Partially Directed Search for Prices*

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Abstract

I analyse a model of partially directed search where buyers decide which firm to visit based on correct, but incomplete, information about firms' prices. I show that the unique symmetric pure-strategy equilibrium is in price distributions. A firm's equilibrium price distribution assigns positive mass to prices below the marginal cost ("deals"). If some buyers are "shoppers" (i.e., better at searching than the others), then a larger fraction of shoppers can make the best deals worse. This effect can be so large that the welfare of nonshoppers is reduced.

JEL codes: D43, D83, L13
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1 Introduction

In many instances, buyers decide about which firm to visit first based on partial, but correct, price information. For example, a person may visit a specific supermarket first because his weekly shop was cheap there last week, despite knowing that this week he needs a different basket of groceries. Another might start looking for a laptop in the firm where she previously bought a cheap smartphone. And a third may start searching for flights on the website where her friend got a great deal. In all these cases, the buyer knows the price that a specific firm has charged (potentially for another product), but also knows that the price offer that she gets today can be different.

I model the idea that a buyer can get partial but correct information about firms' prices prior to costly search by allowing firms to post (possibly degenerate)

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price distributions and buyers to direct their search based on a sample of price draws from the firms' distributions. A price distribution can be interpreted as a distribution across different products, product baskets, or a short time interval. A buyer's partial information about prices can be interpreted as stemming from his own past search, potentially for a different product, or from information from other people. I show that the unique symmetric pure-strategy equilibrium is in price distributions and, in contrast to standard search models with price dispersion, that shoppers can increase the lowest quoted prices.

In my main model, buyers sequentially search for a product at a low price.[1] The product can be thought of as homogeneous or horizontally differentiated. A fixed number of homogeneous firms sell the product. Firms post, potentially degenerate, price distributions. Prior to search, each buyer gets a sample of price signals for free, one randomly drawn price from each firm's distribution. By assumption, a buyer partially directs her search: she visits first the firm with the lowest price in her sample.[2] When the buyer visits a firm, however, she cannot buy at the price based on which she directed search, but draws a new price offer from the firm's price distribution. Each price offer costs a fixed amount for the buyer. In Section 4, I let some buyers be "shoppers", i.e., better at searching than the others ("nonshoppers"). Shoppers can be interpreted as buyers with a lower opportunity cost of time (for example, people who are on pension, unemployed, or have low rather than high income) or as buyers with access to a price-comparison website. Shoppers can buy at any price in their free sample of price signals, but are otherwise exactly like nonshoppers.[3]

The results of the model are as follows. First, in the unique symmetric pure-strategy equilibrium firms commit to nondegenerate price distributions (even in the absence of shoppers).[4] A single-price equilibrium fails to exist because a firm profits from setting two prices if others set one. A profitable deviation is to set one price marginally below the proposed equilibrium price and the other equal to the buyers' continuation value (that exceeds the proposed equilibrium price by the search cost), with equal probabilities. The firm attracts half of the buyers and makes on average higher profits on them than in the proposed equilibrium.

[1]I discuss three extensions to the model in Section 5 (as described below).

[2]This assumption has multiple interpretations. First, it is the natural extension to a model where firms are restricted to setting a single price each and buyers direct their search based on prices. An alternative interpretation is that buyers use the rule to break their indifference in a symmetric equilibrium. Finally, in case of a symmetric equilibrium, buyers can be thought of as boundedly rational and not realising that firms set the same price distributions in equilibrium.

[3]I argue in Section 4 that this way of modelling shoppers is in the spirit of Stahl (1989), but the exact way that I model them is unimportant.

[4]An analogous result is in Spiegler (2006), but it deserves greater emphasis in the context of search literature.

Second, I characterise the equilibrium price distribution. The highest posted price equals the buyers' cutoff price: a buyer just accepts this price rather than continues to search. The lowest prices are (weakly) below the marginal cost, prices that I call "deals". If the number of firms increases, as in Spiegler (2006), buyers' welfare is unaffected and price dispersion increases. If the search cost decreases, buyers' welfare increases and price dispersion decreases, as in standard search models that follow Stahl (1989). If the search cost goes to zero, the market converges to the Bertrand outcome. I summarise the testable predictions implied by the comparative statics in Table 1 (on p. 14).

Third, in the model with shoppers, I show that an increase in the fraction of shoppers leads to an rise in the lowest prices in my model, in contrast to other search models with price dispersion that follow Stahl (1989). In the other models, shoppers make the market more competitive, so that the entire price distribution shifts down and nonshoppers' welfare increases. In my model, shoppers are buyers who often execute deals, which are unprofitable for a firm. A larger fraction of shoppers in general makes the best deals worse. This effect can be so large that the average price increases, reducing nonshoppers' welfare. The model, thus, suggests that average prices may be higher in low-income neighbourhoods, which is in line with the mixed empirical evidence.[5] It also suggests that the welfare effect of introducing a price-comparison website is ambiguous.

Finally, in Section 5 I show that the equilibrium in price distributions does not change much under three extensions. If only a fraction of nonshoppers directs search, the equilibrium exists as long as the fraction is large enough. If shoppers are replaced by supershoppers, i.e., buyers with zero search cost (who inspect an infinite number of prices), firms no longer offer deals in equilibrium. If products are horizontally differentiated, i.e., a buyer's value from a product is positive with a probability less than one, the equilibrium distribution is unchanged except that the effective search cost, search cost divided by the match probability, matters.

Literature. My paper contributes to the consumer search literature, by showing that if buyers partially direct search on prices, the only symmetric pure-strategy equilibrium is in price distributions. The equilibrium distribution puts a positive weight on below-marginal-cost prices. In contrast to results in earlier literature, these lowest prices can increase in the fraction of shoppers.

In most other sequential search models where firms sell a homogeneous product that feature price dispersion, the dispersion is across rather than within firms.[6] An exception is Salop (1977), where a monopolist posts a price distribution to

[5]See, for example, Hayes (2000) and Talukdar (2008).

[6]See, for example, Stahl (1989), Janssen and Moraga González (2004), Lester (2011), and Huang (2016).

discriminate between buyers with different search costs. In the main example of Salop (1977), the distribution has two-point support and both prices exceed the marginal cost, whereas in my model the distribution is continuous and the lowest prices are below the marginal cost. In the models that follow Stahl (1989), nonshoppers search randomly across firms and price dispersion stems from firms using mixed strategies. Instead, I show that if nonshoppers partially direct search on prices, then in the symmetric pure-strategy equilibrium firms set price distributions. In the resulting market, in the above models no firm posts below-marginal-cost prices, whereas in my model firms post such prices with positive probability. In addition, in these models of across-firm price dispersion, adding more shoppers leads to a decrease in the lowest price in the market, while in my model in general leads to an increase in the lowest price.

Other papers have studied partially directed search. In the consumer search framework, the models assume that buyers direct their search based on non-price information (which is exogenous, as in ordered search, or endogenous) and/or full price information.[7] In advertising literature, an ad from a seller reveals its product's characteristics and/or price at no cost to the buyer.[8] In other setups, Menzio (2007) allows job-seekers to direct search based on cheap-talk messages from firms. Bethune et al. (2018) and Lentz et al. (2019) are mixture models where a person directs search perfectly with some probability and searches randomly with the remaining probability. In all these models, a firm is assumed to commit to a single price, which the buyer pays if she buys from this firm. In contrast, in my model a buyer may pay a different price than the one she directs search on.

In other search models where a single firm sets different prices for different units of the same product, a firm can discriminate between buyers based on their characteristics. For example, in one model in Armstrong and Zhou (2011), firms can charge different prices to new and repeat customers. Fabra and Montero (2017) and Fabra and Reguant (2018) explicitly study price discrimination. In contrast, in my model firms cannot discriminate between buyers.

Other models where firms charge a below-marginal-cost price to attract rational buyers and another price to make profits are Gerstner and Hess (1990), Weinstein and Ambrus (2008), and Chen and Rey (2012).[9] None of these pa-

[7]See, for example, Arbatskaya (2007), Armstrong et al. (2009), Wilson (2010), Armstrong and Zhou (2011), Haan and Moraga González (2011), Mayzlin and Shin (2011), Zhou (2011), Armstrong (2017), Choi et al. (2018), Ding and Zhang (2018), García and Shelegia (2018), Haan et al. (2018), Moraga González (2018), Shen (2015), Choi and Smith (2016), Anderson and Renault (2017), and Teh and Wright (2018).

[8]See, for example, Butters (1977), Grossman and Shapiro (1984), Stegeman (1991), Robert and Stahl (1993), for early papers.

[9]Other models where firms use such strategies, but where the lowest price exceeds the marginal cost, are, for example, Salop and Stiglitz (1977), Varian (1980), and Lazear (1995).

per analyses truly sequential search, including the effect of shoppers. In Spiegler (2006), a buyer gets price signals, visits first the firm with the lowest price signal, and firms set price distributions in equilibrium, as in my model. In Spiegler (2006), a buyer must purchase at the first firm she visits. In contrast, she can continue searching in my model, which allows me to analyse the effects of a lower search cost and more shoppers.

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 shows that if buyers partially direct search, then single-price equilibria fail to exist and characterises the equilibrium in price distributions. Section 4 introduces shoppers to the model and shows that increasing their fraction can make the best offers worse, and that to the extent that the welfare of nonshoppers decreases. Section 5 analyses three extensions. All proofs are in the Appendix.

2 Model

A number $n \geq 2$ of profit-maximising firms produce a homogeneous product at zero marginal costs. Firm i commits to a potentially degenerate *price distribution* $F_i(p)$.[10]

A unit mass of homogeneous buyers with a unit demand each are looking for the product at a low price. A buyer's valuation for the unit of the product is $v = 1$. Buyers can always get zero utility by not buying. Buyers are *partially informed* about the firms' prices. In particular, a buyer gets a sample of n price signals, one signal per firm. Signal k in a buyer's sample is a random draw from firm k's price distribution. The samples of signals are independent across buyers. One interpretation of this information is that the buyer hears about prices at which her friends bought at the different firms. Another interpretation is that she remembers the prices at the different firms from a time in the past when she sought a different product. Buyers *direct* their search based on the price sample: a buyer visits first the cheapest firm in her sample. If she finds an acceptable price at the firm, she buys and leaves the market. If she finds a price that she does not like at that firm, she can continue to search for a lower price at that firm or at a different firm. Getting a price offer costs $c > 0$ for a buyer. If a buyer's expected value from starting the search process is negative, she does not start searching.

I look for symmetric pure-strategy equilibria where firms post the same price

[10] An interpretation of the price distribution is that the firm sells product baskets or different varieties of the same product at different prices. Another interpretation is that the firm commits to prices for a short time interval, say, a week, and buyers arrive at different days of the week. Infrequent price changes could be the result of algorithmic pricing, a high cost of reoptimising or a high cost of changing prices.

distribution $F(p)$ and buyers use the same cutoff rule.[11] Buyers accept all prices below their optimal cutoff $\bar{p}$. I assume that a buyer buys if she is indifferent between buying and continuing to search.

3 Equilibrium in price distributions

I first describe a buyer's and then a firm's problem. Then I describe the equilibrium and provide the comparative statics' results.

Suppose that a buyer has received her set of free price signals and decided to visit firm k first because the lowest price signal was from firm k. When she visits k, she gets some price offer p' from $F_k(p)$. Should she buy at price p' or continue to search (at the same or a different firm i)? Her continuation value is

$$V = F_i(\bar{p})\mathbb{E}_{F_i}[1 - p|p \leq \bar{p}] + (1 - F_i(\bar{p}))V - c.$$

The buyer accepts the new price offer p if it falls below her cutoff $\bar{p}$ and rejects the offer otherwise. If she accepts p, she gets net utility $v - p = 1 - p$. If she rejects p, she continues to search. The buyer has to pay the search cost c per offer. The optimal cutoff price for the buyer is such that she is indifferent between accepting price $p = \bar{p}$ and continuing: $1 - \bar{p} = V$. The buyer is better off by not starting to search if all prices in the market exceed her cutoff $\bar{p}$.

Firm i's expected profit from setting a price distribution $F_i(p)$ is

$$\pi_i = \mathbb{E}_{F_i}[D_i(p)|p \leq \bar{p}]\mathbb{E}_{F_i}[p|p \leq \bar{p}],$$

where $D_i(p) := \Pi_{j \neq i} 1 - F_j(p)$ is the probability that a signal p from firm i is the lowest received by a buyer. If p is the lowest price in a buyer's sample, the buyer visits firm i first. But upon visiting, she draws a new price p'' from $F_i(p)$ and buys if $p'' \leq \bar{p}$. $D_i(p)$ can be interpreted as firm i's expected demand at price p.

Offering prices above $\bar{p}$ is strictly dominated for the firms: shifting the mass from prices above $\bar{p}$ to $\bar{p}$ would increase demand and not affect revenue. Thus, in a symmetric equilibrium a firm's expected profit simplifies to

$$\pi = \mathbb{E}_F[D(p)]\mathbb{E}_F[p]. \tag{1}$$

[11] In a symmetric equilibrium each firm posts the same price distribution so there is no financial reason for a buyer to visit any particular firm before another. One interpretation of the assumption that these buyers nevertheless direct search is that they use the sample of prices to break their indifference, another is that they are boundedly rational, and, finally, it suffices if firms expect these buyers to direct search in this manner.

A buyer's problem simplifies to

$$\bar{p} = \mathbb{E}_F[p] + c. \tag{2}$$

The expected price is just the cutoff price $\bar{p}$ plus the search cost so $\bar{p}$ directly measures (the negative of) a buyer's welfare.

The expectations and the expected demand $D(p)$ in equations (1) and (2) depend on the equilibrium distribution F. The derivation of the distribution differs from the standard method used in papers of search with price dispersion where an individual firm mixes across prices, but posts a single price. In these models, a firm is indifferent across all the prices in the support of the equilibrium price distribution. Here, a firm is not necessarily indifferent across the different prices in the support of its distribution. I use the methods in Spiegler (2006) to solve for the equilibrium.

I first show that a pure-strategy equilibrium where firms post a single price cannot be an equilibrium. An analogous result is present in Spiegler (2006), but is worth a greater emphasis in the context of search literature.

Proposition 1. *Suppose that firms post a single price p^* in equilibrium. Then a firm has an incentive to deviate to a distribution of prices that has at least two-point support.*

In the proposed single-price equilibrium each firm expects to get $1/n$ of the buyers and price p^* from each. One profitable deviation for a firm is to use a "bait-and-switch" strategy: with equal probabilities, to set one price marginally lower and the other price discretely higher than the proposed equilibrium price p^*. The firm can set the higher price at p^*+c: a price that leaves the buyers just indifferent between buying and continuing. Such a deviation is always profitable because the low price attracts half of the buyers, but once there, half of them pay the higher price, which is strictly above the proposed equilibrium price. In the context of search literature, this means that if (firms expect that) buyers direct their search based on price information, single-price pure-strategy equilibria break down if firms are allowed to set nondegenerate price distributions. In other words, price dispersion is generated in a search model with homogeneous buyers and sellers.

The unique symmetric pure-strategy equilibrium is summarised in

Proposition 2. *A firm's equilibrium price distribution is $F(p) = 1-\left(\frac{p_{max}-p}{p_{max}-p_{min}}\right)^{\frac{1}{n-1}}$ with support $[p_{min}, p_{max}]$ where $p_{min} = -c(n-2)$ and $p_{max} = \bar{p} = 2c$. The equilibrium exists if $c \leq \frac{1}{2}$.*

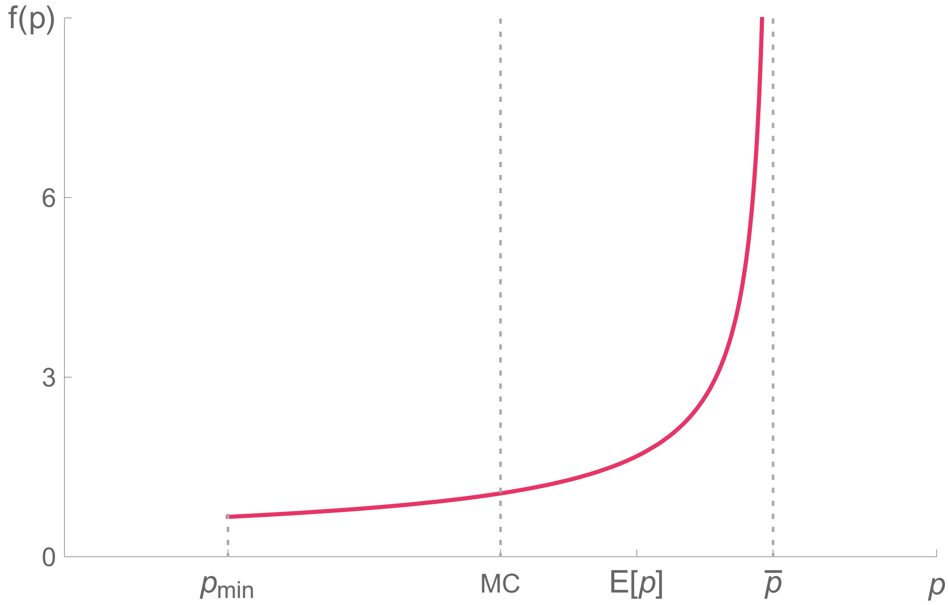

Figure 1: *The equilibrium pdf of prices;* $n = 4$*,* $c = \frac{1}{8}$.

In equilibrium, sellers post a smooth distribution of prices, which is illustrated in Figure 1. The highest price in the support of the distribution leaves a buyer just indifferent between accepting the price and continuing to search. The firms offer "deals", i.e., prices below the marginal cost, with positive probability if there are at least three firms. These low prices attract buyers and make losses, but the firm knows that buyers rarely end up paying such prices. The best deals are the better the more firms there are (i.e., p_{min} decreases in n) because of stiffer competition. Interestingly, the probability that a firm offers deals (i.e., $F(0)$) is nonmonotone in n: if the number of firms is small, a single firm can attract many buyers by offering deals. But, as competition increases, the additional demand is so small and the best deals so costly that a firm is better off by reducing the probability of deals. The nonmomontonicity is illustrated in Figure 2.

The comparative statics' results are summarised in

Corollary 1. *As the search cost, c, decreases,*

(i) the dispersion of prices (i.e., $p_{max} - p_{min}$) decreases: the lower bound of the distribution shifts up and the upper bound down.

(ii) buyers' welfare increases (i.e., $\mathbb{E}[p]$ decreases).

As the number of firms, n, increases,

(i) the dispersion of prices increases: the upper bound of the distribution does not change, but the lower bound shifts down.

(ii) buyers' welfare is unaffected.

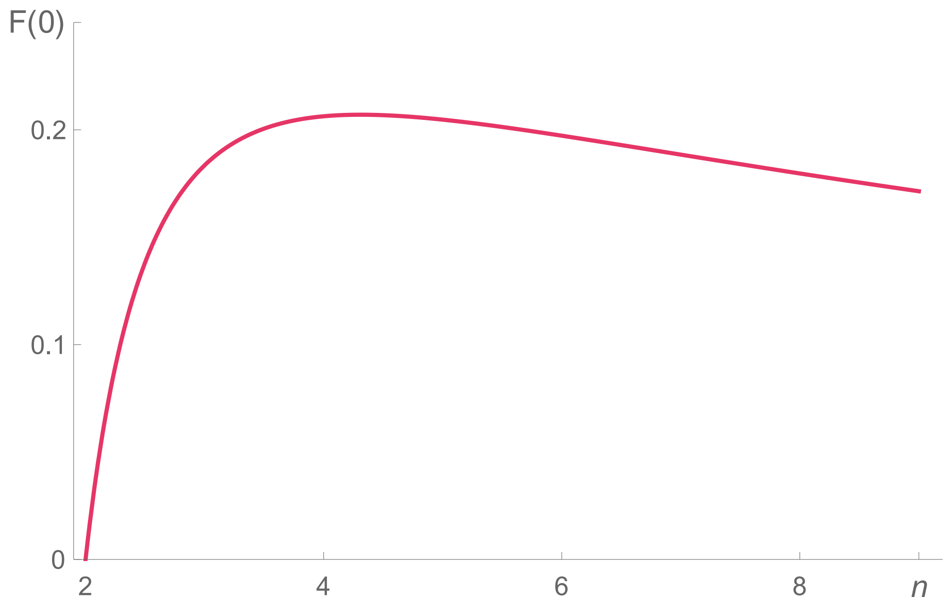

Figure 2: *The probability of a deal (i.e., $p < MC = 0$) is nonmonotone in the number of firms, n.*

The offered prices are less dispersed as the search cost decreases. If the equilibrium distribution did not change, buyers would decrease their optimal cutoff (which shifts down the upper bound of prices). But if sellers have to lower the highest prices they offer, then they optimally also raise the lowest prices they offer (i.e., worsen the best deals). In line with intuition, buyers' welfare increases as search cost decreases. These comparative statics generate testable predictions that differ from those in Spiegler (2006).

The comparative statics with respect to the number of firms n are as in Spiegler (2006). Firms offer better deals as the number of firms increases due to stiffer competition. The number of firms does not affect buyers' welfare. If there are more firms, they offer better deals in equilibrium, but at the same time shift more weight to the highest prices in the support of the distribution. The optimal way of doing this happens to be such that buyers' welfare is unaffected.

4 Adding shoppers

To compare my model's results to the class of search models with price dispersion that follows Stahl (1989), in this Section I let some buyers be "shoppers". Shoppers are better at searching than the rest of the buyers ("nonshoppers").

In Stahl (1989), the search cost of shoppers is zero, which means in his model that shoppers inspect the price that each firm charges; altogether n prices. In my model, if a buyer's search cost is zero, she can search through all prices of all n

firms. Since each firm charges a distribution of prices, this means that a buyer of this sort (who I call a "supershopper") inspects infinitely many prices.

Supershoppers are buyers with an extreme (and perhaps unrealistic) knack for shopping.[12] Therefore, I model shoppers as buyers who see one price from each firm at no cost, as in Stahl (1989), but have to pay the same search cost c as nonshoppers to get further price offers. I show that as the fraction of shoppers increases, firms in general offer worse deals. This effect can be so strong that the expected price increases so that nonshoppers' welfare decreases.

The details of how I model shoppers, i.e., buyers who are better at searching than nonshoppers, is unimportant. Shoppers' effect on the equilibrium distribution is qualitatively the same as long as they are more likely to buy at low prices than nonshoppers. Without shoppers, the lowest prices are below the marginal cost (i.e., unprofitable in expectation). If the likelihood that buyers actually buy at such prices grows, a firm raises these prices. That is, adding shoppers makes the best deals worse. The effect can be large enough to reduce nonshoppers' welfare.

4.1 Model

Consider a model where a fraction $1-\mu$ of the buyers are as before (nonshoppers) and a fraction $\mu < 1$ are shoppers. Shoppers are otherwise like nonshoppers, except that a shopper can also purchase at prices in her free sample of price signals (that nonshoppers can only use for directing search). In other words, a shopper first gets n price offers, one from each firm, for free. She can buy at any of these prices or continue searching for a lower one. Further price offers cost c each, just like for nonshoppers.

4.2 Equilibrium in price distributions

I first describe a shopper's and then a firm's problem. A nonshopper's problem is exactly as before. Then I describe the equilibrium.

After receiving the n free price offers, a shopper can continue to search if she wishes. Suppose that she uses a cutoff $\tilde{p}$ if she continues. Her continuation value $\tilde{V}$ is

$$\tilde{V} = F(\tilde{p})\mathbb{E}_F[1-p|p \leq \tilde{p}] + (1-F(\tilde{p}))\tilde{V} - c,$$

because she has to pay the search cost c for any further offer. But then her continuation problem is identical to a nonshopper's problem and her optimal cutoff

[12] I analyse my model with a positive fraction of supershoppers in Section 5.2. Their presence has a discrete effect on the equilibrium price distribution: firms no longer offer deals.

must also be the same: $\tilde{p} = \bar{p}$. The optimal cutoff solves

$$\bar{p} = \mathbb{E}_F[p] + c. \tag{3}$$

As a result, offering prices above $\bar{p}$ is again dominated for a firm. This, in turn, means that all prices in the set of a shopper's free price offers are below her cutoff. A shopper, thus, optimally accepts the lowest price in her set of free offers.

Given the optimal behaviour of nonshoppers and shoppers, firm i's expected profit when posting price distribution $F(p)$ is

$$\pi = (1-\mu)\mathbb{E}_F[D(p)]\mathbb{E}_F[p] + \mu\mathbb{E}_F[D(p)p], \tag{4}$$

where $D(p) = \Pi_{j\neq i} 1 - F_j(p)$ is the probability both that the signal p from firm i is the lowest one received by a nonshopper and that the offer p from firm i is the lowest among the free offers received by a shopper. The profit from nonshoppers is as before: if firm i's price signal is the lowest among the nonshopper's signals, she visits firm i first and then accepts any *new* price offer. The profit from shoppers is different. A shopper buys from firm i if its offer p is the best among the shopper's free offers, in which case she pays the *same* price p. Thus, there are two expectation operators in the expected profit associated with nonshoppers and only one in the part associated with shoppers. All expectations are taken with respect to the endogenous distribution $F(p)$.

In the model with shoppers and nonshoppers, the symmetric equilibrium in price distributions is summarised in

Proposition 3. *A firm's equilibrium price distribution is* $F(p) = 1 - \left[\frac{p_{max}-p}{p_{max}-p_{min}+\gamma(p-p_{min})}\right]^{\frac{1}{n-1}}$ *with support* $[p_{min}, p_{max}]$ *where* $p_{min} = \frac{c}{\gamma+n}[n-(n-1)K]$ *and* $p_{max} = \bar{p} = \frac{c}{\gamma+n}[n+(1+\gamma)K]$, *with* $K := \left[1 - \int_0^1 \frac{1-z^{n-1}}{1+\gamma z^{n-1}}\,\mathrm{d}z\right]^{-1}$ *and* $\gamma := n\frac{\mu}{1-\mu}$. *The lowest price* $p_{min} < 0$ *if* $\mu < \bar{\mu}(n)$ *where* $\mu(n)$ *solves* $n = (n-1)K(\mu = \mu(n))$. *The equilibrium exists if* $c \leq \frac{\gamma+n}{(1+\gamma)K+n}$.

I can no longer use the methods in Spiegler (2006) to solve for the equilibrium because the profit function has a different form. In the proof, I use the property that if $F(p)$ is optimal, then a firm cannot profitably reallocate mass from any price within the support of $F(p)$ to other prices. I first show that a firm does not want to move a small amount of mass from (the neighbourhood of) any $p \in$ *support*$[F(p)]$ to another *particular* price $p'(p) \in$ *support*$[F(p)]$ only if the expected demand $D(p)$ has a specific form. I then use the above necessary condition to derive a necessary condition that the lowest price in the support of $F(p)$ has to satisfy such that a firm does not want to move a small amount of mass from any $p \in$ *support*$[F(p)]$ to *any* other price $p' \in$ *support*$[F(p)]$.

In Proposition 3, the term within the squared brackets in $F(p)$ looks like a combination of the equivalent terms in Proposition 2 and in the unit-demand version of Stahl (1989) (see, for example, Proposition 2 in Janssen and Moraga González (2004)). I use a clever trick in Janssen and Moraga González (2004) to get the expression for the cutoff price $\bar{p}$. Three or more firms is no longer sufficient for firms to offer deals because shoppers make deals less attractive for firms. Firms offer deals in equilibrium if the fraction of shoppers is not too large.

The comparative statics with respect to the fraction of shoppers are summarised in

Proposition 4. *As the fraction of shoppers, μ, increases,*

(i) *the dispersion of prices, $p_{max} - p_{min}$, decreases.*

(ii) *a sufficient condition for the best deal to become worse (i.e., p_{min} to increase) is $\mu < \bar{\mu}(n)$.*

(iii) *a sufficient condition for the nonshoppers' welfare to decreases is that $\mu \to 0$ and $n \geq 4$.*

An increase in the fraction of shoppers reduces the dispersion of prices. The best deal p_{min} gets worse as the fraction of shoppers increases because firms do not want to make unprofitable offers if these offers are frequently realised. The region of parameters for which this holds is not small: for example, $\bar{\mu}(3) = 0.78$ and $\bar{\mu}(4) = 0.96$.[13] The effect of the best deals getting worse may be so large that nonshoppers' welfare decreases (or, $\bar{p}$ increases).

Figure 3 depicts the price distribution for two different fractions of shoppers μ. Figure 3 illustrates three features. First, as in other models a la Stahl (1989), firms place more mass on prices close to p_{min} if μ is larger. A firm has an incentive to set low prices more frequently because these attract a lot of the shoppers. Second, unlike in other models a la Stahl (1989), the best offers are worse if μ is larger. Third, the highest price $\bar{p}$ is higher, thus, nonshoppers' welfare lower, if μ is larger.

Figure 4 illustrates that the lowest price in general increases in the fraction of shoppers μ. The highest price $\bar{p}$ can increase in μ, and that for a larger range of μ than perhaps Proposition 4 seems to suggest. The Figure also illustrates that the effect of shoppers on the highest price can be large: if the fraction of shoppers changes from zero to a half, the highest price rises by 24%.[14]

[13] The cutoff $\bar{\mu}(n)$ is illustrated in Figure 5 in the Appendix, p. 23. For very large μ, the comparative statics are similar to those in the unit-demand version of Stahl (1989), where both the highest price and expected price decrease in μ.

[14] If $n = 8$ and $c = 1/8$, at $\mu = 0$, $\bar{p} = 0.25$ and at $\mu = 1/2$, $\bar{p} = 0.31$.

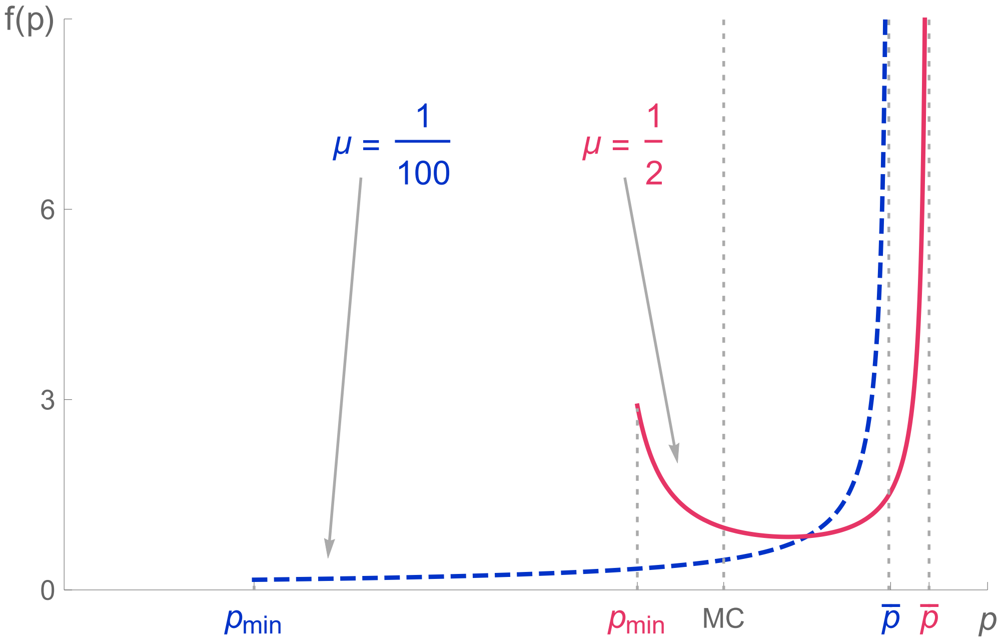

Figure 3: *Equilibrium pdf of prices if the fraction of shoppers is low* ($\mu = \frac{1}{100}$, *dashed) or high* ($\mu = \frac{1}{2}$, *solid);* $c = \frac{1}{8}$, $n = 8$.

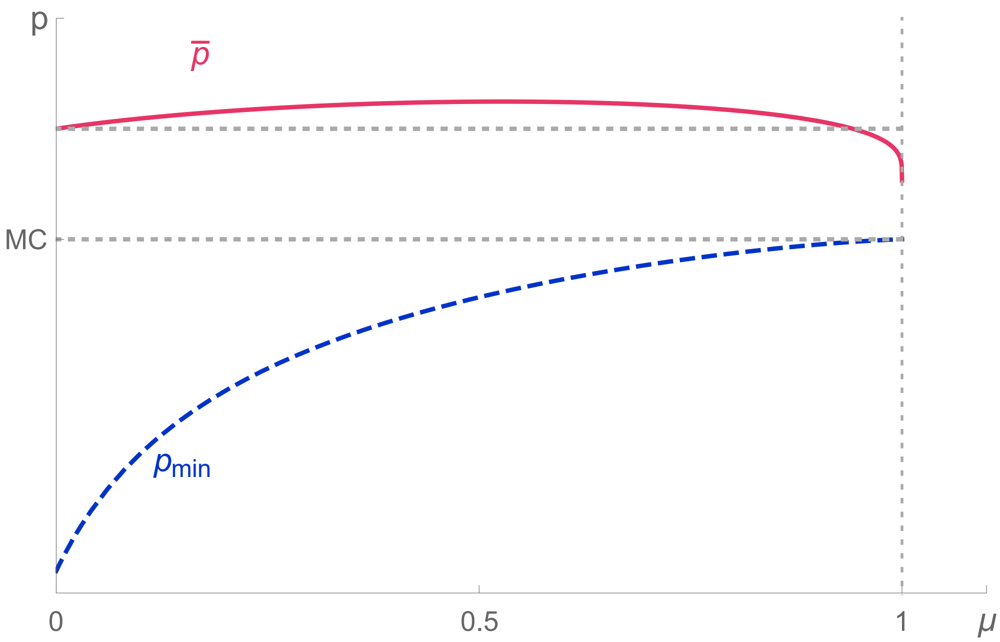

Figure 4: *The lowest price (dashed) and highest price (solid) as functions of the fraction of shoppers,* μ*;* $c = \frac{1}{8}$, $n = 8$.

	Stahl (1989)	This model
Single firm sets	p	$F(p)$
Presence of $p < MC$	No	Yes
Lower search cost: $c \downarrow$	$p_{min} \downarrow$ and $\mathbb{E}[p] \downarrow$	$p_{min} \downarrow$ and $\mathbb{E}[p] \downarrow$
More shoppers: $\mu \uparrow$	$p_{min} \downarrow$ and $\mathbb{E}[p] \downarrow$	$p_{min} \uparrow$ and $\mathbb{E}[p] \uparrow$ as $\mu \to 0$

Table 1: *Testable predictions of this model differ from those of Stahl (1989); $n \geq 4$ and $\mu \leq \bar{\mu}(n)$.*

4.3 Testable predictions

Table 1 summarises for $n \geq 4$ and $\mu \leq \bar{\mu}(n)$ how the testable predictions of my model differ from the models following Stahl (1989). The most notable prediction is that the best offers get worse (or, equivalently, the lowest prices increase) as the fraction of shoppers increases. This is in direct contrast to the results of other models that build on Stahl (1989).

My model, thus predicts that if we compare a market with many shoppers to one with few, the market with many shoppers can exhibit worse deals. For example, if buyers with access to a price-comparison website are interpreted as shoppers, then my model suggests that a market with such a website can have worse best deals than a comparable market without a price-comparison website. Also, if buyers with a lower opportunity cost of time are interpreted as shoppers, then my model suggests that average prices may be higher in low-income neighbourhoods than in high-income neighbourhoods.

5 Extensions

I discuss three extensions to the main model. The main results remain unchanged. I assume that there are no shoppers ($\mu = 0$) throughout this section for clarity.

5.1 Some buyers do not receive price signals

A fraction $\lambda > 0$ of the buyers are partially informed about the firms' prices as before. The rest of the buyers, fraction $1 - \lambda$, are uninformed buyers who do not receive price information. The uninformed buyers have the same search cost c as the partially informed buyers. I assume that the uninformed buyers visit each firm first with an equal probability. They can continue to search just like the partially informed buyers.

After visiting the first firm, the continuation problem of a partially informed and an uninformed buyer looks identical. Thus, they optimally use the same cutoff $\bar{p}$. A buyer's continuation value is, as before,

$$V = F(\bar{p})\mathbb{E}[1 - p|p \leq \bar{p}] + (1 - F(\bar{p}))V - c.$$

Firm i's expected profit is now

$$\pi = \lambda \mathbb{E}[D(p)|p \leq \bar{p}]\mathbb{E}[p|p \leq \bar{p}] + \frac{1-\lambda}{n}\mathbb{E}[p|p \leq \bar{p}],$$

where $D(p) = \Pi_{j \neq i} 1 - F_j(p)$ is the probability that the signal p from firm i is the lowest received by a buyer who partially directs search. Offering prices above $\bar{p}$ is strictly dominated for the firms. The reason is twofold. First, no buyer buys if the price that she is offered exceeds $\bar{p}$ so shifting the mass from prices above $\bar{p}$ to $\bar{p}$ does not affect profits per buyer. Second, a fraction λ of buyers direct their search based on the offer distribution and shifting the mass from prices above $\bar{p}$ to $\bar{p}$ increases demand from them. Thus, a firm's expected profit simplifies to

$$\pi = \lambda \mathbb{E}[D(p)]\mathbb{E}[p] + \frac{1-\lambda}{n}\mathbb{E}[p],$$

and a buyer's problem to

$$\bar{p} = \mathbb{E}[p] + c.$$

The equilibrium is summarised in

Proposition 5. *Suppose that a fraction $1 - \lambda < 1$ of the buyers are uninformed. A firm's equilibrium price distribution is $F(p) = 1 - \left(\frac{p_{max}-p}{p_{max}-p_{min}}\right)^{\frac{1}{n-1}}$ with support $[p_{min}, p_{max}]$ where $p_{min} = -\left(n - \frac{1+\lambda}{\lambda}\right)c$ and $p_{max} = \bar{p} = \frac{1+\lambda}{\lambda}c$. The equilibrium exists if $c \leq \frac{\lambda}{1+\lambda}$.*

Firms do not offer deals in equilibrium if partially informed buyers are rare. If partially informed buyers are rare, then firms compete less fiercely for them and instead focus on making higher profits on the uninformed buyers.

Corollary 2. *As the fraction of partially informed buyers, λ, increases,*

(i) the dispersion of prices stays the same, but the support shifts down: the lower and upper bound of the distribution shift down by the same amount.

(ii) buyers' welfare increases.

The comparative statics' results with respect to the fraction of partially informed buyers are intuitive because these buyers are price-sensitive: their demand is determined by the firms' prices. Thus, if their amount increases, competition increases, leading to a lower expected price and better best offers.

5.2 Supershoppers

Let a fraction $1 - \sigma$ of the buyers be nonshoppers with positive search cost c who partially direct search, as before, and a fraction σ be supershoppers with zero search costs. Supershoppers get as many draws as they like from each firm's price distribution. As a result, they always pay the minimum price available in the market. Without loss of generality, assume that supershoppers start searching across firms in a random order.

A firm's profit in this version of the model is

$$\pi = (1 - \sigma)\mathbb{E}[D(p)|p \leq \bar{p}]\mathbb{E}[p|p \leq \bar{p}] + \sigma p_{min},$$

if the lowest price in the firm's support p_{min} is below the market's minimum,

$$\pi = (1 - \sigma)\mathbb{E}[D(p)|p \leq \bar{p}]\mathbb{E}[p|p \leq \bar{p}] + \frac{\sigma}{n} p_{min},$$

if p_{min} is equal to the market's minimum, and otherwise is

$$\pi = (1 - \sigma)\mathbb{E}[D(p)|p \leq \bar{p}]\mathbb{E}[p|p \leq \bar{p}].$$

In a symmetric equilibrium, all firms post the same price distribution. The next Lemma shows that the lowest price offered in equilibrium must be equal to the marginal cost.

Lemma 1. *The lowest price that firms post in equilibrium is $p_{min} = 0$.*

No firm wants to post the lowest price in the market if it falls below the marginal cost because then a deviation to a slightly higher minimum price would be profitable. All supershoppers (who generate losses) would buy from a different firm after the deviation. Conversely, if the lowest price in the market is above zero, a deviation to a slightly lower minimum price would be profitable. All supershoppers (who now generate profits) would buy from the deviating firm. Thus, in a symmetric equilibrium firms post $p_{min} = 0$ and make zero profits on supershoppers. Firms essentially compete only over nonshoppers in equilibrium.

Given Lemma 1 and the fact that no firm offers prices above $\bar{p}$, a firm's expected equilibrium profit is

$$\pi = (1 - \sigma)\mathbb{E}[D(p)]\mathbb{E}[p].$$

The equilibrium is characterised in

Proposition 6. *Suppose that a fraction $\sigma \in (0,1)$ of the buyers are supershoppers. An equilibrium in price distributions exists for all $c \leq 1/2$. In equilibrium $p_{min} = 0$ and $\bar{p} = 2c$. The equilibrium characterisation is the same for all $\sigma < 1$.*

The equilibrium price distribution can be derived in a similar way as in Spiegler (2006).[15] Supershoppers have a discrete effect on the equilibrium distribution of prices: the lowest price is $p_{min} = 0$. The equilibrium distribution now has a mass point on the lowest price if the search cost is large. However, an increase in the amount of supershoppers has no further effect on the equilibrium. In particular, the fraction of supershoppers does not affect the expected price, thus, the welfare of nonshoppers.

5.3 Horizontally differentiated products

Suppose that a buyer has a match with a firm's product with probability $\beta \in (0,1]$. If she has a match with the product, she gets utility $v = 1$ from purchasing the product and utility zero otherwise. The buyer observes if she has a match with a product upon visiting the firm. This modification has no effect on the equilibrium except that a buyer's effective search cost becomes c/β.

A firm offers positive value to a fraction β of the buyers so its expected profit can be written as

$$\pi = \beta \mathbb{E}[D(p)|p \leq \bar{p}]\mathbb{E}[p|p \leq \bar{p}].$$

A buyer accepts a firm's offer only if she has a match with the firm's product and the price is at most $\bar{p}$. Otherwise, she continues to search. Her continuation value is

$$V = \beta F(\bar{p})\mathbb{E}[1 - p|p \leq \bar{p}] + [1 - \beta + \beta(1 - F(\bar{p}))]V - c,$$

and her net utility at the optimal cutoff, $1 - \bar{p}$, is equal to the continuation value.

By the same argument as in the main model, a firm never offers prices above the buyer's cutoff $\bar{p}$. A firm's problem simplifies to

$$\pi = \beta \mathbb{E}[D(p)]\mathbb{E}[p],$$

and a buyer's problem to

$$\bar{p} = \mathbb{E}[p] + \frac{c}{\beta}.$$

Thus, the problem looks exactly like in the main model except that the effective search cost of a buyer is c/β. The equilibrium is described in

[15]The full characterisation is in the Appendix.

Corollary 3. *Suppose that a buyer has a match with a firm's product with probability* $\beta \in (0,1]$. *A firm's equilibrium price distribution is* $F(p) = 1 - \left(\frac{p_{max}-p}{p_{max}-p_{min}}\right)^{\frac{1}{n-1}}$ *with support* $[p_{min}, p_{max}]$ *where* $p_{min} = -(n-2)\frac{c}{\beta}$ *and* $p_{max} = \bar{p} = 2\frac{c}{\beta}$. *The equilibrium exists if* $c \leq \frac{\beta}{2}$.

The comparative statics with respect to the probability of a match β are opposite to those with respect to the search cost c, summarised in Corollary 1.

A Appendix

Here are the proofs omitted from the paper. In the proofs, I deal with the net utilities that firms offer to buyers, $u := 1-p$, instead of prices to simplify the derivations. Let the distribution of net utilities that firm i offers be denoted by $G_i(u)$. The exact results in the main body of the paper follow when we substitute back $u = 1-p$, $u_{max} = 1 - p_{min}$, $u_{min} = 1 - p_{max}$, and $G(u) = 1 - F(p)$.

A.1 Main model

Proof. (Proposition 1.) Suppose that all firms but i post a single net utility $u^* \geq 0$ in equilibrium. In order for firms to make positive profits, $u^* \leq 1$ must hold in such an equilibrium. If all firms post this utility, then a buyer's cutoff is $\bar{u} = \max\{u^* - c, 0\}$. A firm's expected equilibrium profit is $\pi^* = \frac{1-u^*}{n}$.

A profitable deviation for a firm is to set a distribution G' with $P_{G'}(u = u^* + \varepsilon) = \frac{1}{2}$, $\varepsilon > 0$ small, and $P_{G'}(u = u^* - c) = \frac{1}{2}$. The deviating firm would attract half of the buyers, those who get signal $u^* + \varepsilon$ from it. Half of them pay $1 - (u^* + \varepsilon)$ and half $1 - (u^* - c)$ to the firm. The profit from deviating is thus $\pi' = \frac{1}{2}\frac{2-2u^*+c-\varepsilon}{2}$. The deviation is profitable if $u^* \leq 1$ and $\varepsilon < c$. □

Proof. (Proposition 2.) The proof is a special case of the proof of Proposition 5, on page 24, where $\lambda = 1$. □

A.2 Adding shoppers

Proof. (Proposition 3.) Let $H(u) := \Pi_{j \neq i} G_j(u)$ denote the probability that firm i's offer u is the best net utility draw that a buyer sees among n draws, where one draw comes from each firm's distribution. I can no longer use the method developed in Spiegler (2006) because, as I show below, $H(u)$ is not linear in u. In Step 1, I show that a single-u equilibrium never exists. In Step 2, I derive the properties that $G_i(u)$ must have in any equilibrium and in Step 3, the properties of a symmetric-equilibrium $G(u)$. In Step 4 I argue that such an equilibrium exists.

Step 1: An equilibrium in degenerate distributions $G(u)$ does not exist.

Suppose that all firms set $u = \hat{u}$ in equilibrium with probability one. The proposed equilibrium profits are $\hat{\pi} = \frac{1-\hat{u}}{n}$, where $\hat{u} \leq 1$ must hold for weakly positive profits. If all firms set $u = \hat{u}$ in equilibrium, then a buyer's expected value is $\mathbb{E}[u] - c = \hat{u} - c$ so she accepts any first offer.

I show that firm i has an incentive to deviate to a dispersed distribution G_i' such that $P'(u = \hat{u} + \varepsilon) = \frac{1}{2}$ and $P'(u = \hat{u} - c) = \frac{1}{2}$ for $\varepsilon > 0$ small.

Firm i's profit from this deviation is

$$\pi' = \frac{\mu}{2}(1 - \hat{u} - \varepsilon) + \frac{1-\mu}{2}\left[\frac{1}{2}(1 - \hat{u} - \varepsilon) + \frac{1}{2}(1 - \hat{u} + c)\right].$$

Firm i sells to half of the shoppers (those, who get the offer $u = \hat{u} + \varepsilon$ from it) and gets revenue $1 - \hat{u} - \varepsilon$ from each of them. It also attracts half of the nonshoppers (those, who get the signal $u = \hat{u} + \varepsilon$ from it), but gets revenue $1 - \hat{u} - \varepsilon$ from half of them and $1 - \hat{u} + c$ from the rest of them. This deviation is profitable if $\pi' > \hat{\pi}$, or,

$$2n(1 - \hat{u}) + n[(1-\mu)c - (1+\mu)\varepsilon] > 4(1 - \hat{u})$$

which holds for all $n \geq 2$ as long as $\varepsilon < \frac{1-\mu}{1+\mu}c$.

Step 2: Let T_i denote the support of G_i, $u_{min} := \inf(T_i)$ and $u_{max} := \sup(T_i)$. I established in the main part of the paper that $p_{max} \leq \bar{p}$, i.e., that $u_{min} \geq \bar{u}$. I need to derive the rest of $G(u)$. Recall that a firm's expected profit is

$$\pi = (1-\mu)\mathbb{E}[H(u)]\mathbb{E}[1-u] + \mu\mathbb{E}[H(u)(1-u)],$$

and a buyer's cutoff solves is

$$\bar{u} = \mathbb{E}[u] - c.$$

Step 2a: I derive $H_i(u)$ by using the property that if $G_i(u)$ is optimal, it must be unprofitable to reallocate mass within its support T_i.

First suppose $G_i(u)$ assigns positive mass to utilities $u \in T_i$, u_{min} and u_{max}. I derive a necessary condition that has to be satisfied by $H_i(u)$ in equilibrium. In particular, I find a particular fraction $\alpha(u)$ for every u such that firm i does not want to reallocate a small amount of mass $\varepsilon \neq 0$ from u and put mass $\alpha(u)\varepsilon$ on u_{min} and mass $(1-\alpha(u))\varepsilon$ on u_{max} (where the movement of mass is from u if $\varepsilon > 0$ and to u if $\varepsilon < 0$).

Removing a small probability mass $\varepsilon \neq 0$ from u and putting mass $\alpha\varepsilon$ on u_{min} and mass $(1-\alpha)\varepsilon$ on u_{max} changes firm i's profit by

$$\Delta\pi = (1-\mu)\{\Delta\mathbb{E}[H(u)]\mathbb{E}[1-u] + \mathbb{E}[H(u)]\Delta\mathbb{E}[1-u]\} + \mu\Delta\mathbb{E}[H(u)(1-u)]$$

$$= \varepsilon \,\langle (1-\mu)\{[\alpha H(u_{min}) + (1-\alpha)H(u_{max}) - H(u)](1-m)$$

$$+ d[\alpha(1-\bar{u}) + (1-\alpha)(1-u_{max}) - (1-u)]\}$$

$$+ \mu[\alpha H(u_{min})(1-u_{min}) + (1-\alpha)H(u_{max})(1-u_{max}) - H(u)(1-u)]\rangle\,,$$

where $m := \mathbb{E}[u]$ and $d := \mathbb{E}[H(u)]$ for brevity. If $G(u)$ is optimal, $\Delta\pi \leq 0$ has to hold for all $\varepsilon \neq 0$. Since $\Delta\pi$ is proportional to ε, $\Delta\pi \leq 0$ holds for both $\varepsilon < 0$ and $\varepsilon > 0$ only if the term in the triangular brackets, $\frac{\Delta\pi}{\varepsilon}$, is equal to zero.

Noting that $H(u_{min}) = 0$ and $H(u_{max}) = 1$, we can simplify $\frac{\Delta\pi}{\varepsilon} = 0$ to

$$0 = (1-\mu)\{[(1-\alpha) - H(u)](1-m) + d[u - \alpha u_{min} - (1-\alpha)u_{max}]\}$$

$$+ \mu[(1-\alpha)(1-u_{max}) - H(u)(1-u)], \tag{5}$$

which can be rearranged to

$$H(u) = 1-\alpha+\frac{(1-\mu)d[u-u_{min}-(1-\alpha)(u_{max}-u_{min})] - \mu(1-\alpha)(u_{max}-u)}{\mu(1-u)+(1-\mu)(1-m)}.$$

Then $H(u) = 1-\alpha(u)$ iff the numerator of the second term is equal to zero (I verify later that the denominator is positive for all u, i.e., that $u \leq 1 + \frac{(1-\mu)(1-m)}{\mu}$), which can be rearranged to give

$$1-\alpha(u) = \frac{(1-\mu)d(u-u_{min})}{(1-\mu)d(u_{max}-u_{min}) + \mu(u_{max}-u)}.$$

Thus, a necessary condition that $H(u)$ has to satisfy in equilibrium is that

$$H(u) = \frac{(1-\mu)d(u-u_{min})}{(1-\mu)d(u_{max}-u_{min}) + \mu(u_{max}-u)}, \tag{6}$$

where $d := \mathbb{E}[H(u)]$.

The generalisation to a smooth $G(u)$ follows when one considers moving mass from the close neighbourhood of any $u \in (u_{min}, u_{max})$ to the neighbourhood of u_{min} and u_{max}.

Step 2a': I derive u_{max} by using again the property that if $G_i(u)$ is optimal, it must be unprofitable to reallocate mass within its support T_i, and the necessary condition (6) on $H(u)$.

I derive a necessary condition that has to be satisfied by u_{max} in equilibrium. In particular, I show that if u_{max} satisfies the condition, then, for *any* $\alpha \in [0,1]$, firm i does not want to reallocate a small amount of mass $\varepsilon \neq 0$ from $u \in (u_{min}, u_{max})$, $u \in T_i$, and put mass $\alpha\varepsilon$ on u_{min} and mass $(1-\alpha)\varepsilon$ on u_{max} (where the movement of mass is from u if $\varepsilon > 0$ and to u if $\varepsilon < 0$). Plugging equation (6) into (5) gives

$$0 = [(1-\mu)(1-m) + \mu(1-u_{max}) - H(1-\mu)(u_{max}-u_{min})]\times$$

$$\{-d(1-\mu)(u-u_{min}) + (1-\alpha)[(1-\mu)d(u_{max}-u_{min}) + \mu(u_{max}-u)]\},$$

which holds iff either the term in the first squared brackets or the term in the curly brackets is equal to zero. The latter, however, depends on α. Thus, the equality holds for all α if the first term in the squared brackets is equal to zero. This can be rearranged to give

$$u_{max} = 1 + (1-\mu)\frac{1-m-H(1-u_{min})}{(1-\mu)d+\mu}, \tag{7}$$

where $d := \mathbb{E}[H(u)]$ and $m := \mathbb{E}[u]$. Note that the condition that I assumed when deriving $H(u)$, i.e., that $u \leq 1+\frac{(1-\mu)(1-m)}{\mu}$, is satisfied because $u_{max} < 1 + \frac{(1-\mu)(1-m)}{\mu}$ (since $m < 1$ must hold for weakly positive profits in equilibrium).

If $H_i(u)$ satisfies condition (6) and u_{max} condition (7), then moving mass from any $u_1 \in T_i$ to another $u_2 \in [u_{min}, u_{max}]$ is unprofitable. This is because I can choose $u = u_1$ and rewrite u_2 as $u_2 = \alpha u_{min} + (1-\alpha)u_{max}$ for some $\alpha \in [0,1]$ so the above analysis applies. Thus, if $H_i(u)$ satisfies (6) and u_{max} satisfies (7), then $G_i(u)$ is a best response by firm i.

Steps 2b to 3a are almost exactly the same as in the proof of Proposition 5, on page 24, (where $\lambda = 1$) so I skip them here to save space. The only difference is that if changes in $\mathbb{E}[u]$ (or $\mathbb{E}[H(u)]$) are infitessimal, then so are changes in $\mathbb{E}[H(u)u]$ so the arguments in Spiegler (2006) apply.

Step 3: Let G be the symmetric equilibrium strategy and T be the support of G. Let $u_{max} := \sup(T)$.

Step 3b: I derive $\bar{u}$ by using a clever trick from Janssen and Moraga González (2004) (see p. 1097).

First not that in asymmetric equilibrium, $\mathbb{E}[H(u)] = \frac{1}{n}$ and let $\gamma := n\frac{\mu}{1-\mu}$. Then rewrite equation (6) as

$$u = u_{max} - (u_{max} - u_{min})\frac{1-D}{1+\gamma D}, \tag{8}$$

where $D := H(u)$. Then I define a new variable $z := G(u)$ so that I can write

$$\mathbb{E}[u] = \int_{u_{min}}^{u_{max}} u \, \mathrm{d}G(u) = \int_0^1 u \, \mathrm{d}z. \tag{9}$$

From the buyer's optimisation problem, we know that $\mathbb{E}[u] = \bar{u} + c$ and in a symmetric equilibrium, $D = z^{n-1}$. Thus, I can rewrite (9) using (8) as

$$\bar{u} + c = \int_0^1 \left[u_{max} - (u_{max} - u_{min})\frac{1-z^{n-1}}{1+\gamma z^{n-1}} \right] \mathrm{d}z,$$

and because $u_{min} = \bar{u}$,

$$u_{max} - u_{min} = c\left[1 - \int_0^1 \frac{1-z^{n-1}}{1+\gamma z^{n-1}} \, \mathrm{d}z\right]^{-1}, \tag{10}$$

where the right-hand side of (10) depends only on exogenous variables. It is straightforward to see that $u_{max} - u_{min}$ decreases in γ (equivalently, μ). Using equations (7) and (10), I can solve for $\bar{u}$:

$$\bar{u} = 1 - \frac{c}{\gamma + n}[K(1+\gamma) + n], \tag{11}$$

where $K := \left[1 - \int_0^1 \frac{1-z^{n-1}}{1+\gamma z^{n-1}} \, \mathrm{d}z\right]^{-1}$. Note that K decreases in γ and $K > 1$ for all γ. The equilibrium exists if the nonshoppers' value from searching is positive, i.e., if $\bar{u} > 0$, or $c < \frac{\gamma+n}{K(1+\gamma)+n}$.

I use the explicit form for $\bar{u}$ in (7) to get an explicit form for u_{max}:

$$u_{max} = 1 + (\gamma + n)^{-1}c[(n-1)K - n].$$

Firms offer deals (or $u_{max} > 1$) if $(n-1)K > n$, which is the hardest to satisfy for the smallest K, i.e., the largest $\gamma = n\frac{\mu}{1-\mu}$. As $\gamma \to +\infty$, $\lim_{\gamma \to n} K = 1$ so there exists $\bar{\gamma}$ such that for $\gamma > \bar{\gamma}(n)$, $u_{max} < 1$. That is, firms stop offering deals if there are many shoppers on the market. Note that $\bar{\gamma}(n)$ increases in n: $(n-1)K(\gamma = \bar{\gamma}) = n$ can be rearranged as $1 = n\int_0^1 \frac{1-z^{n-1}}{1+\gamma z^{n-1}} \, \mathrm{d}z$, where the right-hand side increases in n and decreases in γ. Let $\bar{\mu}(n)$ satisfy $\bar{\gamma}(n) = \frac{n\bar{\mu}(n)}{1-\bar{\mu}(n)}$. Figure 5 illustrates the critical fraction

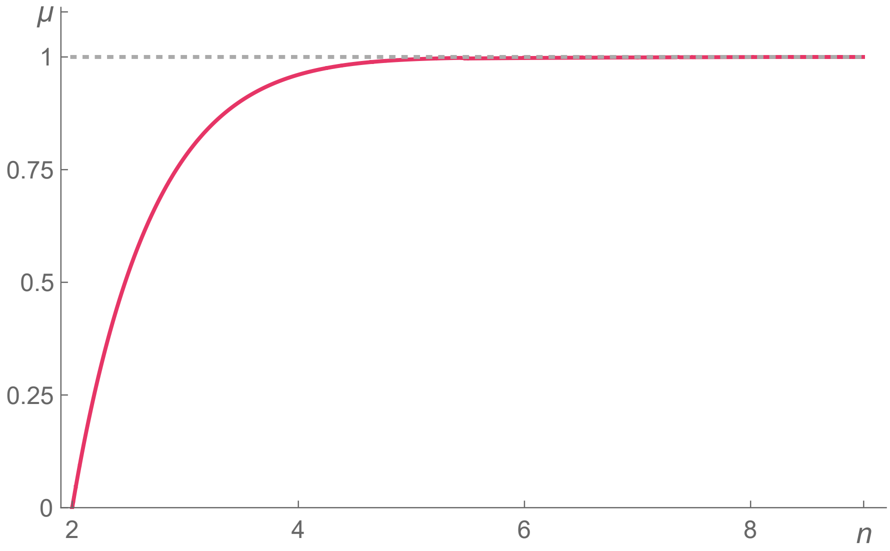

Figure 5: *Critical fraction of shoppers $\bar{\mu}(n)$.*

of shoppers $\bar{\mu}(n)$. For example, $\bar{\mu}(3) \approx 0.78$ and $\bar{\mu}(4) \approx 0.96$.

Step 4: An equilibrium as described in Step 3 exists.

Suppose all firms but i use $G(u)$ as described in Step 3. Then we know that $H_i(u)$ satisfies (6) so firm i cannot improve its profits by using a different distribution than $G(u)$. Thus, all firms using $G(u)$ is an equilibrium. □

Proof. (Proposition 4.) The result with respect to the range of the distribution, $u_{max} - \bar{u}$, follows from inspecting equation (10). The result with respect to nonshoppers' welfare follows from differentiating equation (11) with respect to $\gamma = n\frac{\mu}{1-\mu}$. The result is

$$\frac{\partial \bar{u}}{\partial \gamma} = \frac{c}{\gamma + n}\left[n - (n-1)K - (\gamma + n)(1+\gamma)K'\right].$$

As $\mu \to 0$, $\gamma \to 0$ with $\lim_{\gamma \to 0} K = n$ and $\lim_{\gamma \to 0} K' = -\frac{n(n-1)}{2n-1}$ so that

$$\lim_{\gamma \to 0} \frac{\partial \bar{u}}{\partial \gamma} = \frac{c}{2n-1}\left[(4-n)n - 2\right],$$

which is negative for all $n \geq 4$.

The result with respect to u_{max} follows from differentiating the equation for u_{max} with respect to $\gamma = n\frac{\mu}{1-\mu}$:

$$\frac{\partial u_{max}}{\partial \gamma} = \frac{c}{\gamma + n}\left[n - (n-1)K - \gamma K'\right],$$

where $K' < 0$ so that a sufficient condition for $\frac{\partial u_{max}}{\partial \gamma} < 0$ is that $u_{max} \geq 1$, i.e., if $\gamma \leq \bar{\gamma}(n)$ (where from above, we know $\gamma < \bar{\gamma}(n)$ corresponds to approximately $\mu \leq 0.78$ for $n = 3$ and to $\mu \leq 0.96$ for $n = 4$). $\square$

A.3 Some buyers do not receive price signals

Proof. (Proposition 5.) A fraction $\lambda > 0$ of the buyers are as before and the rest, fraction $1 - \lambda$, are uninformed buyers who do not receive price information.

I solve for the equilibrium using the method developed in Spiegler (2006). In Step 1, I show that a single-u equilibrium never exists. In Step 2, I derive the properties that $G_i(u)$ must have in any equilibrium and in Step 3, the properties of a symmetric-equilibrium $G(u)$. In Step 4 I argue that such an equilibrium exists.

Let $H(u) := \Pi_{j \neq i} G_j(u)$ denote the probability that firm i's offer u is the best net utility draw that a buyer sees among n draws, where one draw comes from each firm's distribution.

Step 1: An equilibrium in degenerate distributions $G(u)$ does not exist.

Suppose that all firms set $u = \hat{u}$ in equilibrium with probability one. The proposed equilibrium profits are $\hat{\pi} = \frac{1-\hat{u}}{n}$. For weakly positive profits in equilibrium, it must be that $\hat{u} \leq 1$. If all firms set $u = \hat{u}$ in equilibrium, then a buyer's expected value is $\mathbb{E}[u] - c = \hat{u} - c$ so she accepts any first offer. I show that firm i has an incentive to deviate to a dispersed distribution G_i' such that $P'(u = \hat{u} + \varepsilon) = \frac{1}{2}$ and $P'(u = \hat{u} - c) = \frac{1}{2}$ for $\varepsilon > 0$ small.

Firm i's profit from this deviation is

$$\pi' = \left[\frac{\lambda}{2} + \frac{1-\lambda}{n}\right] \left[\frac{1}{2}(1 - \hat{u} - \varepsilon) + \frac{1}{2}(1 - \hat{u} + c)\right]$$

because it attracts half of the buyers who partially direct search (those, who get the signal $u = \hat{u} + \varepsilon$ from it) and its fair share of uninformed buyers. This deviation is profitable if $\pi' > \hat{\pi}$, or,

$$[2 + \lambda(n-2)] \left[2(1 - \hat{u}) + c - \varepsilon\right] > 4(1 - \hat{u}),$$

which holds for all $n \geq 2$ as long as $\varepsilon < c$.

Step 2: Let T_i denote the support of G_i and $u_{min} := \inf(T_i)$. I established in the main part of the paper that $p_{max} \leq \bar{p}$, i.e., that $u_{min} \geq \bar{u}$. I need to derive the rest of $G(u)$. Recall that a firm's expected profit is

$$\pi = \left(\lambda \mathbb{E}[H(u)] + \frac{1-\lambda}{n}\right) \mathbb{E}[1 - u],$$

and a buyer's problem is

$$\bar{u} = \mathbb{E}[u] - c.$$

Step 2a: All the points $\{(u, H_i(u)) | u \in T_i\}$ lie on a straight line, except possibly for a zero-measure subset.

The proof follows directly from the proof of Lemma 1 in Spiegler (2006).

A Corollary to Step 2a, which I use below, is

Corollary 4. *Given G_{-i}, firm i is indifferent between $G_i(u)$ and $G_i^*(u)$ such that $\mathbb{E}_G[u] = \mathbb{E}_{G^*}[u]$ and $T_i^* \subseteq T_i$.*

Proof. Because $H_i(u)$ is linear, firm i's expected profit is the same under G_i and G_i^* if $\mathbb{E}_G[u] = \mathbb{E}_{G^*}[u]$. □

Step 2b: For any $\lambda > 0$, in any Nash equilibrium $G(u)$ is continuous on $(\bar{u}, \infty)$.

The proof follows directly from the proof of Lemma 2 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 2c: In any Nash equilibrium, $u_{min} = \bar{u}$.

I only need to show that $u_{min} > \bar{u}$ cannot hold in equilibrium. The proof follows directly from the proof of Lemma 3 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 3: Let G be the symmetric equilibrium strategy and T be the support of G. Let $u_{max} := \sup(T)$.

Step 3a: For any $\lambda > 0$, $T = [u_{min}, u_{max}]$ and G is continuous over $[u_{min}, u_{max}]$.

The proof follows directly from Steps 1 and 2 in the proof of Proposition 1 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 3b: If $\frac{1+\lambda}{\lambda} c < 1$, then $\bar{u} = 1 - \frac{1+\lambda}{\lambda} c$, $u_{max} = 1 + \left(n - \frac{1+\lambda}{\lambda}\right) c$, and $G(u) = \left(\frac{u-\bar{u}}{u_{max}-\bar{u}}\right)^{\frac{1}{n-1}}$.

The proof follows from Step 3 in the proof of Proposition 1 in Spiegler (2006). I use Corollary 4 to derive the equilibrium. Let G^* be such that $T^* = \{\bar{u}, u_{max}\}$ with $P^*(u = \bar{u}) = \alpha$ and $P^*(u = u_{max}) = 1 - \alpha$ with $\mathbb{E}_G[u] = \mathbb{E}^*[u]$, where $\mathbb{E}^* := \mathbb{E}_{G^*}$. Then the expected profit under G^*, π^*, is

$$\pi^* = \left(\lambda \mathbb{E}^*[H(u)] + \frac{1-\lambda}{n}\right) \mathbb{E}^*[1-u]$$

$$= \left[\lambda(1-\alpha) + \frac{1-\lambda}{n}\right][1 - u_{max} + \alpha(u_{max} - \bar{u})],$$

where the latter equality follows from writing out the expectations explicitly and from the fact that $H(\bar{u}) = 0$ and $H(u_{max}) = 1$. The first-order condition with respect to α yields

$$2\lambda(u_{max} - \bar{u})(1-\alpha^*) = \lambda(1-\bar{u}) - (u_{max} - \bar{u})\frac{1-\lambda}{n}. \tag{12}$$

Since $\mathbb{E}^*[H(u)] = 1 - \alpha^*$ and, in a symmetric equilibrium, $\mathbb{E}[H(u)] = \frac{1}{n}$, equation (12) can be rearranged to give

$$u_{max} = \bar{u} + \frac{\lambda n(1-\bar{u})}{1+\lambda}. \tag{13}$$

From the buyer's optimisation problem, we know that $\mathbb{E}[u] = \bar{u} + c$. But $\mathbb{E}_G[u] = \mathbb{E}^*[u] = \bar{u} + (1-\alpha^*)(u_{max} - \bar{u})$ so that

$$\bar{u} = 1 - \frac{1+\lambda}{\lambda}c.$$

Plugging this back to (13) yields $u_{max} = 1 + \left(n - \frac{1+\lambda}{\lambda}\right)c$. Note that $u_{max} > 1$ if $n > \frac{1+\lambda}{\lambda}$ (which collapses to $n > 2$ if $\lambda = 1$).

Since $H(u)$ is linear and continuous, $H(u) = \frac{u_{max} - u}{u_{max} - \bar{u}}$ and in a symmetric equilibrium, $G(u) = (H(u))^{\frac{1}{n-1}}$.

Step 3c: If $\frac{1+\lambda}{\lambda}c \geq 1$, then $\bar{u} = 0$ and buyers choose not to search.

If buyers' optimal cutoff is zero, a distribution G^* as described in Step 3a still exists. Thus, u_{max} can be derived in the same way, which gives $u_{max} = \frac{n\lambda}{1+\lambda}$. Note that $\mathbb{E}^*[u] = (1-\alpha^*)u_{max} = \frac{\lambda}{1+\lambda}$ so that $\mathbb{E}[u] - c \leq 0$ (and $\bar{u} = 0$) if $\frac{1+\lambda}{\lambda}c \geq 1$.

Step 4: An equilibrium as described in Step 3a-b exists.

Suppose all firms but i use $G(u)$ as described in Step 3a-b. Then any cdf $G_i(u)$ with expectation equal to $\mathbb{E}_G[u]$, including $G(u)$, is a best reply for firm i because $H_i(u)$ is linear. Thus, all firms using $G(u)$ is an equilibrium.

□

A.4 Supershoppers

Proof. (Lemma 1.) The proof is by contradiction. Suppose first that all firms set $u_{max} > 1$ so that firms are making losses on their sales to supershoppers. Consider a deviation by firm i to $u'_{max} = u_{max} - \varepsilon$, for some small $\varepsilon > 0$. The firm loses a negligible fraction of searchers from this deviation (while increasing the expected price from these buyers). But the firm saves the loss associated with supershoppers because all of them will now buy at some other firm. Thus, the deviation is profitable and in a symmetric equilibrium, $u_{max} \leq 1$.

Suppose instead that all firms set $u_{max} < 1$ in equilibrium. Consider a deviation by firm i to $u'_{max} = u_{max} + \varepsilon$, for some small $\varepsilon > 0$ such that $u'_{max} < 1$. The firm gains a negligible fraction of searchers from this deviation, but gets a slightly lower expected price from them. But the firm gains the custom of all supershoppers and they all yield a profit. Thus, the deviation is profitable. Altogether, in a symmetric equilibrium, $u_{max} = 1$. □

For completeness, I provide a full characterisation of the equilibrium price distribution here.

Proposition 7. *Suppose that a fraction $\sigma \in (0, 1)$ of the buyers are supershoppers.*

(i) If the search cost is small, $c \in (0, \frac{1}{n}]$, the equilibrium price distribution is

$$F(p) = 1 - \left(\frac{p_{max} - p}{p_{max} - p_{min}}\right)^{\frac{1}{n-1}},$$

with support $[p_{min}, p_{max}]$ where $p_{min} = 0$ and $p_{max} = \bar{p} = 2c$.

(ii) If the search cost is large, $c \in (\frac{1}{n}, \frac{1}{2}]$, the equilibrium price distribution is

$$F(p) = \begin{cases} 1 - A & \text{for } p \in [p_{min}, p_{med}) \\ 1 - \left(\frac{p_{max} - p}{p_{max} - p_{min}}\right)^{\frac{1}{n-1}} & \text{for } p \in [p_{med}, p_{max}], \end{cases}$$

with support $\{p_{min}\} \cup [p_{med}, p_{max}]$ where $p_{min} = 0$, $p_{max} = \bar{p} = 2c$, $p_{med} = c(2 - nA^{n-1})$, and A solves $cA^{n-1} - c + 1 - A = 0$.

The equilibrium characterisation is the same for all $\sigma \in (0, 1)$.

Proof. (Propositions 6 and 7.) Let the size of the mass point be denoted $1 - A$ and the support of the net utility distribution be denoted $T = [\bar{u}, 1 - b] \cup \{1\}$ The proof closely follows Spiegler (2006) (see Proof of Proposition 7). The differences in my proof arise because of the search cost. The expected net utility is derived as in Spiegler (2006), but is here $\mathbb{E}[u] = 1 - c$ so that $\bar{u} = 1 - 2c$ and $H(u) = \frac{2}{n}\frac{u - \bar{u}}{1 - \bar{u}}$.

The equation that defines b becomes

$$b = c\left(2 - nA^{n-1}\right).$$

The equation that defines the end of the size of the mass point becomes

$$cA^n - A + 1 - c = 0. \tag{14}$$

I now show that equation (14) only has a solution $A \in (0,1)$ if $c > \frac{1}{n}$. Note that I know that equation (14) has at most two positive roots because of the Descartes' rule of signs. Also, I know that $A = 1$ is a root for all c and n. Since the equation cannot have a single positive root, we know it has exactly two positive roots. Thus, equation (14) has at most one suitable root $A \in (0,1)$.

To see that a suitable root exists only if c is large enough, define variable $B := A - 1$ and rewrite (14) as

$$c(B+1)^n - B - c = 0. \tag{15}$$

If equation (15) has no positive roots, then equation (14) has a root $A \in (0,1)$. I can expand (15) as

$$c\beta_1 B^n + c\beta_2 B^{n-1} + ... + c\beta_n B + c\beta_{n+1} - B - c = 0,$$

where $\beta_i \geq 1$ is the ith entry in the nth row of the Pascal's triangle. Since $\beta_{n+1} = 1$, in order to apply the Descartes' rule of signs again, I need to divide the expanded equation by B. I get

$$c\beta_1 B^{n-1} + c\beta_2 B^{n-2} + ... + c\beta_{n-1} B + (c\beta_n - 1) = 0,$$

which has no positive roots if $c\beta_n - 1 > 0$. But the nth element in the nth row of the Pascal's triangle is n so equation (15) has no positive roots, or, equivalently, (14) has one root $A \in (0,1)$ if $c > \frac{1}{n}$. □

Perhaps counterintuitively, the mass point on the lowest price exists for larger search costs, but not for smaller ones. The reason is that it is competition between the firms that induces firms to place a lot of mass on the lowest possible price, not the nonshoppers' ability to shop. To see that, note that also in the absence of supershoppers, the lowest price increases as the search cost decreases.

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