}_{-1}\mathbf{G}^{2'} - f^2\mathbf{S}^{2'} - \mathbf{z}\mathbf{g}^{2'})]$$

G^2, S^2 and g^2 are parameter matrices and vectors for Brand 2's price rule, which correspond to those for Brand 1 (Equation (4)); f^2 is an $n \times 2$ vector of forecasts of Brand 2; z is an $n \times 1$ vector of ones. V_2 is the (scalar) variance of the price variable for follower Brand 2, while Σ is the covariance matrix of the sales vector. All other matrices are as in (A.1).

(3) Next we consider the model referred to as SPO, which is the sales and price model with optimizing conditions that we propose. For Brand 2, the Lagrangian based on the log likelihood function and set of three solution equations (5)–(7) which act as constraints is:

$$L_{SPO} = L_{\widetilde{SPNO}} - \mathrm{Tr}\cdot[\Omega(G^2 + (B_2'B_2)^{-1}B_2'(A + B_1G^1)]$$
$$- \mathrm{Tr}\cdot[\omega(S^2 + G^2(A + G^{2\prime}B_2' + G1'B_1')\Sigma\Gamma_2^{-1})]$$
$$- \phi'(g^2 - (B_2'B_2)^{-1}B_2'(q^{2*} - B_1g^1)) \qquad (A.3)$$

Ω, ω and ϕ are the Lagrangian multipliers corresponding to the equations that yield solutions for G^2, S^2 and g^2. The follower Brand 2 takes the price of 1 as given, substitutes OLS estimates of the leader's price rule $\hat{G}^1$, $\hat{S}^1$, $\hat{g}^1$ in (A.3), and maximizes its generalized Lagrangian function.

If the decision makers play noncooperatively, in a Nash game, the Lagrangian to be maximized is more complex, but the estimation problem is simpler. We use a likelihood function as in Equation (A.4), based on joint normally distributed errors from the two price equations and two sales equations:

$$L_{\widetilde{Nash}} = \text{const} - \frac{n}{2}\log|\Sigma| - \frac{n}{2}\log V_1 - \frac{n}{2}\log V_2$$
$$- \frac{1}{2}\mathrm{Tr}\cdot[\Sigma^{-1}(q' - Aq_{-1}' - B_1p^{1\prime} - B_2p^{2\prime}$$
$$\times (q - Aq_{-1} - B_1p^1 - B_2p^2)]$$
$$- \frac{1}{2}[V_1^{-1}(p^{1\prime} - G^1q_{-1}' - S^1f^1 - g^1z')$$
$$\times (p^1 - q_{-1}G^{1\prime} - S^1f^1 - z'g^{1\prime})]$$
$$- \frac{1}{2}[V_2^{-1}(p^{2\prime} - G^2q_{-1}' - S^2f^2 - g^2z')$$
$$\times (p^2 - q_{-1}G^{2\prime} - S^2f^2 - zg^{2\prime})] \qquad (A.4)$$

V_1 is the (scalar) variance of the price variable for leader brand 1, (similar to V_2 for the follower). All other terms are as defined previously.

Equation (A.4) is maximized subject to two sets of constraints imposed simultaneously. One set consists of Equations (5)–(7) of §3, and the other set is identical except for the superscripts 1 and 2 interchanged. A generalized Lagrangian is obtained, similar to (A.3).

<u>**Dimensions of variables and parameters:**</u>

$q = 2 \times 1$; $p^1 = 1 \times 1$; $p^2 = 1 \times 1$;

$A = 2 \times 2$; $B_1 = 2 \times 1$; $B_2 = 2 \times 1$;

$G^1 = 1 \times 2$; $G^2 = 1 \times 2$;

$S^1 = 1 \times 2$; $S^2 = 1 \times 2$; $g^1 = 1 \times 1$; $g^2 = 1 \times 1$;

$f^1 = 2 \times 1$; $f^2 = 2 \times 1$; $\Omega = 2 \times 1$; $\omega = 2 \times 1$; $\phi = 1 \times 1$

$\Gamma_1 = 2 \times 2$; $\Gamma_2 = 2 \times 2$; $\Sigma = 2 \times 2$;

$u = 2 \times 1$; $q^{2*} = 1 \times 1$; $q^{1*} = 1 \times 1$.

References

Bagchi, A. and T. Basar, "Stackelberg Strategies in Linear-Quadratic Stochastic Differential Games," *J. Optimization Theory and Applications*, 35 (1981), 443–464.

Bensoussan, A., A. Bultez and P. Naert, "Leader's Dynamic Marketing Behavior in Oligopoly," *TIMS Studies in the Management Sciences*, 9 (1978), 123–145.

Breusch, T. S., "Maximum Likelihood Estimation of Random Effects Models," *J. Econometrics*, 36 (1987), 383–389.

Carpenter, G. S., L. G. Cooper, D. M. Hanssens and D. F. Midgley, "Modeling Asymmetric Competition," *Marketing Sci.*, 7 (1988), 393–412.

Choi, S. C., W. S. DeSarbo and P. T. Harker, "Product Positioning Under Price Competition," *Management Sci.*, 36 (1990), 175–199.

Chow, G. C., *Analysis and Control of Dynamic Economic Systems*, Wiley, New York, 1975.

Consumers Union, *Consumer Reports, Special Issue on Automobiles*, Mt. Vernon, NY, 1989.

Dixit, A., "A Model of Duopoly Suggesting a Theory of Entry Barriers," *The Bell J. Economics*, 10 (1979), 20–32.

Dockner, E. and S. Jorgensen, "Optimal Pricing Strategies for New Products in Dynamic Oligopolies," *Marketing Sci.*, 7 (1988), 315–334.

Dolan, R. J. and A. P. Jeuland, "Experience Curves and Dynamic Demand Models; Implications for Optimal Pricing Strategies," *J. Marketing*, 45 (1981), 52–62.

Eliashberg, J. and A. P. Jeuland, "The Impact of Competitive Entry in a Developing Market Upon Dynamic Pricing Strategies," *Marketing Sci.*, 5 (1986), 20–36.

Erickson, G. M., "Dynamic Pricing in Oligopolistic New Product Markets," Working Paper, University of Washington, Seattle, 1983.

Fruhan, W. Jr., "Pyrrhic Victories in Fights for Market Share," *Harvard Business Review*, 50 (1972), 100–107.

Gasmi, F., J. J. Laffont and Q. H. Vuong, "Econometric Analysis of Collusive Behavior in a Soft Drink Market," Working Paper, University of Southern California, Los Angeles, 1988.

——, —— and ——, "A Structural Approach to Empirical Analysis of Collusive Behavior," *European Economic Review*, 34 (1990), 513–523.

Granger, C. W. J., "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," *Econometrica*, 37 (1969), 424–438.

Hanssens, D. M., L. J. Parsons and R. L. Schultz, *Market Response Models: Econometric and Time Series Analysis*, Kluwer, Norwell, MA, 1990.

Johnson, N. L. and S. Kotz, "Tables of Distributions of Positive Definite Quadratic Forms in Central Normal Variables," *Sankhya*, Series B (1969), 303–314.

Kahn, B. E., M. U. Kalwani and D. G. Morrison, "Measuring Variety Seeking and Reinforcement Behavior Using Panel Data," *J. Marketing Research*, 23 (1986), 89–100.

Kalish, S., "Monopolist Pricing with Dynamic Demand and Production Cost," *Marketing Sci.*, 2 (1983), 135–160.

Lanzillotti, R. F., "Pricing Objectives in Large Companies," *American Economic Review*, 48 (1958), 921–940.

Nagle, T. T., *The Strategy and Tactics of Pricing*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

Narasimhan, C., "Incorporating Consumer Price Expectations in Diffusion Models," *Marketing Sci.*, 8 (1989), 343–357.

Rao, R. C. and F. M. Bass, "Competition, Strategy and Price Dynamics: A Theoretical and Empirical Investigation," *J. Marketing Research*, 22 (1985), 283–296.

Robinson, V. and C. Lakhani, "Dynamic Pricing Models for New Product Planning," *Management Sci.*, 21 (1975), 1113–1132.

Roy, A., "Optimal Pricing with Demand Feedback in a Hierarchical Market," Unpublished doctoral dissertation, University of California Los Angeles, 1990.

Saghafi, M. M., "Market Share Stability and Marketing Policy: An Axiomatic Approach," in J. Sheth (Ed.), *Research in Marketing*, JAI Press, Greenwich, CT, 1987.

——, "Optimal Pricing to Maximize Profits and Achieve Market-Share Targets for Single-Product and Multiproduct Companies," in T. M. Devinney (Ed.), *Issues in Pricing*, Lexington Books, Lexington, MA, 1988.

Shmueli, A. and C. S. Tapiero, "Optimal Stochastic Control and Stabilization of the Israeli Meat Sector," *Applied Stochastic Control in Econometrics and Management Science*, North-Holland, 1980, 79–113.

Spitzer, J. J., "A Fast and Efficient Algorithm for the Estimation of Parameters in Models with the Box-and-Cox Transformation," *J. American Statistical Association*, 77 (1982), 760–766.

Thompson, G. L. and J. T. Teng, "Optimal Pricing and Advertising Policies for New Product Oligopoly Models," *Marketing Sci.*, 3 (1984), 148–168.

Urban, G. L., J. R. Hauser and J. H. Roberts, "Prelaunch Forecasting of New Automobiles," *Management Sci.*, 36 (1990), 401–421.

Vuong, Q. H., "Likelihood Ratio Tests for Model Selection and Non-nested Hypotheses," *Econometrica*, 57 (1989), 307–333.

Wall Street Journal, "Chrysler Plans To Raise Prices By 3% to 5%," July 28, 1992, A4.

Ward's Communications, *Ward's Automotive Yearbook*, Detroit, MI, 22–48 (1960–86).

Webster, F. Jr., "Top Management's Concerns About Marketing: Issues for the 1980's," *J. Marketing*, 45 (1981), 9–16.

Accepted by Jehoshua Eliashberg; received February 20, 1992. This paper has been with the authors 5 months for 2 revisions.