Online Retail Platform, Consumer Search, and Filtering

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Abstract

Consumers often search different sellers' products on a retail platform, which typically charges a percentage referral fee for the sellers' sales. We study how consumers' search cost and filtering on the retail platform affect the market outcome. We show that, given the platform's referral fee, a lower search cost reduces equilibrium retail prices but can raise sellers' profits by attracting more consumers to the platform. If the platform can optimally adjust its referral fee, a lower search cost will always increase the platform's profit. As search cost decreases, if demand elasticity on the platform increases significantly, the platform should reduce its fee, potentially resulting in an *all-win* for the platform, sellers and consumers. By contrast, if the decrease in search cost does not significantly increase the demand elasticity, the equilibrium referral fee and the retail prices can increase. Furthermore, we find that the availability of filtering on the platform will in expectation make consumers search fewer products but buy products with higher match levels. Moreover, filtering can either increase or decrease the equilibrium retail prices.

Key words: search, retail platform, pricing, e-commerce, competition, filtering, channel

INTRODUCTION

U.S. online retail sales as a percent of total retail sales have more than tripled from 2.7% in 2006 to 9.5% in 2018, and the estimated sales on online retail platforms in U.S. (e.g., Amazon.com and eBay.com) reached $124 billion in the first quarter of 2018.[1] In a survey covering 19 countries and territories, more than 95% of some 19,000 respondents have online shopping experience, and more than half of them shop online at least once a month (PwC 2015). Consumers often buy products from independent sellers on retail platforms. For example, Amazon has over two million independent sellers, contributing to 83% of its revenue.[2] A retail platform's profit typically comes from referral fees (i.e. commissions paid by independent sellers), which are most commonly a percentage of the product prices.[3]

Consumers search for products that better match their preferences or have lower prices. Consumers' search cost on online retail platforms has significantly dropped in recent years. One driving force is technology advancements. Wider adoptions of high-speed Internet have significantly reduced the amount of time of loading webpages. The increasing popularity of user-friendly smartphones and tablet computers has also lowered consumers' search cost. In addition, many retail platforms invest in reducing search cost for their customers. Amazon has been continually improving its website design, thereby enabling its independent sellers to demonstrate their products with multimedia content formats (e.g., info-graphics, audios, and videos). For example, Amazon permitted some qualified independent sellers to upload "video shorts" to their product pages in 2014, and offered this function to all independent sellers in 2017.[4] In 2009, it debuted the camera search function in its mobile app, enabling consumers to shoot a photo of an item to search for similar products on Amazon. Amazon also partnered with Samsung to incorporate camera search in the camera app of Samsung's smartphones. More recently, Amazon has been developing its augmented-reality view technology, which allows consumers to see, for example, how a watch on Amazon would look around their wrists before purchase.[5] Retail platforms invest billions of dollars in reducing consumers' search cost. For instance, in the first quarter of 2016, Amazon spent over $1.2 billion on augmented reality and virtual reality, and a considerable fraction of this investment was intended for improving consumers' search experience.[6]

Empirical research also suggests that consumers' online search cost has decreased over time. Many studies have estimated consumers' online search cost for hotels, books, tablets, etc. (Bajari et al. 2003, Blake et al. 2016, Chen and Yao 2017, De los Santos 2008, De los Santos et al. 2017, Ghose et al. 2018, Hong and Shum 2006, Jiang et al. 2017, Jolivet and Turon 2017, Koulayev 2014, Moraga-Gonzaíez and Wildenbeest 2008, Moraga-González et al. 2013, Santos et al. 2012). Most of these studies indicate that consumers' online search cost is between US$1 and US$10 per item. Figure 1 plots the logarithm of their estimated search costs against the years of their datasets used for estimation.[7] Overall, the estimated search costs tend to decrease over time, although these estimates are obtained for different product categories on different platforms using different methods.[8]

[Insert Figure 1 about here.]

It is not obvious why retail platforms spend billions of dollars on reducing consumers' search cost. Lowering search cost can have opposing effects on the platform. On the positive side, reduced search cost can help the platform attract more customers. On the negative side, it will intensify the seller competition and hence reduce the platform's referral fees per unit sale. Extant literature provides little explanation on when a platform should invest in lowering consumers' search cost.

Extant literature also provides little insight on how the consumer's search cost will influence the platform's optimal referral-fee decisions. Most studies either assume the sellers sell directly to consumers—completely ignoring the retail platform—or treat the platform's referral fee as an exogenous variable. Because the referral fee is a major contributor to a retail platform's profit, it is practically important for a platform to understand how to optimally adjust its referral fee when the consumer's search cost changes.

Besides reducing the consumer's cost for acquiring and understanding product information, online retail platforms have also utilized other instruments to make consumer search easier. One main feature on many platforms is that consumers can filter the search results based on certain product attributes, e.g., shoe size and color when searching for shoes. Consumers still need to search information for other unfilterable attributes, e.g., design and style. To the best of our knowledge, the extant literature has not studied how filtering affects the platform, independent sellers, and consumers, or how the effect of filtering differs from that of a reduction in search cost.

In summary, several important questions have not been well studied by the extant research.

- Why have many retail platforms been heavily investing in reducing the consumer's search cost, given that lowering search cost has opposing effects on the platform?
- How will the consumer's search cost affect optimal pricing decisions of the sellers and the platform? How will their profits be affected?
- How will filtering affect the platform, the sellers, and consumers? How is its impact different from that of a reduction of the consumer's search cost?

We develop a game-theoretic framework to address these research questions. In our model, many independent sellers sell horizontally differentiated products on a retail platform where consumers sequentially search for these products. As in Wolinsky (1986) and Anderson and Renault (1997), consumers ex ante do not know the product's price or match level—how well a product matches their preferences—and can learn the price and match level after incurring a search cost.

The retail platform sets a percentage referral fee (also known as "commission" or "final value fee" in practice) that is charged to sellers. The sellers simultaneously set their retail prices. We also model the filtering feature on the platform. A product's aggregate match level consists of two parts: the filterable match level and the unfilterable match level. Filtering allows consumers to costless filter products by filterable match levels, but they still need to search to learn a product's unfilterable match level. We derive the equilibria in two scenarios. In the first scenario, the platform's referral fee is exogenous, i.e., the platform keeps its fee unchanged when the consumer's search cost changes. This setting is reasonable in the short term. In the second scenario, the referral fee is endogenously decided by the platform, i.e., the platform optimally chooses its referral fee as the consumer's search cost changes. This setting is more reasonable in the long term. By comparing the equilibrium outcomes in these two scenarios, we can determine how the effects of search cost differ in the short term versus in the long term.

We highlight several main findings. First, we show that, though a lower search cost will reduce sellers' profit margins due to intensified competition among them, it can actually increase the sellers' profits, because a decrease in search cost can attract more consumers to the platform (rather than choosing their outside options) and hence expand the market demand.

Second, we show that under exogenous referral fees, a decrease in search cost can either increase or reduce the platform's profit, but it will strictly increase the platform's profit if the platform optimally sets its referral fee. When the referral fee is exogenous, a lower search cost will attract more consumers to the platform, but also intensify price competition among sellers and reduce the platform's referral fee per unit sold. Therefore, the platform's equilibrium profit can either increase or decrease depending on which effect dominates. By contrast, by optimally adjusting its referral fee, the platform can expand its demand without reducing its profit margin too much, so its profit will always increase as search cost decreases. Our finding potentially explains why many platforms make large investments to reduce the consumer's search cost in reality.

Third, we show how the platform should optimally adjust its referral fee as the consumer's search cost decreases. One may intuit that because a lower search cost tends to intensify seller competition and reduce the platform's profit margin, the platform should compensate for it by raising its referral fee. Interestingly, the platform can find it optimal to lower its referral fee because the reduction in search cost may significantly increase the demand elasticity. In practice, if a decrease in search cost will attract new customers who are more price-sensitive relative to the platform's existing customers, the platform will have a more elastic demand and will thus tend to reduce its referral fee. In this case, a lower search cost can make the platform, the sellers, and the consumers all better off. By contrast, when the decrease in search cost does not significantly increase the demand elasticity, the platform will find it optimal to *increase* its referral fee, in which case, the decrease in search cost can counterintuitively lead to *higher* equilibrium retail prices. This result provides a possible explanation to the empirical puzzle that in very competitive markets prices much higher than marginal costs can still be maintained even when search cost is very low (Clemons et al. 2002, Hortaçsu and Syverson 2004).

Fourth, we show that filtering on the platform makes consumers more likely to stop searching and buy the product after a search and thus consumers will search fewer items in expectation. This is because filtering resolves some uncertainty about the product match level with consumers, so conducting a search will give consumers less information and potential benefit. We also find that, although filtering reduces the expected benefit of conducting a search, in equilibrium filtering will increase consumers' aggregate-match-level threshold for purchasing, above which they will stop searching and purchase a product. This is because filtering can help consumers rule out products with low match levels for filterable attributes, so the products searched by consumers tend to have higher aggregate match levels.

Fifth, filtering can either increase or decrease the equilibrium retail prices, depending on how it will reshape the sellers' demand function. Intuitively speaking, if filtering makes the product demand function to have a fatter (thinner) tail, competition among sellers will tend to be alleviated (intensified) and the equilibrium retail prices will more likely increase (decrease). In other words, filtering can either strengthen or soften the seller competition. These results suggest that filtering can have very different marketing implications compared with a search-cost reduction, which induces consumers to search *more* products and makes seller competition *more* intense.

Lastly, we show that our results are robust when consumers have heterogeneous search costs, when products have heterogeneous quality levels, and when the platform charges a fixed per-unit (instead of percentage) referral fee. We also provide guidance on how sellers should optimally choose their prices when consumers have heterogeneous search costs, and investigate how the distribution of the consumer's outside options will affect the profits of the platform and sellers, as well as the platform's benefit from a reduction in the consumer's search cost.

LITERATURE REVIEW

Our research is closely related to the literature on consumer search. Extensive literature has studied how a decrease in the consumer's search cost can influence product prices, firm profits, and consumer welfare. Many theoretical studies show that prices will drop to the marginal cost when the consumer's search cost diminishes to zero (Anderson and Renault 1999, Salop and Stiglitz 1977, Stahl 1989, Stigler 1961, Wolinsky 1986). Their predictions are supported by several empirical studies showing that the Internet has intensified price competition and reduced prices in markets for insurance (Brown and Goolsbee 2002), books and CDs (Brynjolfsson and Smith 2000), and prescription drugs (Sorensen 2000). To hinder consumer search, some sellers obfuscate their product information (Ellison and Ellison 2009). However, some evidence shows that many firms' prices are still considerably higher than marginal costs even in very competitive markets with low search costs (Clemons et al. 2002, Hortaçsu and Syverson 2004). More recent analytical research finds that a decrease in search cost may not always lead to more intense competition or lower profits. Kuksov (2004) suggests that a decrease in search cost may facilitate product differentiation and lead to higher prices and industry profit. Cachon et al. (2008) show that when consumers' search cost decreases, sellers may expand their product assortments, which can increase consumers' willingness-to-pay for their most-desired products, leading to higher equilibrium price and profit. Some research examines different features in search markets. Armstrong et al. (2009) study the implication of "prominence" in a search market, where all consumers will first search the prominent seller. They show that the highest-quality sellers will have the highest incentive to become prominent. Some research (Branco et al. 2012, 2016, Ke et al. 2016) has considered consumers' decision on how much product information to search before purchase. Dukes and Liu (2016) examine a platform's optimal choice for search cost when consumers decide the number of sellers to simultaneously search and how deeply to evaluate each seller.

This article complements the above literature on consumer search by examining how the consumer's search cost and filtering on the retail platform affect the platform's optimal referral fee and the competition among sellers. First, we contribute to the extant literature by studying filtering—a practically common feature on retail platforms—and providing relevant managerial insights. Second, we study how the consumer's search cost affects the platform's and the sellers' optimal pricing decisions whereas the extant literature discussed earlier has typically neglected to study the sellers' or the platforms' pricing decisions. One exception in that literature is Janssen and

Shelegia (2015), who examine how the consumers' lack of knowledge about the manufacturer's wholesale price will affect consumers' search and the manufacturer's and the retailers' pricing decisions in a traditional distributional channel. By contrast, we study how the consumer's search cost affects the platform's optimal referral fee and the competition among sellers in a retail platform setting. Moreover, in our framework, there is a large number of sellers offering *differentiated* products through the platform and consumers ex ante know neither the match levels nor the prices of the products, whereas in Janssen and Shelegia (2015) the monopoly manufacturer has only one product (of known valuation), whose retail price is ex ante not known.

Our analysis of filtering is related to Zhong (2018), who studies targeted search which allows the platform to customize each consumer's search results to display only the products of match levels above an exogenous threshold. He shows that if the threshold is low, an increase in the threshold has an effect equivalent to that of a decrease in search cost, which lowers the equilibrium retail prices. However, if that threshold is very high, consumers will in equilibrium search only one product, making sellers de facto monopolists and thus increasing equilibrium prices. Our research shows that filtering is very different from such targeted search. Filtering resolves some uncertainty about match levels of the products, so conducting a search will now give consumers less information and potential benefit. Hence, filtering will make consumers search *fewer* products in expectation. By contrast, targeted search can increase the probability that the consumers' next search will find a product having a higher match value than their previously searched products, thus targeted search makes consumers search *more* products in expectation. Moreover, filtering and targeted search can have different effects on the equilibrium prices. Targeted search will *lower* equilibrium prices in general except in the extreme case of de facto monopoly sellers, where all consumers in equilibrium search only one product. By contrast, filtering can *increase* equilibrium prices in very general situations where some consumers search multiple products in equilibrium.

MODEL SETUP

There are n independent sellers selling differentiated products on an online retail platform, which has zero marginal cost and charges sellers a percentage referral fee (r). We focus on the case of a large n such that consumers have negligible probability of running out of sellers to search. A consumer j's utility of buying seller i's product is $u_{ij} = M_{ij} - p_i$, where M_{ij} is the "match level" of product i to consumer j, measuring how well product i matches consumer j's preference, and p_i is the price of product i. Each consumer buys at most one product. All sellers have the same marginal production cost c, and ex ante they have no differences. Consumer j has an outside option with utility u_{0j}, distributed with the cumulative distribution function (c.d.f.) $F_0(u)$ and the probability distribution function (p.d.f.) $f_0(u)$. Without loss of generality, we normalize the total number of consumers to 1.

Filtering

Many retail platforms allow customers to filter products based on some product attributes. For example, if a consumer wants to buy a pair of Nike Lunarglide 8 running shoes, Amazon allows her to filter the search outcomes based on attributes such as shoe size, width, and color (See Figure 2 for an example.). Filtering enables consumers to find products that match their preferences better on these attributes. We refer to these attributes as a product's *filterable attributes*. In practice, however, filtering usually cannot exhaust all product attributes. There are some *unfilterable attributes*, e.g., shoe design and style, which consumers can learn only from search.

[Insert Figure 2 about here.]

We decompose a consumer's match level, M_{ij}, into two parts: $M_{ij} = \mu_{ij} + m_{ij}$. The filterable match level, μ_{ij}, captures how well the filterable attributes match a consumer's preference. The unfilterable match level, m_{ij}, reflects how well the unfilterable attributes match the consumer's preference. Filtering on the platform enables consumers to costless learn the filterable match level, μ_{ij}, of the products, but consumers still need to search for a product's unfilterable match level, m_{ij}. We assume that μ_{ij} and m_{ij} are independently and identically distributed across products and consumers. To avoid confusion, we henceforth refer to M_{ij} as the product's *aggregate* match level.

In practice, filters on the platform usually classify each filterable attribute into a finite number of categories. For example, Amazon classifies the color of men's running shoes into 12 categories. In our model, μ_{ij} follows a discrete distribution with $K(\geq 2)$ possible outcomes; μ_{ij} is equal to the k-th-lowest-possible filterable match level, μ_k, with probability ϕ_k, where $\mu_1 < \mu_2 < \cdots < \mu_K$ and $\sum_{k=1}^{K} \phi_k = 1$. The c.d.f. and p.d.f. of m_{ij} are $F(m)$ and $f(m)$, respectively. Without loss of generality, we assume $\mathbf{E}[\mu_{ij}] = 0$, which implies that $\mu_1 < 0$ and $\mu_K > 0$. In the main analysis, we assume that consumers can filter products on the platform.

We make several technical assumptions on the distributions of m_{ij} and u_{0j}. First, $f(m)$ and $f_0(u)$ are twice continuously differentiable and strictly positive on the supports $(m_{\min}, m_{\max})$ and $(u_{\min}, u_{\max})$, respectively, where $-\infty \leq m_{\min} < m_{\max} \leq +\infty$ and $-\infty \leq u_{\min} < u_{\max} \leq +\infty$. Second, to guarantee the existence of a unique symmetric pure-strategy equilibrium, $F(m)$ is assumed to have a decreasing inverse hazard rate $h(m) = \frac{1-F(m)}{f(m)}$. Many commonly-used distributions, e.g., normal distributions, uniform distributions, exponential distributions, and logistic distributions, satisfy these properties above. Third, we assume $\tau < \mathbf{E}[m] - \max\{m_{\min}, u_{\min}\}$ to exclude the trivial case in which no consumers will search after the first search.

Consumer Search

Each consumer will either purchase from a seller on the platform or choose the outside option (not buying on the platform). A consumer *a priori* does not know a product's unfilterable match level, m_{ij}, nor its price, p_i. We assume that consumers ex ante do not know p_i since in practice many online sellers do not immediately show a specific product's final transaction price to a consumer until she visits the product pages or the product is put in the shopping cart. For example, Ellison and Ellison (2009) document that many online sellers of computer parts on retail platforms post low prices to attract consumers to visit their product pages and show consumers the shipping-and-handling cost, taxes or add-on fees later on. The final transaction price can be much higher than the posted price that consumers see initially. Moreover, a seller may set different prices for different variants of a product (e.g., the same design of shoes with different colors), and often the platform will put all these variants together under one product listing, showing only the price range or the minimum price of all these variants. Consumers can find the exact price of a specific product variant only after conducting further search. Note also that in practice filtering and sorting by price are often ineffective at identifying the exact prices of the products because of the above reason. In addition, filtering and sorting by price often put many irrelevant results on the top of a page, e.g., if one sorts the search results of "Nike Lunarglide 8" by prices, 20 out of the top 24 results are actually other products (see Figure 3).

[Insert Figure 3 about here.]

Each consumer j *a priori* knows the utility of her own outside option u_{0j} and the filterable match level of the product, μ_{ij}. To find out the exact values of p_i and m_{ij} for a product, the consumer needs to incur a search cost τ. A consumer's search cost can include her time and effort in reading product descriptions/specifications, understanding the information, evaluating the product, and identifying additional fees, as well as the related psychological cost (e.g., search fatigue). A consumer will first decide whether to shop on the platform. If she does, she will search the sellers sequentially. After each search, the consumer learns the exact value of m_{ij} and p_i for the searched product i, then she can decide whether to buy it. If the consumer decides to buy a product, she will stop searching, otherwise she can continue searching another seller or leave the market without buying anything. If the consumer continues searching, one can show that her next search will be a seller with the highest μ_{ij} not yet searched. If multiple unsearched sellers have the highest μ_{ij}, they will be equally likely searched next.

Sellers, Retail Platform and Time Sequence

The timing of the game is as follows. First, the retail platform sets a percentage referral fee $r \in (0,1)$. Second, given r, all sellers simultaneously decide their retail prices. For each unit of product i sold at retail price p_i, the retail platform earns rp_i and seller i's profit is $(1-r)p_i - c$. Last, consumers make search and purchase decisions. In the Extensions section, we will study an alternative case when the platform charges a fixed per-unit referral fee instead of a percentage fee.

Using backward induction, we determine the symmetric pure-strategy Nash equilibrium: given the platform's referral fee r, all sellers will in equilibrium charge the same retail price, p^*. Let $p^*(r)$ denote the seller's equilibrium retail price given the referral fee r, and let r^* denote the platform's optimal percentage referral fee. Consumers have rational expectations about the sellers' prices, i.e., prior to a search they expect that the next seller's retail price will be $p^*(r)$.

EQUILIBRIUM ANALYSIS

We start by analyzing the consumer's searching and purchasing strategies. Suppose that the platform charges a referral fee r and the equilibrium retail price is $p^*(r)$. Let product i be the previous product that consumer j has just searched, which will give her utility of u_{ij}. Because the number of sellers (n) is large, consumers will search only sellers with the highest possible filterable match level, μ_K. It follows from Wolinsky (1986) that consumer j's optimal sequential-search strategy is to stop searching and purchase the *last* searched product i if and only if conducting another search will increase her expected utility beyond u_{ij} by an amount lower than search cost τ; otherwise she will continue searching. In equilibrium, a consumer will never buy a product that she had searched before product i. We have the following two results.

RESULT 1. *In a symmetric equilibrium where all sellers charge prices $p^*(r)$, if consumer j faces product i with $p_i = p^*(r)$ and m_{ij}, she will purchase product i if and only if $m_{ij} \geq \bar{m}(\tau)$, where $\bar{m}(\tau)$ is defined by $\int_{\bar{m}(\tau)}^{m_{\max}} (m - \bar{m}(\tau)) dF(m) = \tau$.*

RESULT 2. *$\bar{m}(\tau)$ strictly decreases in the consumer's search cost τ.*

Result 1 suggests that $\bar{m}(\tau)$ is essentially the equilibrium acceptance threshold for the unfilterable match level. Note that the threshold is uniquely determined by search cost, τ. In the rest of the article, for conciseness, we use $\bar{m}$ to represent $\bar{m}(\tau)$. Result 2 shows that when search cost (τ) decreases, searching another product becomes less costly to consumers. Therefore, they will have a higher purchase threshold $\bar{m}$.

Similar to Wolinsky (1986), one can derive seller i's profit as a function of its price p_i:

$$\pi_i^S = \underbrace{\frac{\phi_K F_0(\bar{m} + \mu_K - p^*(r))}{\phi_K n \cdot (1 - F(\bar{m}))}}_{\text{Prob(a consumer visits seller } i)} \cdot \underbrace{[1 - F(\bar{m} - p^*(r) + p_i)]}_{\text{Prob(Consumer buys from seller } i)} \cdot \underbrace{[(1 - r)p_i - c]}_{\text{Profit per sale}}, \quad (1)$$

where the superscript S represents the seller.

The platform's total demand, the number of consumers buying on the platform, is $D = F_0(\overline{m} + \mu_K - p^*(r))$. The expected consumer surplus is $CS(\tau, r) = \mathbf{E}_{u_0}[\max\{u_0, \overline{m} + \mu_K - p^*(r)\}]$.

Exogenous Referral Fee

We first analyze the benchmark case where the referral fee r is exogenous, that is, the platform's referral fee does not change with search cost. We use "~" over variables to indicate the exogenous-referral-fee case. In practice, platforms do not adjust the referral fees frequently. One reason is that platforms usually need to make announcements months before the fee change becomes effective. For example, on November 9, 2016, Amazon announced that it would increase its referral fee for media products starting March 1, 2017.[9] Similarly, on February 26, 2017, eBay announced an increase in its fee for certain products to be effective after May 1, 2017. Another reason is that frequently changing the referral fee will increase sellers' risk of selling on the platform and lead to confusion, distrust, and frustration among them. Therefore, it is reasonable to assume an exogenous referral fee in the short term.

LEMMA 1. *Given the platform's referral fee* r*, the sellers' equilibrium retail price is* $\tilde{p}^* = \frac{c}{1-r} + h(\overline{m})$*, a seller's per-unit profit and total profit are* $(1-r)h(\overline{m})$ *and* $\tilde{\pi}_i^{S*} = \frac{F_0\left(\overline{m}+\mu_K-\frac{c}{1-r}-h(\overline{m})\right)}{n}(1-r)h(\overline{m})$*, respectively. The total demand on the platform is* $\widetilde{D}^* = F_0\left(\overline{m} + \mu_K - \frac{c}{1-r} - h(\overline{m})\right)$.

Lemma 1 summarizes the equilibrium retail price, the sellers' profit, and the total demand given the percentage referral fee r. As r increases, the equilibrium retail price will increase. Thus, the total market demand on the platform and the sellers' profit will both decrease. From Lemma 1, we see that a seller's per-unit profit increases with $h(\overline{m})$. Our analysis shows that if a consumer has searched a seller and found the seller charges the equilibrium price, $\tilde{p}^*$, the consumer will buy from this seller if and only if $m_{ij} > \overline{m}$. If this seller's price is a small amount Δp above $\tilde{p}^*$, then the consumer will buy the product if and only if $m_{ij} > \overline{m} + \Delta p$. So, the small increase in price will reduce the seller's unit sales approximately by a fraction of $\frac{f(\overline{m})\Delta p}{1-F(\overline{m})} = \frac{\Delta p}{h(\overline{m})}$. Thus, a higher $h(\overline{m})$ gives a seller stronger incentives to increase its price because doing so will reduce sales only slightly. Hence, intuitively speaking, a higher $h(\overline{m})$ corresponds to less intense competition among sellers.

LEMMA 2. *As the consumer's search cost decreases, for a consumer who searches on the platform, her expected number of products to search before purchase will increase, and the expected aggregate match level of the product that she will purchase will also increase.*

Note that when search becomes less costly, a consumer will stop searching at a higher match-level threshold, $\overline{m}$. Thus, as Lemma 2 shows, as search cost decreases, the consumer will search more products and purchase a product with higher M_{ij} in expectation.

PROPOSITION 1. *When the referral fee (r) is exogenous, as the consumer's search cost (τ) decreases, (1) the equilibrium price $\tilde{p}^*$ decreases ($\frac{\partial \tilde{p}^*}{\partial \tau} > 0$), (2) the equilibrium total market demand strictly increases ($\frac{\partial \tilde{D}}{\partial \tau} < 0$), (3) each seller's expected profit $\tilde{\pi}_i^{S*}$ and the platform's expected profit $\tilde{\pi}^{P*}$ can either increase or decrease.*

Proposition 1 summarizes how the equilibrium retail prices, the market demand, and the sellers' profits will change with search cost (τ). One may intuit that a lower search cost will intensify the competition among sellers and reduce their prices and profits. We find that when search cost decreases, although sellers' equilibrium prices decrease, they may receive higher profits. This is due to a market-expansion effect—a decrease in search cost will make consumers more likely to shop on the platform rather than choosing the outside option, so the seller's market demand will increase. The platform's equilibrium profit can either increase or decrease as the consumer's search cost decreases.

Example 1. To explore in detail how the consumer's search cost (τ) affects the sellers and the platform, we consider a special example where $\mu_K = 0.5$, m_{ij} follows the uniform distribution between -0.5 and 0.5, and the utility of the outside option u_{0j} is uniformly distributed between 0 and 1. The inverse hazard function of m_{ij} is $h(m) = 0.5 - m$. We focus on the case of $\tau < \frac{\left(1-\frac{c}{1-r}\right)^2}{8}$, otherwise all consumers will search at most one product in equilibrium. One can show that $\bar{m} = 0.5 - \sqrt{2\tau}$ and the equilibrium retail price is $\tilde{p}^* = \frac{c}{1-r} + \sqrt{2\tau}$.

A seller's expected profit increases with τ when $\tau < \frac{\left(1-\frac{c}{1-r}\right)^2}{32}$ and decreases with τ when $\frac{\left(1-\frac{c}{1-r}\right)^2}{32} \leq \tau < \frac{\left(1-\frac{c}{1-r}\right)^2}{8}$. The platform's profit increases with τ when $\tau < \frac{\left(1-\frac{3c}{1-r}\right)^2}{32}$, and decreases with τ when $\frac{\left(1-\frac{3c}{1-r}\right)^2}{32} \leq \tau < \frac{\left(1-\frac{c}{1-r}\right)^2}{8}$. Therefore, under exogenous referral fee r, both the platform's profit and the sellers' profit first increase and then decrease with τ, as illustrated in Figure 4.

[Insert Figure 4 about here.]

Endogenous Referral Fee

In the long term, the platform can strategically adjust its referral fee when search cost changes. So, in this subsection, we analyze the case where the retail platform endogenously chooses its referral fee to maximize its expected profit. Let $r^*(\tau)$ denote the platform's optimal percentage fee and $\pi^{P*} = \pi^P(r^*(\tau))$ denote the platform's optimal profit given the consumer's search cost τ.

PROPOSITION 2. *When the platform endogenously sets the referral fee r, a decrease in search cost will strictly increase the platform's expected profit, i.e.,* $\frac{d\pi^{P*}}{d\tau} < 0$.

Intuitively, a lower search cost can have either positive or negative impacts on the platform's profit. On the one hand, a lower search cost can benefit the platform because it increases the market demand. On the other hand, it tends to intensify competition among sellers, which can reduce the platform's per-unit profit. Proposition 1 shows that under exogenous referral fees, a search-cost reduction can either increase or reduce the platform's profit. In contrast, Proposition 2 shows that under endogenous referral fees, a lower search cost will *always* benefit the platform because it can choose a referral fee to increase the demand without too significantly reducing its profit margin.

Proposition 2 suggests that if the retail platform can optimally adjust its referral fee, reducing the consumer's search cost will benefit the platform as long as doing so is not too costly. The result potentially explains why retail platforms, such as Amazon.com, have invested billions of dollars in improving consumers' search experience and reducing their search cost on the platform. Although the platform may see a profit drop in the short term (with the referral fee fixed), it can improve its profit in the long term when it optimally adjusts its referral fees.

Next, we study how the retail platform should optimally adjust its referral fee with the consumer's search cost. Intuitively, a reduction in search cost tends to intensify the competition among sellers, which reduces the platform's profit margin and hence gives the platform an incentive to raise its referral fee to mitigate its profit-margin decrease. However, Proposition 3 shows that under some conditions, it may be optimal for the platform to *reduce* its referral fee as the consumer's search cost decreases. Let $\epsilon_{D,r}(r) \equiv \frac{\partial D}{\partial r} \cdot \frac{r}{D}$ be the platform's demand elasticity of referral fee r.

PROPOSITION 3. *When the consumer's search cost* τ *decreases (i.e., when* $\overline{m}$ *increases), the platform's optimal referral fee* r^* *will increase if* $\frac{\partial|\epsilon_{D,r}(r^*)|}{\partial\overline{m}} < \frac{-cr^*(\overline{m})h'(\overline{m})}{\left(cr^*(\overline{m})+h(\overline{m})\right)^2}$*, and decrease if* $\frac{\partial|\epsilon_{D,r}(r^*)|}{\partial\overline{m}} > \frac{-cr^*(\overline{m})h'(\overline{m})}{\left(cr^*(\overline{m})+h(\overline{m})\right)^2}$.

Proposition 3 characterizes the conditions under which the platform should raise or reduce its referral fee as search cost decreases. If a decrease in search cost significantly increases the (magnitude of) demand elasticity, the platform will find it optimal to reduce its referral fee because doing so can significantly increase the total demand. Note that the lowered referral fee will reduce the equilibrium retail price but can actually increase the sellers' net profit margin. As a result, a lower search cost can be *all-win* for the platform, sellers and consumers if the platform reduces its referral fee. Example A1 in the Appendix provides a numerical example where the platform finds it optimal to reduce the referral fee as search cost decreases. By contrast, if a decrease in search cost does *not* significantly raise the (magnitude of) demand elasticity, then the platform will find it optimal to raise its referral fee. In this case, a decrease in search cost can reduce sellers' profits and increase the equilibrium retail prices, potentially making the consumers worse off as well. This result gives a possible explanation to the empirical puzzle that in very competitive markets prices much higher than marginal costs can still be maintained even when search cost is very low.

Proposition 3 suggests that when consumers' search cost decreases, whether the retail platform should reduce or raise its referral fee depends on how the demand elasticity changes. If a decrease in search cost will attract new customers who are more price-sensitive relative to the platform's existing customers, the platform will have a more elastic demand, so it will tend to reduce its referral fee. In reality, this can happen for a high-end platform with a wealthier, price-insensitive customer base. By lowering consumers' search cost, the platform can attract some marginal consumers, who are more price-sensitive than its existing customers. In this case, the platform's demand elasticity will increase, so the platform may find it optimal to lower its referral fee to boost its sales volume. By contrast, if the decrease in search cost will attract new customers who are less price-sensitive compared with the existing customer base, the platform's demand will tend to become more inelastic, so its optimal referral fee will tend to increase. In reality, this is more likely to happen for a low-end platform with a price-sensitive customer base. A decrease in search cost will help the low-end platform to attract some wealthier customers, which reduces its overall demand elasticity.

In practice, it is easy for the platform to reduce its referral fees. However, the platform may receive complaints from sellers and consumers when raising the referral fees. In practice, when the platform finds it optimal to raise the referral fees, it should try to alleviate the sellers' and consumers' dissatisfaction. For example, the platform may want to announce a soon-to-be-in-effect fee increase at the same time as it introduces new search features and technologies on its website (e.g., augmented-reality technologies), and it can communicate to sellers and consumers that the fee increase will enable the platform to cover the cost of developing and offering such new technologies.

Example 2. To further study how the search cost's impact may be different when the referral fee is endogenous versus when it is exogenous, we consider the numerical example with the same distributions and parameters as those in Example 1 (the exogenous-referral-fee case), but allow the platform to endogenously choose its referral fee.

[Insert Figure 5 about here.]

Figure 5(a) plots how the platform's optimal referral fee and the equilibrium retail price change with search cost τ. In Example 1 where the referral fee is exogenous, the reduction in τ will always reduce the equilibrium retail price, p^*, because it intensifies seller competition. By contrast, in Example 2 with endogenous referral fee, when τ is already low, further reduction in τ can lead to a higher p^*. This is because the platform will increase its referral fee when τ becomes lower, forcing the sellers to charge higher prices to cover their cost. Despite the increase in retail price, the sellers' net profit per unit sold will decrease. In other words, the reduction in the consumer's search cost may intensify the double-marginalization problem in the channel and lead to higher retail prices. Figure 5(b) depicts how the platform's and the sellers' profits will change with the consumer's search cost, τ. Echoing Proposition 2, although the decrease in τ can lower the platform's profit under exogenous referral fee, the platform's profit will always increase if it can optimally adjust r.

Our analysis also sheds light on how the impacts of a lower search cost may differ when the sellers sell directly to consumers (without any intermediary/platform) versus when they sell through a strategic retail platform. Note that the direct-selling scenario is a special case of our exogenous-referral-fee model with the referral fee set to zero ($r = 0$). Hence, we can see how removing the platform will change the results by comparing the outcomes of the exogenous-referral-fee model (with $r = 0$) and the endogenous-referral-fee models. We find that the market outcome can be very different in these two scenarios. In a direct-selling model without the platform, a lower search cost will always reduce the equilibrium retail price, making consumers better off. By contrast, in the model with a strategic retail platform, a lower search cost can *increase* the equilibrium retail price (due to the platform's strategic increase in its referral fee), potentially making consumers worse off.

EFFECTS OF FILTERING

Note that both a lower search cost τ and the filtering feature can make consumer search easier on a retail platform. In the previous section, we have examined how τ affects the market outcome. Next, we study the effects of filtering on the consumer's search decisions and the sellers' pricing strategies, and show that the effects of filtering can differ from that of a lower search cost. To this end, we need to analyze a model with no filtering feature on the platform and compare the equilibrium outcome with that of the main model where filtering is available.

Suppose that filtering is not available on the platform, so consumers do not observe a product's filterable match level (μ_{ij}) before searching. Searching a product will inform a consumer of the aggregate match level $M_{ij} = \mu_{ij} + m_{ij}$. Let $F_M(M)$ and $f_M(M)$ denote the c.d.f. and p.d.f. of M_{ij}, respectively. Note that $\mu_{ij} = \mu_k$ with probability ϕ_k and the c.d.f. of m_{ij} is $F(m)$, so

$F_M(M) = \Pr\left(\mu_{ij} + m_{ij} \leq M\right) = \sum_{k=1}^{K} \Pr\left(\mu_{ij} = \mu_k\right) \cdot \Pr(m_{ij} \leq M - \mu_k) = \sum_{k=1}^{K} \phi_k F(M - \mu_k)$,

or equivalently, $F_M(M) = \mathbf{E}_\mu[F(M - \mu)]$.

This equation characterizes the relationship among the distributions of M_{ij}, μ_{ij}, and m_{ij}. Intuitively, when the filterable match level is μ_k, the aggregate match level (M_{ij}) is smaller than M if and only if the corresponding unfilterable match level (m_{ij}) is lower than $M - \mu_k$. Hence, the overall probability of $M_{ij} \leq M$ will be the expected probability of $m_{ij} \leq M - \mu$, where the expectation is taken with respect to μ. Note that $F_M(\cdot)$ is a convex combination of $F(\cdot)$, so $F_M(M)$ will be greater (smaller) than $F(M)$ when $F(\cdot)$ is "convex (concave) enough" around M. Similarly, the p.d.f. of M satisfies $f_M(M) = \sum_{k=1}^{K} \phi_k f(M - \mu_k) = \mathbf{E}_\mu[f(M - \mu)]$, which will be greater (less) than $f(M)$ when $f(\cdot)$ is "convex (concave) enough" around M. The inverse hazard rate of M is

$$h_M(M) = \frac{1-F_M(M)}{f_M(M)} = \frac{\mathbf{E}_\mu[1-F(M-\mu)]}{\mathbf{E}_\mu[f(M-\mu)]} = \frac{\mathbf{E}_\mu[1-F(M-\mu)]}{1-F(M)} \cdot \frac{f(M)}{\mathbf{E}_\mu[f(M-\mu)]} \cdot h(M). \tag{2}$$

We assume that $F_M(M)$ is twice continuously differentiable and $h_M(M)$ is decreasing.[10] From Equation 2, one can see that $h_M(M) > h(M)$ if $1 - F(\cdot)$ is "sufficiently convex" (such that $\frac{\mathbf{E}_\mu[1-F(M-\mu)]}{1-F(M)}$ is sufficiently large) and $f(\cdot)$ is "sufficiently concave" (such that $\frac{f(M)}{\mathbf{E}_\mu[f(M-\mu)]}$ is sufficiently large), i.e., when $f'(\cdot)$ and $f''(\cdot)$ are sufficiently small. Similarly, $h_M(M) < h(M)$ if

$f'(\cdot)$ and $f''(\cdot)$ are sufficiently large. We show later that the magnitude of $h_M(\cdot)$ relative to $h(\cdot)$ is important in determining how filtering affects the sellers' equilibrium prices.

Consumer's Optimal Search Strategy

Let us define $M_{\max} = m_{\max} + \mu_K$, the maximum possible value of M. We will use a subscript N to indicate the no-filtering case. Without filtering on the platform, the consumers will stop searching and buy the current product they have searched if and only if its aggregate match level, M_{ij}, exceeds the equilibrium acceptance threshold, $\bar{M}_N$, which is uniquely and implicitly defined by $\int_{\bar{M}_N}^{M_{\max}} (M - \bar{M}_N) dF_M(M) = \tau$. By contrast, if $M_{ij} \leq \bar{M}_N$ for the current product, consumers will continue searching other products. Note that with filtering on the platform, the consumer's equilibrium acceptance threshold for the aggregate match level is $\bar{M} = \mu_K + \bar{m}$ since consumers will search only products with the highest filterable match level (μ_K). Lemma 3 compares $\bar{M}$ and $\bar{M}_N$ and illustrates how filtering affects the consumer's optimal searching strategy.

LEMMA 3. *Filtering will increase the consumer's acceptance threshold for the aggregate match level by less than* μ_K*, i.e.,* $0 < \bar{M} - \bar{M}_N < \mu_K$.

Filtering has two opposing effects on the consumer's acceptance threshold for a product's aggregate match level. First, filtering allows consumers to know a product's filterable match level and narrows their consideration set down to those products having the best match levels (with $\mu_{ij} = \mu_K$) for filterable attributes. This effect raises the consumer's acceptance threshold for the aggregate match level by μ_K. Second, filtering also has a side effect of reducing the direct benefit of searching. Because filtering has already revealed the filterable match level and resolved some of the consumer's uncertainty, conducting a search will uncover less information about match levels and thus give the consumer less benefit, *ceteris paribus*. Thus, the consumer is more likely to stop searching at a lower acceptance threshold. Overall, the first effect dominates the second effect, increasing the consumer's equilibrium acceptance threshold for M_{ij} but by less than μ_K.

Our discussion above suggests that filtering will reduce consumers' expected benefit from conducting a search, making consumers more likely to stop searching and buy the product after a search. As a result, with filtering on the platform, consumes will search fewer products in expectation. Proposition 4 documents these results.

PROPOSITION 4. *In equilibrium, filtering increases a consumer's probability of buying a product after searching it, i.e.,* $1 - F(\bar{m}) > 1 - F_M(\bar{M}_N)$, *and reduces the consumer's expected number of products to search.*

Sellers' Pricing Decisions

Next, we investigate how filtering affects the sellers' pricing. We show that the equilibrium retail price in the no-filtering case is $\tilde{p}_N^* = \frac{c}{1-r} + h_M(\bar{M}_N)$, as compared with $\tilde{p}^* = \frac{c}{1-r} + h(\bar{m})$ when filtering is available. Filtering will reduce the equilibrium retail price ($\tilde{p}^* < \tilde{p}_N^*$) if and only if $\frac{h_M(\bar{M}_N)}{h(\bar{m})} = \frac{h_M(\bar{M}_N)}{h(\bar{M}_N)} \cdot \frac{h(\bar{M}_N)}{h(\bar{m})} > 1$. Note that, as discussed before, when $f'(\cdot)$ and $f''(\cdot)$ are lower, $\frac{h_M(\bar{M}_N)}{h(\bar{M}_N)}$ is higher, making the above inequality more likely to hold. Thus, intuitively speaking, filtering tends to reduce (increase) the equilibrium retail price when $f'(\cdot)$ and $f''(\cdot)$ are low (high).

To formalize the above reasoning, let us consider the "marginal" effect of filtering on the seller's equilibrium price by assuming that filtering reveals only an infinitesimal amount of match-level information to consumers. In such a scenario, the magnitude of the filterable match level, μ_{ij}, is very small such that $\max_{1<k\leq K} |\mu_k| \to 0$. Proposition 5 characterizes how filtering affects the equilibrium retail price.

PROPOSITION 5. *Suppose* $\max_{1<k\le K} |\mu_k| \to 0$.

(1) If $h^2(\bar{m})f''(\bar{m}) + 2h(\bar{m})f'(\bar{m}) + f(\bar{m}) > 0$, *filtering will increase the equilibrium retail prices, i.e.,* $\tilde{p}^* > \tilde{p}_N^*$.

(2) If $h^2(\bar{m})f''(\bar{m}) + 2h(\bar{m})f'(\bar{m}) + f(\bar{m}) < 0$, *filtering will decrease the equilibrium retail prices, i.e.,* $\tilde{p}^* < \tilde{p}_N^*$.

What do the conditions about $f(\cdot)$, $f'(\cdot)$ and $f''(\cdot)$ in Proposition 5 mean? Mathematically, a high $f''(\cdot)$ indicates that $f'(\cdot)$ will not decrease fast within a certain range. Hence, if $f(x)$, $f'(x)$ and $f''(x)$ are high at point x, $f(x)$ is high at x and will stay relatively high as x increases. This suggests that the distribution of a product's unfilterable match level (m_{ij}) has a relatively fat tail. Note also that when filtering is available, the demand for a product is proportional to $1 - F(\bar{m} - p^*(r) + p_i)$, so the product's demand tends to have a relatively fat tail when $f(x)$, $f'(x)$ and $f''(x)$ are high. In this case, filtering tends to reduce seller competition and lead to higher equilibrium prices. An example product category is one-of-a-kind or niche products (e.g., paintings or decor) that have unfilterable unique features (e.g., design aesthetic) for which some consumers have very high valuations. For these products, the platform will be more likely to allow consumers to filter the search results based on their filterable attributes (e.g., size or material) since doing so can increase the sellers' retail prices and their referral fees paid to the platform.

By contrast, when $h^2(\bar{m})f''(\bar{m}) + 2h(\bar{m})f'(\bar{m}) + f(\bar{m}) < 0$, i.e., when $f(\cdot)$, $f'(\cdot)$ and $f''(\cdot)$ are relatively low, the distribution of unfilterable match levels and the demand function for a product tend to have relatively thin tails. In this case, filtering tends to decrease the equilibrium price. This is more likely to hold for products whose major features can be filtered and whose unfilterable attributes have limited differentiation values. For such products, the platform should be cautious about introducing filtering on all filterable attributes, because it will likely trigger fierce competition among sellers, which may reduce the profits of the sellers and the platform.

Although both filtering and a reduction in search cost make consumer search easier, our analysis above suggests that they can have very different marketing implications. For example, a reduction in search cost will always encourage consumers to search more products, but filtering will reduce the benefit of search, making consumers search *fewer* products in expectation. Moreover, a decrease in search cost will intensify competition among sellers, leading to lower retail prices (for a given referral fee). By contrast, filtering can soften seller competition and lead to *higher* retail prices, which may be desirable to the platform.

EXTENSIONS

Heterogeneous Search Cost

In reality, consumers may differ in their search efficiency, product knowledge, and opportunity costs for searching. In this extension, we analyze a model in which consumers have heterogeneous search cost, τ. Specifically, let $\Phi(\tau)$ denote the c.d.f. of τ, where $0 < \tau_{\min} \leq \tau \leq \tau_{\max}$, and $\tilde{p}^*$ denote the equilibrium price when consumers have heterogeneous search costs.[11] In addition, let $\tilde{p}_\tau^* = \frac{c}{1-r} + h(\overline{m}(\tau))$ denote the equilibrium retail price in the homogeneous-search-cost case (as derived earlier) where all consumers have the same search cost τ.

How should a seller choose its price when facing consumers with heterogeneous search costs? A naïve approach is to "weigh each consumer equally" and set the price to $\mathbf{E}_\tau[\tilde{p}_\tau^*]$, the average of the homogeneous-search-cost prices (p_τ^*) weighted by the probability density of τ. Proposition 6 shows that this naïve choice of price is suboptimal for the seller and that the optimal price should give more weight to consumers with low search costs, resulting in $\tilde{p}^* < \mathbf{E}_\tau[\tilde{p}_\tau^*]$.

PROPOSITION 6. *When consumers have heterogeneous search costs, the seller's optimal retail price is a weighted average of the homogeneous-search-cost equilibrium prices. Mathematically,* $\tilde{p}^* = \mathbf{E}_\tau[a(\tau)\tilde{p}_\tau^*]$, *where the weighting function* $a(\tau)$ *decreases with the consumer's search cost,* [12] *i.e.,* $a'(\tau) < 0$. *Moreover,* $\tilde{p}^* < \mathbf{E}_\tau[\tilde{p}_\tau^*]$.

Intuitively, a consumer with lower search costs is more likely to shop on the platform instead of choosing the outside option, so she is more likely to visit a seller than a consumer with higher search costs. Therefore, the distribution of a seller's visitors is skewed towards those with low search costs, which gives the seller an incentive to lower its price to target these consumers.

To check the robustness of our main insights from the main model, we examine an example with the commonly used, analytically trackable, uniform distribution for search costs: $\tau \sim \text{Uniform}(0, \bar{\tau})$. Our analysis shows that all the major results in our model with homogeneous search costs remain qualitatively the same in the model with uniformly distributed search costs. Please refer to Online Appendix for details.

Heterogeneous Product Quality

In reality, some sellers may have better product or service quality than others. To parsimoniously capture the quality heterogeneity, let us assume that there is one premium seller offering a premium product for which consumers on average have higher match levels than the non-premium products sold by the other $n-1$ non-premium sellers. Without loss of generality, we assume that the seller $i = 1$ sells the premium product, and sellers $i = 2, \dots, n$ sell the non-premium products. For analytical tractability, we consider the case where all product attributes are unfilterable, i.e., $\mu_i = 0$, $\forall i$. Let the aggregate match level of the premium product ($i = 1$) be $M_{1j} = \Delta q + m_{1j}$, where Δq is the premium product's quality advantage, i.e., the difference between the average aggregate match level of the premium product and that of non-premium products. The distribution of m_{1j} is the same as that of a non-premium product. The marginal costs of these two types of products are c_H and c_L, respectively. We assume $c_H - c_L < \Delta q$, i.e. the premium seller is relatively cost-effective.

In the Online Appendix, we show that a consumer's optimal search strategy depends on her outside options. If a consumer's utility from the outside option is low, she will search the premium product first and, if its match level is low, she will then search non-premium products. If a consumer's outside option has a high utility, she will choose the outside option and not search on the platform. However, the search pattern for consumers with intermediate utilities for their outside options is different from the main model where all products have homogeneous quality levels. These consumers will search only the premium product and if its match level is low, they will choose the outside option. For these consumers, searching the premium product is expected to be better than directly choosing the outside option, but the expected benefit of searching a non-premium product does not justify the search cost.

Next, we examine the sellers' pricing strategies. Let $\tilde{p}_1^*$ and $\tilde{p}^*$ be the equilibrium prices of the premium product (product 1) and the non-premium products, respectively. In the Online Appendix, we prove $\tilde{p}^* < \tilde{p}_1^*(r) < \tilde{p}^* + \Delta q$, i.e., the premium seller charges a higher price than non-premium sellers, but the price difference is smaller than their quality difference, so consumers will in equilibrium search the premium seller first. This result is similar to Armstrong et al. (2009), who find that higher-quality sellers have a stronger incentive to become prominent, i.e., being searched first by consumers. However, they assume that consumers will always search the prominent seller first, even though they may be better off in expectation to search other items first. By contrast, the search sequence in our model is not exogenously imposed but endogenously determined by consumers' optimal decisions. Next, we examine the non-premium sellers' pricing strategies.

RESULT 3. *The equilibrium retail price of the non-premium products is* $\tilde{p}^*(r) = \frac{c}{1-r} + h(\bar{m})$, *which is independent of the quality premium of the advantage product,* Δq.

Result 3 suggests that $\tilde{p}^*$ is independent of Δq. The intuition is as follows. The consumer's optimal search strategy indicates that consumers will always search the premium seller before they search any non-premium ones. In equilibrium, the premium seller will choose its price according to its quality advantage (Δq) to induce consumers to search the premium product first. Only consumers who have already searched the premium product but found low match levels will continue to search the non-premium products. Thus, for consumers who have decided to search non-premium sellers, the premium product's quality and price will no longer affect their probability of buying from a specific non-premium seller. Note that consumers who have not searched non-premium sellers cannot observe any given non-premium seller's price, so a deviation (e.g., a drop) in any non-premium seller's price will not affect the consumer's probability of finding this seller. Put differently, in equilibrium, the distribution of aggregate match levels for non-premium products for the consumers who will search the non-premium products does not depend on the premium seller's quality advantage and price. Thus, the equilibrium prices of non-premium products do not depend on the price and quality of the premium product.

We also analyze a numerical example and show that both the premium seller's and the non-premium sellers' equilibrium prices will decrease when the consumer's search cost (τ) decreases. When the platform's referral fee is exogenous, the profits of the premium seller, the non-premium sellers, and the platform can either increase or decrease with search cost. When the platform endogenously chooses r, its optimal profit will always increase when τ decreases. Thus, our results in the main model are robust when products have heterogeneous quality levels.

Fixed Per-unit Referral Fee

Our previous analyses have assumed that the platform charges sellers a *percentage* referral fee as is common on most retail platforms. However, one might wonder how our results are affected if the platform charges a *fixed* per-unit referral fee. In the Online Appendix, we have analyzed the case where the platform charges the sellers a *fixed* referral fee of $d > 0$ for each unit sold (independent of retail prices). Other aspects of the model are the same as the main model.

Our analysis shows that most results from the main model (under percentage referral fees) remain qualitatively the same in the current setting with fixed per-unit referral fees. More specifically, a decrease in search cost will reduce the equilibrium retail prices and increase the demand on the platform; a seller's profit can either increase or decrease. Moreover, when the platform can endogenously choose the referral fee d, a lower search cost will always increase the platform's profit. We find one result to be different in the fixed-referral-fee setting. When the platform's referral fee is exogenous, a decrease in search cost will always improve the platform's profit under a fixed per-unit referral fee, but it can *reduce* the platform's profit under a percentage referral fee. This is because when the referral fee is fixed (independent of retail price), a lower search cost will intensify competition among sellers and reduce the retail prices, but will not reduce the platform's profit (d) from each sale.

The result above suggests a potential benefit of fixed referral fees to the platform under the exogenous fees. However, our analysis in the Online Appendix reveals that if the platform can *endogenously* choose its referral fee, its profit will be strictly higher with a percentage fee than with a fixed per-unit fee. The reason is that a percentage fee will increase sellers' incentives to lower their prices, because the decrease in price will also reduce the sellers' referral fee paid to the platform. By contrast, with a fixed per-unit referral fee, any decrease in price will reduce only the sellers' unit profit margin and *not* their referral fees. Hence, a percentage fee, relative to a fixed per-unit fee, will alleviate the double-marginalization problem in the channel, which will increase the platform's profit. This result provides a potential explanation for why most retail platforms (e.g., Amazon, eBay, and Etsy) use percentage fees as their main referral fees.

Outside Option and Retail Platform Competition

In practice, there may be multiple retail platforms that consumers can visit and shop. Our main model has analyzed the consumer's search behavior on a focal retail platform, which can be thought of as the dominant retail platform in reality. Note that, in our model, the consumer's outside option can be considered as the consumer's shopping on other competing retail platforms. In this sense, our analytical framework can partially capture the competition among retail platforms (e.g., Amazon.com versus eBay.com) though we do not explicitly consider the competitor's strategic pricing decisions. When the focal platform faces stronger competition, consumers are more likely to have a better outside option. We analyze how the distribution of outside options affects consumers, sellers, and the platform. For analytical tractability, we assume that the platform will endogenously choose its fixed per-unit referral fee, d. We adopt the distribution assumptions in Example 1 and 2: $\mu_K = 0.5$ and $m_{ij} \sim \text{Uniform}(-0.5, 0.5)$. The consumer's outside option, u_0, follows a uniform distribution between l and $1 + l$. A larger l indicates that the outside option tends to be more attractive, i.e., the competing platform is more competitive. We consider the nontrivial case with $-2\sqrt{2\tau} - c < l < 1 - 2\sqrt{2\tau} - c$, otherwise either no consumers or all consumers will shop on the focal platform.

We show that in equilibrium, the platform's fixed referral fee is $d^* = \frac{1-2\sqrt{2\tau}-c-l}{2}$, a seller's profit is $\pi_i^{S*} = \frac{\left(1-2\sqrt{2\tau}-c-l\right)}{2n}\sqrt{2\tau}$, and the platform's profit is $\pi^{P*} = \frac{\left(1-2\sqrt{2\tau}-c-l\right)^2}{4}$. When the platform faces stronger competition (i.e., a larger l), its optimal referral fee will decrease, and its profit as well as the sellers' profit will decrease. The platform can always benefit from a lower search cost: $\frac{\partial \pi^{S*}}{\partial \tau} = -\frac{1-2\sqrt{2\tau}-c-l}{\sqrt{\tau}} < 0$.

Note that the absolute value of the above derivative decreases with l. Or put differently, when the platform competes with stronger competitors, it will receive *less* benefit from a decrease in the consumer's search cost. One may have intuitively conjectured that a platform facing stronger competition would be more inclined to lower its search cost in order to poach customers from competitors. However, interestingly, our result suggests that it is the platform facing *weaker* competitors (with a smaller l) that has a stronger incentive to reduce its search cost. This is because such a platform has a higher per-unit profit and can benefit more from acquiring an additional customer. This result may help explain why large retail platforms (e.g., Amazon.com and Overstock.com) tend to invest heavily in search-cost-reduction technologies, e.g., augmented-reality view, whereas smaller shopping sites usually use less advanced search technologies. This can create a Matthew effect, allowing larger platforms to offer better search experience and attract even more customers.

CONCLUSION

This article studies how the consumer's search cost and filtering affect the retail platform, independent sellers, and consumers. Though a lower search cost will reduce sellers' profit margins due to intensified competition, it can actually increase the sellers' profits because a lower search cost can attract more consumers to shop on the platform. Under exogenous referral fees, a decrease in search cost can either increase or reduce the platform's profit, but it will strictly increase the platform's profit if the platform optimally adjusts its referral fee. This result implies that the platform should reduce the consumer's search cost as long as doing so is not too costly.

Our analysis shows that when the consumer's search cost decreases, the platform can find it optimal to lower its referral fee because the reduction in search cost may significantly increase the demand elasticity. In this case, a lower search cost can make the platform, the sellers, and the consumers all better off. By contrast, when the decrease in search cost does not significantly raise the demand elasticity, the platform will find it optimal to *increase* its referral fee, in which case the decrease in search cost can counterintuitively lead to *higher* retail prices. This implies that a retail platform can improve its profit by setting different referral fees for different product categories with different consumers' search costs. Ceteris paribus, a platform should charge lower referral fees for product categories with higher search costs unless the product demands in these categories are significantly more inelastic than those in product categories with lower search costs.[13]

We have also analyzed how filtering affects the consumer's search and the sellers' pricing decisions. With filtering on the platform, consumers will be more likely to stop searching and buy the product after a search and hence will search fewer products in expectation. This is because filtering resolves some uncertainty about the product match level, so conducting a search will now uncover less information and bring less benefit to consumers. Moreover, filtering can either increase or decrease the equilibrium retail prices, depending on how it will reshape the sellers' demand. Intuitively speaking, if filtering makes the product's demand function to have a fatter (thinner) tail, competition among sellers will tend to be alleviated (intensified) and the equilibrium retail prices will more likely increase (decrease). In other words, filtering can either strengthen or soften the seller competition. These results suggest that filtering can have very different marketing implications compared with a search-cost reduction, which induces consumers to search *more* products and makes seller competition *more* intense.

Even though we assume that sellers sell differentiated products, our model can conceptually apply to the situation where some sellers sell a common branded product. In essence, a "product" in our model is the totality of the core product and any seller attributes. Hence, the differentiation among "products" comes from not only the core product attributes (e.g., shoes of different brands, style, color, size, etc.) but also the sellers' attributes (e.g., services, return policy, existence of physical stores close to the customers, etc.). For example, sellers on the east coast of the U.S. can ship their products to New York City within one day, but it may take longer to ship to Los Angeles. Similarly, sellers on the west coast of the U.S. can ship products to Los Angeles faster than to New York City. Furthermore, whether the sellers have physical stores close to the customers can differentiate the sellers, because consumers may have more convenient options for product returns or exchanges if the seller has a physical store nearby.

This article mainly focuses on the online retail setting, which is conceptually very different from the offline setting with a brick-and-mortar store or stores-within-a-mall setting, e.g., in Gu and Liu (2013). Let us compare Amazon (the online retail platform) and Walmart (a brick-and-mortar store) to more clearly see the differences. On Amazon, a large number of third-party sellers choose the retail prices for their products and will pay a percentage fee to Amazon for each sale. In Walmart stores, Walmart chooses the retail prices for a large number of products that it buys from a large number of different manufacturers or suppliers on wholesale prices. So, conceptually, in terms of market structure, the online platform Amazon (in case of third-party selling) is an upstream firm dealing with a large number of downstream firms (the third-party sellers), whereas Walmart is the downstream firm dealing with a large number of upstream firms (the manufacturers or suppliers). Note that the retail pricing decision rights reside with the downstream firms, and that the typical contracts in both settings are different. In addition, this article analyzes how filtering will impact the consumers, sellers, and the platform. The filtering feature is unique to the online retail environment because filtering can rarely be applied to offline stores.

We conclude by discussing a few caveats about our model and directions for future research. First, our model does not explicitly consider how the consumer's search cost affects the sellers' entry decisions. One can extend our model, e.g., by assuming that a seller needs to incur some fixed cost to sell on the platform. If in our model a seller's expected profit increases when the consumer's search cost decreases, then in that parameter region, sellers will be more likely to enter the market. Similarly, if a lower search cost reduces the seller's expected profit, then some sellers will likely exit the market. Second, we have assumed that each seller sells only one product. In practice, a seller may sell multiple differentiated products on the platform. One can study how the consumer's search cost affects the seller's product-assortment decisions. Cachon et al. (2008) show that firms that sell directly to consumers will increase their product assortments when the consumer's search cost decreases. We conjecture that if sellers sell their products through a retail platform, a decrease in the consumer's search cost will increase the sellers' optimal product assortments when the referral fee is exogenous. However, when the platform endogenously adjusts its referral fee, a lower search cost may induce the platform to increase its referral fee, which may reduce the sellers' optimal assortments. Lastly, explicitly modeling the strategic interactions between competing platforms may bring forth additional insights about the effects of search costs and filtering. We leave that exploration to future research.

APPENDIX

Example A1. This example presents a case where the platform's optimal percentage referral fee r can decrease when the consumer's search cost decreases. Suppose that $\mu_K = 2, c = 1$, u_{0j} follows the student-t distribution with degree of freedom 5, and m_{ij} follows a logistic distribution with c.d.f. $F(m) = \frac{1}{1+e^{-5m}}$. The platform's optimal referral fee is shown in Figure A1. When $\bar{m} <$ 3.5, the platform's optimal referral fee will decrease as the consumer's search cost decreases.

[Insert Figure A1 about here.]

REFERENCES

Anderson, Eric T., Nathan M. Fong, Duncan I. Simester, and Catherine E. Tucker (2010), "How Sales Taxes Affect Customer and Firm Behavior: The Role of Search on the Internet," *Journal of Marketing Research*, 47(2), 229–239

Anderson, Simon and Regis Renault (1999), "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *The RAND Journal of Economics*, 30(4), 719–735

Armstrong, Mark, John Vickers, and Jidong Zhou (2009), "Prominence and Consumer Search," *The RAND Journal of Economics*, 40(2), 209–233

Bajari, Patrick, Ali Hortaçsu, and Ali Hortacsu (2003), "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from EBay Auctions," *The RAND Journal of Economics*, 34(2), 329–355

Blake, Thomas, Chris Nosko, and Steven Tadelis (2016), "Returns to consumer search: Evidence from eBay" In *Proceedings of the 2016 ACM Conference on Economics and Computation* (pp. 531–545) New York, USA: ACM Press

Branco, Fernando, Monic Sun, and J. Miguel Villas-boas (2012), "Optimal Search for Product Information," *Management Science*, 58(11), 2037–2056

Branco, Fernando, Monic Sun, and J. Miguel Villas-Boas (2016), "Too Much Information? Information Gathering and Search Costs," *Marketing Science*, 35(4), 608–618

Bronnenberg, Bart J., Jun B. Kim, and Carl F. Mela (2016), "Zooming In on Choice: How Do

Consumers Search for Cameras Online?," *Marketing Science*, 35(5), 693–712

Brown, Jeffrey R. and Austan Goolsbee (2002), "Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry," *Journal of Political Economy*, 110(3), 481–507

Brynjolfsson, Erik and Michael D. Smith (2000), "Frictionless Commerce? A Comparison of Internet and Conventional Retailers," *Management Science*, 46(4), 563–585

Cachon, Gérard P., Christian Terwiesch, and Yi Xu (2008), "On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market," *Marketing Science*, 27(3), 461–473

Chen, Yuxin and Song Yao (2017), "Sequential Search with Refinement: Model and Application with Click-Stream Data," *Management Science*, 63(12), 3999–4446

Clemons, Eric K., Il-Horn Hann, and Lorin M. Hitt (2002), "Price Dispersion and Differentiation in Online Travel: An Empirical Investigation," *Management Science*, 48(4), 543–549

De los Santos, Babur I. (2008), "Consumer Search on the Internet," Working Paper.

De los Santos, Babur I., Ali Hortaçsu, and Matthijs R. Wildenbeest (2017), "Search with Learning for Differentiated Products: Evidence from E-Commerce," *Journal of Business & Economic Statistics*, 35(4), 626–641

Dellaert, Benedict G.C. and Gerald HÄubl (2012), "Searching in Choice Mode: Consumer Decision Processes in Product Search with Recommendations," *Journal of Marketing Research*, 49(2), 277–288

Dukes, Anthony and Lin Liu (2016), "Online Shopping Intermediaries: The Strategic Design of Search Environments," *Management Science*, 62(4), 1064–1077

Ellison, Glenn and Sara Ellison (2009), "Search, Obfuscation, and Price Elasticities on the Internet," *Econometrica*, 77(2), 427–452

Ghose, Anindya, Panagiotis Ipeirotis, and Beibei Li (2018), "Modeling Consumer Footprints on Search Engines: An Interplay with Social Media," *Management Science*, 35(3), 1363–1385

Gu, Zheyin (Jane); and Yunchuan Liu (2013), "Consumer Fit Search, Retailer Shelf Layout, and

Channel Interaction," *Marketing Science*, 32(4), 652–668

Hong, Han and Matthew Shum (2006), "Using Price Distributions to Estimate Search Costs," *The RAND Journal of Economics*, 37(2), 257–275

Hortaçsu, Ali and Chad Syverson (2004), "Product Differentiation, Search Costs, and Competition in the Mutual Fund Industry : A Case Study of S&P 500 Index Funds," *The Quarterly Journal of Economics*, 119(2), 403–456

Janssen, Maarten and Sandro Shelegia (2015), "Consumer Search and Double Marginalization," *The American Economic Review*, 105(6), 1683–1710

Jiang, Zhenling, Tat Chan, Hai Che, and Youwei Wang (2017), "Consumer Search and Purchase: An Empirical Investigation of Search-Based Retargeting," Working Paper.

Jolivet, Grégory and Hélène Turon (2017), "Consumer search costs and preferences on the Internet"

Ke, T. Tony, Zuo-jun Max Shen, and J. Miguel Villas-boas (2016), "Search for Information on Multiple Products," *Management Science*, 62(12), 3576–3603

Kim, J. B., P. Albuquerque, and B.J. Bronnenberg (2010), "Online Demand Under Limited Consumer Search," *Marketing Science*, 29(6), 1001–1023

Kim, Jun B., Paulo Albuquerque, and Bart T. Bronnenberg (2011), "Mapping Online Consumer Search," *Journal of Marketing Research*, 48(1), 13–27

Koulayev, Sergei (2014), "Search for Differentiated Products: Identification and Estimation," *The RAND Journal of Economics*, 45(3), 553–575

Kuksov, Dmitri (2004), "Buyer Search Costs and Endogenous Product Design," *Marketing Science*, 23(4), 490–499

Moraga-Gonzaíez, Joseuis and Matthijs R. Wildenbeest (2008), "Maximum Likelihood Estimation of Search Costs," *European Economic Review*, 52(5), 820–848

Moraga-González, José Luis, Zsolt Sándor, and Matthijs R. Wildenbeest (2013), "Semi-Nonparametric Estimation of Consumer Search Cost," *Journal of Applied Econometrics*, 28(7), 1205–1223

PwC (2015), "Total Retail 2015: Retailers and the Age of Disruption"

Salop, Steven and Joseph E. Stiglitz (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *The Review of Economic Studies*, 44(3), 493–510

Santos, B. De Los, Ali Hortacsu, and Matthijs R. Wildenbeest (2012), "Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior," *The American Economic Review*, 102(1983), 2955–2980

Sorensen, Alan T. (2000), "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs," *Journal of Political Economy*, 108(4), 833–850

Stahl, Dale O. (1989), "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review*, 79(4), 700–712

Stigler, George (1961), "The Economics of Information," *Journal of Political Economy*, 69(3), 213–225

Wolinsky, Asher (1986), "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101(3), 493–511

Zhang, Shuo, Zhenling Jiang, and Hai Che (2019), "Online Shopping with Endogenous PC and Mobile Channel Choice," Working Paper.

Zhong, Zemin (2018), "A Model of Search Design," Working Paper.

Footnotes

[1] https://www2.census.gov/retail/releases/historical/ecomm/17q1.pdf

[2] https://www.businessinsider.com/third-party-merchants-drive-amazon-grow-2015-10

[3] Amazon charges independent sellers 6%-20% of their selling prices (depending on product categories), and eBay charges 10% of the selling prices for most categories. The percentage fee is the main component of the fees paid to the platforms for most of sellers. Sometimes, in addition to the percentage referral fees, Amazon also charges a $0.99 per transaction fee for casual sellers with very low sales volume (below 40 units per month). In some situations, for low-value products with very low prices, Amazon simply charges a fixed $0.99 referral fee for the sale rather than the typical percentage fee when that percentage fee will be lower than $0.99.

[4] https://www.amzproductvideo.com/pages/how-it-works

[5] https://www.geekwire.com/2017/augmented-reality-shopping-phone-patent-hints-amazons-aspirations/

[6] https://nichehunt.com/Amazon-augmented-reality-patent-could-reshape-the-future-of-e-commerce/

[7] Different articles present estimated search costs in different formats. We use the median of their estimates whenever possible because the distributions of many estimates have long tails for very high search costs. For articles only reporting the range of the estimated search cost, we use the midpoint of the range. We take natural logarithm to mitigate the impact of extreme values.

[8] Some other empirical research (e.g., Anderson et al. (2010), Bronnenberg et al. (2016), Dellaert and Haubl (2012), Kim et al. (2010), Kim et al. (2011), Zhang et al. (2019)) studies the consumer's search behaviors online but either does not estimate the consumer's search cost or estimates search cost as a function of variables.

[9] https://www.sellerlabs.com/blog/Amazon-policy-update-november-9th-2016-3-changes-selling-media/

[10] An example of the M distribution satisfying that $h_M(M)$ is decreasing on $(-\infty, +\infty)$ is one corresponding to the m_{ij} that follows the standard normal distribution, $K = 2$, $\mu_1 = -1$, $\mu_2 = 1$, and $\phi_1 = \phi_2 = 0.5$.

[11] Without loss of generality, we assume that the consumer with the largest search cost ($\tau = \tau_{max}$) will search on the platform with a positive probability.

[12] The expression of $a(\tau)$ is given in the Online Appendix.

[13] We thank an anonymous reviewer for suggesting this discussion.

Figure 1 log(Estimated Search Cost) vs Dataset Time

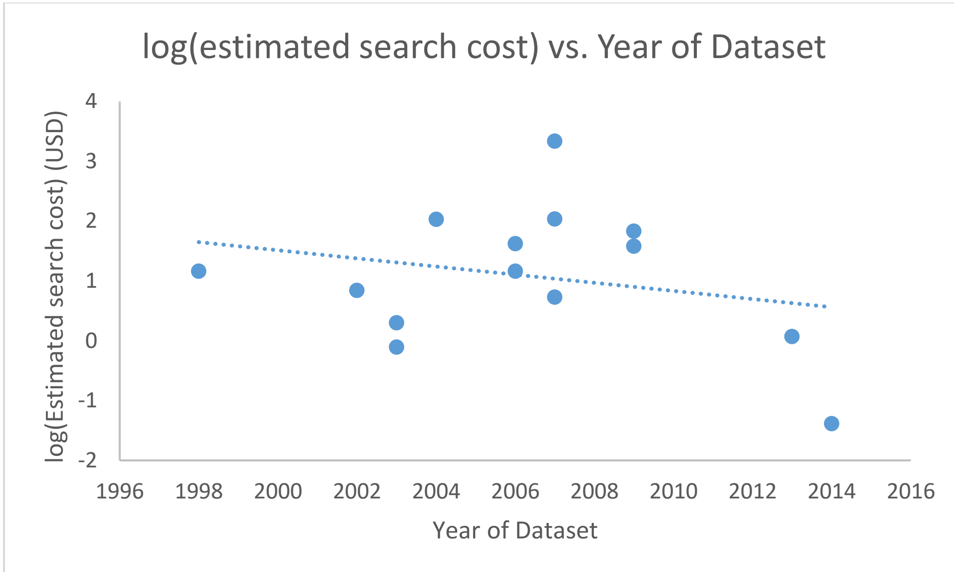


Figure 2 Screenshot of Search Results of “Nike Lunarglide 8” on Amazon.com

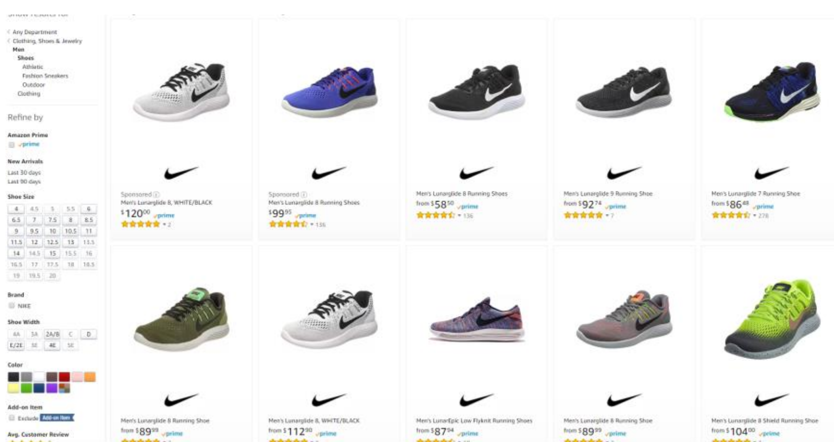


Figure 3 Screenshot of Search Results of "Nike Lunarglide 8" on Amazon.com, Sorted by price (Only Circled Results Are Nike Lunarglide 8)

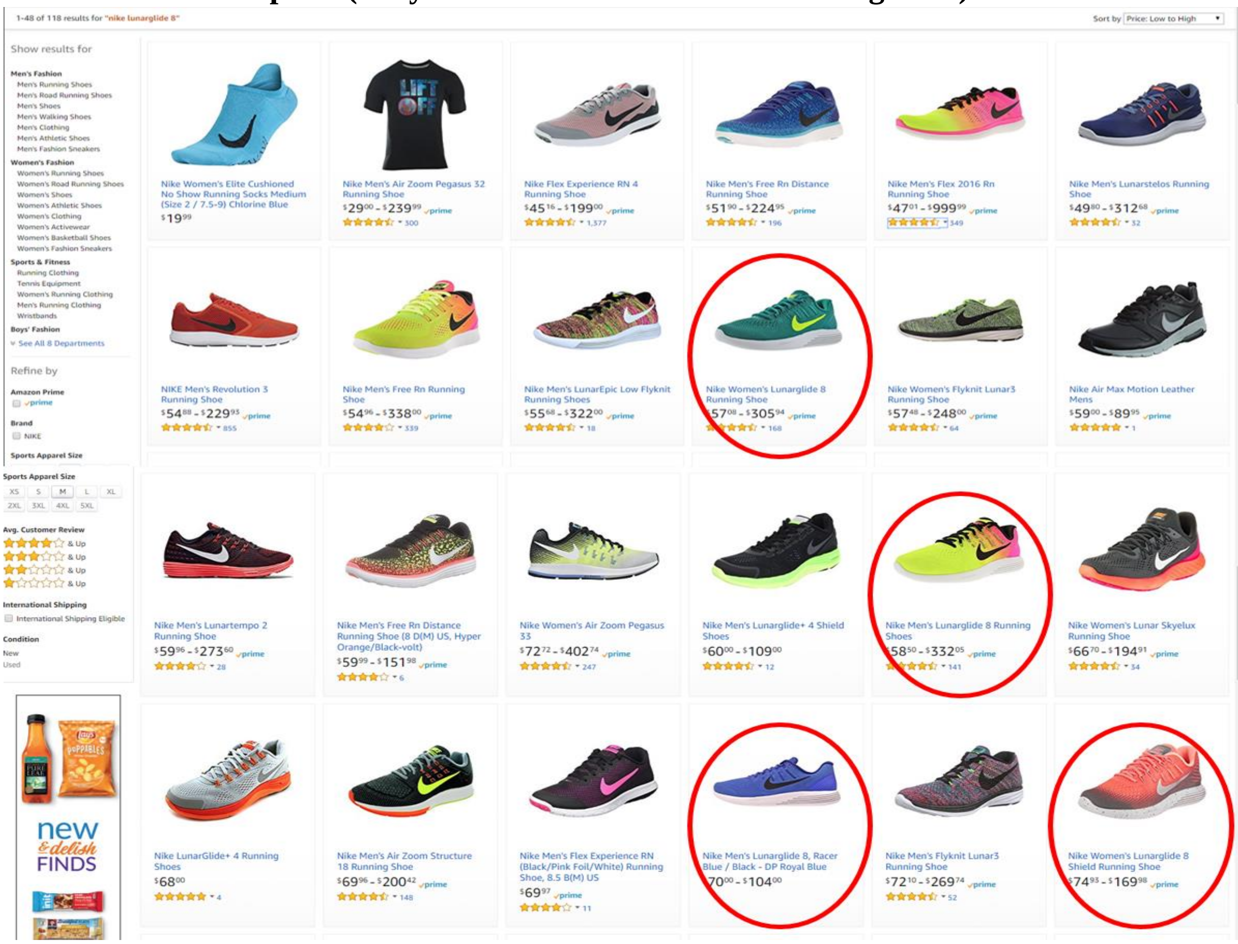
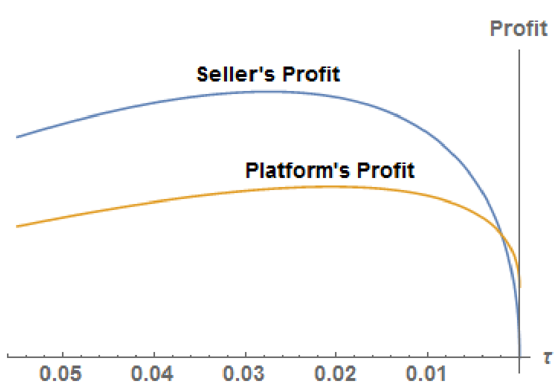
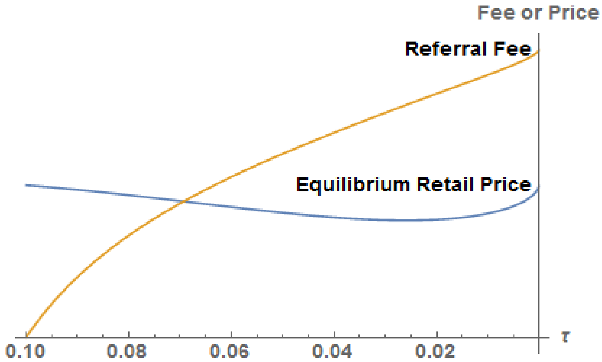


Figure 4 Seller's and Platform's Profits

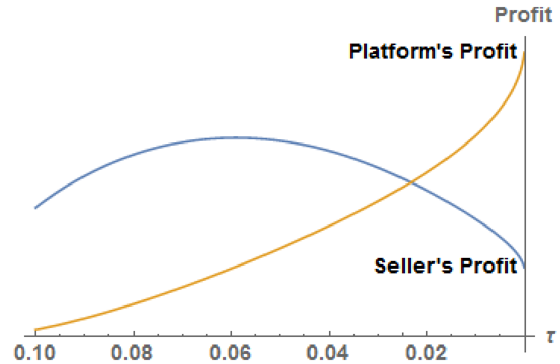


Note. This figure is plotted using $c = 0.05$, and $r = 0.2$. The curves are rescaled to be fit in the same figure.

Figure 5 Numerical Example with Endogenous Referral Fee



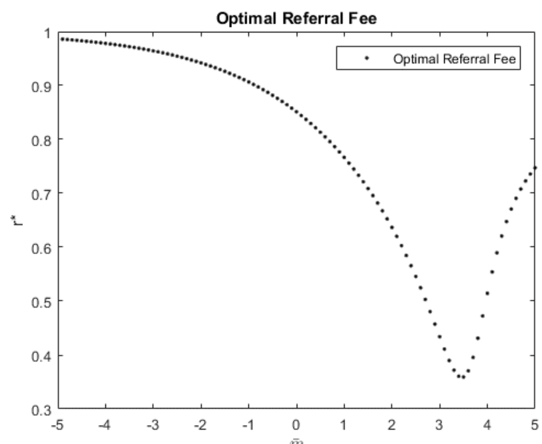
(a) Equilibrium Referral Fee and Retail Price



(b) Seller's and Platform's Profit

Note. This figure is plotted using $c = 0.05$. The curves are rescaled to be fit in the same figure.

Figure A1 Optimal Referral Fee



TECHNICAL APPENDIX

PROOF OF LEMMA 1. Suppose that the referral fee is r and all sellers' retail prices are $p^*(r)$. In equilibrium, seller i will not change its retail price p_i away from $p^*(r)$. Seller i's profit is

$$\pi_i^S(p_i; r) = \frac{F_0(\bar{m} + \mu_K - p^*(r))}{n(1 - F(\bar{m}))} \cdot [1 - F(\bar{m} - p^*(r) + p_i)] \cdot [(1 - r)p_i - c].$$

The first-order condition (FOC) is $\frac{d\pi_i^S(p_i;r)}{dp_i} = \frac{F_0(\bar{m}+\mu_K-p^*(r))}{n(1-F(\bar{m}))} \cdot \{[1 - F(\bar{m} - p^*(r) + p_i)](1 - r) - f(\bar{m} - p^*(r) + p_i) \cdot [(1 - r)p_i - c]\} = 0$, i.e., $p_i = \frac{c}{1-r} + h(\bar{m} - p^*(r) + p_i)$. In a symmetric equilibrium, $p_i = p^*(r)$, therefore, $p^*(r) = \frac{c}{1-r} + h(\bar{m})$. We also need to check the second-order condition, i.e., $\frac{d^2\pi_i^S(p_i;r)}{dp_i^2}|_{p_i=p^*(r)} < 0$. This is equivalent to $-2f(\bar{m}) - f'(\bar{m})h(\bar{m}) < 0$. This is true because $h'(\bar{m}) < 0$ implies $-\frac{f(\bar{m})+h(\bar{m})f'(\bar{m})}{f(\bar{m})} < 0$, which implies $-2f(\bar{m}) - f'(\bar{m})h(\bar{m}) < 0$.

Substituting $p^*(r) = \frac{c}{1-r} + h(\bar{m})$ into expressions of $\tilde{\pi}_i^S$ and $\widetilde{D}$, we have $\tilde{\pi}_i^{S*} = \frac{F_0\left(\bar{m}+\mu_K-\frac{c}{1-r}-h(\bar{m})\right)}{n}(1 - r)h(\bar{m})$ and $\widetilde{D} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right)$. ■

PROOF OF PROPOSITION 1. These results are straightforward because $\frac{\partial \tilde{p}^*}{\partial \tau} = h'(\bar{m}) \cdot \frac{\partial \bar{m}}{\partial \tau} > 0$ and $\frac{\partial \widetilde{D}}{\partial \tau} = f_0\left(\bar{m} + q - \frac{c}{1-r} - h(\bar{m})\right)\left(1 - h'(\bar{m})\right) \cdot \frac{\partial \bar{m}}{\partial \tau} < 0$. ■

PROOF OF PROPOSITION 2. Since $\frac{\partial \bar{m}}{\partial \tau} < 0$, it is sufficient to show $\frac{d\pi^{P*}(\bar{m})}{d\bar{m}} > 0$. $\pi^{P*}(\bar{m}) = \pi^P\left(r^*(\bar{m})\right) = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) \cdot r^*(\bar{m}) \cdot \left(\frac{c}{1-r^*(\bar{m})} + h(\bar{m})\right)$. Because $r^*(\bar{m})$ maximizes the platform's profit π^P, it must follow the FOC that $\frac{\partial \pi^{P*}}{\partial r}|_{r=r^*} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right)\left[\frac{c}{(1-r)^2} + h(\bar{m})\right] - \frac{rc}{(1-r)^2}\left(\frac{c}{1-r} + h(\bar{m})\right)f_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right) = 0$.

When consumer search becomes easier, $\bar{m}$ increases. By the envelope theorem, $\frac{d\pi^{P*}(\bar{m})}{d\bar{m}} = \frac{\partial \pi^P}{\partial \bar{m}}|_{r=r^*(\bar{m})} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) r^*(\bar{m}) h'(\bar{m}) + f_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right)\left(1 - h'(\bar{m})\right) \cdot r^*(\bar{m}) \cdot \left(\frac{c}{1-r^*(\bar{m})} + h(\bar{m})\right) = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) r^*(\bar{m}) h'(\bar{m}) + F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right)\left(1 - h'(\bar{m})\right) \cdot \frac{\frac{c}{(1-r^*(\bar{m}))^2} + h(\bar{m})}{\frac{c}{(1-r^*(\bar{m}))^2}} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) \cdot \left[\left(1 + \frac{h(\bar{m})(1-r^*(\bar{m}))^2}{c}\right) - h'(\bar{m})\left(1 - r^*(\bar{m}) + \frac{h(\bar{m})(1-r^*(\bar{m}))^2}{c}\right)\right] > 0.$ ■

PROOF OF PROPOSITION 3. The platform's profit is $\pi^P = D(r,\bar{m}) \cdot (\frac{cr}{1-r} + rh(\bar{m}))$. The FOC is $\frac{\partial \pi^{P*}}{\partial r}|_{r=r^*(\bar{m})} = 0$ for all $\bar{m}$, thus $0 = \frac{d\frac{\partial \pi^P}{\partial r}|_{r=r^*(\bar{m})}}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r^2}|_{r=r^*(\bar{m})} \cdot \frac{\partial r^*(\bar{m})}{\partial \bar{m}} + \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})}$. Because the second-order condition guarantees $\frac{\partial^2 \pi^P|_{r=r^*(\bar{m})}}{\partial r^2} < 0$, we know $\frac{\partial r^*(\bar{m})}{\partial \bar{m}} > 0$ if and only if $\frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} > 0$. Moreover, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}}{d\bar{m}} = \frac{d\left(\frac{\partial \pi^P}{\partial r} \cdot \frac{r}{\pi^P}\right)}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}} \cdot \frac{r}{\pi^P} + \frac{\partial \pi^P}{\partial r} \cdot \frac{-r \cdot \frac{\partial \pi^P}{\partial \bar{m}}}{(\pi^P)^2}$. Thus $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} \cdot \frac{r^*(\bar{m})}{\pi^P(r^*(\bar{m}))} + 0 = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} \cdot \frac{r^*(\bar{m})}{\pi^P(r^*(\bar{m}))}$. Hence, $\frac{\partial r^*(\bar{m})}{\partial \bar{m}} > 0$ if and only if $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$.

Note that $\frac{\partial ln\pi^P}{\partial lnr} = \epsilon_{D,r}(r,\bar{m}) + \frac{\partial\left(\frac{cr}{1-r} + rh(\bar{m})\right)}{\partial r} \cdot \frac{r}{\frac{cr}{1-r} + rh(\bar{m})} = \epsilon_{D,r}(r,\bar{m}) + \frac{c + h(\bar{m})(1-r)^2}{c + h(\bar{m})(1-r)}$. Thus, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$ if and only if $\frac{\partial \epsilon_{D,r}(r^*,\bar{m})}{\partial \bar{m}} > \frac{ch'(\bar{m})r^*(\bar{m})}{\left(c + h(\bar{m})(1-r^*(\bar{m}))\right)^2}$. Because $\epsilon_{D,r}(r^*,\bar{m}) < 0$, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$ if and only if $\frac{\partial|\epsilon_{D,r}(r^*,\bar{m})|}{\partial \bar{m}} < \frac{-ch'(\bar{m})r^*(\bar{m})}{\left(c + h(\bar{m})(1-r^*(\bar{m}))\right)^2}$. ■

PROOF OF LEMMA 3. Let $G(x) = \int_x^{m_{\max}} (m - x) f(m) dm$. Note that $G(x)$ is strictly decreasing and convex since $G'(x) = F(x) - 1 < 0$ and $G''(x) = f(x) > 0$. When filtering is not available, the consumer's acceptance aggregate match level threshold, $\bar{M}$, satisfies $\int_{\bar{M}_N}^{M_{\max}}(M - \bar{M}_N)f_M(M)dM = \tau$, where $M_{\max} = m_{\max} + \mu_K$. Note that $f(m) = 0$ when $m > m_{\max}$. Note that $f(x) = 0$ when $x > m_{\max}$, so the left-hand-side can be written as:

$\tau = \int_{\bar{M}_N}^{m_{\max}+\mu_K}(M - \bar{M}_N)f_M(M)dM = \sum_{k=1}^{K}\phi_k \int_{\bar{M}_N-\mu_k}^{m_{\max}+(\mu_K-\mu_k)}[m - (\bar{M}_N - \mu_k)]f(m)dm = \sum_{k=1}^{K}\phi_k \int_{\bar{M}_N-\mu_k}^{m_{\max}}[m - (\bar{M}_N - \mu_k)]f(m)dm = \sum_{k=1}^{K}\phi_k G(\bar{M}_N - \mu_k)$.

Because $G''(x) > 0$ and $\sum_{k=1}^{K}\phi_k\mu_k = 0$, $G(\bar{M}_N) = G(\sum_{k=1}^{K}\phi_k(\bar{M}_N - \mu_k)) < \sum_{k=1}^{K}\phi_k G(\bar{M}_N - \mu_k) = \tau$.

When filtering is available, consumers will search products with $m_{ij} = \mu_K$ and will buy a product if and only if $m_{ij} > \bar{m}$, where $\bar{m}$ is determined by $G(\bar{m}) = \int_{\bar{m}}^{m_{\max}}(m - \bar{m})f(m)dm = \tau > G(\bar{M}_N)$. Because $G(x)$ is a strictly decreasing function, $\bar{M}_N > \bar{m}$. Thus, $\bar{M} - \bar{M}_N = \mu_K + \bar{m} - \bar{M}_N < \mu_K$. Moreover, $G(\bar{m}) = \tau = \sum_{k=1}^{K}\phi_k G(\bar{M}_N - \mu_k) < \sum_{k=1}^{K}\phi_k G(\bar{M}_N - \mu_K) = G(\bar{M}_N - \mu_K)$, so $\bar{M}_N - \mu_K < \bar{m}$, i.e., $\bar{M} - \bar{M}_N > 0$. ■

PROOF OF PROPOSITION 4. It is sufficient to show $F(\bar{m}) < F_M(\bar{M}_N) = \mathbf{E}_\mu[F(\bar{M}_N - \mu)]$.

First, note that $G(x) = \int_x^{m_{\max}}\left(1 - F(m)\right)dm$, so $\tau = \int_{\bar{m}}^{m_{\max}}\left(1 - F(m)\right)dm = \mathbf{E}_\mu[\int_{\bar{M}_N-\mu}^{m_{\max}}\left(1 - F(m)\right)dm]$. This implies $\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\left(1 - F(m)\right)dm\right] = 0$.

Second, because $h(x)$ is a decreasing function, we know $(h(m) - h(\bar{m}))\cdot f(m) > 0$ when $m < \bar{m}$, and $(h(m) - h(\bar{m}))\cdot f(m) < 0$ when $m > \bar{m}$. Hence, $\int_x^{\bar{m}}\left(h(m) - h(\bar{m})\right)\cdot f(m)dm > 0, \forall x \neq \bar{m}$.

This implies

$$\begin{aligned}
0 &< \mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\left(h(m) - h(\bar{m})\right)\cdot f(m)dm\right] \\
&= \mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\left(h(m)f(m) - h(\bar{m})f(m)\right)dm\right] \\
&= \mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\left(\left(1 - F(m)\right) - h(\bar{m})f(m)\right)dm\right]
\end{aligned}$$

$$= \mathbf{E}_\mu \left[\int_{\bar{M}_N-\mu}^{\bar{m}} (1 - F(m)) dm\right] - h(\bar{m}) \mathbf{E}_\mu \left[\int_{\bar{M}_N-\mu}^{\bar{m}} f(m) dm\right]$$
$$= 0 - h(\bar{m}) \mathbf{E}_\mu [F(\bar{m}) - F(\bar{M}_N - \mu)].$$

Hence, $\mathbf{E}_\mu [F(\bar{m}) - F(\bar{M}_N - \mu)] < 0$, i.e., $F(\bar{m}) < \mathbf{E}_\mu [F(\bar{M}_N - \mu)] = F_M(\bar{M}_N)$. ■

PROOF OF PROPOSITION 5. Consider the marginal impact of filtering on the sellers' equilibrium price. Let $\max_k |\mu_k| \to 0$. Let $\sigma_\mu^2 = \mathbf{Var}(\mu)$.

First we determine the relationship between $\bar{M}_N$ and $\bar{m}$ using second-order Taylor's expansion. Observe that:

$$\tau = G(\bar{m}) = \mathbf{E}_\mu G(\bar{M}_N - \mu) = \mathbf{E}_\mu G\big(\bar{m} + (\bar{M}_N - \bar{m} - \mu)\big)$$
$$= \mathbf{E}_\mu \left[G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m} - \mu) + \frac{G''(\bar{m})}{2} (\bar{M}_N - \bar{m} - \mu)^2 + o((\bar{M}_N - \bar{m} - \mu)^2)\right]$$
$$= G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{G''(\bar{m})}{2} \left[(\bar{M}_N - \bar{m})^2 + \sigma_\mu^2\right] + o((\bar{M}_N - \bar{m})^2) + o\big(\sigma_\mu^2\big)$$
$$= G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{G''(\bar{m})}{2} \sigma_\mu^2 + o(\bar{M}_N - \bar{m}) + o\big(\sigma_\mu^2\big) .$$

Thus $\bar{M}_N - \bar{m} \cong -\frac{G''(\bar{m})}{G'(\bar{m})} \cdot \frac{\sigma_\mu^2}{2} = \frac{\sigma_\mu^2}{2} \cdot \frac{f(\bar{m})}{1-F(\bar{m})} = \frac{\sigma_\mu^2}{2} \cdot \frac{1}{h(\bar{m})}$

Substituting the above into the expression of $\frac{h(\bar{m})}{h_M(\bar{M})}$, we have

$$\frac{h(\bar{m})}{h_M(\bar{M}_N)} = h(\bar{m}) \cdot \frac{\mathbf{E}_\mu f(\bar{M}_N-\mu)}{\mathbf{E}_\mu[1-F(\bar{M}_N-\mu)]}$$
$$= h(\bar{m}) \cdot \frac{\mathbf{E}_\mu f\big(\bar{m}+(\bar{M}_N-\bar{m}-\mu)\big)}{\mathbf{E}_\mu\big[1-F\big(\bar{m}+(\bar{M}_N-\bar{m}-\mu)\big)\big]}$$
$$= h(\bar{m}) \cdot \frac{\mathbf{E}_\mu\left[f(\bar{m})+f'(\bar{m})(\bar{M}_N-\bar{m}-\mu)+\frac{f''(\bar{m})}{2}(\bar{M}_N-\bar{m}-\mu)^2+o(\bar{M}_N-\bar{m})+o\big(\sigma_\mu^2\big)\right]}{\mathbf{E}_\mu\left[\big(1-F(\bar{m})\big)-f(\bar{m})(\bar{M}_N-\bar{m}-\mu)-\frac{f'(\bar{m})}{2}(\bar{M}_N-\bar{m}-\mu)^2+o(\bar{M}_N-\bar{m})+o\big(\sigma_\mu^2\big)\right]}$$
$$\cong \frac{1-F(\bar{m})}{f(\bar{m})} \cdot \frac{f(\bar{m})+f'(\bar{m})(\bar{M}_N-\bar{m})+\frac{f''(\bar{m})}{2}\sigma_\mu^2}{\big(1-F(\bar{m})\big)-f(\bar{m})(\bar{M}_N-\bar{m})-\frac{f'(\bar{m})}{2}\sigma_\mu^2}$$
$$= \frac{1+\frac{f'(\bar{m})}{f(\bar{m})}\cdot\frac{\sigma_\mu^2}{2}\cdot\frac{f(\bar{m})}{1-F(\bar{m})}+\frac{f''(\bar{m})}{f(\bar{m})}\cdot\frac{\sigma_\mu^2}{2}}{1-\frac{f(\bar{m})}{1-F(\bar{m})}\cdot\frac{\sigma_\mu^2}{2}\cdot\frac{f(\bar{m})}{1-F(\bar{m})}-\frac{f'(\bar{m})}{1-F(\bar{m})}\cdot\frac{\sigma_\mu^2}{2}}$$
$$\cong 1 + \frac{\sigma_\mu^2}{2}\left[\frac{f''(\bar{m})}{f(\bar{m})} + \frac{f(\bar{m})}{1-F(\bar{m})} \cdot \frac{f(\bar{m})}{1-F(\bar{m})} + \frac{2f'(\bar{m})}{1-F(\bar{m})}\right]$$

The above expression is greater than 1 if and only if $f''(\bar{m}) \cdot h^2(\bar{m}) + 2f'(\bar{m}) \cdot h(\bar{m}) + f(\bar{m}) > 0$. ■

PROOF OF RESULT 3. According to Lemma 4, the consumers will search the non-premium products if and only if $u_0 < -p^*(r) + \bar{m}$ and $m_{1j} < p_1 - p^*(r) - \Delta q + \bar{m}$. Following Wolinsky (1986), the profit of a non-premium seller's profit is $\pi_i^S(p_i) = F_0(\bar{m} - p^*(r)) \cdot F(p_1 - p^*(r) - \Delta q + \bar{m}) \cdot \frac{[1-F(\bar{m}-p^*(r)+p_i)]\cdot[(1-r)p_i-c]}{n-1}$. Note that the first two terms are positive and independent of p_i, so the optimal p_i should satisfy $\frac{\partial[1-F(\bar{m}-p^*(r)+p_i)]\cdot[(1-r)p_i-c]}{\partial p_i} = 0$. The FOC implies $p^*(r) = \frac{c}{1-r} + h(\bar{m})$. ■

PROOF OF PROPOSITION 6. A seller's profit as a function of its price p_i is given by $\pi_i = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-p^*(r))}{n(1-F(\bar{m}(\tau)))}[1 - F(\bar{m}(\tau) - p^* + p_i)][(1-r)p_i - c]\right]$.

The seller's optimal price $\tilde{p}^*$ satisfies the FOC, i.e., $\frac{\partial \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{n\left(1-F(\bar{m}(\tau))\right)}[1-F(\bar{m}(\tau)-\tilde{p}^*+p_i)][(1-r)p_i-c]\right]}{\partial p} = 0$ when $p_i = \tilde{p}^*$. Rearranging terms, we get $0 = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{\frac{\left(1-F(\bar{m}(\tau))\right)}{f(\bar{m}(\tau))}}(\tilde{p}^* - \tilde{p}_\tau^*)\right] = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}(\tilde{p}^* - \tilde{p}_\tau^*)\right]$. Let $a(\tau) = \frac{\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}}{\mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}\right]}$, which is a strict decreasing function of τ, because $F_0(\bar{m} + \mu_K - p^*(r))$ increases with $\bar{m}$, $h(\bar{m})$ decreases with $\bar{m}$, and $\bar{m}$ decreases with τ. Note $\mathbf{E}_\tau[a(\tau)] = 1$, so $\tilde{p}^* = \mathbf{E}_\tau[a(\tau)\tilde{p}_\tau^*]$. Note that $\tilde{p}_\tau^*$ strictly increases with τ, so $\tilde{p}^* = \mathbf{E}_\tau[a(\tau)\tilde{p}_\tau^*] = \mathbf{E}_\tau[a(\tau)]\mathbf{E}_\tau[\tilde{p}_\tau^*] + \mathbf{Cov}_\tau(a(\tau), \tilde{p}_\tau^*) < \mathbf{E}_\tau[\tilde{p}_\tau^*]$. ■

HETEROGENEOUS SEARCH COST: SPECIAL CASE. Consider the case with $\tau \sim \text{Uniform}(0, \tau_{\max})$, where $\tau_{\max} < \frac{1}{8}\left(1 - \frac{c}{1-r}\right)^2$. The FOC can be simplified as $\int_0^{\tau_{\max}} \frac{1}{\tau_{\max}} \cdot \frac{1-\sqrt{2\tau}-\tilde{p}^*}{\sqrt{2\tau}}\left(\tilde{p}^* - \frac{c}{1-r} - \sqrt{2\tau}\right) d\tau = 0$. Let $s = \sqrt{2\tau}$, so $\int_0^{\sqrt{2\tau_{\max}}}(1 - s - \tilde{p}^*)\left(\tilde{p}^* - \frac{c}{1-r} - s\right) ds = 0$. One can derive that $\tilde{p}^* = \frac{1-\frac{c}{1-r}}{2} - \sqrt{\frac{1}{3}\left(\sqrt{2\tau_{\max}} - \frac{3}{2}\frac{1-\frac{c}{1-r}}{2}\right)^2 + \left(\frac{1-\frac{c}{1-r}}{2}\right)^2}$, which decreases with $\tau_{\max}$. ■

FIXED REFERRAL FEE. Seller i 's profit is given by $\pi_i^S(p_i; d) = \frac{F_0(\bar{m}+\mu_K-p^*(d))}{n(1-F(\bar{m}))} \cdot [1 - F(\bar{m} - p^*(d) + p_i)] \cdot [p_i - d - c]$.

The equilibrium retail price is $\tilde{p}^*(d) = d + c + h(\bar{m})$, the seller's equilibrium profit is $\tilde{\pi}_i^{S*} = \frac{F_0(\bar{m}+\mu_K-d-c-h(\bar{m}))}{n} h(\bar{m})$. The total demand on the retail platform is $\widetilde{D} = F_0\big(\bar{m} + \mu_K - d - c - h(\bar{m})\big)$. The platform's profit is $\tilde{\pi}^{P*} = F_0\big(\bar{m} + \mu_K - d - c - h(\bar{m})\big) \cdot d$.

If the platform endogenously chooses its referral fee, the platform will earn a strictly higher profit when it charges a percentage referral fee than when it charges a fixed per-unit referral fee. The proof is below.

Suppose that in the case with a fixed per-unit referral fee, the platform's optimal referral fee is d^*, so its profit is $F_0\big(\bar{m} + \mu_K - d^* - c - h(\bar{m})\big) \cdot d^*$. Let us consider the case with a percentage referral fee and set the referral fee to $r = \frac{d^*}{c+d^*}$. The platform's profit is given by $r \cdot \left(\frac{c}{1-r} + h(\bar{m})\right) F_0 \left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right) = F_0\big(\bar{m} + \mu_K - d^* - c - h(\bar{m})\big) \cdot d^* \cdot \frac{c+d^*+h(\bar{m})}{c+d^*} > F_0\big(\bar{m} + \mu_K - d^* - c - h(\bar{m})\big) \cdot d^*$. Thus, the platform's profit under the optimal percentage referral fee is strictly higher than its profit under the optimal fixed per-unit referral fee.