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Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure

This paper is concerned mainly with the effects of corporate income taxes on the relationship between capital structure and valuation. If the interest tax savings cease once a firm has gone bankrupt, it is apparent that the issue of additional debt will have two effects on the value of the firm: on one hand, it will increase the tax savings to be enjoyed so long as the firm survives; on the other hand, it will reduce the probability of the firm's survival for any given period. Depending on which of these conflicting influences prevails, the value of the firm may increase or decrease as additional debt is issued. The option pricing framework is used to relate the value of a levered firm to the value of an unlevered firm, the amount of debt, and the time to maturity of the debt.

I

Most aspects of the theory of capital structure and valuation in perfect capital markets with no corporate income tax have by now been investigated.[1] Somewhat less attention, however, has been directed to the effects of the corporate income tax on the relationship between capital structure and valuation,[2] although it is the corporate income tax which lends most of the interest to the optimal capital structure problem and although the conclusions reached by Modigliani and Miller (1963) were much more tentative when corporate income taxes were introduced.

Analysis of the effects of the corporate income tax is of interest not only because corporate income taxes do in fact exist but because the analysis appears to lead to the conclusion that an optimal capital structure will consist almost entirely of debt. This conclusion leads to inconsistency between the premise that managements act so as to maximize the wealth of stockholders and the empirical observation that most firms eschew highly levered capital structures. Modigliani and Miller themselves attribute this discrepancy be-

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1. Durand (1952), Modigliani and Miller (1958), Solomon (1963), Stiglitz (1969, 1972), Smith (1972), and Milne (1975).
2. See, however, Hamada (1969), Kraus and Litzenberger (1973), and Rubinstein (1973).

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tween the predictions of their model and the observed reality to the effects of the personal income tax, which may make retained earnings a cheaper source of finance than debt, and to "the need for preserving flexibility," which is "not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety" (1963, p. 443). Subsequent writers have attempted to explain the low levels of leverage observed by resort to "bankruptcy costs" (Robichek and Myers 1965; Kraus and Litzenberger 1973), though the nature and empirical significance of these costs has received scant attention (however, see Baxter 1967; Warner 1975), suggesting that they may perhaps be no more than a convenient hypothesis to reconcile the theory with the data. Finally, observed capital structures may be explained by the hypothesis that managements are more interested in their own job security and other agency costs, and hence in the avoidance of bankruptcy, than they are in the maximization of the wealth of stockholders (Donaldson 1963; Jensen and Meckling 1976).

This paper is concerned mainly with relaxing the assumption that the tax savings due to debt issuance constitute a "sure stream." Modigliani and Miller themselves acknowledge that "some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets" (1963, n. 5). They attribute this uncertainty to two causes: first, the possibility of future changes in the tax rate and, second, the possibility that at some future date the firm may have no taxable income against which the interest payments on the debt may be offset.

In this paper we consider the latter possibility, noting in particular that once a firm has gone bankrupt the interest tax savings will cease. Once this possibility is acknowledged, it is apparent that the issue of additional debt will have two effects on the value of the firm: on one hand, it will increase the tax savings to be enjoyed so long as the firm survives; on the other hand, it will reduce the probability of the firm's survival for any given period. Depending on which of these conflicting influences prevails, the value of the firm may increase or decrease as additional debt be issued. It seems reasonable to suppose a priori that, as additional debt is issued from a small base, the survival probabilities of the firm will not be substantially affected, so that the former influence will outweigh the latter and the value of the firm will increase, but that at high initial levels of debt further increments of debt may so affect the survival probabilities that the value of the firm will actually decrease. If such is the case, then an optimal capital structure may exist even without the existence of bankruptcy costs.

The analysis of this paper rests on the Modigliani-Miller (1958) risk-class assumption and compares the value of the levered firm with an otherwise-identical unlevered firm. By assuming that the levered and unlevered firms are identical in all respects save their capital structures, we are neglecting the possibility that the managers of the levered firms will be induced to alter their investment decisions by the structure of the firm's liabilities. This may occur first because managers are concerned to avoid bankruptcy and its attendant costs and second because in making investment decisions they take account of redistribution effects between bondholders and stockholders.[3] Following Modigliani and Miller, we take as given the existence of an unlevered firm and do not enquire why such a firm should exist if there is any advantage to the issue of debt. The value of the levered firm is assumed to be a function of the value of the unlevered firm, the amount of debt outstanding, and the time to maturity of the debt. This specification, together with the assumption that the value of the unlevered firm follows a Gauss-Wiener process, enables us to derive a differential equation relating the values of the unlevered firm and the levered firm. This differential equation is identical with that derived by Black and Scholes (1973) and Merton (1973) for the option-pricing problem. Specification of appropriate boundary conditions then permits a solution of the differential equation, yielding the value of the unlevered firm in terms of the value of the levered firm, the par value of the outstanding debt (and the interest rate on the debt), and the maturity of the debt. Since the methodology employed is sufficiently flexible to allow for bankruptcy costs also, without significant additional complications, and since many authors have regarded these as important, they are included in the model formulation and in one of the numerical examples which follow.

II

It is assumed that the market value of a levered firm, V, may be written solely as a function of the value of an otherwise identical unlevered firm, U, which belongs to the same risk class and has the same earnings and investment policy; the face value of the debt outstanding, B, and the coupon rate on the debt, i; and time, t, which enters the valuation expression because the debt is assumed to have a finite maturity, T. Thus the value of the levered firm may be written as

$$V \equiv V(U,B,i,t). \tag{1}$$

Simple equilibrium considerations dictate that the ratio V/U must be a function solely of the leverage ratio[4] B/U, i, and t, so that (1) may be written as

$$V = UV(1,B/U,i,t). \tag{2}$$

3. Note that the firm-value-maximizing investment policies will be dependent on capital structure to the extent that bankruptcy costs are significant. For a detailed discussion of the distinction between firm-value-maximizing investment decisions and equity-value-maximizing decisions, see Galai and Masulis (1976).

4. This follows from the basic homogeneity property that two levered firms with the same leverage ratio have twice the value of one.

Equation (2) will prove useful when we wish to examine the relationship between V and the leverage ratio for a given value of U. We shall actually solve (1) for different values of U and a fixed value of B. Equation (2) then permits us to derive from that solution V in terms of B for a given value of U.

Since we shall be considering the relationship between V and U for fixed values of B and i, it will be convenient to write (1) as

$$V \equiv V(U,t). \tag{3}$$

In between dividend payments which are assumed to occur at discrete intervals, the value of the unlevered firm is assumed to follow the Gauss-Wiener process[5]

$$\frac{dU}{U} = \mu\, dt + \sigma\, dz, \tag{4}$$

where dz is a Gauss-Wiener process and $E[dz] = 0$, $E(dz)^2 = dt$.

Then, assuming that trading takes place continuously, it can be shown that arbitrage considerations imply that the value of the levered firm between dividend dates must follow the differential equation

$$\tfrac{1}{2}\sigma^2 U^2 V_{uu} + rUV_u + V_t - Vr = 0, \tag{5}$$

where r is the known, constant, risk-free rate of interest and subscripts denote partial derivatives. Equation (5) is derived by forming a zero net investment portfolio consisting of investments in the levered firm, the unlevered firm, and the risk-free asset such that the return on this portfolio must be nonstochastic; its return must therefore be zero.

We consider next the boundary conditions which must be satisfied by the differential equation.

First, at the maturity of the debt, T, we have

$$\begin{aligned} V(U,T) &= U \text{ for } U \geq B, \\ &= U\text{-}C(U), \text{ for } U < B, \end{aligned} \tag{6}$$

where $C(U)$ is the bankruptcy costs that will be incurred if a firm whose unlevered value is U files for bankruptcy. To consider first the case in which $U \geq B$ so that bankruptcy at maturity is avoided, the value of the levered firm is equal to the value of the unlevered firm. This is because we are considering the effect on firm value of a single issue of debt, so that when that debt matures, if bankruptcy is avoided, levered and unlevered firms are in all respects identical and must therefore command the same market value. No loss of generality is involved in this assumption, since the date at which the levered firm reverts to being an

5. This stochastic process has been frequently employed in models of this kind (Black and Scholes 1973; Merton 1973); it implies that the value of U at any future instant follows the lognormal distribution.

unlevered firm may be made as distant as desired.[6] If $U < B$, the levered firm must file for bankruptcy, in which case costs $C(U)$ are incurred which serve to reduce the value of the levered firm below that of the corresponding unlevered firm.

Since the assets of the levered and unlevered firms are at all times identical, we assume that they both pay identical constant dividends,[7] D. Then at a dividend date the following boundary condition holds:

$$V(U,t^-) = V(U\text{-}D,t^+) + D, \tag{7}$$

where t^- and t^+ denote the instants before and after the dividend, respectively. Equation (7) expresses the cum-dividend value of the levered firm as the sum of its ex-dividend value and the value of the dividend paid. Note that the dividend payment causes the value of the unlevered firm to fall by the amount D.

The dividend may be made a function of the firm's economic fortunes by making it a function of U and t. Moreover, a negative value of D may be introduced to represent the issue of additional stock. In the interests of simplicity, we assign a fixed value to D.

In addition to paying dividends, the levered firm must also make periodic coupon payments, iB. Each coupon payment will reduce the value of the firm's assets by $iB(1\text{-}\tau)$ after giving effect to the concomitant tax savings, where τ is the corporate tax rate. Since the assets of the levered firm are by assumption identical with those of the unlevered firm which makes no such coupon payments, it is necessary to assume that the assets of the levered firm are restored by the issue of stock in the amount $iB(1\text{-}\tau)$. Then at a coupon payment date, t,

$$\begin{aligned} V(U,t^-) &= V(U,t^+) + iB - (1\text{-}\tau)iB, \\ &= V(U,t^+) + \tau iB. \end{aligned} \tag{8}$$

Equation (8) is derived by expressing the precoupon value of the levered firm as the sum of its postcoupon value and the coupon received, less the value of the stock sold to make the coupon payment $(1\text{-}\tau)iB$.

To simplify subsequent numerical examples, it is assumed that dividend and coupon payments are made on the same day, so that, if (7) and (8) are combined, the joint effects of coupon and dividend payments are represented by the boundary condition

$$V(U,t^-) = V(U\text{-}D,t^+) + D + \tau iB. \tag{9}$$

The remaining boundary condition relates to the conditions under which the firm becomes bankrupt and the value of the firm when it becomes bankrupt. The value of a bankrupt levered firm is equal to the

6. This point will be illustrated later in conjunction with figs. 5 and 6.

7. We could have assumed equivalently that the levered firm retired a corresponding amount of debt and equity as long as the ratio B/U was maintained.

value of an identical unlevered firm less the costs of bankruptcy, since the assets of the levered firm, to be taken over by the creditors, are by definition identical with those of the unlevered firm before allowance is made for the costs of bankruptcy.

It is assumed that the firm will become bankrupt if, on a coupon date, the value of its assets (i.e., the corresponding value of an unlevered firm) is less than some critical value. The lowest conceivable critical value is $(1-\tau)iB$, the net interest obligation; a more reasonable one, and the one adopted here, is the par value of the outstanding bonds, B, so that the firm is assumed to become bankrupt when its value falls to the par value of its outstanding bonds.

Then, if we combine this condition with (9), the value of the firm on a coupon/dividend date is given by

$$\begin{aligned} V(U,t^-) &= V(U\text{-}D,t^+) + D + \tau iB \quad \text{for } U \geqslant B, \\ &= U\text{-}C(U) \qquad\qquad\qquad\quad \text{for } U < B. \end{aligned} \tag{10}$$

In summary, the value of the levered firm is given by the solution to the differential equation (5), subject to the boundary conditions (6) and (10). One further boundary condition is required for the solution algorithm: This is

$$\lim_{U \to \infty} V_u = 1. \tag{11}$$

Equation (11) follows from the consideration that, as the value of a levered firm becomes indefinitely large for a given level of debt commitments, the probability of default becomes arbitrarily small, so that the tax savings become essentially riskless, and the formula analysis of Modigliani and Miller (1963) applies, which implies (11).

Although there exists no closed-form solution to the differential equation subject to the foregoing boundary conditions, a solution may be readily obtained by the use of numerical methods (Brennan and Schwartz 1977). The following section presents the results of such numerical analysis.

III

The data for the basic example, given in table 1, relate to a 25-year bond issue by a firm which pays no dividends and would issue no stock if it were not debt financed.[8] The differential equation was solved for a given level of B ($200), yielding V as a function of U. Equation (2) was then used to derive the ratio V/U as a function of the ratio B/U, and in figure 1 V/U is plotted as a function of the leverage ratio B/V. As is evident from this figure the conjecture is borne out that the possibility

8. To avoid confounding the effect of investment policy and financing policy, it will be recalled that we assumed that the levered firm issues sufficient stock to offset its net interest payment; see eq. (8).

TABLE 1 Parameters for Basic Example

Debt maturity (T)	25 years
Debt par value (B)	$200
Debt coupon rate (i)	.07/year
Risk-free interest rate (r)	.06/year
Corporate tax rate (τ)	.50
Unlevered-firm variance rate (σ^2)	.05/year
Aggregate annual dividend (D)	$0
Bankruptcy cost (as fraction of firm value at bankruptcy) (BC)	.00

of a bankruptcy, with the resulting uncertainty of tax savings, is sufficient to lead to an optimal capital structure even in the absence of bankruptcy costs. For the given parameters, the optimal leverage ratio, B/V, is 54%, and at the optimum the value of the levered firm exceeds that of the unlevered firm by 23%—the maximum leverage premium.[9]

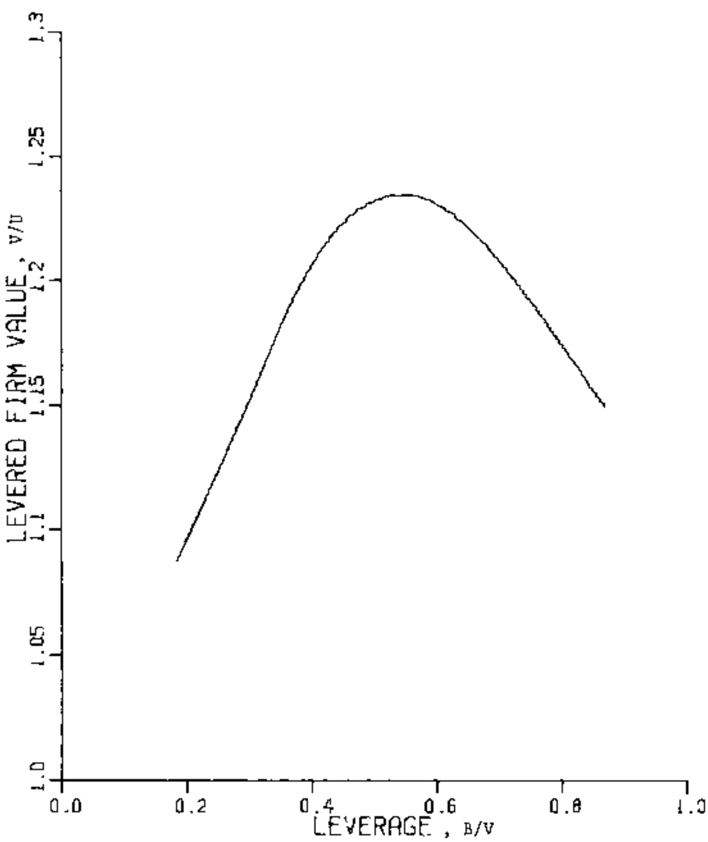

Fig. 1.—Firm value as a function of leverage: basic example

9. Care should be taken in interpreting this leverage ratio, which is the ratio of the book value of a 7% bond to the market value of the firm. It is likely that this ratio understates the true market-value leverage ratio for low values of the ratio and overstates it for high values of the ratio, since the relationship between the bond's market and book values will depend on the risk of the bond and hence on the leverage ratio.

Figure 2 illustrates the effect of business risk on the value of the levered firm and the optimal leverage ratio. Different levels of business risk are represented by different variance rates for the unlevered firm. As would be expected, the more risky the firm (the higher the variance rate), the less the advantage of debt and the lower the optimal leverage ratio, because of the higher probability of bankruptcy for a given leverage ratio. Thus for a variance rate of 0.02 the optimal leverage ratio is 60% and the maximum leverage premium is 30%, while for a variance rate of 0.08 these figures drop to 51% and 22%, respectively.

In figure 3 the relationship between value and leverage is shown for different payout policies. This figure should not be construed as indicating that dividend policy matters in the Modigliani-Miller sense (Miller and Modigliani 1961), for in this model dividends are not offset by stock issues while investment remains constant: rather, the different dividends represent different investment policies, so that a bigger dividend corresponds to a reduction in the exogenously determined asset growth rate. Not surprisingly, the reduced asset growth rate, like

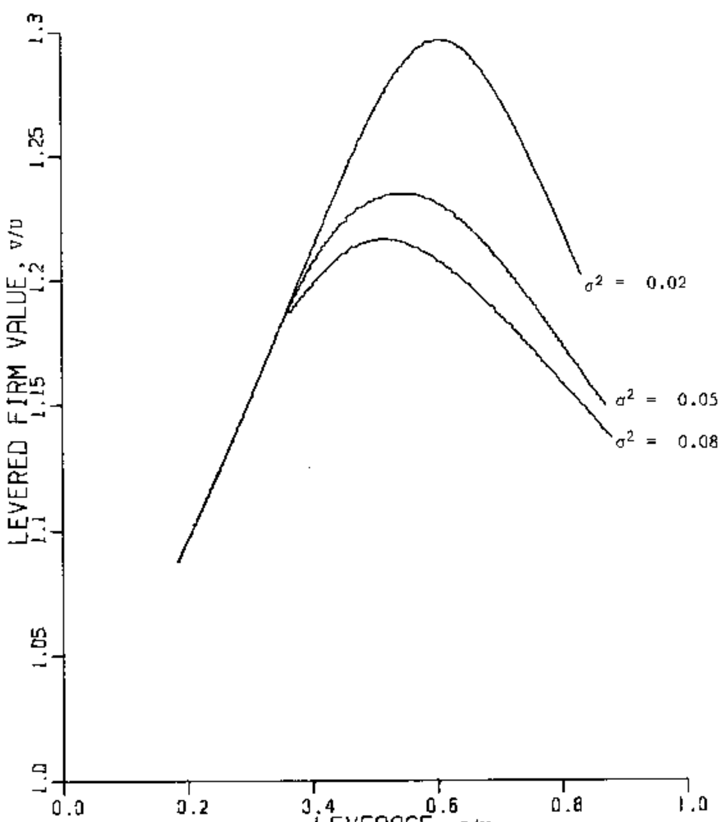

FIG. 2.—Business risk, firm value, and leverage

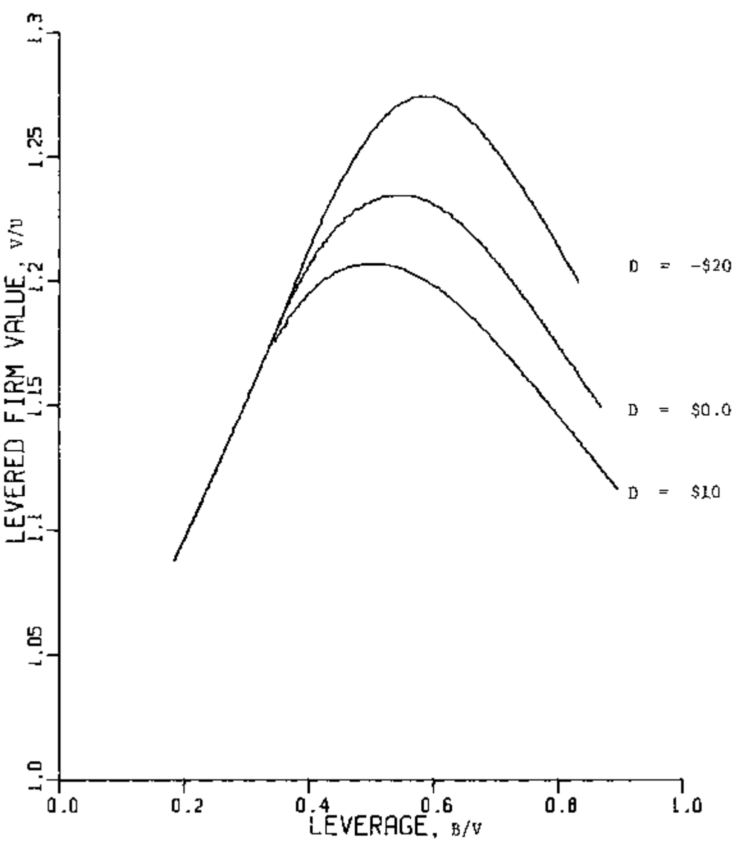

FIG. 3.—Payout policy, firm value, and leverage

increased business risk, raises the probability of bankruptcy for a given leverage ratio, so that the greater the dividend, the less the advantage of debt and the lower the optimal leverage ratio. The negative dividend of \$20 represents an annual stock issue of \$20 (10% of the par value of the debt); this exogenous infusion of funds enhances the security of the debt and reduces the probability of bankruptcy. For $D = -\$20$ the optimal leverage ratio is 59% and the maximum leverage premium is 27%, while for $D = \$10$ the corresponding figures are 50% and 21%. Clearly an important determinant of the value of a levered firm and of its optimal capital structure is its investment policy (stock issues less dividends): the lower this net investment, the lower the value of the firm and the optimal leverage ratio.

In figure 4, the influence of proportional bankruptcy costs is depicted; the differential bankruptcy costs are assumed to be proportional to the value of the unlevered firm at the time of bankruptcy. The additional influence of bankruptcy costs over and above the tax effect is perhaps surprisingly small at the optimum. Even with a

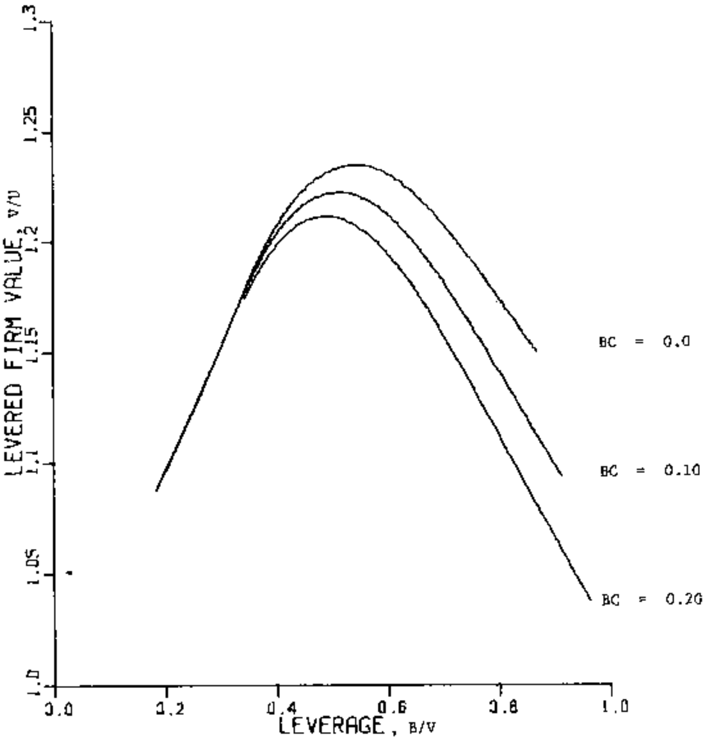

FIG. 4.—Bankruptcy cost ratio, firm value, and leverage

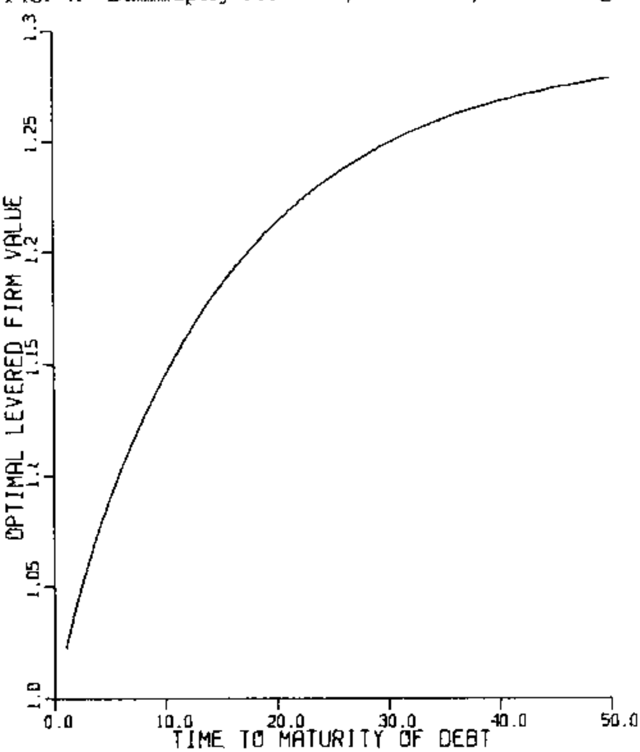

FIG. 5.—Optimal levered-firm value and time to maturity of debt

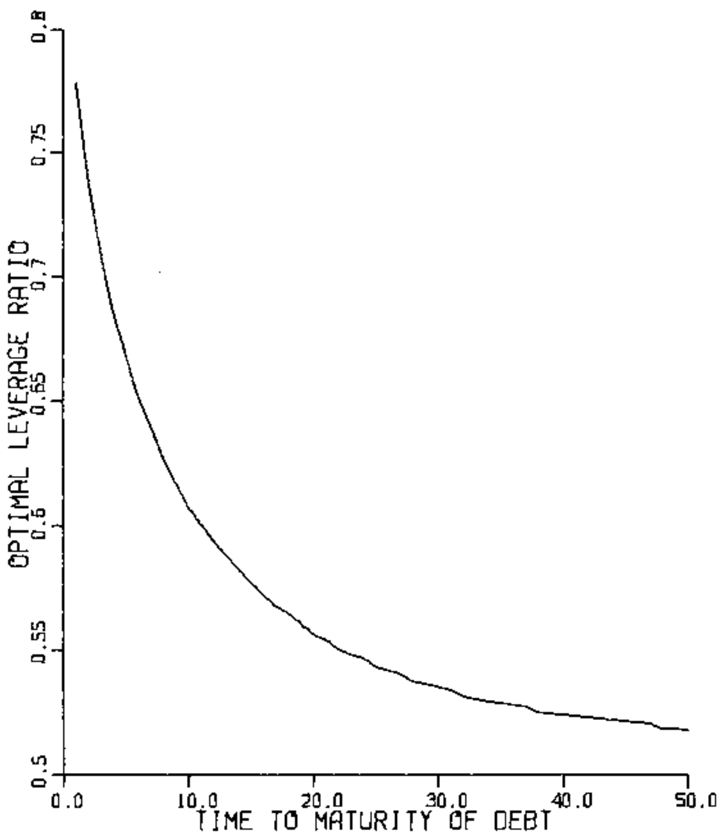

FIG. 6.—Optimal leverage ratio and time to maturity of debt

differential bankruptcy cost ratio (*BC*) of 20%, the optimal leverage ratio only falls from 54% to 49%, while the maximum leverage premium falls from 23% to 21%.

Finally, figures 5 and 6 show the effect of the maturity of the debt on the optimal value of the levered firm and the optimal leverage ratio, respectively, for the basic example. As the maturity of the debt increases, the corresponding maximum leverage premium tends asymptotically to 28% and the optimal leverage ratio to 52%. We see that, by making the date at which the levered firm reverts to being an unlevered firm distant enough, we can obtain results which approximate the permanent (infinite maturity) debt case considered by Modigliani and Miller.

Note that, due to the terminal boundary condition (6), the optimal leverage ratio and firm value depend on the maturity of the debt. This is because we are considering the incremental effects of a single debt issue. In reality, short-term debt may be rolled over, and, in the absence of transactions costs, it will be optimal to issue and redeem debt continuously, for in this way bankruptcy may be avoided while the tax savings are still enjoyed.

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