

Determinants of the Price-to-Earnings Ratio

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Abstract

We examine a firm's P/E ratio in the context of a model wherein firms make sequential investments in production capacity. In addition to investment growth, our analysis seeks to identify pricing power in the firm's product markets and accounting conservatism as determinants of the P/E ratio. A benchmark result demonstrates that, given unbiased accounting, the firm's P/E ratio can be expressed as a convex combination of the P/E ratios suggested respectively by the permanent earnings model and the Gordon growth model. The relative weight to be placed on these two endpoints is captured entirely by Tobin's q . Relative to this benchmark result, we examine the behavior of the P/E ratio when the applicable accounting rules are conservative. In particular, the impact of higher past growth in investments on the P/E ratio is shown to depend on the degree of accounting conservatism.

1 Introduction

The forward Price-to-Earnings (P/E) ratio is commonly calculated as the market value of a firm's common stock at a particular date divided by the firm's earnings in the following year. The P/E ratio is ubiquitous in part because it is viewed as a first gauge for validating the pricing of a particular stock.¹ While there does not seem to be a universally accepted benchmark value for the "normal" forward P/E ratio, textbooks in financial statement analysis typically describe several standard models, such as the permanent earnings model, the Gordon growth model and the abnormal earnings growth model (Ohlson and Juettner-Nauroth, 2005).² In deriving a particular P/E formula, each one of these model frameworks typically represents economic fundamentals as certain "parameters" in the time-series of accounting earnings.

Our objective in this paper is to provide insights into the structure of the P/E ratio by modeling explicitly transactions of the firm as well as the accounting rules that are employed to represent these transactions. Our framework allows for the P/E ratio to be determined by a number of variables, including pricing power that the firm enjoys in its product markets, anticipated future demand in those markets and growth in past investments. A particular focus of our study is how investment growth, both past and future, and the applicable accounting rules interact in shaping the P/E ratio.

According to the permanent earnings model, the normal forward P/E ratio is equal to the inverse of the firm's cost of capital.³ The permanent earnings model holds for fair value accounting, which requires the book value of the firm's assets at each point in time to equal the present value of the firm's future cash flows.⁴ If the firm's investments have zero net present value, fair value accounting is consistent with historical cost accounting. In the presence of positive NPV investments, however, fair value accounting is generally inconsistent with the practice of recognizing operating assets at their acquisition cost at the time of purchase.⁵

An alternative benchmark value for the P/E ratio suggested in earlier literature is the Gordon growth model which states that this ratio is equal to the inverse of the difference

¹See, for example, Basu (1977) and Jaffe, Keim, and Westerfield (1989).

²Penman (2010), (2011) provides a detailed discussion of these three models.

³Among others, Fairfield (1994), Ohlson (1995), Feltham and Ohlson (1995), Feltham and Ohlson (1996), Penman (1996), Ohlson and Juettner-Nauroth (2005) have provided justifications for the permanent earnings model.

⁴The permanent earnings model also holds if book values understate fair values by a constant amount at all dates.

⁵See, for instance, Feltham and Ohlson (1996).

between the firm's cost of capital and the anticipated growth rate in earnings.⁶ This benchmark can be justified in a setting where firm sales grow at a constant rate and the firm has only variable cash operating expenses. Accounting earnings are presumed equal to cash flow and, as a consequence, firm value can be expressed as a multiple of forward earnings. The assumptions of the Gordon growth model are likely to be violated in the presence of capital investments that generate benefits in several periods, especially if such investments do not increase over time at the same rate as revenues.⁷

In our model setting, firms make sequential investments in identical capital assets and use the capacity of those assets to deliver output to the product markets. To the extent that the product selling price exceeds the long-run average cost of production, a firm enjoys pricing (monopoly) power in the product market. In the context of capacity investments, there is a natural notion of unbiased accounting which requires that newly acquired assets be capitalized at their acquisition cost. In subsequent periods, unbiased accounting requires that these assets be depreciated so that their book value at each point in time is equal to the present value of cash flows they would generate if output was sold precisely at the long-run average cost of production. Alternatively, unbiased accounting can be interpreted as carrying operating assets at a book value that reflects the value of these assets in a hypothetical competitive rental market for capacity services. Accordingly, we refer to this depreciation rule as *replacement cost accounting* and to the corresponding book values as the replacement cost of assets in place.⁸

One implication of unbiased accounting in our model is that residual income measures the firm's economic profit, defined as the premium that the firm earns in the product market above the average long-run cost of production. Future economic profits are determined by the firm's pricing power in the product market and by the anticipated future demand for its product(s). The firm's market value can then be expressed as the sum of the replacement cost of assets in place and the anticipated present value of future economic profits.⁹ Given replacement cost accounting, we obtain a benchmark value for the forward P/E ratio which

⁶See, for example, Beaver and Morse (1978), Zarowin (1990), and Damodaran (2006, p. 245).

⁷A related model states that the forward P/E ratio, on average, should approximate the inverse of the risk-free rate. This benchmark is known as the Fed model and it follows from the Gordon's growth formula under the additional assumption that the growth rate in earnings approximates the risk premium implicit in the firm's cost of capital (i.e., "if all growth is risky"). Thomas and Zhang (2009), among others, provide an empirical analysis of the Fed model at the aggregate (stock market) level.

⁸Our model framework of capacity investments and replacement cost accounting has been used in a number of recent studies, spanning managerial performance evaluation (Rogerson (2008) and Dutta and Reichelstein (2010)) and financial statement analysis (McNichols, Rajan and Reichelstein (2012), and Nezlobin (2012)).

⁹See also McNichols et al. (2012) and Nezlobin (2012).

can be interpreted as a “convex combination” of the permanent earnings and the Gordon growth formulas. The relative weight to be placed on these two “endpoints” turns out to be entirely a function of Tobin’s q , defined as the ratio of the firm’s market value to the replacement cost of its assets. In the finance and economics literature, Tobin’s q is usually interpreted as a measure of the firm’s ability to earn abnormal economic profits in the future. Accordingly, this ratio is equal to one for a firm operating in a competitive environment.¹⁰

For a firm operating in a competitive environment (Tobin’s $q = 1$), our benchmark value for the forward P/E ratio reduces to the permanent earnings model. This finding is consistent with the observation that for a competitive firm replacement cost accounting corresponds to fair value accounting. In contrast, as the firm’s pricing power in its product market increases, our benchmark value for the P/E ratio approaches that predicted by the Gordon growth model. When the firm charges a relatively high premium in the product market, its valuation becomes more determined by its revenues than by capital costs. If the product market is expanding, revenues will increase correspondingly and the Gordon’s growth formula tends to provide a better approximation of the P/E ratio.

Our model highlights the potentially opposite effects that past and future growth have on the forward P/E ratio. Higher future growth corresponds ceteris paribus to a higher forward P/E ratio under replacement cost accounting, simply because the value of future growth opportunities is reflected in the firm’s stock price, but not in the forward earnings. But higher past investment growth translates into a *lower* Tobin’s q and a *lower* forward P/E ratio under replacement cost accounting.¹¹ Intuitively, a firm with higher past investment growth has newer assets and a correspondingly greater replacement cost of assets in place. However, greater replacement cost of assets in place also increases the firm’s market value by a corresponding amount, as the discounted value of future economic profits is independent of the firm’s investment history. Therefore, higher past investment growth translates into equal increases in the numerator and denominator of Tobin’s q and brings the overall ratio closer to one. As a consequence, our benchmark P/E ratio is also decreasing in past growth provided the accounting is unbiased.

Generally accepted accounting principles for operating assets are widely viewed as a source of unconditional conservatism. Investments in most intangible assets cannot be capitalized in the first place and, in addition, depreciation rules like the common straight-line

¹⁰Consistent with this interpretation, Lindenberg and Ross (1981) conclude that q “*exceeds one by the capitalized value of the Ricardian and monopoly rents which the firm enjoys.*”

¹¹In formulating this result, we consider two situations that differ only in past investment growth rates and assume that all other parameters of the model, including future demand growth, are the same. Past investment growth in our model can be different from future demand growth either because the product market has expanded at a different rate in the past, or because the firm has been replacing its aging assets.

method will in many circumstances be accelerated relative to the economic decline of the asset. For an assessment of the P/E ratio, it is therefore essential to understand the impact of asset valuation rules that are more conservative than replacement cost accounting. As a starting point, it is useful to recall that for firms in a steady state (constant investment levels over the relevant horizon) the P/E ratio must be equal to the benchmark value identified under replacement cost accounting. Put differently, the “Canceling Errors” Theorem implies that all P/E curves, obtained from alternative asset valuation rules, must yield the same benchmark value for a steady state firm. As a consequence, one obtains a “Pivot” point through which all P/E ratios, when viewed as a function of past growth, must pass.

Growth is generally viewed as a key variable in evaluating or predicting P/E ratios (Penman 2010, 2011). It is also generally accepted that more conservative accounting (more accelerated depreciation) tends to result *ceteris paribus* in a higher P/E ratio, provided the firm’s investments have expanded in the past. The opposite effect obtains for firms characterized by declining investments over the relevant horizon. This leads to a pattern of P/E ratios that rotate counter-clockwise through the Pivot point.¹² Our analysis examines the impact of higher growth on the P/E ratio, supposing the accounting is conservative. In sharp contrast to our finding for replacement cost accounting, we find that for sufficiently conservative depreciation rules, including the straight line rule, higher past growth will increase the P/E ratio. Furthermore, this monotonic relation becomes more accentuated for more conservative accounting rules. One prediction of our analysis therefore is that the impact of past growth on the P/E ratio will be more pronounced for industries characterized by a higher degree of conservatism, possibly due to a high proportion of intangible investments.

The remainder of the paper is organized as follows. Section 2 presents the capacity model and the financial ratios representing Tobin’s q and the P/E ratio. Section 3 derives our benchmark values for these ratios under the hypothetical assumption of replacement cost accounting. The effects of conservative accounting on the P/E ratio are examined in Section 4. We examine the particular setting of infinitely lived assets that are subject to a geometric decay pattern in Section 5. The geometric decline setting entails additional aggregation properties and thus leads to a sharper characterization of the P/E ratio. We conclude in Section 6.

¹²This findings is conceptually related to the “quadrant result” obtained in connection with the Accounting Rate-of-Return: see for instance, Salamon (1985), Fisher and McGowan (1983) and Rajan et al.(2007). In contrast to those studies though, we find that the quadrants for the P/E ratio are generally delimited by a non-linear coordinate system.

2 Model Description

Consider a single-product firm that makes sequential investments in productive capacity. Such capacity is generated by operating assets that can be purchased in each period at a constant unit cost. The useful life of operating assets is T periods. Specifically, a unit of asset purchased in period t adds capacity to produce x_τ units of the product in periods $t + \tau$ for $1 \leq \tau \leq T$. Without loss of generality, the acquisition cost of one asset unit is set equal to one. Denoting by I_t the investment in period t , the aggregate capacity available in period t , K_t , is determined by the investments over the past T periods:

$$K_t = x_1 \cdot I_{t-1} + x_2 \cdot I_{t-2} + \dots + x_T \cdot I_{t-T},$$

where $\mathbf{x} = (x_1, x_2, \dots, x_T)$ will be referred to as the asset's productivity pattern and $\boldsymbol{\theta}_t = (I_t, I_{t-1}, \dots, I_{t-T+1})$ as the relevant investment history at date t . The productivity of assets is assumed to decline weakly over their useful life, possibly reflecting physical wear and tear or increasing maintenance requirements:

$$1 = x_1 \geq x_2 \geq \dots \geq x_T > 0.$$

In the special case of undiminished capacity, all x_τ are equal to one. In the regulation literature, this scenario is commonly referred to as the *one-hoss shay* pattern, e.g., Laffont and Tirole (2000) and Rogerson (2011). In contrast, parts of the finance literature have focused on settings in which assets remain productive indefinitely and their capacity declines geometrically over time:

$$x_\tau = (1 - \alpha) \cdot x_{\tau-1},$$

where $0 < \alpha \leq 1$; see, for instance, Berk et al. (1999) and Biglaiser and Riordan (2000). We refer to such an asset decay pattern as the *geometric decline* scenario.¹³

In period t , the firm can produce and sell Q_t units of product, subject to the capacity constraint, $Q_t \leq K_t$. The firm will then realize revenues of

$$R_t(Q_t) = P_t(Q_t) \cdot Q_t,$$

where $P_t(Q_t)$ denotes the net revenue (sales revenue minus variable production costs) per unit of output as a function of the quantity supplied. The net-revenue functions, $R_t(\cdot)$, are assumed to be strictly increasing and concave. Profit maximization requires the firm to operate at capacity in every period, i.e., $Q_t = K_t$. In addition, we assume that the inverse demand functions have the following structural property.

¹³Section 5 below focuses on the geometric decline scenario.

Assumption (A1): Demand for the firm's product expands over time proportionately at all price levels, such that:

$$P_{t+1}(Q \cdot (1 + \mu_{t+1})) = P_t(Q),$$

where $\mu_{t+1} \geq 0$.¹⁴

The significance of Assumption (A1) is that the firm can increase its sales volume by the factor $1 + \mu_{t+1}$ in period $t+1$ while maintaining the same product price. As shown in Nezlobin, Rajan and Reichelstein (2012), assumption (A1) can be met by standard functional forms for demand curves, including linear or constant elasticity demand curves. For simplicity of exposition, our model focuses on an all-equity firm that disburses all free cash flows to its owners immediately. Cash flows either arise from investment expenditures or the net-revenues from sales.

Since assets are assumed to have a useful life of T periods, we examine the firm's forward P/E ratio at date T , taking into consideration both the history and the anticipated expansion of investments. The firm's market value at date T is defined as the present value of its future cash flows under the optimal investment policy:

$$P_T \equiv \max_{\{I_{T+i}\}_{i=1}^{\infty}} \sum_{i=1}^{\infty} [R_{T+i}(K_{T+i}) - I_{T+i}] \cdot \gamma^i, \quad (1)$$

subject to

$$I_{T+i} \geq 0. \quad (2)$$

where $\gamma = \frac{1}{1+r}$ and r denotes the firm's cost of (equity) capital.

Arrow (1964) has shown that the infinite-horizon investment problem in (1) - (2) can effectively be decomposed intertemporally. The key concept in this decomposition is the unit cost of capacity. It represents the cost to the firm of increasing its capacity in a single period by one unit, holding capacity levels in future periods unchanged. Arrow (1964) shows that c is given by:

$$c = \frac{1}{\gamma \cdot x_1 + \dots + \gamma^T \cdot x_T}. \quad (3)$$

The marginal cost of capacity can also be viewed as the rental price that a hypothetical supplier of rental services would charge if such a supplier were to make zero economic profits, i.e., if the rental market were perfectly competitive. Indeed, if the firm buys one unit of the

¹⁴The current model can be extended to accommodate a declining product market ($\mu_t \leq 0$), provided the decline in the incumbent capacity is sufficiently large so that a value maximizing firm would still have to make new investments in order to maintain its optimal capacity level.

asset in period t and rents out the available capacity in the following T periods, the net present value of this project is:

$$-1 + \gamma \cdot c \cdot x_1 + \dots + \gamma^T \cdot c \cdot x_T = 0. \quad (4)$$

The main result in Arrow (1964) is that the optimal investment policy in problem (1)-(2), I_t^* , maximizes

$$\pi_t = R_t(K_t) - c \cdot K_t$$

in every period. This decomposition requires that the non-negativity constraint in (2) does not bind. In our framework this condition is met on account of Assumption (A1). We refer to $c \cdot K_t$ as the *economic cost of capacity*, and to $R_t(K_t) - c \cdot K_t$ as the *economic income* of the firm in period t .

Let K_t^* denote the optimal capacity levels, given by the first-order conditions: $R_t'(K_t^*) = c$. It follows directly from Assumption (A1) that the optimal product price is constant over time and the optimal capacity levels grow at rates μ_t :

$$K_t^* = (1 + \mu_t) K_{t-1}^*.$$

Let $p^* = P_{T+1}(K_{T+1}^*)$ denote the optimal product price, and π_t^* denote the optimized economic income in period t . For future reference, we note that given Assumption (A1), economic income grows at the same rate as the optimal capacity levels:

$$\pi_t^* = p^* \cdot K_t^* - c \cdot K_t^* = (1 + \mu_t) \pi_{t-1}^*. \quad (5)$$

If $p^* = c$, the net present value of the firm's investments is zero, and, therefore, we refer to such a firm as operating in a competitive environment. Pricing power and monopoly rents correspond to values of p^* exceeding the marginal cost of capacity.

Accruals in our model arise only from the depreciation of operating assets. Depreciation expense is recognized according to some schedule $\mathbf{d} = (d_1, \dots, d_T)$, where d_τ is the share of investment that is expensed in period τ of its useful life, and the vector \mathbf{d} satisfies:

$$\sum_{\tau=1}^T d_\tau = 1.$$

The aggregate depreciation expense in period t is then given by:

$$D_t = d_1 \cdot I_{t-1} + \dots + d_T \cdot I_{t-T}.$$

Let $\mathbf{bv} = (bv_0, \dots, bv_T)$ denote the corresponding asset valuation rule, so that the aggregate book value at date t is:

$$BV_t = bv_0 \cdot I_t + bv_1 \cdot I_{t-1} + \dots + bv_T \cdot I_{t-T}.$$

We impose the usual clean surplus condition that the book value of existing assets changes by the amount of depreciation expense recognized in a given period and that assets are fully expensed by the end of their useful life:

$$bv_\tau = bv_{\tau-1} - d_\tau \text{ and } bv_T = 0.$$

One natural depreciation rule to consider is the one that allocates the cost of an asset over its useful life in proportion to capacity the asset generates in different periods. We refer to this rule as *proportional depreciation* rule.¹⁵ Formally, the corresponding depreciation charges are given by

$$d_\tau^p = \frac{x_\tau}{x_1 + \dots + x_T}.$$

In particular, this rule calls for straight-line depreciation of assets conforming to the one-hoss shay pattern (i.e., if all x_τ are equal). Straightforward algebra shows that in the case of geometrically declining productivity ($x_\tau = (1 - \alpha) \cdot x_{\tau-1}$), proportional depreciation takes the form:

$$d_\tau^p = \alpha(1 - \alpha)^{\tau-1}.$$

These depreciation charges add up to one and decline over time at the same rate as the asset's productivity.

Given any depreciation rule \mathbf{d} , the firm's *Earnings* in period t , E_t , are equal to the difference between revenues and the aggregate depreciation expense:

$$E_t = R_t(K_t) - D_t.$$

Residual Income subtracts from earnings the imputed interest charge for the current book value of assets:

$$RI_t = E_t - r \cdot BV_{t-1} = R_t(K_t) - D_t - r \cdot BV_{t-1}.$$

We refer to the sum of depreciation expense and the interest charge on the book value of assets as the historical cost of capacity, H_t . The historical cost of capacity is determined by the history of past investments:

$$H_t = D_t + r \cdot BV_{t-1} = z_1 \cdot I_{t-1} + \dots + z_T \cdot I_{t-T},$$

where $z_\tau = d_\tau + r \cdot bv_{\tau-1}$.

¹⁵Assumption (A1) implies that future product prices are constant on the optimal capacity path, and therefore the revenues generated by an asset are proportional to its productive capacity. The proportional depreciation rule then allocates the cost of investment according to the (nominal) cash flows the asset generates, ignoring the time value of those cash flows.

In order to represent future product market growth by a one-dimensional parameter, we shall assume that after period $T + 1$, demand grows at a constant rate μ :¹⁶

$$\mu \equiv \mu_{T+2} = \mu_{T+3} = \dots$$

Note that we use period $T+1$ (not period T) as a baseline in our definition of the parameter μ to ensure that μ represents the growth in the product market that has not yet been reflected in the investment history at date T . The parameter μ can, therefore, be interpreted as a measure of the firm's future growth opportunities.

Finally, the forward P/E ratio at date T is defined as the ratio the firm's market value at date T to earnings in period $T + 1$:

$$PE_T = \frac{P_T}{E_{T+1}}. \quad (6)$$

To summarize, in our model the P/E ratio is determined by the firm's investment history, the depreciation rule in use, and investors' anticipation of future growth opportunities in the firm's product market.

3 Replacement Cost Accounting

3.1 The P/E Ratio and Tobin's q

This section establishes a benchmark value for the P/E ratio by considering a hypothetical scenario of unbiased depreciation rules. The notion of unbiased accounting is that book values are equal to the *replacement cost* of assets in the sense that incumbent assets are valued in according to what they would trade for in a hypothetical competitive rental market for capacity services. As shown below, this accounting rule implies that economic income, as defined above, coincides with residual accounting income. In that sense, we refer to replacement cost accounting as *unbiased* in the context of our model.

To begin with, it is instructive to contrast replacement cost accounting with *fair value accounting*. If the firm under consideration has pricing power in the product market, the present value of cash flows generated by the incumbent assets will exceed the replacement

¹⁶If demand growth after period $T + 1$ is not constant, our results hold if one defines μ such that

$$\frac{1}{r - \mu} \equiv \gamma + \gamma^2 \cdot (1 + \mu_{T+2}) + \gamma^3 \cdot (1 + \mu_{T+3}) + \dots,$$

Thus μ is defined such that the discounted value of the sequence growing according to rates $\mu_{T+\tau}$ is equal to the discounted value of a sequence growing at the constant rate μ .

cost of those assets. Fair value accounting would require that the firm's book value captures the equity market value, thus resulting in a Market-to-Book ratio equal to one.¹⁷ This would require that, at the time of acquisition, operating assets be recorded at the present value of cash flows they will generate, not at their historical cost. In addition, if the product market is expanding, the firm will optimally increase its capacity by additional operating assets in future periods. With pricing power in the product market, each future asset purchase will represent a positive net present value investment. To equate book values with market values at any point in time, the net present value of future anticipated investments would therefore need to be capitalized *before* those investments are made. The informational requirements for such treatment of operating investments would obviously be rather demanding. In addition, such an approach would generally be inconsistent with the generally accepted notion that assets can be recognized only to the extent that they originate from *past* transactions.¹⁸

Under replacement cost accounting, assets are initially recorded at their acquisition cost. Depreciation is calculated such that at each point in time the remaining book values reflects what the used asset would trade for if capacity services could be obtained on a rental basis under competitive conditions. Let $\mathbf{bv}^* = (bv_0^*, \dots, bv_T^*)$ denote the corresponding sequence of replacement cost values. The corresponding break-even conditions are:

$$bv_\tau^* = \gamma \cdot c \cdot x_{\tau+1} + \dots + \gamma^{T-\tau} \cdot c \cdot x_T, \quad (7)$$

with c as defined in (3). Thus, the aggregate book value of assets under replacement cost accounting is given by

$$BV_t^* = bv_0^* \cdot I_t + \dots + bv_{T-1}^* \cdot I_{t-T+1}.$$

The depreciation charges under replacement cost accounting, $\mathbf{d}^* = (d_0^*, \dots, d_T^*)$, must yield historical cost charges that reflect current replacement cost values:

$$z_\tau^* \equiv d_\tau^* + r \cdot bv_{\tau-1}^* = c \cdot x_\tau.$$

It is readily verified that for assets exhibiting the one-hoss shay productivity pattern, replacement cost accounting amounts to the annuity depreciation rule with depreciation charges compounding at the cost of capital. For a geometrically declining productivity pattern, replacement cost accounting coincides with proportional depreciation:

$$d_\tau^* = d_\tau^p = \alpha (1 - \alpha)^{\tau-1}.$$

¹⁷This is effectively the notion of unbiased accounting in Feltham and Ohlson (1996), Zhang (2000) and Ohlson and Gao (2006).

¹⁸See, for instance, FASB Concepts Statement No. 6

Lastly, the straight-line depreciation rule corresponds to replacement cost accounting for assets with linearly declining productivity (Rajan and Reichelstein, 2009).¹⁹

Let E_t^* and RI_t^* denote the firm's earnings and residual income, respectively, under replacement cost accounting. The key property then is that the historical cost of capacity properly reflects its economic cost for any history of investments (Rogerson, 2008).²⁰ Therefore, RI_t^* is equal and to the firm's economic income in every period:

$$RI_t^* = E_t^* - r \cdot BV_{t-1}^* = (p^* - c) \cdot K_t^* = \pi_t^*.$$

This result is useful in expressing the firm's equity value at date T . By the residual income valuation formula, the equity value at date T is equal to the book value of the firm's assets at date T plus the present value of future residual earnings:

$$P_T = BV_T + \sum_{i=1}^{\infty} \gamma^i RI_{T+i},$$

irrespective of the accounting rules as long as earnings are measured comprehensively. Under replacement cost accounting, residual income grows at the same rate as demand for the firm's product. Referring to equation (5), we find that after period $T+1$, residual income will grow at the market growth rate μ :

$$RI_{T+i+1}^* = (1 + \mu) \cdot RI_{T+i}^*, \quad (8)$$

for $i \geq 1$. Therefore, firm value at date T can be expressed as:

$$P_T = BV_T^* + \frac{RI_{T+1}^*}{r - \mu}. \quad (9)$$

Equation (9) has an appealing economic interpretation: firm value is equal to the replacement cost of the firm's operating assets plus the capitalized value of future abnormal economic profits.²¹

¹⁹Specifically, the productivity pattern is given by

$$x_\tau = 1 - \frac{r}{1 + r \cdot T} \cdot (\tau - 1).$$

²⁰In particular, one obtains that for any history of investments,

$$\begin{aligned} H_t^* &= D_t^* + r \cdot BV_{t-1}^* = z_1^* \cdot I_{t-1} + \dots + z_T^* \cdot I_{t-T} \\ &= c \cdot (x_1 \cdot I_{t-1} + \dots + x_T \cdot I_{t-T}) = c \cdot K_t. \end{aligned}$$

²¹See Nezlobin (2012) for a formal derivation of this result. This decomposition is also consistent with similar ones obtained in the finance and economics literature, see, for example, Thomadakis (1976) and Lindenberg and Ross (1981).

Tobin's q is defined as the ratio of the market value of the firm to the replacement cost of its assets. This ratio is usually interpreted as a measure of future abnormal economic profits.²² In particular, Lindenberg and Ross (1981) state: "...for firms engaged in positive investment, in equilibrium, we expect q to exceed one by the capitalized value of the Ricardian and monopoly rents which the firm enjoys." In the context of our model, Tobin's q can be expressed as:

$$q = \frac{P_T}{BV_T^*}.$$

For a competitive firm, this ratio is equal to one for any history of investments. If the firm has pricing power in the product market, in the sense that $p^* > c$, equation (9) implies that Tobin's q will indeed exceed one by the ratio of capitalized future economic profits relative to the competitive firm value. Our first result states a benchmark value for the P/E ratio in terms of Tobin's q .

Proposition 1 *Given replacement cost accounting, the forward P/E ratio is equal to*

$$\frac{P_T}{E_{T+1}^*} = \frac{1}{r - \mu \cdot \frac{q-1}{q}}. \quad (10)$$

Three special cases of equation (10) are of particular interest. First, for a competitive firm ($q = 1$), the benchmark ratio in (10) reduces to

$$\frac{P_T}{E_{T+1}^*} = \frac{1}{r},$$

because E_{T+1}^* satisfies the permanent earnings model. This finding makes intuitive sense insofar as with a competitive product market, replacement cost accounting is equivalent to fair value accounting, which then yields the permanent earnings model.

Second, for a firm that enjoys a strong degree of monopoly power (q is large), equation (10) approximates the Gordon growth formula:

$$\frac{P_T}{E_{T+1}^*} \approx \frac{1}{r - \mu}.$$

As the firm's monopoly profits increase, capacity costs become relatively less important, and the firm's value is largely determined by the discounted value of future revenues. Growth in the product market directly translates into revenue growth and, therefore, increases the capitalization factor according to the Gordon growth formula.

²²See, for example, Lindenberg and Ross (1981) and Salinger (1984).

Finally, the permanent earnings model is obtained if demand for the firm’s product is anticipated to remain at the same level in all future periods ($\mu = 0$). This result holds irrespective of the firm’s monopoly power and, in particular, when replacement cost accounting is different from fair value accounting ($q > 1$). Recall that book values under replacement cost accounting understate market values by the present value of future monopoly profits. When the product market is stationary, the present value of future monopoly profits does not change over time, and BV_t^* understates P_t by a constant amount at all times. The “Canceling Errors” Theorem then implies that earnings will be the same under replacement cost accounting and fair value accounting. As a consequence, the permanent earnings model applies.

An essential feature of replacement cost accounting is that firm value can be expressed as a linear combination of forward earnings and current book value. Equation (9) can be restated as:

$$P_T = BV_T^* + \frac{E_{T+1}^* - rBV_T^*}{r - \mu} = \frac{E_{T+1}^*}{r - \mu} - \frac{\mu}{r - \mu}BV_T^*, \quad (11)$$

or equivalently:

$$P_T = \frac{E_{T+1}^*}{r} + \frac{\mu}{(r - \mu) \cdot r}RI_{T+1}^*. \quad (12)$$

This characterization recovers Proposition 1 in the Ohlson-Juettner (2005) valuation model, where price is given by capitalized forward earnings plus the capitalized value of future abnormal earnings growth ($RI_{T+i+1}^* - RI_{T+i}^* = \mu \cdot RI_{T+i}^*$). We emphasize, however, that in the Ohlson-Juettner model the “persistence” parameter, which requires abnormal earnings to grow (or decline) at a constant rate, does not correspond to an immediate accounting or economic construct. One would expect the magnitude of this parameter generally to depend on both economic fundamentals of the firm and the applicable accounting rules. In contrast, the parameter μ in our model stands only for growth in the product market. As demonstrated in the next section, residual income will no longer grow at a constant rate once the accounting rules entail biases relative to the benchmark of replacement cost accounting. In particular, conservative accounting will then also lead to a shift in the benchmark value for the P/E ratio.

3.2 The Effects of Growth

We now proceed to analyze the effects of growth on the forward P/E ratio, maintaining our focus on replacement cost accounting. It is immediate that higher *future growth* in the product market, μ , will ceteris paribus translate into a higher Tobin’s q and, by Proposition

1, also into a higher forward P/E ratio. The present value of future growth opportunities is reflected in the firm's market value, but not in the replacement cost of its assets in place at date T . Accordingly, Tobin's q is strictly increasing in μ , unless the firm is perfectly competitive, that is, all current and future investments have zero net present value. For a firm operating in a competitive environment, Tobin's q is equal to one, irrespective of future growth. Furthermore, the firm's earnings in period $T + 1$ do not reflect future growth in the product market. The forward P/E ratio under replacement cost accounting is therefore strictly increasing in μ , except when the firm is competitive, in which case the P/E ratio is equal to $1/r$ for all values of μ .

To examine the impact of past growth, we will focus on realized growth rates in investments rather than growth in the aggregate capacity levels. The investment growth rates will be denoted by λ_t , so that

$$I_t = (1 + \lambda_t) \cdot I_{t-1}.$$

The growth rates $\boldsymbol{\lambda} \equiv (\lambda_2, \dots, \lambda_T)$ are, therefore, determined by the investment history at date T , $\boldsymbol{\theta}_T = (I_T, I_{T-1}, \dots, I_1)$. Intuitively, higher values of λ_t correspond to situations where a larger share of current capacity is generated by newer assets.

It will be convenient to restate the expression for Tobin's q in equation (9) :

$$q = 1 + \frac{RI_{T+1}^*}{(r - \mu) BV_T^*}.$$

Since residual income is equal to economic income under replacement cost accounting, it follows that:

$$q = 1 + \frac{(p^* - c) K_{T+1}^*}{(r - \mu) BV_T^*}. \quad (13)$$

Recalling the definition of bv_τ^* in equation (7), we obtain the following equivalent representation of BV_t^* :

$$BV_t^* = \gamma \cdot c \cdot K_{t+1} + \gamma^2 \cdot c \cdot K_{t+2}^o + \dots + \gamma^T \cdot c \cdot K_{t+T}^o, \quad (14)$$

where K_{t+i}^o is the firm's capacity level in period $t + i$ assuming that no new assets are purchased after period t . Tobin's q then becomes:

$$q = 1 + \left(\frac{1}{r - \mu} \right) \left(\frac{p^* - c}{c} \right) \left(\frac{K_{T+1}^*}{\gamma \cdot K_{T+1}^* + \gamma^2 \cdot K_{T+2}^o + \dots + \gamma^T \cdot K_{2T}^o} \right). \quad (15)$$

Equation (15) indicates that Tobin's q will exceed one by the product of three terms. The first term, $1/(r - \mu)$, reflects the firm's cost of capital and future growth in demand

for the firm's product. The term $(p^* - c)/c$ captures, on a percentage basis, the optimal markup that the firm charges in the product market above its long-run marginal cost. For a firm generating positive economic profits, this markup will reflect the degree of the firm's monopoly power, with zero as the benchmark for a firm operating under competitive conditions.²³ The third term determining Tobin's q in (15) is the ratio of capacity that the firm's assets in place will generate in the next period to the discounted value of capacity that those assets will generate over their remaining lifetime. Ceteris paribus, the impact of past growth on Tobin's q is captured through this last term.

Assumption (A2):

$$\frac{x_{\tau-1}}{x_{\tau}} \leq \frac{x_{\tau}}{x_{\tau+1}} \quad (16)$$

for all $1 \leq \tau \leq T$, where $x_0 \equiv x_{T+1} \equiv 0$.

Assumption (A2) states that the rate of capacity degradation increases with asset age. Clearly, this assumption is satisfied for linearly and geometrically declining productivity patterns. For future reference, we also note that in the geometric setting with indefinite useful lives, all inequalities in (A2) are met as equalities because capacity declines at a constant rate regardless of the asset's age. If the useful life is finite, then at least the last inequality, the one corresponding to $\tau = T$, will be strict. Assumption (A2) has the following implication for replacement cost accounting.

Lemma 1 *Given (A2),*

$$\frac{bv_{\tau-1}^*}{x_{\tau}} \text{ is decreasing in } \tau \text{ and } \frac{d_{\tau}^*}{x_{\tau}} \text{ is increasing in } \tau.$$

The first statement in Lemma 1 supports the intuition that older assets should have a lower replacement cost per unit of capacity generated in the next period. The second statement follows from the first one by recalling that given replacement cost accounting, the historical cost of capacity is equal to its economic cost:

$$d_{\tau}^* + r \cdot bv_{\tau-1}^* = c \cdot x_{\tau}.$$

²³It is readily verified that $(p^* - c)/c$ can be expressed as a monotone transformation of the Lerner index of monopoly power, $L \equiv (p^* - c)/p^*$ (Martin, 2002):

$$\frac{p^* - c}{c} = \frac{L}{1 - L}.$$

In particular for a demand curve exhibiting constant price elasticity of demand, say ϵ , L will be equal to $\frac{1}{\epsilon}$.

Dividing both sides by x_τ , we obtain

$$\frac{d_\tau^*}{x_\tau} + r \cdot \frac{bv_{\tau-1}^*}{x_\tau} = c.$$

Therefore, if the second term in the left-hand side decreases in τ , the first term must be increasing in τ . We are now in a position to state the following formal result.

Proposition 2 *Given Assumptions (A1)-(A2), both Tobin's q and the P_T/E_{T+1}^* ratio are decreasing in each λ_t for $1 < t \leq T$.*

The intuition for Proposition 2 can be captured by considering two firms operating at the same capacity in period $T + 1$ and facing equivalent future product market conditions. Suppose also that one of these firms has *newer assets* in the sense that the histories of investment growth rates for the two firms are the same except for one λ_t for some $1 < t \leq T$. The firm with the higher investment growth rate in period t can be viewed as having newer assets, since a larger share of its capacity is generated by assets purchased in period t or later.²⁴ Given assumption (A2), the firm with newer assets will have a higher replacement cost of assets in place. Since future economic profits are equal for the two firms, the difference between their market values will be exactly equal to the difference in the replacement cost of their assets in place. Therefore, Tobin's q for the firm with older assets becomes:

$$q^{(o)} \equiv \frac{P_T^{(o)}}{BV_T^{*(o)}} = \frac{BV_T^{*(o)} + \Delta}{BV_T^{*(o)}},$$

where Δ represents the present value of future economic profits. For the firm with newer assets we then obtain:

$$q^{(n)} = \frac{BV_T^{*(o)} + M + \Delta}{BV_T^{*(o)} + M},$$

where M represents the increase in the replacement value of assets due to higher growth. The increment M in both the numerator and the denominator of $q^{(n)}$ pushes the q -ratio towards unity and therefore $q^{(o)} > q^{(n)}$. Finally, the claimed monotonicity of the P_T/E_{T+1}^* ratio in past growth follows directly from Proposition 1.

²⁴To have equal capacity in period $T + 1$, the firms then must have different investments in the first period, I_1 . Note, however, that I_1 cancels out from the calculation of Tobin's q in equation (15), since

$$K_{T+1} = I_1 \cdot \left(x_1 \cdot \prod_{\tau=2}^T (1 + \lambda_\tau) + \dots + x_{T-1} \cdot (1 + \lambda_2) + x_T \right),$$

and all K_{T+i}^0 are similarly proportional to I_1 . Therefore, Tobin's q does not depend on the absolute value of I_1 , but rather on investment growth rates λ_t for $1 < t \leq T$.

To conclude this section, we note that Tobin’s q and the P_T/E_{T+1}^* ratio are generally *strictly* decreasing in each λ_t except in the following special cases. First, for a competitive firm ($p^* = c$), Tobin’s q is equal to 1 and the P_T/E_{T+1}^* ratio is equal to $1/r$ regardless of the history of investments. This finding is consistent with the observation that in the competitive scenario replacement cost accounting coincides with fair value accounting and thus the permanent earnings model holds.

Second, if the firm’s product market is stationary ($\mu = 0$), Proposition 1 shows that the P_T/E_{T+1}^* ratio will be equal to $1/r$ regardless of past investment growth. In this case, future economic profits do not change over time, and the firm’s market value is anticipated to exceed the replacement cost of its assets in place by the same amount in all future periods. While Tobin’s q at date T will generally depend on λ_t for $1 < t \leq T$, the “Canceling Errors” Theorem implies that E_{T+1}^* will be equal to permanent earnings for any history of past growth rates.²⁵

4 Accounting Conservatism and the P/E Ratio

This section examines the interaction between accounting rules and past investment growth in shaping the forward P/E ratio, holding economic profitability and future market growth constant. Current financial reporting rules arguably differ from our baseline scenario of replacement cost accounting in at least two respects. First, some expenditures that arguably generate cash returns in future periods, such as those in research and development, are not recognized as assets and are expensed as incurred. Second, the depreciation rules that are applied under current financial reporting rules to amortize capitalized assets usually ignore the time value of money. Our model framework interprets both of these factors as making current accounting practice more conservative than replacement cost accounting. Therefore, we are interested in studying how accounting conservatism alters the relation between the P/E ratio and its other determinants.

By the “Canceling Errors” Theorem, earnings are unaffected by the accounting rules in use for a firm that operates in a steady state of no growth. In addition, it has been observed that conservative accounting results in lower earnings if investments have been increasing over the relevant history. Conversely, if investments have followed a declining trajectory, conservative accounting produces higher net income and a correspondingly lower forward P/E ratio.²⁶ These observations are explained by the effect that conservative accounting

²⁵Another example where Tobin’s q and the P_T/E_{T+1}^* ratio do not depend on past investment growth rates is the geometric decline scenario considered in Section 5 below.

²⁶See, for example, Penman (2010), p. 580.

results in relatively high depreciation charges for newer assets. If the firm has been increasing its investments in operating assets, then higher depreciation charges will be applied to larger (more recent) investments, leading to a higher aggregate depreciation expense and a lower net income.

Our model framework allows us to verify that these conjectures do hold subject to some mild regularity conditions. We then proceed to address the dual question: Given conservative accounting, what is the impact of higher growth in past investments on the forward P/E ratio. We begin with the following partial ranking of depreciation rules in terms of their degree of conservatism.

Definition 1: *Depreciation rule \mathbf{d} is more accelerated than \mathbf{d}' if for any τ :*

$$bv_\tau \leq bv'_\tau.$$

By this criterion, straight-line depreciation is more accelerated than the annuity rule with depreciation charges increasing at rate r (Rajan and Reichelstein, 2009). If the productivity of assets satisfies the one-hoss shay pattern, the straight-line rule corresponds to proportional depreciation, while the annuity rule corresponds to replacement cost accounting. More generally, it turns out that if the productivity pattern satisfies assumption (A2), then proportional depreciation is more accelerated than replacement cost accounting.

Clearly, the notion of more accelerated depreciation also translates to lower aggregate book value of assets independent of the history of investments:

$$BV_T(\mathbf{d}) \leq BV_T(\mathbf{d}').$$

Let

$$PE_T(\boldsymbol{\lambda}, \mathbf{d}) = \frac{P_T(\boldsymbol{\lambda})}{E_{T+1}(\boldsymbol{\lambda}, \mathbf{d})}$$

denote the forward P/E ratio at date T assuming that depreciation rule \mathbf{d} is used and the firm's investment history is given by $\boldsymbol{\lambda}$. To further emphasize that Tobin's q is a function of $\boldsymbol{\lambda}$, we will write it as $q(\boldsymbol{\lambda})$. We refer to a *growing* firm as one where $\lambda_t \geq 0$ for all t . Similarly, a firm will be said to be *declining* in past investments if $\lambda_t \leq 0$. This leads us to the following result relating investment growth, accounting conservatism and abnormal economic profitability.

Proposition 3 *Given (A1)-(A2), the forward P/E ratio for a growing firm satisfies:*

$$PE_T(\boldsymbol{\lambda}, \mathbf{d}) \geq \frac{1}{r - \mu \cdot \frac{q(\boldsymbol{\lambda})-1}{q(\boldsymbol{\lambda})}}, \quad (17)$$

provided the depreciation rule \mathbf{d} is more accelerated than replacement cost accounting. The inequality in (17) is reversed for declining firms.

This result combines our earlier finding in Proposition 1 with the observation that *ceteris paribus* aggregate earnings for a growing firm must decrease as depreciation becomes more accelerated, that is:

$$E_{T+1}(\boldsymbol{\lambda}, \mathbf{d}') \geq E_{T+1}(\boldsymbol{\lambda}, \mathbf{d}),$$

provided $\boldsymbol{\lambda} \geq 0$ and \mathbf{d} is more accelerated than \mathbf{d}' . It is an immediate consequence of the “Canceling Errors” Theorem that the P/E values generated by alternative accounting rules must all pass through the *Pivot Point*:

$$PE_T(\mathbf{0}, \mathbf{d}) = \frac{1}{r - \mu \cdot \frac{q(\mathbf{0})-1}{q(\mathbf{0})}}. \quad (18)$$

For the case of constant growth, that is $\lambda_t = \lambda$, Figure 1 illustrates the effect of more accelerated depreciation: the P/E ratios rotate counter-clockwise relative to the benchmark curve corresponding to replacement cost accounting.

The finding in Proposition 3 is reminiscent of the “quadrant result” obtained in earlier accounting studies for the the accounting Rate-of-Return (RoR). Specifically, if a firm has expanded its investments at a growth rate equal the cost of capital, r , then $RoR = r$, regardless of the accounting rules.²⁷ When the accounting is conservative, the RoR will exceed r if past growth rates have been consistently below r (Rajan, Reichelstein and Soliman, 2007).²⁸ Conversely, $RoR \leq r$ if past growth rates have consistently exceeded r .

For the accounting Rate-of-Return, the Pivot Point is $(\lambda, RoR) = (r, r)$, while the Pivot Point for the P/E ratio is $(\lambda, PE) = (0, [r - \mu \frac{q(\mathbf{0})-1}{q(\mathbf{0})}]^{-1})$. For the RoR function, unbiased accounting delimits the quadrants by a horizontal axis at r . In contrast, the delimitation of the quadrants is given by the downward sloping curve $PE_T(\cdot, \mathbf{d}^*)$ for the P/E ratio. Furthermore, more conservative accounting rotates the P/E ratio, when viewed as a function of λ , in a counter-clockwise fashion. The opposite effect emerges for the RoR, which rotates in a clockwise fashion as the accounting rules become more conservative.

²⁷See, for instance, Salamon (1985), Fisher and McGowan (2003), or Gjesdal (2004).

²⁸These results have been obtained in a “representative project” model which is consistent with our capacity model in the special case of zero economic profits, that is, $p^* = c$.

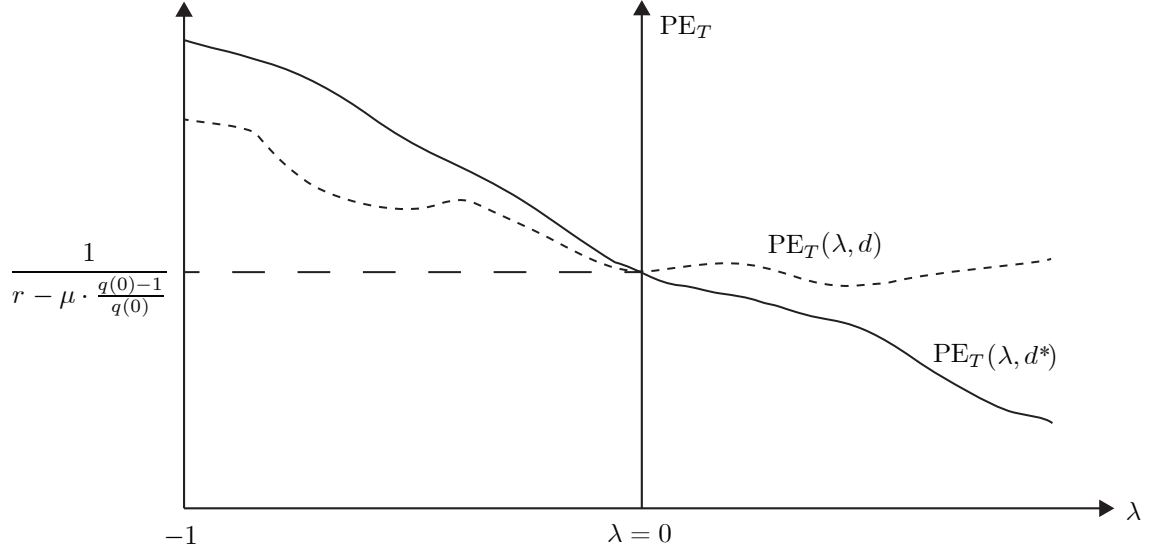


Figure 1: **Impact of Growth on the P/E Ratio**

If the applicable depreciation rules are sufficiently close to replacement cost accounting, the forces that drive $PE_T(\cdot, \mathbf{d}^*)$ to be decreasing in past growth will continue to prevail and, as a consequence of Proposition 3, the benchmark value identified in (18) would then be an upper bound for the P/E ratio for a growing firm. On the other hand if the accounting becomes sufficiently conservative, it is conceivable that the corresponding $PE_T(\cdot, \mathbf{d})$ curve ultimately ceases to be decreasing, as suggested by the dashed line in Figure 1. In exploring this possibility, the proportional depreciation rule,

$$d_\tau^p = \frac{x_\tau}{x_1 + \dots + x_T},$$

will be of particular interest. We recall that this rule reduces to the straight-line rule if assets correspond to the one-hoss shay productivity pattern. It will also be useful to note that proportional depreciation implies that:

$$\frac{d_\tau^p}{d_{\tau+1}^p} = \frac{x_\tau}{x_{\tau+1}}.$$

Our next finding relies on a stronger ordering of alternative depreciation schedules in terms of their degree of acceleration.

Definition 2: *Depreciation rule \mathbf{d} is more conservative than \mathbf{d}' if for any $\tau \leq T - 1$*

$$\frac{d_\tau}{d_{\tau+1}} \geq \frac{d'_\tau}{d'_{\tau+1}}. \quad (19)$$

Thus one depreciation rule is more conservative than another if the depreciation charges decline faster over time. Since their sum must be equal to one for both rules, the more conservative rule must entail greater charges in earlier periods. This implies that book values will be lower under the more conservative rule, and, therefore, the criterion in Definition 2 is stronger than that of Definition 1.

Observation 1: *If \mathbf{d} is more conservative than \mathbf{d}' , then \mathbf{d} is accelerated relative to \mathbf{d}' .*

The two definitions are not equivalent, as illustrated by the following example for assets with a useful life of three periods: $\mathbf{d} = (.9, .04, .06)$ and $\mathbf{d}' = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Then, \mathbf{d} is more accelerated than \mathbf{d}' , but not more conservative than \mathbf{d}' . While book values are always lower under depreciation rule \mathbf{d} , inequality (19) is violated at $\tau = 2$.

In many industries, a major source of unconditional conservatism is that firms directly expense expenditures related to intangible assets. To capture this effect in our results, we say that \mathbf{d}' is obtained from $\mathbf{d} = \mathbf{d} = (d_1, \dots, d_T)$ by increasing the share of investment directly expensed in the first year of operation, if for some $\eta > 0$,

$$d'_1 = (1 - \eta) d_1 + \eta,$$

and

$$d'_\tau = (1 - \eta) d_\tau$$

for $\tau > 1$. Depreciation rule \mathbf{d}' directly expenses an η -share of investment initially, and applies the same depreciation pattern as \mathbf{d} to the remaining book value. The following Observation confirms the intuition that direct expensing does indeed correspond to a higher degree of conservatism according to Definition 2 above.

Observation 2: *Suppose that \mathbf{d}' is obtained from \mathbf{d} by increasing the share of investment directly expensed. Then, \mathbf{d}' is more conservative than \mathbf{d} .*

In particular, a depreciation rule \mathbf{d} will be more conservative than the proportional depreciation rule if:

$$\frac{d_\tau}{d_{\tau+1}} \geq \frac{x_\tau}{x_{\tau+1}} \Leftrightarrow \frac{d_\tau}{x_\tau} \geq \frac{d_{\tau+1}}{x_{\tau+1}}. \quad (20)$$

for all τ . Thus, a depreciation rule is more conservative than proportional depreciation if the average depreciation charge per unit of productive capacity declines with the asset's age. Lemma 1 shows that this relation holds for replacement cost accounting if the productivity pattern satisfies assumption (A1).

Observation 3: *Given Assumption (A2), the proportional depreciation rule is more conservative than replacement cost accounting.*

We are now in a position to characterize the behavior of the P/E ratio in past growth. As in Propositions 2 and 3, we consider two firms that differ in only one of the realized investment growth rates, λ_t . All other parameters of the model, such as future growth in the product market and the firm's monopoly power, are presumed constant and equal across the two firms. As before, the firm with higher realized growth in investments can be viewed as having newer assets. We additionally impose the technical (and innocent) condition that the product price is sufficiently high so as to cover the average depreciation charge per unit of capacity in each period:

$$p^* \geq \max_{\tau} \frac{d_{\tau}}{x_{\tau}}$$

for $1 \leq \tau \leq T$.²⁹

Proposition 4 *Given Assumption (A1)-(A2), the forward P/E ratio $PE_T(\boldsymbol{\lambda}, \mathbf{d})$ is increasing in each λ_t for $1 < t \leq T$, provided \mathbf{d} is at least as conservative as the proportional depreciation rule.*

The intuition behind this result is most transparent in the one-hoss shay scenario. If depreciation is calculated according to the straight-line method, the aggregate depreciation expense will be proportional to the productive capacity regardless of whether the firm's assets are old or new. For example, the depreciation charge would be the same for assets purchased one and $T - 1$ periods ago provided these assets generate the same capacity in period $T + 1$. Therefore, if two firms operate at the same level of capacity, their forward earnings under the straight-line rule will be the same. However, the firm with newer assets will have a greater equity value, since it will have to replace its assets further into the future. The firm with newer assets will, therefore, have a higher P/E ratio.³⁰

Taken together, Propositions 2-4 show that past growth has opposite effects on the P/E ratio as the accounting moves from replacement cost depreciation to a schedule that is at

²⁹If this assumption is not satisfied, the firm's accounting earnings will be negative for certain investment histories that implement the optimal capacity levels. In that case, the forward P/E ratio will sometimes be negative on the optimal investment path. It can still be shown that the earnings yield, or the forward E/P ratio, is monotonic in each λ_t in such situations.

³⁰The logic of this argument is closely related to the so-called *old plant trap* usually associated with biases in the Accounting Rate-of-Return (see, for instance Lundholm and Sloan, 2007). The common feature is that differences in the age of incumbent assets may not be properly reflected in earnings or book values, thus causing an accounting-induced bias in the respective financial ratios.

least as conservative as proportional depreciation. We can further illustrate this finding by rewriting the P/E ratio as:

$$\frac{P_T}{E_{T+1}} = \frac{P_T/K_{T+1}}{E_{T+1}/K_{T+1}}. \quad (21)$$

As λ_t increases, so does the numerator of this ratio, since, by equation (9),

$$\frac{P_T}{K_{T+1}} = \frac{BV_T^* + RI_{T+1}^*/(r - \mu)}{K_{T+1}} = \frac{BV_T^*}{K_{T+1}} + \frac{p^* - c}{r - \mu}. \quad (22)$$

The monotonicity of P_T/K_{T+1} then follows from the first claim of Lemma 1 showing that the average replacement cost per unit of capacity produced is greater for newer assets.

Under replacement cost accounting, the denominator of the ratio on the right-hand side of (21), E_{T+1}^*/K_{T+1} , also increases in λ_t . This result follows from the second claim in Lemma 1. Recall that since the productive capacity of newer assets declines more slowly than that of older assets, the average depreciation charge per unit of capacity produced is lower for newer assets under replacement cost accounting. Therefore, as λ_t increases, the average depreciation expense decreases, and the average forward earnings increase. Proposition 2 then shows that the effect of past growth on the denominator of the ratio in the right-hand side of (21) dominates the effect of past growth on the numerator of that ratio, and, as a consequence, the P_T/E_{T+1}^* ratio decreases in λ_t .

In contrast, with proportional depreciation the numerator effect is the same, while the denominator does not depend on λ_t . This is because the aggregate depreciation expense is given by:

$$D_{T+1} = \frac{x_1}{x_1 + \dots + x_T} I_T + \dots + \frac{x_T}{x_1 + \dots + x_T} I_1 = \frac{K_{T+1}}{x_1 + \dots + x_T}.$$

Accordingly, the average forward *earnings per unit of capacity* are equal to

$$\frac{E_{T+1}(\mathbf{d}^p)}{K_{T+1}} = p^* - \frac{1}{x_1 + \dots + x_T}$$

independently of λ_t . Accordingly, the forward P/E ratio will then be increasing in past growth.

5 The P/E Ratio in the Geometric Setting

This section examines how the P/E ratio is affected by growth and accounting conservatism in the case of geometrically declining productivity. As observed above, this pattern is frequently considered in accounting, finance and economics studies alike, primarily for its analytical convenience.³¹ The crucial property of geometric asset decline is that the firm's capacity

³¹See, for example, Dixit and Pindyck (1994, p.374), Feltham and Ohlson (1996) or Biglaiser and Riordan (2000).

is homogeneous in the sense that it declines at the same rate for assets of all ages. As a consequence, the replacement cost of assets in place can be expressed as a function of current capacity only and does not depend on the age composition of the firm's assets. In particular, the geometric setting is characterized by two main assumptions. First, assets have an indefinite useful life and their productive capacity declines geometrically with age:

$$\mathbf{x} = (1, (1 - \alpha), (1 - \alpha)^2, \dots),$$

where $0 \leq \alpha \leq 1$. The aggregate capacity in period $t + 1$ will then be determined by the most recent investment and the aggregate capacity in the previous period:

$$K_{t+1} = (1 - \alpha) \cdot K_t + I_t. \quad (23)$$

Our second assumption is that depreciation charges per dollar of initial investment also decline geometrically with asset age:

$$d_{\tau+1} = (1 - \delta) \cdot d_\tau.$$

It can be verified that the requirement that assets be fully depreciated over their useful life then implies that $d_1 = \delta$. Therefore, we consider depreciation schedules of the following form:

$$\mathbf{d}^\delta = (\delta, (1 - \delta)\delta, (1 - \delta)^2\delta, \dots). \quad (24)$$

The corresponding asset valuation rule is given by:

$$\mathbf{bv}^\delta = (1, 1 - \delta, (1 - \delta)^2, \dots)$$

For this family of depreciation rules, the aggregate book values and depreciation expense will evolve so that

$$BV_{t+1}^\delta = (1 - \delta) \cdot BV_t^\delta + I_{t+1}, \quad (25)$$

and

$$D_{t+1}^\delta = \delta \cdot BV_t^\delta, \quad (26)$$

where the superscript δ indicates that the accounting numbers are calculated using the depreciation rule \mathbf{d}^δ .

When assets have a finite useful life, the firm's state at a particular date requires the full history of its latest T investments. In the geometric scenario, investments remain productive indefinitely, and, therefore, all investments since the inception of the firm are relevant for calculating future accounting numbers and capacity levels. Given the geometric structure, the firm's investment history at date T can be summarized by two numbers: book value,

BV_T , and the aggregate capacity in period $T + 1$, K_{T+1} . The firm's future capacity levels, depreciation expense and book values can be expressed as functions of these two numbers and the investments made after period T by recursively applying equations (23), (25), and (26).

The marginal cost of capacity in the geometric setting is given by

$$c = \frac{1}{\gamma + (1 - \alpha)\gamma^2 + \dots} = r + \alpha.$$

It is straightforward to verify that replacement cost accounting corresponds to the \mathbf{d}^α depreciation rule, i.e., a declining-balance rule with depreciation charges declining at the same rate as productive capacity. The historical cost charges are then equal to:

$$d_\tau^\alpha + r \cdot bv_{\tau-1}^\alpha = \alpha \cdot (1 - \alpha)^{\tau-1} + r \cdot (1 - \alpha)^{\tau-1} = (\alpha + r) \cdot (1 - \alpha)^{\tau-1} = c \cdot x_\tau,$$

which matches the historical cost charges under replacement cost accounting.

Interestingly, proportional depreciation also corresponds to the \mathbf{d}^α rule, because

$$\frac{d_\tau^\alpha}{x_\tau} = (1 - \alpha) = \frac{d_{\tau+1}^\alpha}{x_{\tau+1}}.$$

Propositions 2 and 4 have shown that the P/E ratio under replacement cost accounting decreases in past growth, while the P/E ratio under proportional depreciation increases in past growth. Since in the geometric setting the two depreciation rules coincide, it has to be that the P/E ratio in the geometric setting does not depend on past investment growth rates if the \mathbf{d}^α rule is applied. Furthermore, Proposition 1 also implies that Tobin's q also cannot depend on λ_t . As the following argument shows, these observations reflect the fact that the age-composition of assets in place is irrelevant in the geometric scenario.

Given the \mathbf{d}^α depreciation rule, the book value of a unit investment of age τ is equal to capacity this investment will generate in the next period:

$$bv_\tau^\alpha = (1 - \alpha)^\tau = x_{\tau+1}.$$

Therefore, the aggregate replacement cost of assets at each date is equal to the productive capacity in the following period:

$$BV_T^* = bv_0^\alpha \cdot I_T + \dots + bv_{T-1}^\alpha \cdot I_1 = x_1 \cdot I_T + \dots + x_T \cdot I_1 = K_{T+1}^*. \quad (27)$$

Intuitively, if two investments histories generate the same amount of capacity in a given period, their replacement costs will be equal regardless of the age composition of assets. The aggregate replacement cost of the firm's assets can then be calculated by assuming that all

assets were purchased in the latest period. Since x_1 is normalized to unity, to generate a capacity of K_{T+1}^* in period $T + 1$, the firm would have to make an investment of K_{T+1}^* in period T , and the replacement cost of this investment at date T would be precisely equal to K_{T+1}^* .

The firm's equity value also takes a particularly compact form in the geometric scenario. To increase the capacity of its assets at rate μ , the firm will need to make an investment of $(\alpha + \mu) K_{T+1}^*$ in period $T + 1$. The firm's total cash flows in period $T + 1$ will then be equal to

$$p^* \cdot K_{T+1}^* - (\alpha + \mu) K_{T+1}^*.$$

From period $T + 1$ onwards, the firm's cash flows will increase at rate μ , since both revenues and investments will increase at this rate. Therefore, the firm's value at date T is given by the capitalized value of the forward free cash flow:

$$P_T = \frac{p^* - \alpha - \mu}{r - \mu} K_{T+1}^*. \quad (28)$$

Equations (27) and (28) imply that Tobin's q reduces to

$$q = \frac{p^* - \alpha - \mu}{r - \mu}, \quad (29)$$

for any history of past growth rates. Since Tobin's q does not depend on λ_t , Proposition 1 implies that the P/E ratio under replacement cost accounting also cannot depend on past investment growth. These findings are summarized in the following observation.

Observation 4: *In the geometric setting, replacement cost accounting amounts to proportional depreciation. Tobin's q and the P_T/E_{T+1}^* ratio are given by*

$$q = 1 + \left(\frac{p^* - c}{c} \right) \left(\frac{r + \alpha}{r - \mu} \right),$$

$$\frac{P_T}{E_{T+1}^*} = \frac{P_T}{E_{T+1}(\mathbf{d}^\alpha)} = \frac{1}{r - \mu \cdot \frac{q-1}{q}}$$

for any history of investments.

Referring back to Figure 1, we find that in the geometric setting one obtains a traditional "quadrant result" in which the horizontal boundary of the four quadrants is indeed flat and given by for the P/E ratio with

$$PE_T = \frac{1}{r - \mu \cdot \frac{q-1}{q}} \quad (30)$$

An immediate consequence of Proposition 3 is that, at least locally near $\boldsymbol{\lambda} = \mathbf{0}$, the ratio $PE_T(\cdot, \mathbf{d})$ must be increasing in λ_t , provided \mathbf{d} is more accelerated than replacement cost accounting. On the other hand, for the geometric setting we observe that a depreciation rule \mathbf{d}^δ is more accelerated (and equivalently more conservative) than \mathbf{d}^α if and only if $\delta > \alpha$. Consistent with Proposition 4, we obtain the following result.

Observation 5: *In the geometric scenario, the forward P/E ratio $PE_T(\boldsymbol{\lambda}, \mathbf{d}^\delta)$ is increasing in each λ_t for $t \leq T$ if the depreciation schedule \mathbf{d}^δ is accelerated relative to the replacement cost rule.*

The geometric setting allows for a further characterization the interaction between past investment growth and accounting conservatism on the forward P/E ratio. The following result will confine attention to a setting where the firm has experienced *constant* investment growth for an infinite history. Specifically,

$$I_T = (1 + \lambda) I_{T-1} = (1 + \lambda)^2 I_{T-2} = \dots$$

for some $\lambda \geq 0$. The interpretation here is that up to period $T + 1$, demand for the firm's product has expanded at the constant rate λ . For notational simplicity, let $PE_T(\lambda, \delta)$ denote the forward PE ratio if the depreciation rule \mathbf{d}^δ is used.

Proposition 5 *In the geometric scenario, assume that investments have grown at a constant rate $\lambda \geq 0$ in the past. The forward P/E ratio $PE_T(\lambda, \delta)$ then satisfies:*

$$\frac{\partial^2 PE_T(\lambda, \delta)}{\partial \lambda \partial \delta} \geq 0,$$

provided the depreciation rule in use is accelerated relative to replacement cost accounting, that is, $\delta > \alpha$.

Figure 2 summarizes our findings for the geometric scenario. In particular, the figure illustrates a “traditional” quadrant result for the P/E ratio and the complementary nature between past investment growth and conservatism: for two different growth rates, λ_1 and λ_2 , the P/E ratio exhibits increasing differences as the accounting becomes more conservative. Thus,

$$PE_T(\lambda_2, \delta_2) - PE_T(\lambda_1, \delta_2) > PE_T(\lambda_2, \delta_1) - PE_T(\lambda_1, \delta_1).$$

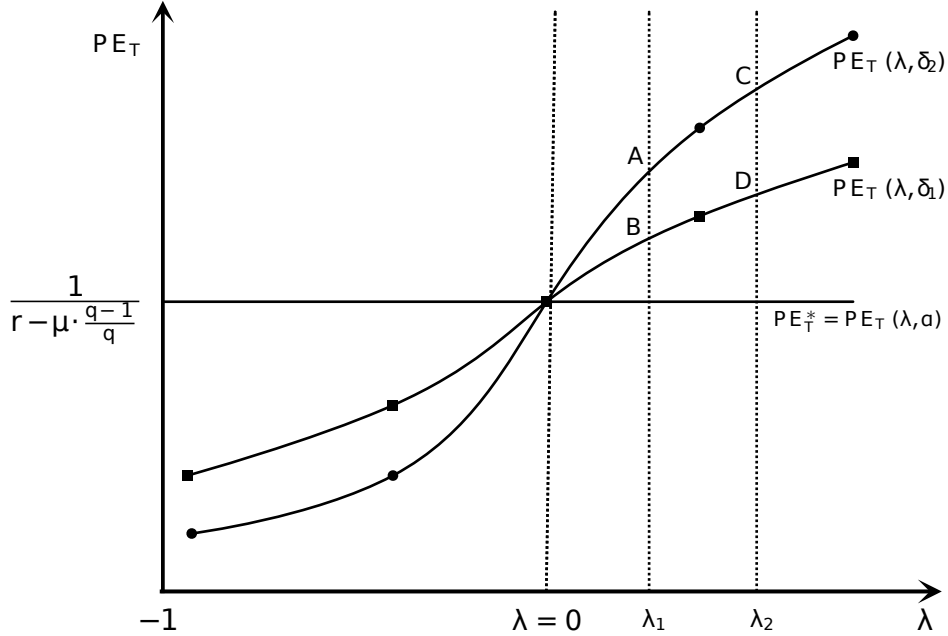


Figure 2: The P/E Ratio in the Geometric Setting ($\delta_2 > \delta_1 > \alpha$)

The key analytical feature of the geometric decline scenario is that the firm’s market value does not depend on the age composition of its assets, so past growth affects the forward P/E only through the denominator of the ratio. A more accelerated depreciation rule is characterized by front-loaded depreciation charges. For a growing firm, these higher depreciation charges are applied to larger (more recent) investments in the calculation of the aggregate depreciation expense in period $T + 1$. As a consequence, the marginal effect of past growth on the forward PE ratio is stronger for more accelerated depreciation rules.

6 Conclusion

The analysis in this paper has examined the interdependence of several variables which jointly determine the magnitude of a firm’s Price-to-Earnings ratio. These variables include the accrual accounting rules in use, the intertemporal pattern of past investments, future growth opportunities in the firm’s product market and the degree of competitiveness for the firm’s products. Our framework of overlapping capacity investments identifies replacement cost accounting as being unbiased insofar as residual income does measure economic profit. While this accounting for operating assets is arguably not descriptive of current financial reporting rules in many industries, it nonetheless allows us to identify a benchmark value for the P/E ratio. This benchmark can be expressed as a “convex combination” between

the permanent earnings and the Gordon growth models. The actual mix between these two “endpoint” P/E ratios is determined entirely by Tobin’s q for the particular firm. For firms in a competitive environment ($q \rightarrow 1$), the P/E ratio will approximate the permanent earnings model, while for a firm with strong pricing power ($q \rightarrow \infty$) the P/E ratio will tend towards that implied by the Gordon growth formula.

Higher growth in future periods in the product market will unambiguously increase the benchmark value for P/E ratio, unless the firm operates in a competitive industry which yield zero economic profits regardless of growth opportunities. In contrast, we show that the impact of higher past growth in investments on the P/E ratio cannot be predicted without reference to the underlying accounting rules. Under replacement cost accounting, higher past growth leads to lower values of Tobin’s q and a lower benchmark value for the P/E ratio. For conservative accounting rules, in contrast, the P/E ratio will exceed its benchmark value for a firm that has been growing in the past, with the opposite being true for a pattern of declining investments in the past. Once the accounting rules become sufficiently conservative, we find that *ceteris paribus* higher growth rates will actually increase the predicted P/E ratio. In particular, this will be true if assets have an undiminished productive use over a finite life span and depreciation is calculated according to the straight line rule.

The findings in this paper suggest a number of promising directions for empirical testing. While for some of our explanatory variables the choice of empirical construct appears rather straightforward, e.g., past growth in investments, a number of empirical proxies come to mind for other variables such as a firm’s current pricing power, anticipated future growth in the product market or the degree of conservatism. Among the main predictions of our model, it would seem particularly important to obtain empirical support for the the suggested relation between the P/E ratio and Tobin’s q in Proposition 1, the quadrant result in Proposition 3, and the predicted interaction between past growth and accounting conservatism, as characterized in Propositions 4 and 5.

Our model has relied on several simplifying assumptions. First, we posited that the firm is always in a position to sell its product at a price which at least covers production costs. Yet, an economically profitable firm may report negative accounting earnings if investments are expensed at a faster rate than their economic value declines. Since the P/E ratio is discontinuous at zero accounting earnings, we restricted attention to settings where the firm’s accounting earnings are positive. Our monotonicity results continue to hold for economically profitable firms with negative accounting earnings if instead of the P/E ratio one considers the forward earnings yield, or the E/P ratio.³² Providing an interpretation for the P/E ratios

³²In his empirical investigation, Penman (1996) also cites continuity considerations for studying the E/P rather than the P/E ratio.

of firms making economic losses would be an interesting direction for future research.

Another major assumption of our model is that the firm operates in a growing product market. If demand for the firm's product can decline over time, myopically optimal investment decisions that equate marginal revenues to the marginal cost of capacity may lead to excess capacity situations in future periods. Anticipating the possible excess capacity problem, a firm will optimally invest less than it would under the myopic rule. Therefore, if a firm faces a declining product market, its optimal investment policy will be different from the one considered in our paper, and our benchmark for the P/E ratio will change.

Finally, our analysis has ignored some of the determinants of the P/E ratio suggested by earlier literature, including leverage and dividend policy. We have also treated the firm's cost of capital as exogenous and independent of the projects undertaken by the firm. Incorporating these additional factors into our modeling framework would lead to a more complete understanding of the P/E ratio and its determinants.

Appendix

Proof of Proposition 1:

Proposition 2 in Nezlobin (2012) shows that for arbitrary future growth rates in the product market, $\mu_{T+i} \geq 0$,

$$P_T = BV_T^* + \alpha_T RI_T^*,$$

where $\alpha_T = \sum_{\tau=1}^{\infty} \gamma^\tau \prod_{i=1}^{\tau} (1 + \mu_{T+i})$. If the product market grows at a constant rate μ after period $T + 1$, the expression for α_T can be simplified as

$$\begin{aligned} \alpha_T &= \gamma(1 + \mu_{T+1}) + \gamma^2(1 + \mu_{T+1})(1 + \mu) + \gamma^3(1 + \mu_{T+1})(1 + \mu)^2 + \dots \\ &= \frac{(1 + \mu_{T+1})}{r - \mu}. \end{aligned}$$

Therefore,

$$\begin{aligned} P_T &= BV_T^* + \frac{(1 + \mu_{T+1})}{r - \mu} RI_T^* \\ &= BV_T^* + \frac{RI_{T+1}^*}{r - \mu} = \frac{E_{T+1}^*}{r - \mu} - \frac{\mu \cdot BV_T^*}{r - \mu}. \end{aligned}$$

Dividing both sides by P_T yields

$$1 = \frac{1}{r - \mu} \frac{E_{T+1}^*}{P_T} - \frac{\mu/q}{r - \mu},$$

which implies

$$\frac{P_T}{E_{T+1}^*} = \frac{1}{r - \mu + \mu/q}.$$

■

In some of the proofs below, we will use the following Lemma. For the proof of this Lemma, see Rajan and Reichelstein (2009), pp. 855-857 (Claim 2 in the proof of Proposition 3 of that paper).

Lemma A *For any numbers a_1, \dots, a_n , positive numbers b_1, \dots, b_n , and growth rates $\xi_2, \dots, \xi_n \geq -1$, the function*

$$f(\xi_1, \dots, \xi_n) = \frac{a_n + (1 + \xi_2) a_{n-1} + \dots + (1 + \xi_2) \dots (1 + \xi_n) a_1}{b_n + (1 + \xi_2) b_{n-1} + \dots + (1 + \xi_2) \dots (1 + \xi_n) b_1}$$

is everywhere increasing (decreasing) in each ξ_i for $2 \leq i \leq n$, if the sequence

$$\frac{a_i}{b_i}$$

is decreasing (increasing) in i .

Proof of Lemma 1:

Note that

$$\frac{bv_{\tau-1}^*}{bv_{\tau}^*} = \frac{\gamma x_{\tau} + \dots + \gamma^{T-\tau+1} x_T}{\gamma x_{\tau+1} + \dots + \gamma^{T-\tau+1} x_{T+1}},$$

where $x_{T+1} = 0$.

Then, we can apply Lemma A to the sequences defined by the following equations:

$$\begin{aligned} (a_1, \dots, a_n) &= (x_T, \dots, x_{\tau}), \\ (b_1, \dots, b_n) &= (x_{T+1}, \dots, x_{\tau+1}), \\ (1 + \xi_2, \dots, 1 + \xi_n) &= (\gamma, \dots, \gamma), \\ (1 + \xi'_2, \dots, 1 + \xi'_n) &= (0, \dots, 0). \end{aligned}$$

Since the productivity pattern satisfies Assumption (A2),

$$\frac{a_{i-1}}{b_{i-1}} \geq \frac{a_i}{b_i}.$$

Therefore, the function f from Lemma A will be increasing in each ξ_i and its value at (ξ_2, \dots, ξ_n) will be greater than its value at (ξ'_2, \dots, ξ'_n) . Hence,

$$\frac{bv_{\tau-1}^*}{bv_{\tau}^*} \geq \frac{\gamma \cdot x_{\tau} + 0 \cdot x_{\tau+1} + \dots + 0 \cdot x_T}{\gamma \cdot x_{\tau+1} + 0 \cdot x_{\tau+2} + \dots + 0 \cdot x_{T+1}} = \frac{x_{\tau}}{x_{\tau+1}},$$

and it follows that the sequence $\frac{bv_{\tau-1}^*}{x_{\tau}}$ is decreasing in τ .

Now recall that

$$\frac{d_{\tau}^* + bv_{\tau-1}^*}{x_{\tau}} = c.$$

Since $\frac{bv_{\tau-1}^*}{x_{\tau}}$ is decreasing in τ , it has to be that the sequence $\frac{d_{\tau}^*}{x_{\tau}}$ increases in τ . ■

Proof of Proposition 2:

From Proposition 1 we know that:

$$\frac{P_T}{E_{T+1}^*} = \frac{1}{r - \mu + \mu/q}.$$

It thus suffices to show that Tobin's q is increasing in λ_t . Tobin's q can be rewritten as

$$\begin{aligned} q &= \frac{P_T}{BV_t^*} = \frac{E_{T+1}^* - \mu \cdot BV_T^*}{(r - \mu) \cdot BV_T^*} = \\ &= \frac{1}{(r - \mu)} \cdot \frac{(x_1 p^* - d_1^* - \mu bv_0^*) I_T + \dots + (x_T p^* - d_T^* - \mu bv_{T-1}^*) I_1}{bv_0^* I_T + \dots + bv_{T-1}^* I_1}. \end{aligned}$$

If we show that

$$\frac{x_{i+1} p^* - d_{i+1}^* - \mu bv_i^*}{bv_i^*}$$

increases in i , then the monotonicity of Tobin's q will follow from Lemma A by setting $n = T$, $a_i = x_i p^* - d_i^* - \mu b v_{i-1}^*$, $b_i = b v_{i-1}^*$, and $\xi_i = \lambda_i$.

Observe that

$$\begin{aligned} \frac{x_{i+1} p^* - d_{i+1}^* - \mu b v_i^*}{b v_i^*} &= \frac{x_{i+1} p^* - d_{i+1}^* - r b v_i^* + (r - \mu) b v_i^*}{b v_i^*} = \\ &= \frac{p^* - c}{(b v_i^* / x_{i+1})} + r - \mu. \end{aligned}$$

It remains to invoke Lemma 1 where it was shown that $b v_i^* / x_{i+1}$ is decreasing in i if the productivity pattern satisfies (A2). \blacksquare

Proof of Proposition 3:

Let D_{T+1} and D'_{T+1} denote the aggregate depreciation expenses under rules \mathbf{d} and \mathbf{d}' , respectively, where \mathbf{d} is more accelerated than \mathbf{d}' . We will show that $D_{T+1} \geq D'_{T+1}$ ($D_{T+1} \leq D'_{T+1}$) if investments I_1, \dots, I_T are monotonically increasing (decreasing). From this it will follow that if \mathbf{d} is more accelerated than replacement cost accounting and the firm is growing, then

$$PE_T(\boldsymbol{\lambda}, \mathbf{d}) = \frac{P_T(\boldsymbol{\lambda})}{E_{T+1}(\boldsymbol{\lambda}, \mathbf{d})} \geq \frac{P_T(\boldsymbol{\lambda})}{E_{T+1}^*} = \frac{1}{r - \mu \cdot \frac{q(\boldsymbol{\lambda}) - 1}{q(\boldsymbol{\lambda})}}.$$

Observe that

$$\begin{aligned} D_{T+1} &= I_1 \cdot d_T + I_2 \cdot d_{T-1} + \dots + I_T \cdot d_1 \\ &= I_1 + (I_2 - I_1) \cdot (1 - b v_{T-1}) + \dots + (I_T - I_{T-1}) \cdot (1 - b v_1). \end{aligned}$$

Therefore,

$$D_{T+1} - D'_{T+1} = (I_2 - I_1) \cdot (b v'_{T-1} - b v_{T-1}) + \dots + (I_T - I_{T-1}) \cdot (b v'_1 - b v_1) \geq 0,$$

if investments are increasing and \mathbf{d} is more accelerated than \mathbf{d}' . \blacksquare

Proof of Observation 1:

Assume that \mathbf{d} is more conservative than \mathbf{d}' . Since

$$\sum d_\tau = 1,$$

we can rewrite d_1 as:

$$d_1 = \frac{1}{1 + \frac{d_2}{d_1} + \dots + \frac{d_2}{d_1} \frac{d_3}{d_2} \dots \frac{d_T}{d_{T-1}}}. \quad (31)$$

Since \mathbf{d} is more conservative than \mathbf{d}' ,

$$\frac{d_{\tau+1}}{d_\tau} \leq \frac{d'_{\tau+1}}{d'_\tau}.$$

Equation (31) then implies that $d_1 \geq d'_1$. Note that if $d_\tau \leq d'_\tau$ for some τ , then $d_{\tau+1} \leq d_\tau \frac{d'_{\tau+1}}{d'_\tau} \leq d'_{\tau+1}$. Applying the same argument iteratively, one can verify that $d_\tau \leq d'_\tau$ implies $d_{\tau+i} \leq d'_{\tau+i}$ for any i .

Let $\delta(i) = bv_i - bv'_i$. We have the following observations:

1. $\delta(1) = bv_1 - bv'_1 \leq 0$.
2. If $\delta(\tau) - \delta(\tau + 1) \leq 0$ for some τ , then $\delta(\tau + i) - \delta(\tau + i + 1) \leq 0$ for any $i \geq 0$.
3. $\delta(T) = 0$.

The function δ is negative at one, and once it becomes increasing it continues to increase up to the end of the useful life. Therefore, $\delta(i)$ can only cross zero once and we know that this happens at $i = T$. The three observations hence imply that $\delta(i) \leq 0$ and $bv_i \leq bv'_i$ for all i . ■

Proof of Observation 2:

Assume that

$$d'_1 = (1 - \eta) d_1 + \eta,$$

and

$$d'_\tau = (1 - \eta) d_\tau$$

for $\tau > 1$ and some $\eta > 0$.

Note that

$$\frac{d_\tau}{d_{\tau+1}} = \frac{d'_\tau}{d'_{\tau+1}}$$

for $\tau > 1$. For $\tau = 1$, we have

$$\frac{d'_1}{d'_2} = \frac{(1 - \eta) d_1 + \eta}{(1 - \eta) d_2} > \frac{d_1}{d_2}.$$

Therefore, \mathbf{d}' is more conservative than \mathbf{d} . ■

Proof of Proposition 4:

We expand the P/E ratio as:

$$\begin{aligned} \frac{P_T}{E_{T+1}} &= \frac{1}{(r - \mu)} \cdot \frac{E_{T+1}^* - \mu BV_T^*}{E_{T+1}} = \\ &= \frac{1}{(r - \mu)} \cdot \frac{I_T(x_1 p^* - d_1^* - \mu bv_0^*) + \dots + I_1(x_T p^* - d_T^* - \mu bv_{T-1}^*)}{I_T(x_1^* p^* - d_1) + \dots + I_1(x_T p^* - d_T)} \end{aligned}$$

To apply Lemma A, we need to show that

$$\frac{x_i p^* - d_i^* - \mu b v_{i-1}^*}{x_i p^* - d_i}$$

is decreasing in i . Note that

$$\begin{aligned} \frac{x_i p^* - d_i^* - \mu b v_{i-1}^*}{x_i p^* - d_i} &= \frac{x_i p^* - d_i^* - r b v_{i-1}^* + (r - \mu) b v_{i-1}^*}{x_i \left(p^* - \frac{d_i}{x_i} \right)} \\ &= \frac{p^* - c + (r - \mu) \frac{b v_{i-1}^*}{x_i}}{p^* - \frac{d_i}{x_i}}. \end{aligned} \quad (32)$$

(A1) implies that $\frac{b v_{i-1}^*}{x_i}$ is decreasing in i , and since $r > \mu$, the numerator is decreasing in i . If \mathbf{d} corresponds to proportional depreciation, then the denominator does not depend on i . If \mathbf{d} is more conservative than the proportional depreciation rule, then $\frac{d_i}{x_i}$ is decreasing in i , and the denominator is increasing in i . Therefore, the ratio (32) is decreasing in i . ■

Proof of Observation 5: Without loss of generality, let date 0 be the firm's inception date and date T be the date at which the P/E ratio is calculated. Let us consider two firms with equal profitability, equal investments in the first period, and having equal past investment growth rates except in period $\tau + 1$, in which the growth rates were equal to $\lambda_{\tau+1}$ and $\lambda'_{\tau+1}$. Since the growth rates after period $\tau + 1$ are equal, the aggregate capacity of the second firm in period $T + 1$, K'_{T+1} , will be equal to

$$K'_{T+1} = \frac{1 + \lambda'_{\tau+1}}{1 + \lambda_{\tau+1}} \left(K_{T+1} - (1 - \alpha)^{T-\tau+1} \cdot K_{\tau} \right) + (1 - \alpha)^{T-\tau+1} \cdot K_{\tau}, \quad (33)$$

where K_{T+1} is the aggregate capacity of the first firm. Similarly, the following equation will hold for the aggregate depreciation expenses in period $T + 1$:

$$D'_{T+1} = \frac{1 + \lambda'_{\tau+1}}{1 + \lambda_{\tau+1}} \left(D_{T+1} - (1 - \delta)^{T-\tau+1} \cdot D_{\tau} \right) + (1 - \delta)^{T-\tau+1} \cdot D_{\tau}. \quad (34)$$

From equations (33) and (34), it follows that

$$\frac{\partial K_{T+1}}{\partial \lambda_{\tau+1}} = \frac{K_{T+1} - (1 - \alpha)^{T-\tau+1} \cdot K_{\tau}}{1 + \lambda_{\tau+1}}$$

and

$$\frac{\partial D_{T+1}}{\partial \lambda_{\tau+1}} = \frac{D_{T+1} - (1 - \delta)^{T-\tau+1} \cdot D_{\tau}}{1 + \lambda_{\tau+1}}.$$

To show that the forward P/E ratio is monotonic it suffices to show that

$$\partial \ln \left(\frac{P_T}{E_{T+1}} \right) / \partial \lambda_{\tau+1} \geq 0.$$

This will imply that $\ln(P_T/E_{T+1})$ increases in $\lambda_{\tau+1}$, and, therefore, P_T/E_{T+1} increases in $\lambda_{\tau+1}$.

Recall that

$$P_T = \frac{p^* - \alpha - \mu}{r - \mu} K_{T+1}$$

and

$$E_{T+1} = p^* K_{T+1} - D_{T+1}.$$

Therefore,

$$\begin{aligned} \partial \ln \left(\frac{P_T}{E_{T+1}} \right) / \partial \lambda_{\tau+1} &= \frac{1}{P_T} \frac{\partial P_T}{\partial \lambda_{\tau+1}} - \frac{1}{E_{T+1}} \frac{\partial E_{T+1}}{\partial \lambda_{\tau+1}} \\ &= \frac{1}{K_{T+1}} \frac{K_{T+1} - (1 - \alpha)^{T-\tau+1} \cdot K_\tau}{1 + \lambda_{\tau+1}} \\ &\quad - \frac{1}{E_{T+1}} \left(p^* \cdot \frac{K_{T+1} - (1 - \alpha)^{T-\tau+1} \cdot K_\tau}{1 + \lambda_{\tau+1}} - \frac{D_{T+1} - (1 - \delta)^{T-\tau+1} \cdot D_\tau}{1 + \lambda_{\tau+1}} \right). \end{aligned}$$

The expression above has the same sign as

$$-\frac{K_\tau}{K_{T+1}} + \frac{p^* K_\tau - \left(\frac{1-\delta}{1-\alpha}\right)^{T-\tau+1} D_\tau}{E_{T+1}},$$

which in turn is non-negative if

$$\frac{(1 - \delta)^{T-\tau+1} D_\tau}{(1 - \alpha)^{T-\tau+1} K_\tau} \leq \frac{D_{T+1}}{K_{T+1}}.$$

Note that

$$\frac{D_{T+1}}{K_{T+1}} = \frac{(1 - \delta)^{T-\tau+1} D_\tau + d_{T-\tau+1} \cdot I_\tau + \dots + d_1 \cdot I_T}{(1 - \alpha)^{T-\tau+1} K_\tau + x_{T-\tau+1} \cdot I_\tau + \dots + x_1 \cdot I_T} \quad (35)$$

and

$$\frac{d_1}{x_1} > \frac{d_2}{x_2} > \dots > \frac{d_{T-\tau+1}}{x_{T-\tau+1}} \quad (36)$$

if $\delta > \alpha$. Since

$$d_i < \delta \cdot x_i$$

for all i , it has to be that

$$\frac{D_\tau}{K_\tau} \leq \delta.$$

Therefore,

$$\frac{d_{T-\tau+1}}{x_{T-\tau+1}} = \delta \frac{(1 - \delta)^{T-\tau}}{(1 - \alpha)^{T-\tau}} \geq \frac{(1 - \delta)^{T-\tau+1} D_\tau}{(1 - \alpha)^{T-\tau+1} K_\tau}. \quad (37)$$

Inequalities (36) and (37) allow us to apply Lemma A to the ratio in (35) to verify that

$$\frac{D_{T+1}}{K_{T+1}} = \frac{(1-\delta)^{T-\tau+1} D_\tau + d_{T-\tau+1} \cdot I_\tau + \dots + d_1 \cdot I_T}{(1-\alpha)^{T-\tau+1} K_\tau + x_{T-\tau+1} \cdot I_\tau + \dots + x_1 \cdot I_T} \geq \frac{(1-\delta)^{T-\tau+1} D_\tau}{(1-\alpha)^{T-\tau+1} K_\tau}.$$

■

Proof of Proposition 5:

Assume that the firm has an infinite history of constant growth:

$$I_T = (1+\lambda) I_{T-1} = (1+\lambda)^2 I_{T-2} = \dots$$

for some $\lambda \geq 0$. Then,

$$D_{T+1} = \delta \left(I_T + \frac{I_{T-1}(1-\delta)}{(1+\lambda)} + \frac{I_{T-1}(1-\delta)^2}{(1+\lambda)^2} + \dots \right) = \frac{(1+\lambda)\delta}{\lambda+\delta} \cdot I_T.$$

Similarly,

$$K_{T+1}^* = \frac{1+\lambda}{\lambda+\alpha} \cdot I_T.$$

It follows from equation (28) that

$$PE_T(\lambda, \delta) = \frac{p^* - \alpha - \mu}{r - \mu} \cdot \frac{K_{T+1}^*}{p^* K_{T+1}^* - D_{T+1}} = A \cdot \frac{1}{p^* - \frac{\delta(\lambda+\alpha)}{\lambda+\delta}},$$

where A does not depend on either λ or δ . Therefore, the cross-derivative of the PE ratio in λ and δ will have the same sign as

$$\frac{\partial^2 \frac{\lambda+\delta}{p^*(\lambda+\delta) - \delta(\lambda+\alpha)}}{\partial \lambda \partial \delta} = \frac{p^* (2\delta\lambda + \alpha(\delta - \lambda)) - \alpha\delta(\alpha + \lambda)}{(p^*(\delta + \lambda) - \delta(\alpha + \lambda))^3}. \quad (38)$$

Since we have assumed that

$$p^* \geq \max_{\tau} \frac{d_\tau}{x_\tau},$$

p^* must exceed δ . Therefore, the denominator of the ratio in the right-hand side of (38) is positive if the depreciation rule is use is accelerated relative to replacement cost accounting ($\delta > \alpha$). Note also that since $p^* \geq \delta > \alpha$,

$$\begin{aligned} p^* (2\delta\lambda + \alpha(\delta - \lambda)) - \alpha\delta(\alpha + \lambda) &\geq \alpha (2\delta\lambda + \alpha(\delta - \lambda)) - \alpha\delta(\alpha + \lambda) \\ &\geq \alpha (\delta\lambda - \alpha\lambda) \geq 0, \end{aligned}$$

and, therefore the ratio in (38) is non-negative. ■

References

Arrow, K. “Optimal Capital Policy, Cost of Capital and Myopic Decision Rules.” *Annals of the Institute of Statistical Mathematics* 1-2 (1964): 21-30.

Basu, S. “Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis.” *Journal of Finance* 32 (1977): 663-82.

Beaver, W., and R. Dukes. “ δ -Depreciation Methods: Some Analytical Results.” *Journal of Accounting Research* 9 (1974): 391-419.

Beaver, W., and D. Morse. “What Determines Price-Earnings Ratios?” *Financial Analysts Journal* 34, 4 (1978): 65-76.

Berk, J., R. Green, and V. Naik. “Optimal Investment, Growth Options and Security Returns.” *Journal of Finance* 54 (1999): 1153–1607.

Biglaiser, G and M. Riordan. “Dynamics of Price Regulation.” *Rand Journal of Economics* 31 (2000): 744-767.

Craig, D., J. Johnson, and M. Joy. “Accounting Methods and P/E Ratios.” *Financial Analysts Journal* 43, 2 (1987): 41- 45.

Damodaran, A. *Damodaran on Valuation*. 2nd ed. Hoboken, New Jersey: John Wiley & Sons, Inc., 2006.

Dixit A. and R. Pindyck. *Investment Under Uncertainty*. Princeton University Press, Princeton NJ. (1994).

Dutta, S. and S. Reichelstein. “Decentralized Capacity Management and Internal Pricing,” *Review of Accounting Studies* 15 (2010). 442-478.

Fairfield, P. M. “P/E, P/B and the Present Value of Future Dividends.” *Financial Analysts Journal* 50, 4 (1994): 23-31.

Feltham, G., and J. Ohlson. “Valuation and Clean Surplus Accounting for Operating and

Financial Activities.” *Contemporary Accounting Research* 11 (1995): 689-731.

Feltham, G., and J. Ohlson. “Uncertainty Resolution and the Theory of Depreciation Measurement.” *Journal of Accounting Research* 34 (1996): 209-234.

Fisher, F., and J. McGowan. “On the Misuse of Accounting Rates of Return to Infer Monopoly Profits.” *American Economic Review* 96 (1983): 82-97.

Financial Accounting Standards Board. “Elements of Financial Statements” *Statement of Financial Accounting Concepts No. 6* (1985), Norwalk, CT: FASB.

Gjesdal, F. “A Steady State Growth Valuation Model: A Note on Accounting and Valuation.” Working paper, University of Bergen (2004).

Jaffe, J., D. B. Keim, and R. Westerfield. “Earnings Yields, Market Values, and Stock Returns.” *Journal of Finance* 44, 1 (1989): 135-48.

Laffont, J. and J. Tirole, *Competition in Telecommunications*. MIT Press. Cambridge MA. (2000).

Lindenberg, E. B., and S. A. Ross. “Tobin’s q Ratio and Industrial Organization.” *Journal of Business* 54 (1981): 1-32.

Lerner, A. “The Concept of Monopoly and the Measurement of Monopoly.” *Review of Economic Studies* 1 (1934): 157–75.

Lundholm, R. and R. Sloan. *Equity Valuation and Analysis with eVal*. McGraw Hill Irwin, New York NY. (2007).

Martin, S. *Advanced Industrial Economics*, Blackwell Publishers, Oxford. 2002.

McNichols, M., M. Rajan, and S. Reichelstein “Decomposition of the Market-to-Book Ratio.” Working Paper (2012), Stanford University.

Nezlobin, A. “Accrual Accounting, Informational Sufficiency, and Equity Valuation.” *Journal of Accounting Research* 50 (2012): 233-273.

Nezlobin, A., Rajan, M. and S. Reichelstein “Dynamics of Rate of Return Regulation.” *Management Science* (2012).

Ohlson, J. “Earnings, Book Values, and Dividends in Equity Valuation.” *Contemporary Accounting Research* (Spring, 1995): 661-687.

Ohlson, J., and B. Juettner-Nauroth. “Expected EPS and EPS Growth as Determinants of Value.” *Review of Accounting Studies* 10, 2-3 (2005).

Ohlson, J. and Z. Gao. “Earnings, Earnings Growth and Value.” *Foundations and Trends in Accounting* 1 (2006): 1-70..

Ohlson, J., and X. Zhang. “Accrual Accounting and Equity Valuation.” *Journal of Accounting Research* 36 (1998): 85-111.

Penman, S. “The Articulation of Price-Earnings Ratios and Market-to-Book Ratios and the Evaluation of Growth.” *Journal of Accounting Research* 34 ,2(1996): 235-259.

Penman, S. *Financial Statement Analysis and Security Valuation*. 4th ed. New York, NY: McGraw-Hill/Irwin, 2010.

Penman, S. *Accounting for Value*. New York, NY: Columbia Business School Press, 2011.

Rajan, M., and S. Reichelstein. “Depreciation Rules and the Relation between Marginal and Historical Cost.” *Journal of Accounting Research* 47 (2009): 823-867.

Rajan, M., S. Reichelstein, and M. Soliman. “Conservatism, Growth, and Return on Investment.” *Review of Accounting Studies* 12 (2007): 325-370.

Rogerson, W. “Inter-Temporal Cost Allocation and Managerial Investment Incentives: A Theory Explaining the Use of Economic Value Added as a Performance Measure.” *Journal of Political Economy* 105 (1997): 770-795.

Rogerson, W. “On the Relationship Between Historic, Forward Looking Cost, and Long Run Marginal Cost.” *Review of Network Economics* 10, 2 (2011): Article 2.

Rogerson, W. "Inter-Temporal Cost Allocation and Investment Decisions." *Journal of Political Economy* 105 (2008): 770-795.

Salamon, G. "Accounting Rates of Return." *The American Economic Review* 75 (1985): 495-504.

Salinger, M. "Tobin's q, Unionization, and the Concentration-Profit Relationship." *Rand Journal of Economics* 15 (1984): 159-170.

Thomadakis, S. "A Model of Market Power, Valuation, and the Firm's Returns." *The Bell Journal of Economics* 7, 1 (1976): 150-162.

Thomas, J., and F. Zhang. "Understanding Two Remarkable Findings about Stock Yields and Growth." *Journal of Portfolio Management* 35 (2009): 158-165.

Zarowin, P. "What Determines Earnings-Price Ratios: Revisited." *Journal of Accounting, Auditing and Finance* 5, 3 (1990): 439-54.

Zhang, X. "Conservative Accounting and Equity Valuation." *Journal of Accounting and Economics* 29 (2000): 125-149.