Communications

Rational Response to the Money Supply

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I. Introduction

Knowledgeable consumers should be attentive to published measures of the money supply. They have an incentive to forecast money's future quantity as accurately as possible, for they must decide which commodities, assets, and labor income should be contracted now, in nominal terms, and which should be left for purchase or sale at future (unknown) prices.

Information about the money supply has a central role in consumption forecasting and choosing because it is commonly and correctly believed to be the most important influence on commodity prices. By forecasting the money supply into the future and estimating its effect on the sequence of commodity prices, the consumer is performing a rational decision calculation, identical in substance with that performed by an investor estimating the effect of predicted future earnings on the sequence of a firm's common stock price.

By the very act of making decisions on the basis of anticipated money stocks, however, consumers decrease the empirical connection between future changes in money and the current price of goods. The efficient commodity market would display a minimal connection between the two because commodity prices would be bid quickly to a discounted level related to the predictable portion of the monetary sequence. Again

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the analogy to a perfect stock market can be made: a firm whose earnings have grown period after period will not find its common stock changing in price after the next earnings increase unless those earnings deviate from the anticipated.

Unlike corporate earnings, the supply of fiat money represents virtually no real resource. It can be manipulated willfully and costlessly, and rules for the distribution of new nominal money balances can be chosen arbitrarily by the fiat issuer. The current distribution rule in the United States involves direct exchange of new money for real goods and services and financial assets; but this is certainly not the only possible rule or even the rule which is most frequently discussed by monetary theorists. They generally suppose that the distribution rule is unimportant and indeed has no bearing whatsoever on the pattern of commodity prices observed in the economy.[1]

In this paper, I hope to argue well for a contrary hypothesis: The nominal money distribution rule is of crucial significance provided that consumers behave by adjusting their current portfolios of money, assets, and consumption on the basis of the anticipated future monetary sequence. Although the rules to be discussed here are primitive, they do illustrate the consequences, for monetary policy, of rational consumer behavior. Hence, they may claim to have a prominence beyond their reality.

II. Asset, Money, and Commodity Demand

On the most disaggregated level, the individual consumer can be envisioned as selecting a feasible consumption program over his lifetime. In the initial period, his choice set contains probability distributions only and does not include a guaranteed future level of consumption. The time-honored method of converting this choice set into a set containing recognizable economic quantities is to suppose he maximizes the expected value of a utility function defined over current and future consumption levels and then to relate such levels to an initial wealth endowment and to quantities of assets selected currently with the purpose of storing wealth for the future.

Throughout the paper, all costs attendant to buying and selling assets and differential tax effects will be neglected. Within these restrictions, the consumer's hope is to maximize $E[U(C_1,\tilde{C}_2, \ldots \tilde{C}_n)]$ while remaining within the constraints imposed by

$$W - M_1 - p_1 A_1 - P_1 C_1 = 0 \tag{1}$$

$$\tilde{P}_j \tilde{C}_j - \tilde{y}_j - (\tilde{M}_{j-1} - \tilde{M}_j) - \tilde{p}_j(\tilde{A}_{j-1} - \tilde{A}_j) = 0; \; j = 2, \ldots n - 1 \tag{2}$$

$$\tilde{P}_n \tilde{C}_n - \tilde{y}_n - \tilde{M}_{n-1} - \tilde{p}_n \tilde{A}_{n-1} = 0 \tag{3}$$

[1] The quantity of nominal money balances is emphasized, of course. It is only the distribution rule which is considered unimportant.

where U = utility; E = mathematical expectation; C_j = goods consumed in period j; P_j = consumption goods price in period j; A_j = quantity of bonds (variable price asset) held from j to $j + 1$; p_j = bond price in period j; M_j = quantity of money[2] held from period j to $j + 1$ (nominal balances); W = initial wealth (human and nonhuman capital);[3] y_j = labor income earned in period j (this is assumed to be unrelated functionally to asset and money holdings); n = maximum possible life span; $\sim$ denotes random variables as of the first (current) period.[4] A solution to this problem requires the "stochastic calculus" of Kushner (1965) because the constraints (2)–(3) contain stochastic elements. Kushner's technique has been discussed in detail in economic applications by Long (1971), and the reader should refer to that paper or to Kushner's own for more information. Here, only the results (first-order extremum conditions) for the solution to the above problem are given. For the asset money, they are[5]

$$E(\tilde{U}_1) = E[\tilde{U}_2/(1 + \tilde{I}_2)]$$

$$\tilde{U}_j = \tilde{U}_{j+1}/(1 + \tilde{I}_{j+1});\ j = 2, \ldots, n - 1 \qquad (4)$$

where subscripted U's indicate partial derivatives of the utility function and I_j is the rate of goods price inflation between periods $j - 1$ and j; $I_j \equiv (P_j - P_{j-1})/P_{j-1}$.

The optimum conditions (4) imply rather simple marginal trade-offs between time preferences and commodity price changes. When one realizes that U_j/U_{j+1} has the units of a real interest rate (i.e., a commodity interest rate), plus unity, the conditions are seen to be the monetary asset's equivalent to the Irving Fisher (1930) relation between real interest rates, nominal interest rates, and the rate of inflation. (For money, the nominal rate of interest is exactly zero.)

There should be as many different U_j's and I_j's in each period, as there are different consumption goods. By not denoting the quantities in (4) like "$U_{j,i}$" and "$I_{j,i}$" for the marginal utility and inflation rate corresponding to good i, I have assumed implicitly a single composite consumption good. This has been done solely for notational ease. Ex-

[2] Money is defined as assets whose nominal values are always fixed; e.g., currency, coin, non-interest-paying demand deposits. Money is assumed to bring no nonpecuniary utility.

[3] For the purpose of this illustration, constraints on the nonmarketability of (human) capital are ignored.

[4] Stochastic prices and labor incomes are realized at the beginning of the period denoted by their subscript. For example, p_3 is the actual bond price exactly at the start of period 3, two full periods after the beginning of the process. Goods purchases and consumption are assumed to occur during each period after prices and income are observed. Repurchases of assets and money also are assumed to occur during each period at the prices observed at its beginning.

[5] The equations in (4) are obtained by combining several first-order conditions to eliminate Lagrange multipliers. There are also conditions on asset holding which derive from the maximization problem but they will not be discussed in this paper.

plicitly accounting for k different available consumption goods in each period would place k equations in (4) for every one there now. This would cause no special difficulty aside from the inevitable problem of selecting the "correct" commodity price index for empirical work.

For $j \geqq 2$, note the relation in (4) of several stochastic quantities whose equilibration is not under the investor's current control. The sense of these equations relates to consumption decisions upcoming in future periods. As they must depend on previous outcomes, the elements of these expressions are obviously stochastic now; but as the investor will choose optimally for any outcomes whatever, the equalities themselves are certain. They are, in fact, the "stochastic calculus" analog to the dynamic programming assumption that all future decisions will be made optimally conditional on the states that have occurred up until the decision period.

The equations in (4) may seem to imply that uncertainty exists only in period 1. This is not the case. At the end of period 1, asset and consumption prices p_2 and P_2 are known and the consumer makes a second round of choices which brings him to the following positions:

$$E(\tilde{U}_1) = E(\tilde{U}_2)/(1 + I_2),$$
$$E(\tilde{U}_2) = E[\tilde{C}_3/(1 + \tilde{I}_2)],$$
$$\tilde{U}_j = \tilde{U}_{j+1}/(1 + \tilde{I}_{j+1});\ j = 3, \ldots, n - 1.$$

These differ from the initial first-order conditions because I_2 now is nonstochastic; marginal utilities $\tilde{U}_1$ and $\tilde{U}_2$ may still be stochastic even though consumption has occurred because the marginal utility of consumption in an early period may be a function of (stochastic) consumption in later periods. For additive utility functions, however, the first equation above would vanish because U_1 would not depend upon later choices.

III. Simple Models of Nominal Money Supply

Helicopters and Other Mints

The lucidness of Friedman's (1969) hypothetical economy recommends its adoption in the following analysis. Friedman supposes a helicopter bearing fiat money to circle over the economy dispensing (or collecting) bills at given rates. No resources are necessary for these flights nor for producing the added (or decremented) currency. Friedman's purpose was to discover the socially optimum frequency of flights, a topic which depends a great deal on the value of money's specific nonpecuniary services.[6] My purpose, however, is to examine the effect of such flights

[6] See Meltzer (1970) for a clear enunciation of this point.

on current commodity prices (and thus on anticipated inflation rates), and for this the nonpecuniary qualities of money simply get in the way, adding extra terms to the derived equations but no extra substance to the economics. Consequently, all nonpecuniary qualities of money except its perfect nominal safety will be ignored here. Of course, Friedman's essay also discussed the process of inflation; but in its concentration on the major issue of optimum money supply, it neglected the rules used by the helicopter's bombardier to distribute money among individuals.

Consider the following three mutually exclusive simple money supply rules. New money is distributed (or collected) (*a*) proportionately to money balances held in the first period;[7] (*b*) proportionately to money balances held at the beginning of each period; (*c*) randomly (and not proportionately to money balances). These rules are so simple that we can just append them to constraints in each individual's demand problem, to equations (1), (2), or (3) of the last section. With rule *a*, money is distributed during all future periods in amounts proportional to M_1, the quantity of money balances held at the beginning of period 1. The individual still intends to maximize expected utility of consumption, $E[U(C_1, \tilde{C}_2, \ldots, \tilde{C}_n)]$, and his first-period wealth constraint is unchanged. But under the assumption of a growth in money at the rate α, distributed in each future period proportionately to first-period holdings, his second- and later-period constraints become[8]

$$\tilde{P}_j\tilde{C}_j - \tilde{y}_j - (\tilde{M}_{j-1} - \tilde{M}_j) - \tilde{p}_j(\tilde{A}_{j-1} - \tilde{A}_j) - M_1[e^{(j-1)\alpha} - e^{(j-2)\alpha}] = 0; j = 2, \ldots, n-1 \quad (2a)$$

$$\tilde{P}_n\tilde{C}_n - \tilde{y}_n - \tilde{M}_{n-1} - \tilde{p}_n\tilde{A}_{n-1} - M_1[e^{(n-1)\alpha} - e^{(n-2)\alpha}] = 0. \quad (3a)$$

A straightforward application of the stochastic calculus to this problem will provide a different set of first-order extremum conditions than that

[7] This is the rule used in Friedman's simplest model (1969, p. 9).

[8] Since, the growth rate of money is α, we have

Beginning of Period	Total Money Supply, M^s	Increment in Supply from Last Period
1	$M_1^s = M_1^s$	...
2	$M_2^s = M_1^s e^{\alpha}$	$M_1^s(e^{\alpha} - 1)$
3	$M_3^s = M_1^s e^{2\alpha}$	$M_1^s(e^{2\alpha} - e^{\alpha})$
j	$M_j^s = M_1^s e^{j\alpha}$	$M_1^s[e^{(j-1)\alpha} - e^{(j-2)\alpha}]$

In period 2, the investor receives $M_1(e^{\alpha} - 1)$ from the helicopter and he has held M_1 from period 1. Thus, his total money balance is $M_1 + M_1(e^{\alpha} - 1) = M_1 e^{\alpha}$ at the beginning of period 2. He chooses to carry the amount M_2 over from period 2 to 3. Thus his money balance in period 3 is M_2 plus the increment he receives from the helicopter, $M_1(e^{2\alpha} - e^{\alpha})$, which is, by the assumed rule, based on first-period balances; etc., for later periods.

formerly obtained with a fixed money supply. Corresponding to equation (4), which depicted the (fixed-supply) marginal trade-off between commodity price inflation and time preference, one will now find

$$E(\tilde{U}_1) = E[\tilde{U}_2/(1 + \tilde{I}_2)]e^{(n-1)\alpha};$$
$$\tilde{U}_j = \tilde{U}_{j+1}/(1 + \tilde{I}_{j+1});\ j > 1. \tag{4a}$$

For the first-period equation, (4a) indicates a rate of commodity price inflation $n - 1$ times the rate of monetary expansion.[9] For later periods, however, the inflation rate is uninfluenced by money's growth even though the two events occur simultaneously. Given neutral time preference by all consumers, these conditions actually imply a zero rate of change in commodity prices between adjacent periods beyond the first two, which implies that money supply rule *a*, linking all future distributions to first-period balances, must have its entire effect during the first period. The helicopter's promised appearance sets off a clamor for period 1 money balances that only ceases when period 1 prices have been bid down (i.e., money's period 1 commodity price has been bid up), to the point that an inflation rate of $e^{(n-1)\alpha}$ is anticipated between periods 1 and 2. The inflation rate between the first two periods is $n - 1$ times the rate of money expansion then because new money to be dispensed later has been discounted in advance, thus raising even further the benefit of holding money in period 1. Under rule *a*, money balances in period 2 are not linked in any way to future distributions of new money so there is no incentive to increase the demand for them similar to the incentive existing for period 1 balances.

Since this conclusion is at variance with Friedman's,[10] it should be worth considering the assumptive differences which must have been responsible. I believe they can be reduced to a single item. Friedman's individuals are assumed to be either myopic or irrational. His "individual is not able to affect the amount of additional cash he receives by altering the amount of cash he holds" (in the first or in any other period) (1969, p. 9). With this presumption, there is no reason for an

[9] With this rule and with the others to follow, the individual first-order optimum relations sometimes are discussed as if they were market equilibrium conditions. This is strictly valid only if all investors have common subjective probability beliefs, i.e., homogeneous expectations. In that case, ratios of marginal utilities between adjacent periods, (U_j/U_{j+1}), must be the same for all investors in equilibrium because individual optimum conditions such as (4a) bind the marginal utility ratios to (commonly held) assessments of future inflation and future money growth. With nonhomogeneous expectations, expressions with the same form as (4a) would be in effect as market-clearing relations; but the $\tilde{U}_j$'s would have to be interpreted as complex weighted averages of individual marginal utilities.

[10] He concludes that a continual monetary expansion or contraction at the rate e^{α} will bring, after initial short-term adjustments, a continual inflation or deflation at the same rate.

individual to forecast future helicopter flights or to perform any portfolio adjustments that would ordinarily bring gains in expected utility. If the individual were not so restricted, he would find it advantageous to increase money balances in period 1 as I have assumed he actually does. It must be admitted, however, that the hypothetical individual of this paper is on the opposite extreme of the myopia spectrum from Friedman's individual; here, he is presumed to realize the helicopter is coming and to understand its distribution rule perfectly.

Given the monetarily prescient consumer, a most interesting feature of rule *b*, which distributes new money proportionately to balances held at the beginning of each period, is an equivalent market phenomenon to that Friedman has found using rule *a* and a myopic or irrational individual. To prove this, notice that constraints (2) and (3) of the basic maximization problem have different forms under rule *b*. Money is now distributed at a growth rate α to holder of balances at the beginning of each period so that notationally, the constraints for future periods are

$$\tilde{P}_j\tilde{C}_j - \tilde{y}_j - e^{\alpha}\tilde{M}_{j-1} + M_j - \tilde{p}_j(\tilde{A}_{j-1} - \tilde{A}_j) = 0; j = 2, \ldots, n-1, \quad (2b)$$

which is the same as under rule *a* only for period $j = 2$; and

$$\tilde{P}_n\tilde{C}_n - \tilde{y}_n - e^{\alpha}\tilde{M}_{n-1} - \tilde{p}_n\tilde{A}_{n-1}. \quad (3b)$$

After performing a few stochastic calculus operations, one now will find the process of inflation described by

$$E(\tilde{U}_1) = E[\tilde{U}_2/(1 + \tilde{I}_2)]e^{\alpha}$$
$$\tilde{U}_j = [\tilde{U}_{j+1}/(1 + \tilde{I}_{j+1})]e^{\alpha}; j = 2, \ldots, n-1, \quad (4b)$$

which, under certainty and with neutral time preference, would imply an inflation rate equal to the money growth rate in every period.[11]

Now consider the third rule, *c*, whereby new fiat money is distributed without regard to initial money balances. Suppose $\tilde{M}_j^s$ is the (random) number of bills received by an individual during period *j*. If these are truly distributed at random without hope of being affected through resource expenditures in earlier periods, future constraints on the

[11] Under rule *b* it is also easy to confirm the Fisherian relation between nominal interest rates, real (commodity) interest rates, and the anticipated rate of inflation. The cause of an inflation premium in the nominal interest rate is quite removed from Fisher's (1930) explanation, however. The helicopter distributes bills to money holders only and the inflation premium is a penalty demanded by bond-holders to compensate for their not benefiting by the helicopter's munificence. Since the holder of a quantity M_j of money at the beginning of period *j* receives $M_j(e^{\alpha} - 1)$ from the helicopter during *j*, he could only be induced to hold bonds if they were expected to appreciate in nominal value by at least the same relative amount.

individual's maximization problem are altered to become

$$\tilde{P}_j\tilde{C}_j - \tilde{y}_j - (\tilde{M}_{j-1} - \tilde{M}_j) + \tilde{M}_j^s - \tilde{p}_j(\tilde{A}_{j-1} - \tilde{A}_j) = 0; \quad j = 2, \ldots, n-1 \tag{2c}$$

$$\tilde{P}_n\tilde{C}_n - \tilde{y}_n - \tilde{M}_{n-1} + \tilde{M}_n^s - \tilde{p}_n\tilde{A}_{n-1}. \tag{3c}$$

The first-order extremum conditions are exactly the same as those of Section II where the money supply was fixed. This is because $\tilde{M}_j^s$ is not a decision variable. It is not under the individual's control and thus cannot be chosen to achieve an optimum asset portfolio or an optimum consumption sequence. Of course, this does not imply a total lack of effect by M^s on sequences of asset and commodity prices. The basic marginal condition relating to inflation rates will still be equations (4), but these relations arise by equating certain random Lagrange multipliers that will be altered by the additions of more money in the future.[12] In fact, one may definitely infer a change in marginal time preference between the equilibrium conditions under absolutely fixed money supply and under rule *c* variation in supply. If investors anticipate future random money increases, they will attempt to decrease current money balances by exchanging them for assets or commodities. This will have at least two effects. It will increase current commodity prices, P_1, and it will decrease the first marginal utility, U_1, to the extent that risk averters use some newly purchased commodities for period 1 consumption.[13] Starting from a position of neutral time preference, $(U_1 = U_j)$, which implies zero inflation,[14] equilibrium first-order conditions will move to a position

[12] Specifically, the first-order conditions responsible for equations (4), with, for example, $n = 2$, are

First-Order Condition	For Decision Variable
$E(\tilde{U}_1 - \tilde{J}_1 P_1) = 0$	C_1
$E(\tilde{J}_1 + \tilde{J}_2) = 0$	M_1
$\tilde{U}_2 + \tilde{J}_2\tilde{P}_2 = 0$	$\tilde{C}_2$

where the J's are random Lagrange multipliers. J_1 corresponds to the wealth constraint for period 1, $W = P_1C_1 + M_1 + p_1A_1$, and J_2 to the constraint for period 2, $\tilde{P}_2\tilde{C}_2 = M_1 + \tilde{P}_2(\tilde{A}_1) + \tilde{M}_2^s + \tilde{y}_2$. The constraints are, of course, first-order conditions with respect to $\tilde{J}_1$ and $\tilde{J}_2$. Since period 2's constraint is altered by the addition of $\tilde{M}_2^s$, its associated Lagrangian, $\tilde{J}_2$, will also be altered; and since $(\tilde{U}_2/\tilde{P}_2) = -\tilde{J}_2$, the first-order condition for $\tilde{C}_2$, and all other first-order conditions involving $\tilde{J}_2$, will also be changed.

[13] This will be mitigated somewhat by consumers carrying over more commodity inventory to future periods.

[14] It should be emphasized again that the implication of zero inflation (brought about by an assumption of neutral time preference) occurs only if money's nonpecuniary benefits are ignored.

of positive inflation, $P_j > P_1$, and negative marginal time preference, $U_j > U_1$, such that equations (4) still hold. Note again the effect of advanced discounting—future money issues change current prices, P_1, as well as future prices, P_j, even though, in this case, the extent of future price changes will be larger than the current price change.

Under a random distribution rule, therefore, the inflation sequence could possibly match the sequence under rule *b*: a continual price change at the rate of monetary expansion. This is not necessary, however, because consumers have the freedom to purchase and hold assets and commodity inventories rather than money. By purchasing inventories initially, for example, just when the helicopter flights and random mintings are first announced, consumers would cause a much larger price change initially than the monetary expansion in period 1. To offset this, later inflation would have to evolve at a rate lower than the rate of money growth. If asset markets arose to offer current claims on future receipts from helicopter flights, individuals would be able to incorporate titles to random future money issues directly into their initial portfolios. Such money "futures" contracts would have the same effect on consumers' decisions as would period 1 money balances under distribution rule *a*; both assets bring future money mintings under the consumer's ownership.

Consideration of Realism

The helicopter mint is a useful pedagogic device, but it bears little resemblance to the actual method of money issuance currently in vogue. In fact, the present issuer uses a sizable fraction of new fiat money to buy helicopters and pilots (and other real goods and services) with the purpose of dispensing objects more lethal than paper bills. The acquisition of real commodities through fiat money issuance complicates life for the economist as well as for the citizen because it necessitates the consideration of commodity production. A realistic money supply rule just cannot be appended to the individual's first-order demand conditions. Rather, it must encompass the rules used by government to select commodities for purchase with newly minted money and the rules used to determine how those commodities will be reallocated among individuals.

In the simple world where fiat money was distributed directly to individuals, a dynamic equilibrium between the rate of monetary expansion and the rate of inflation was found when individuals received money or were relieved of money in proportion to their just previous nominal money balances. This result was brought about by rational and accurate forecasters who discounted in advance the future production of new money and caused current balances to fluctuate in value with changes in their explicit claims to future money issues. The fact of government dispensing new money indirectly, by buying and distributing commodities, does not logically require a change in the presumption of rationality and in the forecasting ability of individuals.[15] Thus, if government-purchased commodities are redistributed to individuals in proportion to their former nominal money balances, and if those commodities are marketable (transferable), advanced discounting of future receipts of goods would increase the current demand for nominal money balances. For example, a situation wherein commodities purchased by the government in each period are distributed proportionately to money balances held at the beginning of the first period is effectively the same as helicopter rule *a* and will bring the same inflation sequence, namely, a very large inflation in the first period and zero inflation thereafter.

Again speaking of things as they actually are, the government neither distributes new money nor taxes old balances proportionately. It may indeed distribute goods and services roughly in relation to wealth; but wealth is composed of many other assets besides money, and for an exact simultaneous relation to exist between money and inflation the link of new money issues to old balances must be exactly proportional.

Little additional time need be devoted to the possibility of private asset supply. Since private supplies are equivalent in effect to short sales, they have been implicitly accommodated on the demand side and thus will alter no conclusions by being explicitly considered. If legal restrictions on privately validated, non-interest-bearing currency were to be removed, their issuers would assume the status of this paper's government, and the time sequences of that currency's commodity value would be determined by the identical mechanism. Interest rates contracted in that currency would be governed similarly.

Finally, we must hedge the present analysis against possible real effects of money. If a position of underemployment (short-run) equilibrium can be alleviated by a monetary expansion, the link between nominal money supply and commodity price level becomes much more tenuous. To put it simply, in a condition of unemployment, newly minted money could be used to coax production from idle capital and labor; at least this seems to be the prevailing opinion among macroeconomists. If this is indeed true, then no inflation tax would inure to money holders, and the resulting rates of inflation would be lower than the rates implied earlier in this paper regardless of the money distribution rule.

[15] However, it most definitely changes the achievable level of utility of each individual since it alters the productive mix of commodities by favoring those the government happens to choose for its reallocation endeavors. Naturally, I have neglected many important matters here such as the production of some goods, say nuclear weapons, which because of difficulty in charging benefits, external effects, nontransferability, etc., would not have existed without the government's demand for them.

References

Fisher, Irving. *The Theory of Interest.* New York: Macmillan, 1930.

Friedman, Milton. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays.* Chicago: Aldine, 1969.

Kushner, Harold J. "On Stochastic Extremum Problems: Calculus." *J. Math. Analysis and Appl.* 10 (April 1965): 354–67.

Long, John B., Jr. "Consumption Investment Decisions and Equilibrium in the Securities Market." In *Studies in the Theory of Capital Markets*, edited by Michael C. Jensen. New York: Praeger, 1971.

Meltzer, Allan H. "Discussion." *J. Finance* 25 (May 1970): 450–53.