

Pricing Options on Agricultural Futures: An Application of the Constant Elasticity of Variance Option Pricing Model

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The pricing of options on futures has generated much recent interest from both an academic as well as a trading perspective. These contingent claims provide new avenues for the allocation of price risk among investors and have been well received by the financial markets. For example, options on Treasury bond futures began trading at the Chicago Board of Trade in 1982 and have been a very successful innovation. This success has assured the commodity exchanges of the value of options trading, and the development of options on other types of futures is being accelerated. Currently, options on certain agricultural futures are being traded in the United States under a three-year pilot program administered by the Commodity Futures Trading Commission.

Many recent academic studies have made significant contributions to option pricing theory using varying asset price behavior assumptions. Among these are the Black-Scholes formula (1973), Roll's American call option formula (1977), Cox's constant elasticity of variance (CEV) formula (1975), Merton's jump-diffusion formula (1976), Binomial pricing method (Cox, Ross, and Rubinstein, 1979), and various numerical methods for option pricing.

Each of the above formulas and methods is based on the continuous-time (or limiting case)¹ no-arbitrage pricing framework of Black and Scholes. Within this

¹The binomial pricing methodology suggested by Sharpe and presented by Cox and Ross (1979) is essentially a discretization of the continuous time sample path of asset prices. As the number of intervals utilized by the binomial method approaches infinity, the pricing results obtained are indistinguishable from those obtained by continuous time methods.

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framework, however, each model makes different assumptions about the explicit stochastic behavior of the underlying asset's price. Consequently, it is the stochastic behavior of the underlying asset's price which determines the proper model to use in option pricing.

Options on futures have drawn special attention because of their unique characteristics. For example, several studies have specifically approached the pricing of options on futures by utilizing a modified form of the Black-Scholes (B-S) formula (e.g., Asay, 1982; Black, 1976). However, this approach may not be appropriate for options on agricultural futures since the dynamic behavior of agricultural futures prices violates the underlying B-S assumptions. Consequently, this article addresses the issue of an appropriate pricing model for options on agricultural futures. Specifically, the objectives of the article are:

- (1) To discuss the stochastic behavior of agricultural futures prices and to identify how these prices violate the underlying B-S assumptions.
- (2) To identify alternatives to the B-S formula which are more consistent with the stochastic behavior of agricultural futures prices.

The remainder of the article is organized in four sections. Section I discusses the stochastic behavior of agricultural futures prices and its ramifications for option pricing. The empirical results focus specifically on soybean futures. Section II presents empirical evidence that the stochastic behavior of soybean futures prices is well represented by Cox's CEV model (1975). Section III applies Cox's CEV closed form solution to options on soybean futures and discusses the implications of differences between the B-S and CEV models. Section IV summarizes the results of the study.

I. STOCHASTIC BEHAVIOR OF AGRICULTURAL FUTURES PRICES

It is of critical importance in pricing contingent claims such as options that the model of stochastic price behavior used conform closely to price behavior actually observed. Empirical evidence suggests there are at least two major differences between the behavior of agricultural and nonagricultural futures prices such as for Treasury bond, GNMA, or stock index contracts, or prices for financial assets such as stocks or Treasury bills.² Both of these differences and their respective implications for option pricing are discussed in this section.

The first major difference between agricultural and nonagricultural prices arises because nonagricultural prices tend to follow a random walk, whereas a number of academic studies have demonstrated that agricultural futures prices follow a seasonal pattern reflecting the annual production-consumption cycle (e.g., Tomek and Robinson, 1981; Working, 1958, 1960; Vaughn, Kelly, and Hochheimer, 1981). This pattern of seasonality would also be present in the first difference of the logarithm of futures prices if seasonality were present in the original price series. Note that the first difference of the logarithm of futures prices,

$$R_t = \ln(F_t) - \ln(F_{t-1}) \quad (1)$$

²See Fama (1976), Tomek and Robinson (1981), Working (1958, 1960), and Vaughn, Kelly, and Hochheimer (1981) for discussions of the empirical evidence.

where F_t and F_{t-1} represent the futures price at time t and $t-1$, can be considered as the analogue of a continuously compounded return. This quantity, R_t , will thus be referred to as the quasi-return³ on a futures contract in this article.

Surprisingly, this seasonal pattern would not be a violation of the fundamental stochastic process assumptions of continuous time option pricing models such as the B-S. This can be seen by briefly reviewing the roles that the expected and unexpected (measured by volatility) components of instantaneous return play in the pricing of contingent claims in continuous time frameworks, which are described below.

In deriving closed form option pricing formulas, Black and Scholes (1973), Merton (1977), and others (Smith, 1976) assume that the dynamic behavior of the underlying asset's price can be described by the following diffusion process in continuous time:⁴

$$\frac{dA}{A} = \xi(A,t)dt + \sigma(A,t)dz \quad (2)$$

where

A ≡ Price of underlying asset.

t ≡ Time.

$\frac{dA}{A}$ ≡ Instantaneous return of underlying asset.

$\xi(A,t)$ ≡ Deterministic (or expected) component of instantaneous return as a function of both A and t ; often referred to as the mean of return.

$\sigma(A,t)$ ≡ Stochastic (or unexpected) component of realized instantaneous return as a function of both A and t ; often referred to as the standard deviation of return.

dz ≡ Standard Wiener process.

Stating the price dynamics as an Ito process allows the use of an arbitrage approach in pricing options since the instantaneous change in the option price is perfectly correlated with the instantaneous change in the underlying asset price (dA) by Ito's Lemma. By choosing an appropriate ratio of options to the underlying asset, a hedged portfolio that is instantaneously risk-free can be constructed. As a result, the constructed hedge portfolio earns the risk-free rate irrespective of the expected return on the underlying asset. Consequently, the expected return of the underlying asset $\xi(A,t)$ does not appear in the solution for option prices.⁵ As a result, any pattern of seasonality in quasi-returns would have no explicit implications for pricing options. However, the unexpected component of return, $\sigma(A,t)$, does enter the

³Since futures contracts are marked-to-market at the end of each trading day, their value is zero at the end of each day (see Black, 1976). Consequently, there is no meaningful sense in which we can discuss the return on a futures contract. However, if an asset or portfolio had a value equal to the futures price each day, then the quasi-return would be the asset or portfolio's return.

⁴For a discussion of stochastic calculus and Ito's Lemma, see Ito and McKean (1964) and Merton (1971).

⁵Smith (1976) presents a simple derivation of the Black-Scholes formula illustrating how the $\xi(A,t)$ term is eliminated from the partial differential equation by the no-arbitrage requirement that risk-free portfolios must earn the risk-free rate in capital market equilibrium.

solution. For example, Black and Scholes assume $\sigma(A,t)$ to be constant in their derivation, with the result that volatility figures prominently in their closed form solution.⁶

The behavior of agricultural and nonagricultural futures prices differs in a second fundamental way. Since the resolution of uncertainty in future commodity prices occurs seasonally, the standard deviation of the quasi-return [or $\sigma(F,t)$] is strongly seasonal. By contrast, nonagricultural and financial asset returns are realistically described as homoskedastic over short periods.⁷

This seasonal trend is documented in Table I which presents the results of a test for seasonality in the monthly volatility of March, July, and November soybean futures prices from 1979 to 1983. The seasonal model tested for each of the three series was:

$$V_t = \beta_0 + \sum_{i=1}^{11} \beta_i D_{it} + \epsilon_t \quad (3)$$

where

V_t \equiv Estimated standard deviation of the quasi-return for month t based on daily data.

D_{it} \equiv Seasonal dummy variables for month t : $i = 1$, June; $i = 11$, April.

ϵ_t \equiv Error term assumed to follow AR(1) process.

As illustrated, each of the seasonal models was estimated by the Cochrane–Orcutt⁸ procedure and was statistically significant at the 95% level based upon an F test. In addition, several of the t statistics for individual coefficients were significant.

This pattern of seasonal volatility violates the constant variance assumption explicit in the B–S formula. Consequently, the use of the B–S formula for pricing options on agricultural futures is inappropriate. In order to correctly price these options using the continuous time framework, the time series pattern of volatility would need to be modeled. By a process similar to that outlined above for the B–S formula,⁹ a risk-free hedge portfolio could be formed resulting in another partial differential equation. In this differential equation, however, the instantaneous standard deviation of the quasi-return $\sigma(F,t)$ would be a time series model rather than a constant. This complexity would in general make it impossible to find a closed form solution for option prices; the differential equation would need to be solved numerically, making this approach of limited practical use.

Fortunately, a closed form solution is available when the seasonality in volatility is of a specific type. Cox's CEV model (1975) assumes that volatility is a function of the price level; that when the price level is high, volatility is also high (or low depending upon the elasticity coefficient) and vice versa. To the extent that the CEV relationship between agricultural futures prices and quasi-return volatility is

⁶Note that the boundary conditions of the partial differential equation are also unaffected by the seasonal quasi-return.

⁷See Fama (1976).

⁸Ordinary least squares generally displayed autocorrelated residuals. Consequently, the Cochrane–Orcutt procedure was employed to yield consistent parameter estimates improving the power of the test.

⁹See the generalized model for pricing contingent claims presented by Merton (1977).

Table I
COCHRANE-ORCUTT ESTIMATES OF SEASONALITY COEFFICIENTS,
MARCH, JULY, AND NOVEMBER SOYBEAN FUTURES,
1979-1983

	March		July		November	
	Coefficient	<i>t</i> Statistic	Coefficient	<i>t</i> Statistic	Coefficient	<i>t</i> Statistic
β_0	13.76	4.9 ^a	13.86	5.4 ^a	12.62	4.4 ^a
β_1	7.04	2.5 ^a	6.33	2.3 ^a	9.74	3.4 ^a
β_2	12.70	3.7 ^a	12.98	4.4 ^a	15.41	4.4 ^a
β_3	10.74	2.9 ^a	10.59	2.9 ^a	13.28	3.6 ^a
β_4	9.38	2.5 ^a	9.19	2.5 ^a	10.39	2.8 ^a
β_5	6.24	1.6	5.06	1.3	6.34	1.7
β_6	6.33	1.6	5.52	1.5	6.39	1.8
β_7	4.41	1.1	4.64	1.2	5.69	1.5
β_8	2.88	0.8	6.25	1.7	5.92	1.5
β_9	3.24	1.0	4.90	1.3	2.88	0.8
β_{10}	5.68	1.8	3.69	1.0	2.67	0.8
β_{11}	4.69	1.7	4.84	1.7	3.48	1.2
	Adj. R^2 : 0.081		Adj. R^2 : 0.132		Adj. R^2 : 0.152	
	S.E. : 5.95		S.E. : 6.29		S.E. : 6.07	
	D.W. ^b : 2.12		D.W. ^b : 2.31		D.W. ^b : 2.09	
	$F_{11, 69}$: 1.57		$F_{11, 70}$: 1.99 ^a		$F_{11, 70}$: 2.19 ^a	

^aSignificant at 95% level.

^bFor residuals after Cochrane-Orcutt estimation.

present, the stochastic process assumptions of Cox's model will be satisfied and will encompass the seasonal behavior of the volatility. Consequently, although the seasonality in volatility violates the B-S Model assumptions, it may be completely consistent with those of the CEV model. The strength of the price volatility relationship and the explanatory power of the CEV model for seasonal volatility is empirically evaluated in the following section.

II. EMPIRICAL TESTS OF THE CONSTANT ELASTICITY OF VARIANCE RELATIONSHIP

The CEV model described by Cox (1976) assumes that price changes in the underlying asset can be described by the following diffusion process:

$$\frac{dA}{A} = \xi(A,t) dt + \sigma A^{\psi-1} dz \quad (4)$$

where:

- A ≡ Price of underlying asset.
 t ≡ Time.
 $\frac{dA}{A}$ ≡ Instantaneous return of underlying asset.
 $\xi(A, t)$ ≡ Deterministic (or expected) component of instantaneous return; a function of both A and t .
 σ ≡ Annualized expected standard deviation of quasi-return when $A_t = 1$.
 ψ ≡ Coefficient of elasticity.
 dz ≡ Standard Wiener Process.

In order to determine the extent to which agricultural futures prices satisfy the assumptions of the CEV model, historical relationships between price and volatility were examined. The strength of the CEV relationship between price level and volatility can be estimated by linearizing the model's parameterization:¹⁰

$$\sigma_t = \sigma F_t^{\psi-1} \quad (5)$$

by a logarithmic transformation yielding:

$$\ln \sigma_t = \ln \sigma + (\psi - 1) \ln F_t \quad (6)$$

where:

- σ_t ≡ Estimated standard deviation of quasi-return for month t .
 F_t ≡ Average futures price for month t .

Note that if $\psi = 1$, then the CEV model reduces to the constant volatility B-S Model.

The linearized relationship was then estimated by the Cochrane-Orcutt methodology given appropriate assumptions about the regression model's error structure.

Table II presents the results of this estimation technique for the March, July, and November soybean futures contracts expiring in 1979 through 1983. As shown, each of the three regressions was statistically significant at the 99% level. This provides strong support for the hypothesis that the assumptions of the CEV model are well satisfied by soybean futures prices. Estimates of elasticity obtained by the Cochrane-Orcutt procedure ranged from 2.19 to 2.51.¹¹

Figures 1 through 3 show graphically the positive relationship between soybean futures price level and the volatility of the quasi-return. In summary, the seasonal volatility in soybean futures quasi-returns appears to be well adapted to the CEV diffusion process assumption of Cox's model.

¹⁰See Jarrow and Rudd (1983) for a discussion of this linearization.

¹¹Since the error terms of the regression models could be correlated across contracts, the equations were reestimated simultaneously by Zellner's seemingly unrelated regression model (1962). This reestimation resulted in essentially identical parameter estimates as the univariate regressions. Although this technique has the potential to increase the efficiency of the parameter estimates, nonoverlapping data (in event time) had to be eliminated for the three contracts. Consequently, there was little gain in efficiency for this specific application.

Table II
COCHRANE-ORCUTT ESTIMATES OF CEV PARAMETERS
1979-1983

	March Soybeans	July Soybeans	November Soybeans
β_0	-4.91	-6.98	-6.32
t_{β_0}	(-2.0) ^a	(-3.4) ^a	(-2.1) ^a
β_1	1.19	1.51	1.40
t_{β_1}	(3.1) ^a	(4.8) ^a	(3.1) ^a
R^2	0.108	0.220	0.114
D. W.	1.94	2.04	2.05
ρ	0.319 ^a	0.117	0.420 ^a

^aSignificant at 95% level.

III. CONSTANT ELASTICITY OF VARIANCE VERSUS BLACK-SCHOLES OPTION PRICES

In the previous section, evidence was presented demonstrating that the CEV model has statistical significance in describing the behavior of soybean futures quasi-returns. In order to explore the implications of the CEV model for pricing options on agricultural futures, both CEV and B-S option prices are calculated for the same set of hypothetical data. This then allows the CEV prices to be contrasted with the more familiar B-S prices.

The well-known B-S European call option pricing formula for futures (1973) is:

$$C = Fe^{-rt} N(d_1) - Ke^{-rt} N(d_2) \quad (7)$$

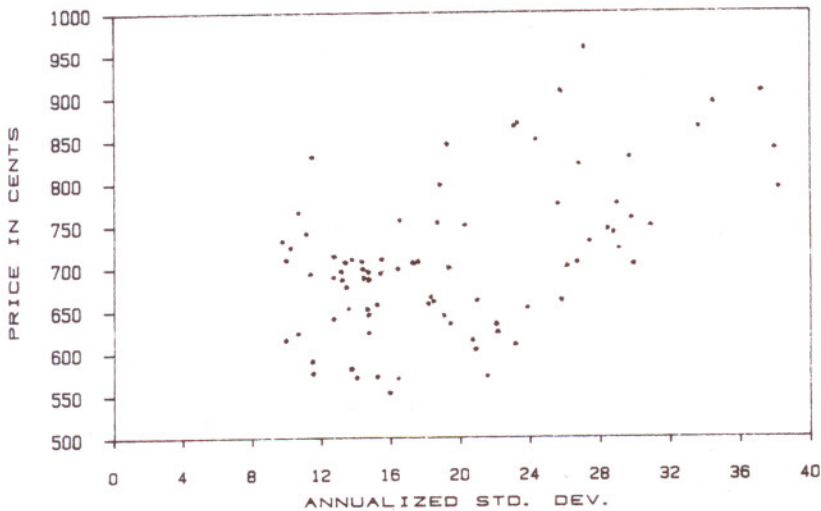


Figure 1

Plot of March soybean prices and quasi-return volatility, 1979-1983.

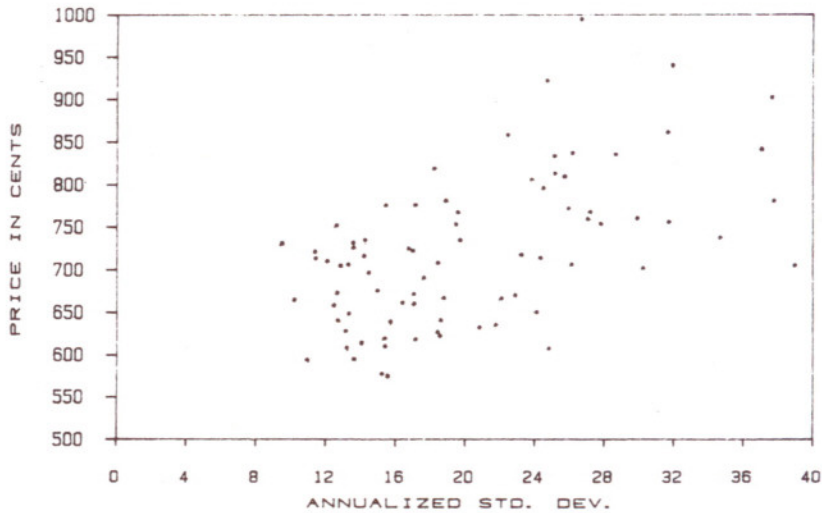


Figure 2

Plot of July soybean prices and quasi-return volatility, 1979-1983.

where:

$$d_1 = (\ln(F/K) + \sigma^2 t / 2) / \sigma \sqrt{t}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

$N(\cdot)$ \equiv Standard cumulative normal distribution function.

F \equiv Current futures price.

t \equiv Time to expiration.

r \equiv Risk-free rate for maturity t .

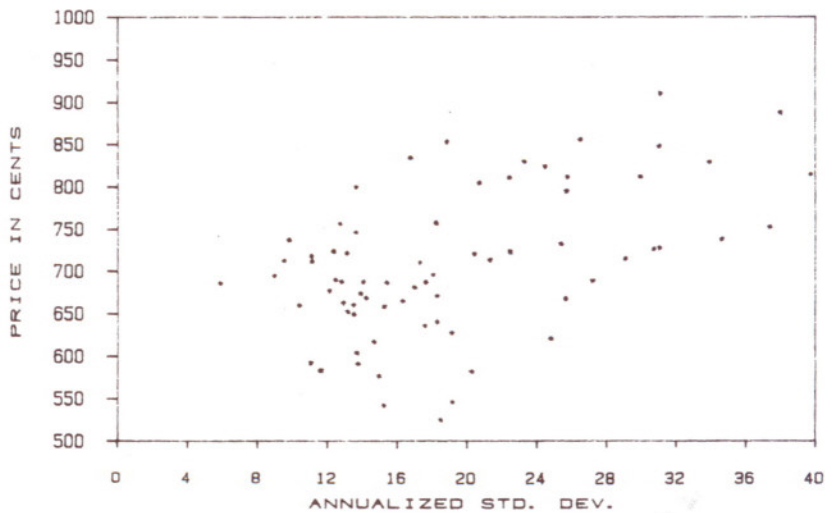


Figure 3

Plot of November soybean prices and quasi-return volatility, 1979-1983.

- K ≡ Exercise price.
- C ≡ Call price.
- σ ≡ Annualized expected standard deviation of quasi-return.

The CEV formula is more complicated in appearance, but can easily be programmed on a small computer. Cox and Rubinstein (1978) give the CEV formula for European calls as:

$$C(F^*, t, r, \sigma, T, K, \psi) = F^* \sum_{n=0}^{\infty} g(\lambda F^{*-\phi}, n+1) G(\lambda (Ke^{-\tau})^{-\phi}, n+1 + \frac{1}{\phi}) - Ke^{-\tau} \sum_{n=0}^{\infty} g(\lambda F^{*-\phi}, n+1 + \frac{1}{\phi}) G(\lambda (Ke^{-\tau})^{-\phi}, n+1) \quad (8)$$

$$\begin{aligned} \phi &\equiv 2\psi - 2 \\ \lambda &\equiv 2r/\sigma^2\phi (e^{\phi\tau} - 1) \\ \Gamma(n) &\equiv \int_0^{\infty} e^{-v} v^{n-1} dv \\ g(z, n) &\equiv e^{-z} z^{n-1} / \Gamma(n) \\ G(w, n) &\equiv \int_0^{\infty} g(z, n) dz \end{aligned}$$

where:

- ψ ≡ Elasticity coefficient < 1 .
- r ≡ Risk-free rate for maturity t .
- t ≡ Time to expiration.
- K ≡ Exercise price.
- C ≡ Call price.
- σ ≡ Annualized expected variance of quasi-return when $F = 1$.
- F^* ≡ Current futures price multiplied by $e^{-\tau}$.

Since the estimated elasticities for agricultural futures tend to be greater than 1, the pricing formula representation can be rewritten as:¹²

$$C(F^*, t, r, \sigma, T, K, \psi) = F^* \left(1 - \sum_{n=0}^{\infty} g(\lambda F^{*-\phi}, n+1 + \frac{1}{\phi}) \times G(\lambda (Ke^{-\tau})^{-\phi}, n+1) \right) - Ke^{-\tau} \times \left(1 - \sum_{n=0}^{\infty} g(\lambda F^{*-\phi}, n+1) G(\lambda (Ke^{-\tau})^{-\phi}, n+1 + \frac{1}{\phi}) \right). \quad (9)$$

¹²See Cox and Rubinstein (1978).

Note that the value of F^* used is the futures price times e^{-rt} , which, as in the B-S case, adjusts the underlying price for the continuous "dividend" of the underlying asset for the option (Black and Scholes, 1973).

Put prices for both models are given by the put/call parity relationship¹³ holding for options on futures.

$$P = C - Fe^{-rt} + Ke^{-rt} \quad (10)$$

where:

P = Put price.

C = Call price.

At the money, put and call prices will be equal.

Hypothetical prices of soybean futures options are presented in Table III for both the B-S and CEV models. As is demonstrated, the CEV model tends to give higher prices than the B-S model for both puts and calls when the futures price is lower than the exercise price. The opposite is true when the futures price equals or exceeds the exercise price. The ratio of the CEV price to the B-S price can vary greatly as parameter values change. At the money, the ratio of the two prices is on the order of 0.94 for the examples in Table III.

Table III
BLACK-SCHOLES VERSUS CEV OPTION PRICES*

		CEV Prices		
		Strike Price (¢)		
Futures Price (¢)		550	700	850
550	Call	46.4	4.9	0.3
	Put	46.4	140.7	271.8
700	Call	141.8	59.1	21.9
	Put	6.1	59.1	157.7
850	Call	271.8	149.0	71.7
	Put	0.4	13.3	71.7
		Black-Scholes Prices		
550	Call	49.4	12.4	2.6
	Put	49.4	148.2	274.1
700	Call	148.2	63.0	21.8
	Put	12.4	63.0	157.5
850	Call	274.1	157.5	76.4
	Put	2.6	21.8	76.4

*Assumes: (1) standard deviation of quasi-return: 25%; (2) one year maturity; (3) 10% risk-free rate of interest; (4) ψ : 2.3.

¹³The put/call parity relationship for options on futures contracts is discussed by Black (1976).

IV. SUMMARY AND CONCLUSIONS

This article has examined the stochastic behavior of agricultural futures price and identified seasonal volatility in soybean futures quasi-returns. This seasonality is clearly inconsistent with the underlying assumptions of the Black-Scholes option pricing model, which are based on constant variance of the quasi-return.

Cox's CEV model was identified as a viable approach to pricing options on agricultural futures in the presence of seasonal volatility, provided the level of volatility was related to price level. The strength of the CEV relationship between volatility and prices was tested by both univariate and multivariate regressions, which were found to be highly significant for soybean futures. Consequently, the CEV model assumptions appear to be well satisfied by the soybean futures data. This suggests that the CEV model is theoretically superior to the B-S model for pricing options on soybean futures.

Since this study's empirical results are based on soybean futures, further research is clearly needed to determine if the CEV relationship holds for other agricultural commodities. Preliminary results suggest that the CEV model may also encompass the stochastic behavior of corn futures prices. Other areas where further research is required include determining the functional form of the quasi-return seasonality. In addition, when sufficient agricultural option price data are available, the B-S and CEV models can be benchmarked against actual market prices to determine which describes option prices more accurately.

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