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The authors propose that a general class of market response models with marketing parameter equations be used for the study of marketing interactions. They inventory alternative model specifications of marketing mix interactions, along with the estimation procedures, and compare two that are relevant in an application to the determinants of U.S. Navy salesforce effectiveness. The authors also illustrate the importance of modeling interactions as a basis for making inferences about marketing mix resource allocation.

Modeling Marketing Interactions with Application to Salesforce Effectiveness

It is well recognized in marketing that the relationship between market performance (e.g., product sales) and marketing efforts is influenced by interaction mechanisms. Advertising may influence sales levels directly, but it also could affect customers' price sensitivity. Likewise, salesforce effectiveness may be a function of the competitiveness of the sales environment. Such interactions were popularized several years ago with the introduction of the "marketing mix" concept, which emphasizes that marketing efforts create sales synergistically rather than independently.

The literature contains substantial empirical support for the presence of marketing interactions. *Advertising effectiveness* reportedly increases with product quality (Kuehn 1962), prior salesperson contact (Swinyard and Ray 1977), retail availability (Kuehn 1962; Parsons 1974), and higher or lower price depending on the ad medium (Prasad and Ring 1976). *Sales call effectiveness* in medical marketing increases with the use of samples and handouts (Parsons and Vanden Abeele 1981). The effects of advertising on consumers' *price sensitivity* are subject to an active debate, with some researchers reporting a positive interaction (e.g., Eskin and Baron 1977; Wittink 1977), a negative relation (e.g., Krishnamurthi and Raj 1985; Lambin 1976), or a relation that depends on the amount of competitive reaction to advertising in the market (Gatignon 1984).

A better understanding of the nature of marketing interactions is important for marketing science and for marketing practice. In spite of the studies cited, we have little systematic knowledge about the *determinants of marketing effectiveness*. This gap is due to the paucity of good quality data (which is changing fast), but also to the lack of a formal model for studying interactions. Furthermore, no one has investigated the implications of marketing interactions for the *optimal allocation of marketing resources,* except under the limited interaction condition built into multiplicative models. For example, if advertising increases customers' price sensitivity, the derivation of an optimal profit margin/advertising ratio is far from trivial.

We introduce and illustrate a class of market response models that formally recognizes the presence of interactions among marketing efforts and with the sales environment. We inventory models that fit into the general class of varying-parameter models and concentrate on those models that incorporate variables explaining the varying nature of the parameters. The latter models are more appropriate for modeling interactions than varying-parameter models in which time is the only explanation for the parameter variations (e.g., Parsons 1975) or models in which the source of parameter variation is purely stochastic (e.g., Wildt and Winer 1983). These models are superior to current practice in that they allow for the formal testing of *a priori* hypotheses about marketing interactions. Furthermore, we demonstrate the importance of modeling interactions by showing that the marketing mix resource allocation is affected considerably by the presence of these interactions. Though more complex, these models are more appropriate for making inferences about optimal marketing behavior.

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ISSUES IN MODELING MARKETING INTERACTIONS

The explicit consideration of interactions in market response is not without cost. An interaction represents the process that drives a response parameter. This process consists of three elements.

1. *Marketing variables.* The marketing mix concept implies that marketing efforts complement each other; for example, selling a product may be easier with stronger advertising support. Thus we generally would expect a positive interaction among marketing mix efforts.
2. *Environmental conditions.* They are typically included as main effects in a market response model, for example, to control for general economic fluctuations in a product market. Besides affecting sales directly, however, changes in environmental conditions over time or across markets may be related to marketing effectiveness. For example, though the main effect of competition on product sales is presumably negative, it may make the company's efforts in personal selling more salient and more effective.
3. *Stochastic elements.* To introduce a random component in marketing effectiveness is consistent with market response modeling and allows for statistical hypothesis testing. The disturbance term can be purely random (i.e., white noise) or may contain systematic factors, such as a gradual change over time. For example, it is often argued that advertising effectiveness declines over the product life cycle (e.g., Parsons 1975).

A General Method for Modeling Market Response with Interactions

To accommodate the three elements, one must specify a multiple-equation system of the following general type.[1]

(1a) $$y = f_1(X, Z; b, g; u)$$

(1b) $$b = f_2(X, Z; c, d; \epsilon)$$

where:

y = a measure of market performance such as product sales,

X, Z = a set of marketing (possibly with lagged effects) and environmental variables hypothesized to influence y,

b, g = the response parameters of the marketing and environmental variables,

c, d = marketing and environmental parameters explaining the response parameters b, and

u, ϵ = disturbance terms obeying the standard assumptions of the general linear model (possibly with an autocorrelation structure).

The system is a comprehensive market model with interactions. Equation 1a is traditional; it describes the relation between marketing effort and market performance and is generally known as the *sales response function*. Equation 1b is a *marketing parameter function* describing the process that generates marketing impact.[2]

In econometric terms, the introduction of a marketing parameter function results in a varying-parameter model with a response equation (1a) and a parameter equation (1b). Varying-parameter models in marketing were reviewed by Wildt and Winer (1983), who found only one application in which the marketing parameter function contained exogenous factors other than time. Most such models have used stochastic parameter equations over time and do not address marketing interactions explicitly.

Several popular market response functions are special cases of the interaction model (1). In the context of linear models, the classical ANOVA model with interactions is one example. It is related to model 1 as follows for a two-variable case.

(2a) $$y = b_0 + b_1X_1 + b_2X_2 + u$$

(2b) $$b_1 = c_0 + c_1X_2$$

Substitution of the marketing parameter function in the sales response function produces a standard ANOVA model. However, this approach is deficient in three ways: the model ignores a stochastic influence on b_1, it assumes a linear response surface that is often inappropriate in a marketing context, and it is observationally equivalent to a reversed interaction model, that is,

(2c) $$b_2 = d_0 + d_1X_1.$$

Next, the multiplicative or constant-elasticity response model,

(3) $$y = b_0X_1^{b_1}X_2^{b_2}e^u,$$

incorporates marketing interactions directly in the response function, as the marginal effect of X_1 on y is a function of the levels of X_1 and X_2. However, this interaction model is also symmetric, that is, the model does not distinguish between, say, advertising level, X_1 (e.g., expenditures), influencing salesforce effectiveness (i.e., the effect on y of one unit of X_2) and salesforce size, X_2, affecting advertising impact (i.e., the effect on y of one unit of X_1). If prior beliefs, based on theory and management, are correct that interactions are not symmetric, the multiplicative model is inappropriate. Also, because there is no explicit marketing parameter function, there

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[1]The marketing mix interaction model is presented in a general form, as it is beyond the scope of the article to specify our state of theoretical knowledge about all marketing mix interactions. The estimation of the parameters of a "complete" general model probably would be difficult (mostly because of multicollinearity) without placing *a priori* restrictions on equation 1b. However, we provide an illustration of the model specification in a particular case.

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[2]The coefficients of the environmental variables also could vary. We present them as constant because the focus of the article is marketing interactions and because such variability in environmental effects would not change the rest of the model specification, the estimation methods, or the results about the impact of marketing mix interactions on resource allocation.

is no allowance for a stochastic influence on the marketing parameters. Though we have pointed out certain limitations, such models can be expanded by transformations and complex error structures. We next propose transformations and error term structures that alleviate the traditional model specification problems and represent marketing mix interactions effectively.

Estimation of Models with Interactions

The estimation of a market model with interactions is greatly facilitated by using the general linear model. This approach is not overly restrictive, as one can often linearize a complex but realistic market model. For example, the constant elasticity model is linearized in the logarithms and the market share attraction model is estimated by logcentering the data (e.g., Nakanishi and Cooper 1974).

Just as the effectiveness of marketing mix variables can be estimated with either time series or cross-section data, interactions can be estimated by specifying the marketing parameter function to vary over time or over cross-sections.[3] When the marketing parameter function varies over time, the model is specified as a pure time-varying parameter model, as discussed by Wildt and Winer (1983). When equation 1b is specified as varying across sections, the model estimation requires both cross-section and time series data. We first discuss the issue of time-varying parameter models versus the cross-sectional and time series models. The estimation procedure for each of these models is complicated by the presence of a disturbance term in the marketing parameter function.

Time series versus cross-section marketing parameter functions. The use of a marketing parameter function implies a causal statement about marketing effects; for example, advertising increases consumers' price sensitivity. When the function is estimated over time, the dynamics of marketing effectiveness can be modeled. However, the variability of the data should be relatively large to avoid collinearity problems in estimating interactions. Cross-sectional data variations are often large, making it appealing to estimate a marketing parameter function across territories or brands. However, this variability may correspond to the fact that different response functions operate. Note that the preceding discussion does not refer to the choice of time series versus cross-section data. It refers to the unit of analysis of the marketing parameter function that describes the variation in marketing mix effectiveness, either over time or over cross-sections.

By incorporating the time or the cross-section subscripts into a marketing mix interaction model, we can

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distinguish between two models, which are estimated by different procedures.

The time-varying parameter model (case I) becomes, with a linear functional form for both equations of the system,

$$(4a) \qquad y_t = \sum_k X_{kt}\beta_{kt} + u_t$$

$$(4b) \qquad \beta_{kt} = \sum_m z_{mt}\alpha_{km} + \epsilon_{kt} \qquad \text{for all } k$$

where:

> y_t = a measure of market performance for time period t,
> X_{kt} = a predictor variable k of performance for time period t,
> z_{mt} = a predictor variable of marketing effectiveness for time period t,
> β's and α's = parameters, and
> u_t, ϵ_{kt} = disturbance terms.

In matrix notation,

$$(5a) \qquad y_t = x_t'\beta_t + u_t$$

$$(5b) \qquad \beta_t = Z_t\alpha_t + \epsilon_t$$

where:

> x_t = vector of predictor variables of performance for time period t,
> β_t = vector of sales response function parameters for time period t,
> α = vector of "marketing parameter function" parameters,
> ϵ_t = vector of disturbance terms (ϵ_{kt}'s),
> $y_t, \beta_t,$ and u_t are as defined before,

$$Z_t = \begin{bmatrix} z_t' & & 0 \\ & z_t' \cdot & \\ 0 & & \cdot z_t' \end{bmatrix}, \text{ and}$$

> z_t = vector of predictors of marketing parameter coefficients.

The variation in response functions over time as specified in equation 5b can be due to certain discrete events, in which case the predictor(s) of marketing parameter coefficients (z) can be represented with dummy variables.

For a cross-sectional marketing parameter function specification (case II), the model is given by

$$(6a) \qquad y_{it} = x_{it}'\beta_i + u_{it}$$

$$(6b) \qquad \beta_i = Z_i\alpha + \epsilon_i \qquad \text{for each cross-section } i$$

where the variables are defined as before, but where i represents a cross-section such as a geographic area.

Stochasticity of the marketing parameter function. We have represented both cases with a disturbance term in the marketing parameter function. The proce-

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[3]Our proposed approach to modeling marketing mix interactions therefore assumes some underlying market process. The issue of the specification of this market process is discussed subsequently.

dures for estimating the parameters of the two models differ whether a disturbance term is specified or not. In theory, the model can be specified with the error term and tests can be performed to infer the presence of such stochastic processes. However, the tests are difficult to carry out, and the estimators obtained with samples of limited size have unknown properties (the statistical properties of the estimators are only asymptotic).

Estimation. In case I, where the model is specified on time series data, the exclusion of the error term in equation 5b renders the estimation straightforward, as OLS estimators are BLUE. The equation to estimate is

$$(7) \qquad y_t = x_t' Z_t \alpha + u_t,$$

which is the standard ANOVA model (as in equation 2).

If the error term is introduced as in 5b, the equation is

$$(8) \qquad y_t = x_t' Z_t \alpha + x_t' \epsilon_t + u_t.$$

Consequently, the estimation procedure with only time series data follows the case of heteroscedasticity where the variances are a function of exogenous variables (Judge et al. 1980).

The cross-sections variation model in equations 6a and 6b can be estimated by following a procedure similar to the one used by Swamy (1970) with the random coefficient model. The procedure, including the case in which a parameter function operates only on certain variables, is discussed fully by Gatignon (1984).

When there is no error term in equation 6b, the estimation is simplified and the model becomes

$$(9) \qquad y_{it} = z_{it}' \alpha + u_{it}$$

where $z_{it}' = x_{it}' Z_i$ for each i, t. However, because of the possibility of contemporaneous covariances ($E[u_{it} u_{jt}] = \sigma_{ij}$), the BLU estimator is obtained by following the procedure of seemingly unrelated regressions.

Specifying Marketing Parameter Functions

A parameter function describes the way in which marketing effectiveness is created, just as a response function describes the way in which market performance is achieved. Therefore, the parameter function specification should reflect theoretical knowledge, just as should the response function specification. Many of the research advances in response modeling apply also to parameter modeling. However, one major difference is that few degrees of freedom for developing observed parameter functions may be available. Another difference is that models with parameter equations do not provide overall tests of significance and easily interpretable fit measures, such as an R^2. This feature does not appear to be a major drawback because, for the allocation of resources to marketing variables, what matters is the magnitude and the significance of the coefficients. If the

model is specified correctly, the estimates of the coefficients are normally distributed and *t*-tests can be performed on individual coefficients. The assumption, however, is that we specify a model *a priori*, on the basis of theoretical knowledge, rather than use adaptive or data-driven modeling.

Specifically, to accommodate prior knowledge and research findings and to be managerially meaningful, a marketing parameter equation should satisfy the following conditions.

1. It should accommodate behavioral hypotheses about marketing interactions (e.g., complementarity between salesforce effectiveness and advertising support). This condition is met by performing a predictive test on the marketing parameter function. For example, a positive regression parameter may imply complementarity.
2. It should be consistent with plausible optimal marketing behavior. This condition requires the use of analytical or numerical optimization techniques; hence, the parameter equation should preferably be continuous and differentiable. The importance of this point is illustrated in the next section, where we show that a model with interaction implies appealing properties about the optimal allocation rules.
3. It should be tractable for statistical parameter estimation. Working with functions that can be transformed to linearity seems desirable so that estimated generalized least squares techniques can be applied.

It is not difficult to specify a marketing parameter function that can test one or more interaction hypotheses and that is estimable. However, because the parameter function acts on the response model, the implied conditions for optimal marketing behavior may be complex. To examine the issue in some detail, consider two marketing activities X_1 and X_2 (measured as expenditures) and one environmental condition Z in the following hypothetical market response model.

$$(10) \qquad Y = c X_1^{b_1} X_2^{b_2} Z^{b_3} e^u$$

$$(11) \qquad b_1 = f_1(X_2, Z, \epsilon_1)$$

$$(12) \qquad b_2 = f_2(X_1, Z, \epsilon_2)$$

where f_1 and f_2 are continuous and differentiable. It can be shown that traditional profit maximization on X_1 and X_2 results in the condition

$$(13) \qquad \frac{X^{f_1 - 1}}{X^{f_2 - 1}} = \frac{f_2 X_1^{f_1} + g_1 \ln (X_1) X_2}{f_1 X_2^{f_2} + g_2 \ln (X_2) X_1}$$

where $g_1 = \delta f_1 / \delta X_2$ and $g_2 = \delta f_2 / \delta X_1$.

This is an application of the Dorfman-Steiner condition. The simplest case, where $f_1 = b_1$ and $f_2 = b_2$, where b_1 and b_2 are constants, results in the well-known condition $X_1 / X_2 = b_1 / b_2$. Now, depending on the complexity of f_1 and f_2, we may or may not be able to derive optimal values for X_1 and X_2. In many cases, an implicit

function between X_1 and X_2 will result, which can be used for simulating resource allocation scenarios and/or for numerical optimization procedures. The addition of a budget constraint ($X_1 + X_2 = B$, the marketing budget) often provides a unique optimal ratio X_1/X_2, as illustrated in the empirical results section.

In conclusion, one should proceed very carefully in the specification of marketing parameter functions. Because of the risk of spurious associations, the functions should be specified with testable marketing hypotheses in mind, and they should be simple to estimate. Most importantly, one should examine the conditions for optimal spending implied by the interaction model, which may necessitate the use of numerical optimization techniques.

We now apply the various stages in marketing interaction modeling to a real-world case of allocating communications efforts between personal selling and advertising in the face of varying competitive conditions.

A CROSS-SECTIONAL INTERACTION MODEL OF SALESFORCE EFFECTIVENESS

Hypotheses

Salesforce resources are a very important element in the communications mix of American companies, yet empirical research in that area is scarce (Weitz 1981). One reason is that salesforce effectiveness is often difficult to measure because of insufficient variation.

Parsons and Vanden Abeele (1981) provide some rare empirical evidence of the interaction between salesforce and other selling efforts. They find that sales call elasticity for a pharmaceutical product increases with the use of samples and handouts. The model they use is based on time series data and corresponds to a specification without a disturbance term in the marketing parameter function. Further, Swinyard and Ray (1977) argue on the basis of experimental data that buyers become more responsive to advertising because of their prior interaction with a salesperson. In both cases, though the direction of causality is reversed, a positive interaction is present.

The environment also is recognized as a crucial factor interacting with the effectiveness of the salesforce (Weitz 1981), but little empirical evidence of this interaction is available. Most research on salesforce effectiveness introduces environmental factors as main effects; for example, competition has a negative main effect in the multiplicative model of Ryans and Weinberg (1979). In general, competition is only one element describing the degree of hostility facing the firm in its marketing task. When the environment is hostile, the selling task becomes more important. We therefore propose that under hostile environmental conditions, salesforce effectiveness increases, *ceteris paribus*.

We examine two hypotheses about the elasticity of the salesforce—the parameter typically used to represent the effectiveness of the marketing variables.

H_1: Salesforce elasticity is related positively to advertising support.[4]

H_2: Salesforce elasticity is higher in hostile environments.

The first hypothesis builds on the marketing mix complementarity notion. In particular, advertising is viewed as facilitating the selling task, as illustrated by Parsons and Vanden Abeele (1981). The second hypothesis implies that salesforce allocations are more effective when the salesforce is really needed because it performs a highly effective market research function and because it is the only marketing mix variable that can adapt readily to individualized needs (Weitz, Sujan, and Sujan 1986). These functions become critical under hostile environmental conditions when management is more pressed than ever to use their most effective marketing variable. This moderating role of the environment is consistent with recent research findings on salesforce effectiveness (Weitz 1981; Weitz, Sujan, and Sujan 1986), which has been argued to be typically the most important promotional tool (Churchill, Ford, and Walker 1985).

Empirical Setting

Since the late 1970s the U.S. armed services, in particular the Navy, have commissioned various research projects in the general area of military manpower recruitment. Several studies have successfully examined recruiting as a marketing mix problem, that is, how the number of volunteer recruits responds to changes in the product (salaries and benefits), the price (length of duty), the advertising support (national and local recruitment advertising), and the size of the salesforce (number of recruiters on active duty) (Carroll et al. 1985; Hanssens and Levien 1983; Morey and McCann 1980). None of these studies, however, models marketing mix variables' interactions. Because military salesforces vary in size continuously over time, they afford a unique opportunity to measure their effectiveness.

Monthly data for the U.S. Navy are available for a cross-section of 43 recruiting districts over a three-year period (1976–1978). These data have been used in prior studies: Morey and McCann investigated the problem of optimal recruitment goal setting, and Hanssens and Levien did an econometric analysis of recruiting effectiveness. We use the latter model as a starting point and refer to the Hanssens and Levien article for details on recruitment marketing.

Salesforce size, advertising expenditures, the civilian unemployment rate, and an attitudinal measure of the

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[4]The reverse relationship of the effect of prior salesforce exposure on advertising elasticity is not investigated in this study as the empirical setting used prevents such a relationship. Hanssens and Levien (1983) demonstrated that advertising serves to generate inquiries and that Navy recruiters are the last contact in the enlisting decision process.

propensity to enlist in the Navy were found to be related positively to three measures of recruiting performance:[5] number of inquiries, immediate-entry contracts, and delayed-entry contracts along with several other variables. Note that the propensity to enlist is a measure of lack of environmental hostility.[6] The econometric model was multiplicative and was estimated across the 43 Navy recruiting districts. For the purpose of our study, the data are aggregated to the six Navy recruiting areas (NRAs) to avoid the estimation problems caused by the large sample resulting from using districts.[7] This change in the unit of analysis causes differences between the model proposed here and the model of Hanssens and Levien (1983). We deleted four environmental variables that have little or no variation across recruiting areas: (1) relative number of blacks (PBLACK), (2) relative urbanization (URBAN), (3) relative number of high school seniors (SENIORS), and (4) attitude toward the military (PROPM). All variables are expressed in per capita ($\times 10^6$) figures, except for the dummy variables and unemployment.

Model Specification

The specification of the measurement equation is based on Hanssens and Levien's model. To consider the effect of territorial aggregation and of the elimination of some variables, a random coefficients version of the model is estimated. The response model is:

$$(14) \quad \ln(\text{SALES}_{i,t}) = c_i + a_{1,i} \ln(\text{UNEMP}_{i,t}) + \sum_{k=2}^{4} a_{k,i} Q_{k,t}$$

$$+ a_{5,i} GI1_t + a_{6,t} GI2_t$$

$$+ b_{1,i} \ln(\text{LEADS}_{i,t-1})$$

$$+ b_{2,i} \ln(\text{ADV}_{i,t})$$

$$+ b_{3,i} \ln(\text{LOCAL}_{i,t-1})$$

$$+ b_{4,i} \ln(\text{REC}_{i,t})$$

$$+ b_{5,i} \ln(\text{SHIPGOAL}_{i,t}) + u_{i,t}$$

where:

SALES = contracts signed for the delayed-entry program, per capita ($\times 10^6$),

UNEMP = civilian unemployment rate,

Q_2, Q_3, Q_4 = dummy variables for second, third, and fourth quarters,

$GI1, GI2$ = dummy variables for GI Bill effects ($GI1_t = 1$ for $t = $ December 1976, 0 otherwise; $GI2_t = 1$ for $t = $ January 1977, February 1977, 0 otherwise),

LEADS = number of inquiries received per capita ($\times 10^6$),

ADV = total national media expenditures per capita ($\times 10^6$),

LOCAL = total local advertising and recruiting aids expenditures per capita ($\times 10^6$),

REC = number of recruiters per capita ($\times 10^6$),

SHIPGOAL = number of immediate-entry contracts that *must* be written to meet this month's recruiting goal, per capita ($\times 10^6$),

and all variations are measured across recruiting areas $i = 1, \ldots 6$ and months $t = 1, \ldots 36$.

Hanssens and Levien (1983) report the reasons for the incorporation of the GI Bill dummy variables ($GI1_t$ and $GI2_t$) and for the selection of the explanatory variable. In particular, the lag structures in the model result from a time series investigation reported in their article. In their model, the log-log specification leads to coefficients that can be interpreted as elasticities. Therefore, the Hanssens and Levien model assumes constant elasticities over time and across recruiting areas. In the model expressed in equation 14, the response functions for each area can differ as the coefficients contain the area subscript i.

The model specification for the marketing parameter equations follows our previously discussed guidelines about behavioral realism, optimal resource allocation plausibility, and estimation tractability. The parameters in the model that do not correspond to an interaction hypothesis follow a simple random process,

$$(15) \qquad\qquad r_i = r' + \epsilon_{r,i}.$$

Behavioral realism. To test our hypotheses, the coefficients that are affected by interactions vary systematically as follows.

1. The intercept, which measures the "base level" of recruiting, is hypothesized to be higher in regions whose environmental characteristics facilitate recruiting. For example, a more positive youth attitude toward the Navy creates more potential candidates from which to recruit. Thus, the specification for the intercept parameter function is

$$(16) \qquad\qquad c_i = c_1' + c_2' \text{PROP}_i^* + \epsilon_{1,i}$$

where PROP_i^* is the average propensity to enlist by area, derived from youth attitude tracking surveys. A linear

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[5]Only the third measure is used here because it is the most important and recruiters have been shown to have strong effects (see Hanssens and Levien 1983 for the modeling of the other variables). Inquiries are used as an explanatory variable; immediate-entry contracts are severely demand-restricted (variable SHIPGOAL), which makes them less appropriate for analysis in this context.

[6]Another aspect of environmental hostility is the intensity of competition from the Army, Air Force, and Marines. However, youth attitude tracking surveys (Market Facts, Inc. 1978) show that military competition is slight in comparison with civilian competition. A high youth propensity to enlist in the Navy accompanies a high propensity to enlist in the military in general.

[7]The rank of the covariance matrix of the disturbances that is needed for the estimation of the coefficients is equal to the number of time periods times the number of cross-sections. When the total number of observations becomes too large, the covariance matrix inversion is very difficult because of computer memory size limitations.

function is used because the natural variation in PROP is small and there are no *a priori* reasons for suspecting nonlinearities.

2. The recruiter elasticity is hypothesized to be affected negatively by the propensity to enlist in an area, which is a surrogate for environmental hostility. When the propensity to enlist is low, competition with civilian employers may be fierce and the addition of Navy recruiters is expected to be more sales effective than under the reverse condition. A linear parameter function is postulated for the same reason as in the intercept equation.

3. Recruiter elasticity is hypothesized to be boosted by advertising support. This is an application of the notion of complementarity in the marketing mix. In the study context, the interaction should occur with respect to local advertising support (local media and recruiter aids, such as flyers and posters), because such support facilitates the recruiting task directly. A reciprocal function is used such that the asymptotes for small amounts and infinite advertising are meaningful. Thus, the complete recruiting parameter function is

(17) $\quad b_{4,i} = b_1' + b_2' \text{PROP}_i^* + b_3'[\text{LOCAL}_i^*]^{-1} + \epsilon_{2,i}$

where PROP_i^* and LOCAL_i^* are the average propensity to enlist and local advertising support in area i.

Optimal resource allocation plausibility. Because of the cross-sectional specification of this marketing parameter equation, the focus is on *territory* or *market* resource allocation. The recruiting model with interactions among personal selling, advertising, and competition leads to some nontrivial conditions for optimal marketing spending. Simplifying the model to express only the variables of interest within an area, we obtain[8]

(18) $\quad E[\text{SALES}]$

$\quad = ke^{c_1' + c_2' \text{PROP}} \text{REC}^{b_1' + b_2' \text{PROP} + b_3'/\text{LOCAL}} \text{ADV}^{b_2} \text{LOCAL}^{b_3}.$

Marketing spending for the Navy is allocated across recruiters (at an approximate average monthly cost r of $3000 per recruiter), national advertising, and local advertising. Between 1976 and 1978 the Navy's average spending proportions were 89%, 5%, and 6%, respectively. Though this average allocation is not totally representative of the allocation in each area, it indicates the extreme emphasis on recruiters. To examine the desirability of such an allocation one must compute marketing elasticities.

(19a) $\quad e_{\text{REC}} = b_1' + b_2' \text{PROP} + b_3'/\text{LOCAL}$

(19b) $\quad e_{\text{ADV}} = b_2$

(19c) $\quad e_{\text{LOCAL}} = b_3 - b_3' \cdot \ln \text{REC}/\text{LOCAL}$

assuming that $\text{LOCAL}_{it} = \text{LOCAL}_i^*$ for any t. Not surprisingly, recruiter and local advertising elasticities vary with changing competitive conditions and with marketing spending. Furthermore, the predictive tests on the process equations ($b_2' < 0$, $b_3' < 0$) imply the following patterns.

—*Ceteris paribus*, a less favorable environment reduces the Navy's recruitment base, but it increases the recruiters' effectiveness. The reduction of the recruitment base comes from the reduction of the scale factor ($c_2' > 0$). However, because of the negative coefficient b_2', the elasticity of recruiters increases.

—More recruiters, in turn, increase the local advertising elasticity ($b_3' < 0$). However, more local advertising depresses its elasticity (decreasing returns to scale).

As a result, the optimal relation between recruiters and local advertising support is complex. The empirical study we report affords examples of such allocation implications. In any given year, the Navy is given a recruiting budget (B) that is set by Congress. We can examine optimal marketing spending by introducing the budget constraint

(20) $\quad \text{ADV} + \text{LOCAL} + r\text{REC} = \text{B}.$

Estimation tractability. As described before, the generalized least squares (GLS) estimator is the minimum variance linear unbiased estimator for the model specified by equations 14 to 17. The covariance matrix of the error term can be estimated by using the cross-sectional data. Therefore, the estimated generalized least squares estimator can be obtained.

The GLS procedure explicitly recognizes the presence of error terms in the parameter functions. In fact, the literature has ignored the stochasticity of the process functions. For comparison, an alternative to the model specification given by equations 14 to 17 is to extend the model of Parsons and Vanden Abeele (1981) to the case of cross-sections and time series. The model is unchanged in the structural form, but there is no error term in the parameter equations. The model is therefore of the type presented in equation 9 and is estimated by a seemingly unrelated regression (SUR) method.[9]

The proposed model of marketing mix interactions is estimable and has important managerial implications in terms of budget allocation. In the next section, we empirically estimate the model and draw conclusions about the optimal allocation of communications expenditures.

Estimation Results

To consider the impact of aggregation and of the elimination of some variables, we estimated a random coef-

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[8]The model specification without disturbance term in the marketing parameter functions is used for computing optimal expenditures. In the case of error terms in the marketing parameter functions, the constant k contains a term representing the variances and covariances in the marketing parameter equation and the response function error variance because of the exponential form of these error terms,

$$E[e^{u_{it} + \Sigma_r r_{ri} \ln X_{rit}}] = E[e^{w_{it}}] = e^{\sigma_{it}^2/2},$$

where x_{rit} represents the variables in the measurement equations and w_{it} is normally distributed with mean zero and variance σ_{it}^2 which is a function of the X_{rit}'s.

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[9]The SUR estimation is performed by the iterative maximum likelihood procedure available in TSP (Time Series Processor).

ficients model. The results are reported in Table 1. All the parameters are significant, and their signs and magnitudes are comparable to those of the reference model. The major differences are the larger estimates of the unemployment effect (.951 vs. .367) and the recruiters' effect (1.589 vs. .618). The new estimate of unemployment elasticity is actually similar to that found in other studies of Navy recruiting effectiveness (Brown 1985). The relative instability of the recruiters' elasticity suggests a parameter function beyond a simple random coefficient.

The full model with the parameter equations gives additional insights about the recruiters' effectiveness. The two models with and without the error terms in the pa-

rameter equations are estimated with the estimated generalized least squares (EGLS) and the seemingly unrelated regression (SUR) methods described before[10] (Table 1). The estimates of the nonvarying coefficients are al-

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[10]In EGLS the emphasis is on the lack of perfect specification of the process equations. The SUR considers the contemporaneous correlations between cross-sections. These differences might explain differences in the coefficients for the two model specifications. A model with both aspects, though theoretically possible, would be difficult to estimate because of the large rank of the covariance matrix to be inverted. Consequently, both estimations should be performed as it is difficult to know a priori which aspect is more crucial for a particular set of data in terms of estimator efficiency.

Table 1
COEFFICIENTS ESTIMATES[a]

	Reference model[b]	Random coefficients[c]	EGLS—model with error term in process equations[c]	SUR—model without error term in process equations[c]
Constant	2.540 (.713)	−.590 (.97)	−56.7 + 388.65 PROP* (27.7) (183.8)	−12.62 + 112.54 PROP* (9.05) (56.0)
Rate of unemployment (UNEMP)	.367 (.051)	.951 (.08)	1.01 (.63)	.81 (.1)
Relative no. blacks (PBLACK)	−.040 (.016)	—	—	—
Relative urbanization of area (URBAN)	−.160 (.048)	—	—	—
Military attitudes (PROPM)	.313 (.088)	—	—	—
Relative no. high school seniors (SENIORS)	.202 (.101)	—	—	—
Second quarter dummy (Q_2)	−.325 (.037)	−.198 (.01)	−.192 (.11)	−.22 (.06)
Third quarter dummy (Q_3)	−.152 (.040)	−.047 (.01)	−.048 (.12)	−.033 (.06)
Fourth quarter dummy (Q_4)	−.052 (.04)	.081 (.02)	.084 (.64)	.040 (.06)
GI Bill dummy for Dec. 1976 (GI1)	.966 (.075)	.962 (.02)	.962 (.18)	.990 (.12)
GI Bill dummy for Dec. 1977 (GI2)	−.158 (.054)	−.152 (.02)	−.154 (.17)	−.089 (.084)
No. inquiries rec'd previous month (LEAD ($t − 1$))	.105 (.020)	.081 (.01)	.083 (.07)	.073 (.033)
Advertising expenditures (ADV)	.027 (.012)	.018 (.005)	.017 (.04)	.034 (.018)
No. recruiters (REC)	.618 (.074)	1.589 (.18)	13.39 − 16.21/LOCAL* − 62.98 PROP* (4.74) (13.8) (31.15)	3.71 − .648/LOCAL* − 19.34 PROP* (1.58) (.176) (9.75)
No. contracts required to meet goals (SHIPGOAL)	−.154 (.014)	−.188 (.02)	−.175 (.13)	−.263 (.04)
Local adv. and recruiting adv. expenditures in previous month (LOCAL ($t − 1$))	.093 (.015)	.028 (.011)	.024 (.10)	.055 (.03)
R^2	.423	N/A[d]	N/A[d]	N/A[d]
N	1505	210	210	210

[a]Standard errors are in parentheses.
[b]Units of analysis are Navy recruiting districts.
[c]Units of analysis are Navy recruiting areas.
[d]Though no R^2 that can be interpreted as a percentage of explained variance is available (Judge et al. 1980), the R^2s for each area are .80, .76, .88, .85, .78, and .90.

most identical to the coefficients obtained when all coefficients are assumed to be random. Even the main effects coefficient of local advertising is not significantly different. Consequently, we can concentrate on the marketing parameter function results. Though the magnitudes of some coefficients differ between the two models and the standard errors are substantially smaller with the SUR procedure, both models provide support for our hypotheses.[11]

First, propensity to enlist is a strong determinant of the scale factor. The parameter function on this coefficient is

$$(21) \qquad c_i = -12.62 + 112.54 \, \text{PROP}_i^*,$$

indicating that recruiting areas showing strong positive attitudes toward the Navy have a larger base of enlistees.

For the parameter function on the recruiters coefficient, both variables hypothesized to interact with recruiters have parameters in the expected direction.

$$(22) \quad b_{4,i} = 3.71 - 19.34 \, \text{PROP}_i^* - .648 \, [\text{LOCAL}_i^*]^{-1}$$

Propensity to enlist, which represents how favorable the environment is in an area, has a strong and significant impact on the effectiveness of recruiters. The higher the propensity to enlist in an area, the lower the effectiveness of the recruiting force. This finding indicates that when competition decreases (in terms of real choice of alternatives for attitudinal reasons), the selling task is easier and there is less need for a large salesforce. This finding also supports our hypothesis based on past research: competition makes the marketing mix variables more effective and acts as a substitute for the other interacting mix variables. This substitution, however, is only at the interaction level because a higher propensity to enlist (and consequently a smaller likelihood of considering other employment alternatives) means a higher scaling factor. In summary, higher propensity to enlist (a favorable environment) has a positive effect on sales but makes the marketing mix variables less effective.

In addition, local advertising, which has a very small effect of its own, is shown to influence positively the effectiveness of recruiters, though the significance level is weaker than for the propensity-to-enlist coefficient. This finding corresponds to the hypothesis of interaction found in the literature. Advertising has an impact beyond the simple effects reported in the literature, especially when the various types of advertising (national vs. local advertising) are separated.

To enhance the value of the substantive findings, we performed validation using a jackknife-like procedure.

Because the limited sample size precluded a cross-validation of the model, we estimated the model several times, taking one cross-section (the data from one Navy recruiting area) out of the sample each time. The six vectors of parameter estimates resulting from this procedure indicate that the results are stable. Across the vectors of parameter estimates and the total sample parameter estimates, the same coefficients are significant and all coefficients have the same signs. The predicted response function elasticities result in coefficients of the same magnitude.

These findings are particularly revealing for the allocation of the recruiting budget across advertising versus salesforce expenditures. Though there is no analytical solution to the optimization problem as generally stated in equation 6, a numerical procedure can be used to solve this nonlinear equation.
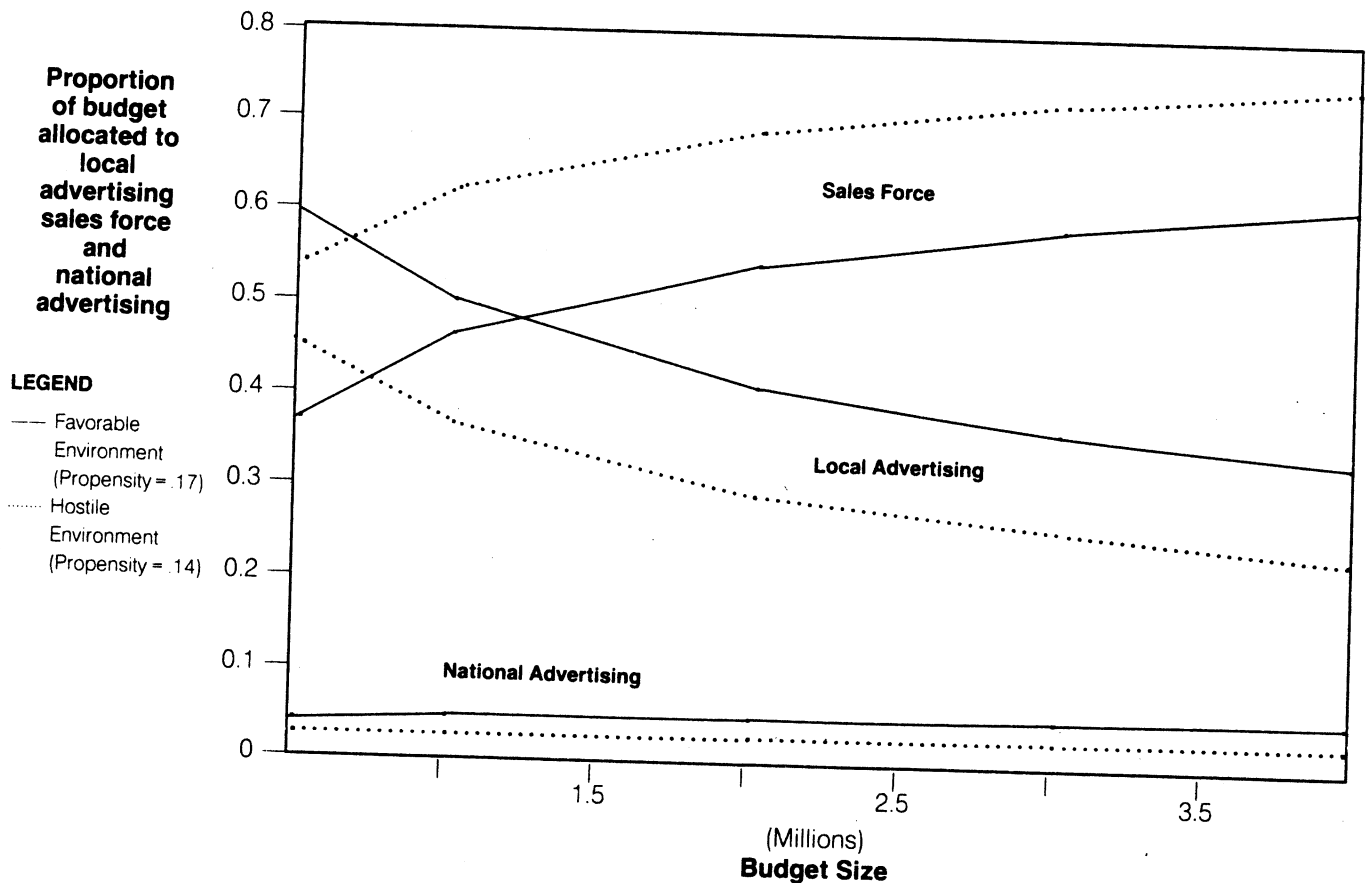
Normative Implications

The Navy recruiting budget is allocated over mass media advertising, local advertising and recruiting aids, and salesforce size. The problem is to maximize the expected value of sales (i.e., enlistment contracts) given a total recruiting budget. An algorithm was developed to find an optimal solution that satisfies the Dorfman-Steiner theorem whereby the elasticities are computed at the optimal solution.[12] To examine the importance of the impact of interaction effects on resource allocation, we derived the optimal communications mix expenditures for different budgets. Figure 1 shows how the allocation differs between areas with different budget levels and how the optimal allocation differs under two different but typical environments for a monthly recruiter cost of $3000 and an area target population of 2 million.

The first impact on allocation is the *environmental condition*. The results show that, for equal budgets, areas favorable to the Navy (less hostile) should have fewer recruiters than areas having very hostile or competitive environments. This result is seen in Figure 1: the proportion of a given budget spent for recruiters is always lower in a favorable environment. Instead, more money should be allocated to local advertising, as recruiters need this support. These results are relative, as a greater budget should be allocated to the favorable areas. Areas with hostile environments should receive a lower proportion of the budget allocated to national advertising programs than areas with favorable environments. The explanation for these results is the fact that recruiters' elasticity is greater in areas with unfavorable environments. Given that according to the Dorfman-Steiner conditions the ratios of expenditure to elasticity should be equal for each marketing variable, the optimization leads to a greater number of recruiters. The Dorfman-Steiner conditions hold for the optimal results shown in Figure 1.

The *budget size* is the second factor affecting the al-

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[11]The coefficients of the SUR estimation without error terms in the process equation are reported in this section because they are used subsequently in the optimization of resource allocation where they provide more realistic sizes of elasticities, that is, the predicted elasticities are plausible within a wider range of values of marketing mix variables. However, the substantive findings in terms of signs and significant coefficients are equivalent, as can be seen by the parameter estimates in Table 1.

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[12]The computational method can be obtained by writing to the authors.

Figure 1
OPTIMAL ALLOCATION OF COMMUNICATIONS MIX

location of communications mix expenditures, contrary to the constant solution implied by a multiplicative response function. In fact, Figure 1 shows that as the budget increases, the share of resources going to recruiters increases while the local advertising share decreases. The national advertising share increases marginally, in spite of its constant elasticity, to satisfy the Dorfman-Steiner conditions. Consequently, the allocation of the budget should not be the same in an area with a high budget as in an area with a lower budget. A larger share of the large budget should go to recruiters' expenditures. The figure illustrates the importance of the interaction, because the ranking of the resource allocation depends on the size of the budget. For lower budgets, local advertising should receive more resources than recruiters, but as the budget increases this order is reversed. In general, these results support the ranking of expenditure sizes for the 1976–1978 budget levels, but not the magnitudes of the allocations. Though the optimal results are computed for a given target population size, environment hostility, and budget, the assumptions reflect an average area. This analysis suggests plausible marketing behaviors discussed before and indicates that the Navy should real-

locate its communications resources by reducing its recruiter force and expanding its local advertising programs. The last recommendation is consistent with the conclusions of Carroll et al. (1985).

CONCLUSIONS

The interactions of marketing mix variables are widely acknowledged as important in understanding and measuring the effectiveness of marketing instruments. Nevertheless, few attempts have been made to build formal marketing interaction models and examine their consequences for marketing decision making. We introduce one general class of interaction models that makes the distinction between market response functions and marketing parameter functions. We argue that such models are better representations of the process generating marketing effectiveness and that they can be estimated by standard generalized least squares procedures on pooled time series and cross-section data.

We illustrate the use of a marketing interaction model in the context of salesforce effectiveness. Using regional monthly data on U.S. Navy volunteer recruitment, we examine two research hypotheses: that salesforce effec-

tiveness increases with advertising support and with environmental hostility. The estimation results confirm the hypotheses and provide interesting and new results about the optimal allocation of communications expenditures. In particular, we demonstrate that the marketing mix resource allocation inferred from models with interactions can be significantly different than that inferred from constant elasticity models. More specifically, two key findings are that the optimal ratio of personal to mass communications expenditures varies with the size of the budget and with the hostility of the sales environment.

The proposed models add complexity to market response analysis. In particular the specification of the marketing parameter function should be guided by marketing theory. We propose three criteria to help develop such models and use them in the salesforce effectiveness study. In the long run, we hope these methods will contribute toward a better understanding and measurement of the determinants of marketing effectiveness and toward a market-driven allocation of the marketing mix.

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