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Advance Booking Discount Programs Under Retail Competition

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We model a situation in which two retailers consider launching an "Advance Booking Discount" (ABD) program. In this program, customers are enticed to precommit their orders at a discount price prior to the regular selling season. However, these precommitted orders are filled during the selling season. While the ABD program enables the retailers to lock in a portion of the customer demand and use this demand information to develop more accurate forecasts and supply plans, the ABD price reduces profit margin. We analyze the four possible scenarios wherein each of the two firms offer an ABD program or not, and establish conditions under which the unique equilibrium calls for launching the ABD program at both retailers.

Key words: retailing; competition; pricing; inventory management
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1. Introduction

Consider an Advance Booking Discount (ABD) program in which the retailer offers a product at a price discount prior to the selling season. If customers accept this offer, then they "reserve" the product to be picked up (or delivered) during the selling season by prepaying the entire discounted price prior to the selling season. While no order cancellation or refund is permitted, the retailer guarantees product availability for precommitted orders. If customers decline the ABD offer, then they can purchase the product at the regular price during the selling season, though the retailer does not guarantee product availability. Retailers typically implement such ABD programs when selling perishable products consumed during a well-defined and concentrated selling season (such as pumpkin pies or fresh turkeys during Thanksgiving, moon cakes during the Chinese Mid-Autumn Festival, or Christmas trees during Christmas), or certain kinds of new durable products with a short-selling season and high demand uncertainty (such as new music CDs, video games, and so forth). Examples of such retailers include Maxim's Bakery in Hong Kong, Amazon.com, Movies Unlimited, Toys R Us, and Electronics Boutique.

When selling a physical product, the ABD program offers three major benefits. First, this program extends the selling season without the need for immediate delivery. This enables the retailer to entice more customers to buy the product over a longer period of time without being constrained by the production capacity. Second, the ABD program offers an opportunity for the retailer to utilize the precommitted orders received during the advance selling period to generate a better demand forecast prior to the start of the selling season. Such improved forecasts allow a more accurate order to be placed at the start of the selling season, which, in turn, reduces the overstock and understock costs.[1] Third, the ABD program allows the retailer to improve cash flows because the retailer receives the payments in advance during the advance selling period. Tang et al. (2004) presents a single-firm model that quantifies the benefits of the ABD program in addition to characterizing the optimal discount price. Because an ABD program can be considered to be a type of discount promotion, the reader is referred to Tang et al. (2004) for a review of the marketing and operations management literature that deals with discount promotion.

To the best of our knowledge, all of the existing research that examines ABD programs are single-firm models or models in which only one firm in an industry can adopt an ABD program. These models solve for the optimal (monopolist) discount price and analyze the benefits of the ABD program. In this paper,

[1] In this paper, we consider the case in which the retailer can place exactly one order at the beginning of the selling season. This situation occurs when the replenishment lead time is longer than the selling season. However, when the replenishment lead time is sufficiently short, the retailer can use the sales data of the early part of the selling season to improve the forecast even further and place an additional order at the middle of the selling season. This specific scenario has been examined by Fisher et al. (2001) and tested at a catalog retailer.

we extend these models to incorporate the competitive nature of retailing by developing a duopoly model to analyze ABD programs under competition. The two retailers in our model sell the same product within a short and concentrated sales season. They each may or may not launch an ABD program resulting in four separate scenarios. We solve for the equilibrium of the competitive metagame formed by the four scenarios, establish when the equilibrium is unique, and analyze how it is affected by a fixed cost of implementing an ABD program. Finally, we provide a sensitivity analysis of the equilibrium ABD discount coefficients. In the next section, we present the basic modeling framework of the ABD program under retail competition.

2. The Model

Consider a situation in which two retailers A and B sell the same (or a similar) product during a short selling season. The unit cost, selling price, and salvage value of this product are c, p, and s, respectively. We consider the case in which the consumer market consists of two segments: (1) one segment intends to buy from retailer A and (2) the other segment intends to buy from retailer B. We assume that each customer buys no more than one unit of the product. The joint distribution of the anticipated demands for retailers A and B, denoted by D_A and D_B, is assumed to be bivariate normal with means μ_A and μ_B, standard deviations σ_A and σ_B, and correlation coefficient $\rho \in (-1, 1)$. To simplify the exposition, we assume that the anticipated demands D_A and D_B have the same coefficient of variation θ, where $\theta = \sigma_A/\mu_A = \sigma_B/\mu_B$. Let μ be the expected total market demand, where $\mu = \mu_A + \mu_B$. Let $\alpha \in (0, 1)$ be the market share of brand A, where $\alpha = \mu_A/\mu$. Given the definition of α and θ, we have $\mu_A = \alpha\mu$, $\mu_B = (1-\alpha)\mu$, $\sigma_A = \theta\alpha\mu$, and $\sigma_B = \theta(1-\alpha)\mu$.

Consider the case in which retailer i, where $i = A, B$, launches the ABD program by offering a discount price $x_i p$ per unit of the product prior to the beginning of the season.[2] The discount coefficient is equal to x_i with $0 \le x_i \le 1$. Notice that firm i can launch an ABD program with $x_i = 1$, corresponding to the case in which firm i allows customers to preorder but offers no discount on the regular season price. If customers accept the ABD offer, they precommit their orders by prepaying $x_i p$ per unit prior to the selling season, and then pick up their orders during the season. If customers decline the ABD offer, they can always attempt to purchase the product during the regular selling season, paying the regular price p per unit, however, the availability of the product will not be guaranteed.

Let $R_{ie}(x_A, x_B) \in [0, 1]$ represent the fraction of customers in segment i who opt to preorder from retailer i. Let $R_{is}(x_A, x_B) \in [0, 1]$ be the fraction of customers in segment i who opt to preorder from competing retailer j. The remainder, $[1 - R_{ie}(x_A, x_B) - R_{is}(x_A, x_B)]$, wait for the regular selling season.[3] To obtain tractable analytical results, we develop a specific functional form for the consumer response functions $R_{ie}(x_A, x_B)$ and $R_{is}(x_A, x_B)$. Consider the situation faced by the customers prior to the selling season. Each consumer in segment i has to compare the discount prices $x_A p$ and $x_B p$ and then decide whether to (1) precommit to retailer i, (2) precommit to retailer j, or (3) wait for the regular selling season. While marketing researchers have considered various functional forms for the consumer's choice function, we employ a modified version of the functional form proposed by Raju et al. (1995). Specifically, we assume that the response functions possess the following form:

$$R_{ie}(x_A, x_B) = \begin{cases} 0 & \text{if retailer } i \text{ does not launch ABD,} \\ (r_{ie}/2)[(1 - a x_i) + b(x_j - x_i)] & \text{for } i = A, B, \end{cases} \tag{1}$$

$$R_{is}(x_A, x_B) = \begin{cases} 0 & \text{if retailer } j \text{ does not launch ABD,} \\ (r_{is}/2)[(1 - a x_j) + b(x_i - x_j)] & \text{for } i = A, B. \end{cases} \tag{2}$$

Suppose neither firm offers an ABD program. Then, (1) and (2) imply that $R_{ie} = R_{is} = R_{je} = R_{js} = 0$, i.e., there are no early orders. If firm i does not launch an ABD program but firm j does, it can be seen from (1) and (2) that $R_{ie} = 0$ and $R_{js} = 0$, respectively. To complete the specification of the response functions in this case, we set $x_i = 1$. Similarly, when firm j does not launch an ABD program but firm i does, (1) and (2) imply that $R_{is} = R_{je} = 0$, and we set $x_j = 1$.

As explained in Raju et al. (1995), the parameter $a \in (0, 1)$ in Equations (1) and (2) represents the price sensitivity of customers, while $b \in (0, 1)$ represents the cross-price sensitivity.[4] This interpretation fits our situation quite well. First, observe from the first equation that the proportion of customers in segment i

[2] To simplify the exposition at this point, we do not include a fixed cost that may incur when a retailer launches the ABD program. This fixed cost is included, however, in a later section.

[3] It is conceivable that aggregate demand may increase as a result of the price discount offered through the ABD program. However, because the products we consider are either durable or perishable in nature and customers receive the product only during the selling season, it seems reasonable to assume that customers do not consume more during the selling season and that consumption does not increase with the level of the discount.

[4] To guarantee that $R_{ie}(x_A, x_B)$ and $R_{is}(x_A, x_B)$ are both nonnegative, we need to impose $a + b < 1$.

who will precommit to retailer i (i.e., $R_{ie}(x_A, x_B)$) increases when retailer i offers a lower discount price $x_i p$ or when retailer i offers a discount price lower than that of retailer j, i.e., when $(x_j - x_i)p > 0$. Second, observe from the second equation that the proportion of customers in segment i who will "switch" and precommit to retailer j (i.e., $R_{is}(x_A, x_B)$) increases when retailer j offers a lower discount price $x_j p$ or when retailer j offers a lower discount price than that of retailer i, i.e., when $(x_i - x_j)p > 0$.

Two features of our response functions that distinguish them from Raju et al. (1995) are the parameters r_{ie} and r_{is}. The first parameter, r_{ie}, represents the consumer's risk aversion to a stockout. The larger r_{ie}, the greater is the chance that a customer will accept the ABD offer to avoid a stockout during the selling season. The second parameter, r_{is}, represents the degree of brand *disloyalty*. The larger r_{is}, the greater the chance that a customer will switch from their regular firm and accept an ABD offer from the competing firm. We bound r_{ie} and r_{is} between 0 and $1/(1+b)$ so as to guarantee that $R_{is} + R_{ie} \leq 1$, ensuring consistency with our assumption that discounts do not increase consumption.

Note that the specification of the response function also allows for the case when both retailers offer an advanced booking program without providing a preseason discount, i.e., AB without D. In this case, the ABD program serves as an early reservation system.

Because each retailer may or may not launch an ABD program, we need to consider four scenarios. In scenario I, neither retailer offers the ABD program. In scenario II, retailer A offers the ABD program while retailer B does not. In scenario III, retailer A does not offer the program while retailer B does. In scenario IV, both retailers offer the ABD program. The effective demand (including the precommitted orders prior to the selling season and the demand occurred during the selling season) for each retailer associated with scenario k, where $k = I, II, III, IV$, is depicted in Figure 1.

Retailer i, $i = A, B$, faces two types of "effective" demands in scenario k: the precommitted orders placed prior to the season $D_{1i}^k(x_A, x_B)$, and the demand that occurs during the regular selling season $D_{2i}^k(x_A, x_B)$. The "effective" demands associated with retailer i can be expressed as follows:

$$D_{1i}^k(x_A, x_B) = R_{ie}^k(x_A, x_B)D_i + R_{js}^k(x_A, x_B)D_j \quad \text{for } j \neq i \quad \text{and} \tag{3}$$
$$D_{2i}^k(x_A, x_B) = \left[1 - R_{ie}^k(x_A, x_B) - R_{is}^k(x_A, x_B)\right]D_i.$$

In scenario k, the total "effective" demand obtained by retailer i is equal to $D_{1i}^k(x_A, x_B) + D_{2i}^k(x_A, x_B) = D_i + R_{js}^k(x_A, x_B)D_j - R_{is}^k(x_A, x_B)D_i$. Retailer i will gain additional demand in scenario k when more customers switch from retailer j to retailer i than those who switch from retailer i to retailer j, i.e., when $R_{js}^k(x_A, x_B)D_j - R_{is}^k(x_A, x_B)D_i > 0$.

Because $D_{1i}^k(x_A, x_B)$ and $D_{2i}^k(x_A, x_B)$ are linear functions of D_A and D_B, it follows that $(D_{1i}^k(x_A, x_B), D_{2i}^k(x_A, x_B))$ also has a bivariate normal distribution. Firm i observes D_{1i}^k before placing the order prior to the selling season (period 2), thus, we are interested in $(D_{2i}^k \mid D_{1i}^k)$, which also has a normal distribution. It follows from (3) that the parameters of these normally distributed demand distributions for firm A, for example, are

$$\mu_{D_{1A}^k(x_A, x_B)} = R_{Ae}^k(x_A, x_B)\alpha\mu + R_{Bs}^k(x_A, x_B)(1-\alpha)\mu, \tag{4}$$

$$\sigma^2_{D_{1A}^k(x_A, x_B)} = (R_{Ae}^k(x_A, x_B))^2[\theta\alpha\mu]^2 + (R_{Bs}^k(x_A, x_B))^2[\theta(1-\alpha)\mu]^2 + 2R_{Ae}^k(x_A, x_B)R_{Bs}^k(x_A, x_B)\rho(\theta\alpha\mu)(\theta(1-\alpha)\mu), \tag{5}$$

$$\mu_{D_{2A}^k(x_A, x_B)} = \left[1 - R_{Ae}^k(x_A, x_B) - R_{As}^k(x_A, x_B)\right]\alpha\mu, \tag{6}$$

$$\sigma^2_{D_{2A}^k(x_A, x_B)} = \left[1 - R_{Ae}^k(x_A, x_B) - R_{As}^k(x_A, x_B)\right]^2[\theta\alpha\mu]^2, \tag{7}$$

$$\mathrm{Cov}\left(D_{1A}^k(x_A, x_B), D_{2A}^k(x_A, x_B)\right) = \alpha R_{Ae}^k(x_A, x_B) + \rho(1-\alpha)R_{Bs}^k(x_A, x_B), \tag{8}$$

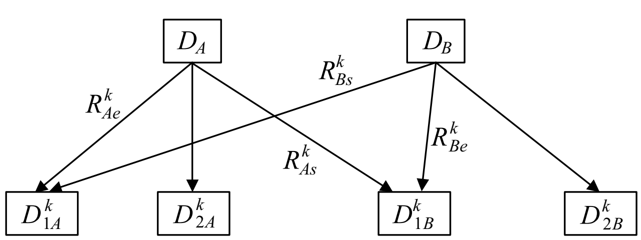
$$\mu_{(D_{2A}^k(x_A, x_B) \mid D_{1A}^k(x_A, x_B) = d)} = \mu_{D_{2A}^k(x_A, x_B)} + \mathrm{Corr}\left(D_{1A}^k(x_A, x_B), D_{2A}^k(x_A, x_B)\right) \cdot \frac{\sigma_{D_{2A}^k}}{\sigma_{D_{1A}^k}}\left[d - \mu_{D_{1A}^k(x_A, x_B)}\right], \tag{9}$$

$$\sigma^2_{(D_{2A}^k(x_A, x_B) \mid D_{1A}^k(x_A, x_B) = d)} = \sigma^2_{D_{2A}^k(x_A, x_B)}\left[1 - \mathrm{Corr}^2(D_{1A}^k(x_A, x_B), D_{2A}^k(x_A, x_B))\right]. \tag{10}$$

Because both firms have the same coefficient of determination θ for the primary demand, deriving the parameters for firm B amounts to substituting A for B and $1-\alpha$ for α in the above set of equations; we omit the details.

Note from (10) that the variance of period 2 demand (i.e., the demand that occurs during the regular selling season) conditioned on observed period 1

Figure 1 **Effective Demands Under Scenario** k, $k = I, II, III, IV$



demand (i.e., the precommitted orders placed prior to the selling season) is less than the variance of the unconditioned period 2 demand (7). This is exactly as should be expected: the information content in period 1 demand reduces the uncertainty for period 2. Furthermore, it can be shown that $\sigma^2_{(D^k_{2A}(x_A,x_B)|D^k_{1A}(x_A,x_B)=d)}$ is concave and decreasing in ρ for $\rho > 0$. Thus, when the primary demands, D_A and D_B, of the two firms are positively correlated, firm A can use the information content, not only from its own early purchasers, $R^k_{Ae}(x_A, x_B)$, but also the information content in the switchers from firm B, $R^k_{Bs}(x_A, x_B)$, to reduce period 2 demand uncertainty.

By utilizing the effective demands associated with retailer i in scenario k (i.e., $D^k_{1i}(x_A, x_B)$ and $D^k_{2i}(x_A, x_B)$), we can determine the optimal discount price that maximizes the retailer's expected profit as follows. First, note that the order is placed at the start of the selling season. Therefore, retailer i can order the exact amount to fulfill the precommitted orders $D^k_{1i}(x_A, x_B)$ observed prior to the selling season. The profit generated from those precommitted orders is equal to $(x_i p - c)D^k_{1i}(x_A, x_B)$. Second, retailer i can utilize the information about $D^k_{1i}(x_A, x_B)$ to update the distribution of $D^k_{2i}(x_A, x_B)$. The retailer orders additional quantity Q (in addition to $D^k_{1i}(x_A, x_B)$) so as to cover the demand during the selling season. The profit generated from the demand $D^k_{2i}(x_A, x_B)$ is equal to $\{p\cdot\min\{Q, D^k_{2i}(x_A, x_B)\} + s[Q - D^k_{2i}(x_A, x_B)]^+ - cQ\}$. Given the discount prices $x_A p$ and $x_B p$, retailer i's expected profit in scenario k can be written as

$$\begin{aligned}\pi^k_i(x_A, x_B) &= E_{D^k_{1i}(x_A,x_B)}\big\{(x_i p - c)D^k_{1i}(x_A, x_B) \\ &\quad + \max_Q E_{[D^k_{2i}(x_A,x_B)|D^k_{1i}(x_A,x_B)]}\big\{p\min\{Q, D^k_{2i}(x_A, x_B)\} \\ &\quad + s[Q - D^k_{2i}(x_A, x_B)]^+ - cQ\big\}\big\}. \qquad (11)\end{aligned}$$

The optimal expected profits attained by retailers A and B in equilibrium in scenario k must satisfy

$$\pi^k_A = \max_{x_A} \pi^k_A(x_A, x_B), \qquad (12)$$

$$\pi^k_B = \max_{x_B} \pi^k_B(x_A, x_B). \qquad (13)$$

Given the optimal within-scenario equilibrium expected profits in each of the four scenarios, we can construct the normal form of the ABD competitive metagame that lists each retailer's strategies and the payoff associated with each scenario k, where $k = I, II, III, IV$.

3. Analysis

To simplify the exposition, we present our analysis as follows. In scenarios I and IV, our analysis will focus on the behavior of firm A: unless otherwise noted, the behavior of firm B is symmetric to that of firm A. The behavior of both firms will be analyzed in scenario II: scenario III is symmetric to scenario II.

3.1. Scenario I: Neither Firm Offers ABD

Because neither firm offers ABD, there is no switching of consumers from one retailer to the other, and there is no period 1 demand: $R^I_{Ae} = R^I_{As} = R^I_{Bs} = R^I_{Be} = 0$ and $D^I_{1A} = D^I_{1B} = 0$. All demand arises in period 2: $D^I_{2A} = D_A$ and $D^I_{2B} = D_B$. In this scenario, there is no competitive interaction between the firms: each firm behaves as a monopolist. The retailer charges p for each unit during the selling season and receives a salvage value s for each unit after the season, where $s < c < p$. The retailer needs to determine the optimal order quantity Q^I_i that maximizes the total expected profit. Let π_i be the optimal expected profit, where

$$\pi^I_i = \max_{Q_i \ge 0} E_{D_i}\{p\min\{Q_i, D_i\} + s[Q_i - D_i]^+ - cQ_i\}, \quad i \in (A, B). \qquad (14)$$

The optimal Q^I_i and π^I_i are given by

$$Q^I_i = \mu_i + w\sigma_i, \quad i \in (A, B), \qquad (15)$$

$$\pi^I_i = (p - c)\mu_i - (p - s)\phi(w)\sigma_i, \quad i \in (A, B), \qquad (16)$$

where $w = \Phi^{-1}((p - c)/(p - s))$, and $\Phi(\cdot)$ and $\phi(\cdot)$ are the cumulative distribution and the density functions of the standard normal distribution, respectively (see Silver et al. 1998).

Rewriting the profit function for retailer A with $\mu_A = \alpha\mu$ and $\sigma_A = \theta\alpha\mu$, we get

$$\begin{aligned}\pi^I_A &= (p - c)\mu_A - (p - s)\phi(w)\sigma_A \\ &= \alpha\mu[(p - c) - (p - s)\phi(w)\theta]. \qquad (17)\end{aligned}$$

Similarly, we can determine the profit function for retailer B. As one would expect in a market share model with fixed prices, where retailer A has share α and retailer B has share $1 - \alpha$,

$$\pi^I_B = \left[\frac{1 - \alpha}{\alpha}\right]\pi^I_A. \qquad (18)$$

3.2. Scenario II: Only Firm A Offers ABD

When firm A offers the item at discounted price $x_A p$ in period 1, and firm B does not offer ABD, the effective demands are given by

$$D^{II}_{1A} = R^{II}_{Ae}(x_A)D_A + R^{II}_{Bs}(x_A)D_B, \qquad (19)$$

$$D^{II}_{1B} = 0, \qquad (20)$$

$$D^{II}_{2A} = [1 - R^{II}_{Ae}(x_A)]D_A, \qquad (21)$$

$$D^{II}_{2B} = [1 - R^{II}_{Bs}(x_A)]D_B. \qquad (22)$$

Firm B, which faces demand only in period 2, has a classic single-period newsvendor problem to solve. From (22), firm B's demand is normally distributed with mean and variance

$$\mu_{D_{2B}^{II}} = [1 - R_{Bs}^{II}(x_A)]\mu_B = [1 - R_{Bs}^{II}(x_A)](1-\alpha)\mu, \quad (23)$$

$$\sigma_{D_{2B}^{II}}^2 = [1 - R_{Bs}^{II}(x_A)]^2\sigma_B^2 = \left[1 - R_{Bs}^{II}(x_A)\right]^2[\theta(1-\alpha)\mu]^2. \quad (24)$$

The optimal expected profit π_B^{II} for firm B is

$$\pi_B^{II} = (p-c)E(D_{2B}^{II}) - (p-s)\phi(w)\sigma_{D_{2B}^{II}}, \quad (25)$$

which can be simplified as

$$\pi_B^{II} = [1 - R_{Bs}^{II}(x_A)](1-\alpha)\mu[(p-c)-(p-s)\phi(w)\theta]. \quad (26)$$

From the definition of $R_{Bs}^{II}(x_A)$ (see (2)), it follows that firm B's optimal profit in scenario II is increasing in x_A, firm A's period 1 discount coefficient. That is, the higher the price charged by firm A in its ABD program, the higher the expected profits for firm B when it does not offer ABD. Comparing the optimal expected profits for firm B in scenario II, (26), with those in scenario I, (18), it is easy to see that

$$\pi_B^{II} \le \pi_B^{I}. \quad (27)$$

That is, when only firm A offers ABD, firm B is worse off than the case where neither firm offers ABD.

Firm A chooses a period 1 price discount coefficient x_A and period 2 order quantity Q_A^{II} to maximize total expected profits π_A^{II}, where

$$\pi_A^{II} = \max_{x_A}\Big\{E_{D_{1A}^{II}}\Big\{(x_A\cdot p - c)D_{1A}^{II} + \max_{Q_A} E_{[D_{2A}^{II}|D_{1A}^{II}]}\big\{p\cdot\min\{Q_A, D_{2A}^{II}\} + s[Q_A - D_{2A}^{II}]^+ - cQ_A\big\}\Big\}\Big\}. \quad (28)$$

Substituting the definitions of the consumer response functions (1) and (2) into the equations for the parameters of the demand distributions (4)–(10), and then optimizing (28), yields the first-order condition for firm A's period 1 discount coefficient x_A^{II}. By simplifying terms, we have

$$x_A^{II} = \frac{p\bar{r}(1+b) + U_2 r_{Ae}(a+b) + c(a+b)\bar{r}}{2(a+b)p\bar{r}}, \quad (29)$$

where

$$\bar{r} = r_{Ae}\alpha + r_{Bs}(1-\alpha), \quad (30)$$

$$U_2 = (p-c)\alpha - (p-s)\phi(w)\theta\alpha T_2, \quad (31)$$

$$T_2 = \sqrt{1 - \mathrm{Corr}(D_{1A}^{II}, D_{2A}^{II})} = (1-\alpha)r_{Bs}\sqrt{1-\rho^2} \cdot \sqrt{\frac{1}{\alpha^2 r_{Ae}^2 + (1-\alpha)^2 r_{Bs}^2 + 2r_{Ae}r_{Bs}\rho\alpha(1-\alpha)}}. \quad (32)$$

In scenario II, the optimal firm A ABD discount coefficient, x_A^{II}, is increasing in cost c, salvage value s, and correlation ρ when $\rho > 0$. Furthermore, x_A^{II} is decreasing in the coefficient of variation θ. These results follow directly from (29) and their interpretation is straightforward. The price charged by firm A in period 1 is $x_A^{II}p$. As the amount of demand uncertainty θ increases, the firm values the information content of the early orders more, hence, it lowers its period 1 price to increase period 1 sales. As cost c increases, the firm has less leeway to reduce price, hence, the optimal period 1 price increases. As the salvage value s increases, there is less downside risk to excess inventory, hence, the value of the information in the early orders decreases and the firm increases period 1 price. Finally, as the demand correlation $\rho > 0$ increases, the information content of each early order increases, and firm A can get the same level of information with fewer period 1 sales, hence, it can raise the period 1 price. In addition, the variance of period 2 demand decreases. The reduced period 2 variance and the higher period 1 price increase overall profits.

Substituting (29) into (28) yields the optimal expected firm A profit in scenario II,

$$\pi_A^{II} = (x_A^{II}p - c)\frac{1}{2}\mu\bar{r}(1+b-(a+b)x_A^{II}) + U_2\mu\left(1 - \frac{r_{Ae}}{2}(1+b) + x_A^{II}\frac{r_{Ae}}{2}(a+b)\right). \quad (33)$$

As previously noted, if firm B does not offer ABD, firm B is better off if firm A also does not offer ABD. As might be expected, but somewhat less easy to see, the opposite is true for firm A: firm A is better off than not by offering an ABD program when firm B does not offer an ABD program. We prove this claim in two steps.[5]

Lemma 1. *If firm B does not offer an ABD program, then offering an ABD program with discount coefficient* 1 *is better for firm A than not having an ABD program, i.e.,* $\pi_A^{II}(1) \ge \pi_A^{I}$.

Lemma 1 proves that if firm B is not offering an ABD program, firm A prefers to start an ABD program with discount coefficient 1 over not having an ABD program.[6] Having an ABD program with the optimal scenario II discount coefficient, x_A^{II}, rather than 1, only improves firm A's situation, i.e., $\pi_A^{II} \ge \pi_A^{II}(1)$. Thus, we have shown

$$\pi_A^{II} \ge \pi_A^{I}. \quad (34)$$

[5] All proofs are provided in McCardle et al. (2005).

[6] Recall that we have not yet included a fixed cost for implementing an ABD program. Such a cost would clearly alter the conclusion of Lemma 1.

The setup and, hence, the results for scenario III are perfectly symmetric with the just presented scenario II. For example, analogous to (29), firm B's period 1, scenario III discount x_B^{III} is given by

$$x_B^{III} = \frac{p\hat{r}(1+b) + U_3 r_{Be}(a+b) + c(a+b)\hat{r}}{2(a+b)p\hat{r}}, \tag{35}$$

where the definitions of $\hat{r}$ and U_3 are similar to (30) and (31). See McCardle et al. (2005) for details.

3.3. Scenario IV: Both Firms Offer ABD

When both firms offer ABD, both potentially poach some of their competitor's customers, as well as entice some of their own customers to purchase early. From (3), we get

$$D_{1A}^{IV} = R_{Ae}^{IV}(x_A, x_B)D_A + R_{Bs}^{IV}(x_A, x_B)D_B, \tag{36}$$

$$D_{1B}^{IV} = R_{Be}^{IV}(x_A, x_B)D_B + R_{As}^{IV}(x_A, x_B)D_A, \tag{37}$$

$$D_{2A}^{IV} = [1 - R_{Ae}^{IV}(x_A, x_B) - R_{As}^{IV}(x_A, x_B)]D_A, \tag{38}$$

$$D_{2B}^{IV} = [1 - R_{Be}^{IV}(x_A, x_B) - R_{Bs}^{IV}(x_A, x_B)]D_B. \tag{39}$$

If firm B uses the price discount coefficient $y < 1$, i.e., charges an ABD price of yp, then firm A faces the maximization problem

$$\pi_A^{IV} = \max_{x_A}\Big\{E_{D_{1A}^{IV}}\Big\{(x_A \cdot p - c)D_{1A}^{IV} + \max_{Q_A} E_{[D_{2A}^{IV}|D_{1A}^{IV}]}\big\{p\cdot\min\{Q_A, D_{2A}^{IV}\} + s[Q_A - D_{2A}^{IV}]^+ - cQ_A\big\}\Big\}\Big\}. \tag{40}$$

Substituting the definitions of the consumer response functions (1) and (2) into the equations for the parameters of the demand distributions (4)–(10), and then optimizing (40), yields the first-order condition for firm A's period 1 discount coefficient $x_A^{IV}(y)$ as a function of firm B's choice y,

$$x_A^{IV}(y) = \frac{1}{2(a+b)p\bar{r}}\cdot\big[p\bar{r}(1+by) + U_2[r_{Ae}(a+b) - r_{As}b] + c(a+b)\bar{r}\big], \tag{41}$$

where $\bar{r}$, U_2, and T_2 are as given in (30)–(32).

Proposition 2. *In scenario* IV, *firm A's best response function $x_A^{IV}(y)$ is increasing in firm B's strategy y. For all $y \le 1$, $x_A^{IV}(y) \le x_A^{II}$, i.e., firm A has a lower ABD price when firm B offers an ABD program than when it does not. Furthermore, $x_A^{IV}(y)$ is decreasing in θ and increasing in c, s, and ρ when $\rho > 0$.*

While firm A's ABD discount in scenario II (only firm A offers ABD) is driven mainly by the period 2 demand uncertainty reduction generated by the early orders, firm A's ABD discount in scenario IV (both firms offer ABD) is also driven by the competitive pressures of firm B's ABD discount. Proposition 2 shows that firm A's ABD discount coefficient $x_A^{IV}(y)$ is increasing in firm B's ABD discount coefficient y. That is, the lower the price charged by firm B in period 1, the lower firm A must charge in period 1 as well. If firm A does not lower its period 1 price in response to a price decrease by firm B, it not only loses a portion of its demand in both periods, it also loses the information content of the lost period 1 demand.

The resulting profit function for firm A, $\pi_A^{IV}(y)$, as a function of firm B's strategy y, is

$$\pi_A^{IV}(y) = (x_A^{IV}p - c)\frac{1}{2}\mu\bar{r}(1+by - x_A^{IV}(a+b)) + U_2\mu\left[\left(1 - \frac{r_{Ae}}{2} - \frac{r_{As}}{2}\right) + x_A^{IV}\left(\frac{r_{Ae}(a+b)}{2} - \frac{r_{As}b}{2}\right) - y\left(\frac{r_{Ae}b}{2} - \frac{r_{As}(a+b)}{2}\right)\right]. \tag{42}$$

The strategy pair (x, y) is in equilibrium if the strategies are best responses to each other.

Proposition 3. *In scenario* IV (*both firms offer ABD*), *there is a unique equilibrium in the discount coefficients (x_A^{IV}, x_B^{IV}). These are given by*

$$x_A^{IV} = \frac{1}{p\bar{r}\hat{r}(2a+3b)(2a+b)}\cdot\big[\bar{r}\hat{r}(2a+3b)(p+c(a+b)) + U_3 b\bar{r}(r_{Be}(a+b) - r_{Bs}b) + 2U_2(a+b)\hat{r}(r_{Ae}(a+b) - r_{As}b)\big],$$

$$x_B^{IV} = \frac{1}{p\bar{r}\hat{r}(2a+3b)(2a+b)}\cdot\big[\bar{r}\hat{r}(2a+3b)(p+c(a+b)) + U_2 b\hat{r}(r_{Ae}(a+b) - r_{As}b) + 2U_3(a+b)\bar{r}(r_{Be}(a+b) - r_{Bs}b)\big].$$

Employing the lattice-theoretic method detailed in Lippman et al. (1986), it follows that the scenario IV equilibrium discount coefficients maintain the same comparative statics regarding the parameters of the model as the best-response functions. That is,

Proposition 4. *The equilibrium discount coefficients, (x_A^{IV}, x_B^{IV}), are decreasing in the level of demand uncertainty θ, and are increasing in cost c, salvage value s, and correlation ρ when $\rho > 0$.*

Corollary 5. *Assume $r_{Ae} = r_{Be} = r_{As} = r_{Bs}$. Then, in the unique scenario* IV *equilibrium, the firm with the larger market share charges the higher discount price xp in period* 1. *That is, $x_A^{IV} \ge x_B^{IV}$ if and only if $\alpha \ge 0.5$.*

Substituting the equilibrium discount coefficients given in Proposition 3 into the profit function for firm A (40) yields the equilibrium expected profit $\pi_A^{IV} = \pi_A^{IV}(x_B^{IV})$ (similarly for firm B).

4. ABD Equilibrium

To determine the equilibrium behavior in the larger game, it is necessary to incorporate the decision of whether or not to have an ABD program. To evaluate if a retailer should offer an ABD program or not, we now compare the expected profit associated with different scenarios and the change in expected profit when moving from one scenario to another.

First, let us suppose that firm B does not offer ABD. In this case, as noted from (34), firm A prefers to offer ABD, i.e., firm A prefers scenario II (only firm A offers ABD) to scenario I (neither firm offers ABD). By symmetry, firm B prefers scenario III (only firm B offers ABD) to scenario I. It remains to examine two situations: when firm B offers ABD, what would firm A prefer to do; and when firm A offers ABD, what would firm B prefer to do. Instead of comparing the within-scenario equilibrium expected profits generated by firm A in scenarios III and IV (or firm B in scenarios II and IV), we examine a slightly different question that enables us to show that scenario IV (both firms offer ABD) yields an unique equilibrium to the ABD game. Specifically, the question we address in this section is, for example, given that firm B offers ABD and firm A does not (scenario III), would firm A want to implement an ABD program? Lemma 6 answers this question in the affirmative.

Lemma 6. *If firm B offers an ABD program with a discount coefficient $y \leq 1$, then offering an ABD program with discount coefficient* 1 *is better for firm A than not having an ABD program, i.e.,* $\pi_A^{IV}(1, y) \geq \pi_A^{III}(y)$.

From Lemma 6, we can conclude that $\pi_A^{IV}(1, x_B^{III}) \geq \pi_A^{III}(x_B^{III})$. This implies that when firm B offers an ABD program, firm A prefers having an ABD program: scenario III does not represent an equilibrium of the larger game. By symmetry, an identical argument can be used to show that $\pi_B^{IV}(x_A^{II}, 1) \geq \pi_B^{II}(x_A^{II})$. This implies that when firm A offers an ABD program, firm B prefers having an ABD program: scenario II does not represent an equilibrium of the larger game.

Based on Lemma 6, firm A reasons as follows: given that firm B has an ABD program with discount coefficient x_B^{III}, firm A is better off with an ABD program with a discount coefficient 1 than without an ABD program; and firm A is even better off if it offers an ABD program with a discount coefficient of $x_A = \arg\max_x \pi_A^{IV}(x, x_B^{III})$.[7] By symmetry, firm B applies the same reasoning to justify the fact that firm B is better off offering an ABD program when firm A has an ABD program with discount coefficient x_A^{II}. Once both firms are offering ABD programs, the resulting equilibrium is as given in scenario IV. This observation is captured in Proposition 7.

[7] Note that we do not claim that the scenario IV firm A expected payoff dominates the scenario III firm A expected payoff.

Proposition 7. *If it is costless to implement an ABD program, the ABD game has a unique equilibrium: both firms offer ABD with discount coefficients as stated in Proposition* 3 *for scenario* IV.

The key argument of our proof hinges on the following four inequalities:

$$\pi_A^{II} \geq \pi_A^{I}, \tag{43}$$

$$\pi_B^{IV}(x_A^{II}, x_B^{IV}(x_A^{II})) \geq \pi_B^{II}, \tag{44}$$

$$\pi_B^{III} \geq \pi_B^{I}, \tag{45}$$

$$\pi_A^{IV}(x_A^{IV}(x_B^{III}), x_B^{III}) \geq \pi_A^{III}. \tag{46}$$

Suppose that instead of being costless, it costs K to implement an ABD program. Then, the expected profits listed in the above inequalities remain the same except the following: firm A's expected profit in scenarios II and IV become $\pi_A^{II} - K$ and $\pi_A^{IV}(x_A^{IV}(x_B^{III}), x_B^{III}) - K$, respectively. Similarly, firm B's expected profit in scenarios III and IV become $\pi_B^{III} - K$ and $\pi_B^{IV}(x_A^{II}, x_B^{IV}(x_A^{II})) - K$, respectively. By considering these effective changes and by rearranging the terms, the above inequalities can be rewritten as

$$\pi_A^{II} - \pi_A^{I} \geq K, \tag{47}$$

$$\pi_B^{IV}(x_A^{II}, x_B^{IV}(x_A^{II})) - \pi_B^{II} \geq K, \tag{48}$$

$$\pi_B^{III} - \pi_B^{I} \geq K, \tag{49}$$

$$\pi_A^{IV}(x_A^{IV}(x_B^{III}), x_B^{III}) - \pi_A^{III} \geq K. \tag{50}$$

For $K = 0$, all four inequalities hold. They are proved via Lemmas 1 and 6, and the material that follows immediately. It follows that for K small enough (near zero), all of these inequalities continue to hold. By applying Lemmas 1 and 6 and the material that follows, we can conclude that there is a unique equilibrium in which both firms offer ABD. On the other hand, if K is large enough, none of these inequalities hold. In this case, we can utilize the same argument as presented in this section to show that there is a unique equilibrium in which neither firm offers ABD. For very large K, whatever benefit that might accrue from having an ABD program is outweighed by the implementation cost K. What will happen for moderate values of K so that some, but not all inequalities hold?

Suppose for some set of parameters, inequality (47) holds, but (48) and (49) do not hold. Then, there is a unique equilibrium in which firm A offers ABD and firm B does not, i.e., scenario II. Similarly, if inequality (49) holds, but (47) and (50) do not hold, then there is a unique equilibrium in which firm B offers ABD but firm A does not, i.e., scenario III. Finally, if both (47) and (49) hold, but (50) and (48) do not hold, there are two equilibria represented by scenarios II and III. That is, it is in equilibrium for either firm to offer ABD, but not both. Numerical examples of each of these cases are provided in McCardle et al. (2005).

5. Concluding Remarks

In this paper, we extended the single-firm ABD model of Tang et al. (2004) to include competition. We showed that, in general, in a two-firm competitive model, if it is optimal for one firm to adopt an ABD program, the unique equilibrium has both firms adopting an ABD program. While the model presented here entailed only two firms, we expect the results (when the cost $K = 0$) to extend to an n-firm competitive model. The difficulty is in the specification of the consumer response functions (1) and (2) when there are more than two firms. There are other limitations to our model, and we plan to use the current model as a basic building block to address them (much as Tang et al. 2004 served as a building block for this paper). For example, we aim to extend the analysis to more than two periods with multiple replenishments; more than one product with joint capacity constraints; to include asymmetries in prices, costs, and salvage values across firms; and to include discounting and price competition in both the pre and regular season.

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