Team Incentives with Non-Verifiable Performance Measures: A Multi-period Foundation for Bonus Pools

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Abstract

A common means of incorporating non-verifiable performance measures in compensation contracts is via bonus pools. We study a prinicpal-multi-agent relational contracting model in which the optimal contract resembles a bonus pool. It specifies a minimum joint bonus the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to increase the size of the pool and to allocate the pool to the agents. The joint bonus floor is useful because of its role in motivating the agents to mutually monitor each other (team incentives). Even when incentive schemes without positive bonus floors could be used to provide team incentives, contracts with positive bonus floors can be less costly because they do a better job of creating strategic complementarity in the agents' payoffs, which is a desirable property of incentive schemes designed to motivate mutual monitoring. When team incentives are not optimal, the contract must be collusion proof. Even when contracts without positive bonus floors would prevent tacit collusion, contacts with joint bonus floors can be less costly because they facilitate strategic independence in the agents' payoffs, which is a desirable property of collusion proof incentives.

1 Introduction

This paper studies discretionary rewards based on non-verifiable performance measures. A concern about discretionary rewards is that the evaluator must be trusted by the evaluates (Anthony and Govindarajan, 1998). In a single-period model, bonus pools are a natural economic solution to the "trust" problem (MacLeod, 2003; Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009). We study a multi-period, prinicpal-multi-agent relational contracting model in which the optimal contract resembles a bonus pool. It specifies a minimum joint bonus the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to increase the size of the pool and to allocate the pool to the agents. Such discretion is common in practice. Murphy and Oyer (2003) find that 42% of their sample of 262 firms gave the compensation committee discretion in determining the size of the executive bonus pool, while 70% had discretion in allocating the bonus pool to individual executives.

The joint bonus floor is useful because of its role in motivating the agents to mutually monitor each other (team incentives). Even when incentive schemes without positive bonus floors could be used to provide team incentives, contracts with positive bonus floors can be less costly because they do a better job of creating strategic complementarity in the agents' payoffs, which is a desirable property of incentive schemes designed to motivate mutual monitoring. When team incentives are not optimal, the contract must be collusion proof. Even when contracts without positive bonus floors would prevent tacit collusion, contacts with joint bonus floors can be less costly because they facilitate strategic independence in the agents' payoffs, which is a desirable property of collusion proof incentives.

The demand for mutual monitoring using implicit (relational) contracts in our model is the same as in Arya, Fellingham, and Glover (1997) and particularly Che and Yoo (2001).[1] The

[1]The difference between explicit and implicit contracts is that the former are enforced by the courts, while the latter must be self-enforcing. There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh's (1993) model of explicit side-contracting can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

agents work closely enough that they observe each other's actions, while the principal observes only individual performance measures that imperfectly capture those actions. The principal can relax the agents' Nash incentive constraints, which are based on the performance measures, by instead using those performance measures to set the stage for the agents to mutually monitor each other. The principal need ensure only that, from the agents' perspective, both working is preferred to both shirking (a group incentive constraint) and that the punishment of playing the stage equilibrium of shirking in all future periods is larger than the one-time benefit of free-riding. The difference between our paper and Che and Yoo (2001) is that, since the performance measures are *non-verifiable* in our model, the principal too has to rely on a relational contract.

The non-verifiability of the performance measures does not entirely rule out explicit contracts. In particular, the principal can specify a joint bonus floor that does not depend on the non-verifiable performance measures. In a single period version of our model, the principal's ability to make promises is so limited that she would renege on any promise to pay more than the minimum, so the joint bonus floor becomes a simple bonus pool arrangement (as in Rajan and Reichelstein 2006; 2009). Under repeated play, relational contracts allow the principal to use discretion not only in allocating the bonus pool between the agents but also to increase its size above the joint bonus floor.

In our model, all players share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important.

When the principal's ability to commit is strong, the optimal contract emphasizes team incentives. Joint performance evaluation (*JPE*) emerges as an optimal means of setting the stage for the agents to mutually monitor each other. *JPE* provides the agents with incentives to monitor each other because their payoffs are intertwined and a means of disciplining each other by creating a punishing equilibrium (a stage game equilibrium with lower payoffs than the agents obtain on the equilibrium path). *JPE* creates a strategic complementarity in the agents' payoffs. The stronger the complementarity the better, since the benefit of free-riding is decreasing in the complementarity. As the principal's reneging constraint becomes a binding constraint, rewarding (joint) poor performance via the bonus floor can be a feature of the optimal compensation arrangement. The role of rewarding poor performance is that it enables the principal to keep incentives focused on mutual monitoring by using *JPE*. The alternative is to use relative performance evaluation (*RPE)* to partially replace mutual monitoring incentives with individual incentives. Here, the potential benefit of *RPE* is not in removing noise from the agents' performance evaluation as in Holmstrom (1979) but rather in relaxing the principal's reneging constraint, since *RPE* has the principal making payments to only one of the two agents. The problem with *RPE* is that it undermines mutual monitoring by reducing the strategic complementarity in the agents' payoffs. As a result, if the principal attempts to replace team incentives with individual incentives using *RPE*, even more individual incentives are needed to makeup for the reduced team incentives. When the agents' ability to commit is relatively strong, the benefit of mutual monitoring is so large that relying entirely on mutual monitoring for incentives is optimal, even when it requires rewarding poor performance. When the agents' ability to commit is instead relatively weak, mutual monitoring's advantage over individual incentives is small, and substituting individual incentives using *RPE* is optimal.

When individual rather than team incentives are optimal, the principal would use *RPE* if she did not have to prevent tacit collusion between the agents. The unappealing feature of *RPE* is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has them alternating between (*work, shirk*) and (*shirk, work*).[2] Paying for poor performance and the agents' payoff strategic independence it facilitates is optimal when the agents' ability to commit is relatively strong and, hence, the cost of preventing collusion is relatively large. When the agents' ability to commit is instead relatively weak, *RPE* is optimal since the cost of preventing collusion is relatively small.

The relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on subjective/non-verifiable performance

[2]Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

measures (e.g., Baker, Gibbons, and Murphy, 1994), but this literature has mostly confined attention to single-agent settings.[3] In this paper, we explore optimal discretionary rewards based on subjective/non-verifiable individual performance measures in a multi-period, principal-multi-agent model. The multi-period relationship creates the possibility of trust between the principal and the agents, since the agents can punish the principal for reneging behavior. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, opportunities for both team incentives/mutual monitoring and collusion between the agents.

Kvaloy and Olsen (2006) also study team incentives in a relational contracting setting. The most important difference between our paper and theirs is that they do not allow for a commitment to a joint bonus floor, which is the focus of our paper. While the principal cannot write a formal contract on the non-verifiable performance measures, it seems difficult to rule out contracts that specify a joint bonus floor. After all, this idea is at the heart of bonus pools. Kvaloy and Olsen also restrict attention to stationary strategies for the agents, while we allow for arbitrary history-dependent strategies.

Our paper is also closely related to Baldenius, Glover, and Xue (2016). In their model, (i) the principal perfectly observes the agents' actions and (ii) there is a verifiable joint performance measure (e.g., firm-wide earnings) that can be explicitly contracted on. Because the principal observes the agents' actions, there is no role for mutual monitoring/team incentives in their model. When mutual monitoring is not optimal, our results are similar to theirs in that strategic independence emerges as an optimal response to collusion.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies implicit side-contracting between the agents. Section 4 specifies the principal's optimization problem and characterizes the optimal contract. Section 5 studies the less demanding collusion constraints used in Kvaloy and Olsen (2006), which impose stationarity (no history dependence). Our results on team on incentive are unchanged, while strategic independence

[3]One exception is Levin (2002), which examines the role multilateral contracting can have in bolstering the principal's ability to commit if the principal's reneging on a promise to any one agent means she will loose the trust of both agents. We make the same assumption. Levin does not study relational contracting between the agents.

no longer plays such a central role in the collusion setting. Section 6 concludes.

2 Model

A principal contracts with two identical agents, $i = A, B$, to perform two independent and ex ante identical projects (one project for each agent) in an infinitely repeated relationship, where t is used to denote the period, $t = 1, 2, 3, \ldots$. Each agent chooses a personally costly effort $e^i \in \{0, 1\}$ in period t, i.e., the agent chooses either "*work*" ($e^i_t = 1$) or "*shirk*" ($e^i_t = 0$). Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to 1. Agent i's performance measure in period t, denoted x^i_t, is assumed to be either high ($x^i_t = H > 0$) or low ($x^i_t = L = 0$) and is a (stochastic) function of only e^i_t. In particular, $q_1 = \Pr(x^i = H|e^i = 1)$, $q_0 = \Pr(x^i = H|e^i = 0)$, and $0 < q_0 < q_1 < 1$. (Whenever it does not cause confusion, we drop sub- and superscripts.) The performance measures are individual rather than joint measures in the sense that each agent's effort does not affect the other agent's probability of producing a high outcome. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e^i_t = 1$) in every period. (Sufficient conditions are provided in the appendix.) The principal's problem is to design a contract that motives both agents to work in every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort in each period. As in Che and Yoo (2001), we assume that the communication from the agents to the principal is blocked and, therefore, the outcome pair (x^i, x^j) is the only signal on which the agents' wage payment can depend. As in Kvaløy and Olsen (2006), we assume the performance measures (x^i, x^j) are unverifiable. Therefore, the principal cannot directly incorporate the performance measures into an explicit contract. Instead, the performance measures can be used in determining compensation only via an *implicit* contract. The implicit contract, which can be interpreted as an oral agreement between the principal and agents, must be self-enforcing.

Unlike Kvaløy and Olsen (2006), we assume the parties can write an *explicit* contract as long as the explicit contract does not depend on the realized non-verifiable performance measures

(x^i, x^j). In particular, the principal can commit to a *bonus floor* $\underline{w}$ – a minimum *total* bonus to paid to both agents independent of the non-verifiable performance measures. It seems difficult to rule out such explicit contracts, since they require only that a court be able to verify whether or not the contractual minimum, $\underline{w} \geq 0$, was paid out. Throughout the paper, we present as benchmarks companion results that assume the principal cannot commit to a joint bonus floor. These benchmarks essentially reproduce the results of Kvaløy and Olsen (2006).[4]

Denote by w^i_{mn} the wage agent i expects to receive according to the *implicit* contract if his outcome is $x^i = m$ and his peer's outcome is $x^j = n$, with $m, n \in \{H, L\}$. For tractability, we assume the ex ante identical agents are offered the same wage schemes, i.e., $w^i_{mn} = w^j_{nm}$, $i \neq j$. As a result, we can drop the agent superscript and denote by $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ the implicit contract the principal promises the agents.[5] The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

$$w_{mn} \geq 0, \forall m, n \in \{H, L\}. \qquad \text{(Non-negativity)}$$

We assume each agent's best outside opportunity provides him with a payoff of 0 in each period. Therefore, any contract that satisfies the limited liability constraints will also satisfy the agents' individual rationality constraints, since the cost of low effort is zero. The individual rationality constraints are suppressed throughout our analysis.

Given the implicit contract $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$, denote by $\pi(k, l; \mathbf{w})$ the expected wage payment of agent i if he chooses an effort level $k \in \{1, 0\}$ while the other agent j chooses effort $l \in \{1, 0\}$ (assuming the principal honors the implicit contract):

$$\pi(k, l; \mathbf{w}) = q_k q_l w_{HH} + q_k(1 - q_l)w_{HL} + (1 - q_k)q_l w_{LH} + (1 - q_k)(1 - q_l)w_{LL}.$$

[4]We say "essentially" because they restrict attention to stationary strategies, while we do not.

[5]Confining attention to symmetric contracts is a restrictive assumption in that asymmetric contracts can be preferred by the principal, as in Demski and Sappington's (1984) single period model. As we will show in Section 4, restricting attention to symmetric contracts greatly simplifies our infinitely repeated contracting problem by reducing the infinite set of possibly binding collusion constraints to two. Without the restriction to symmetric contracts, we know of no way to simplify the set of collusion constraints into a tractable programming problem.

All parties in the model are risk neutral and share a common discount rate r, capturing the time value of money or the probability the relationship will end at each period (the contracting horizon). Denote by H_t the history of all actions and outcomes before period t. Denote by P_t the public profile before period t — the history without the agents' actions. The principal's strategy is a mapping from the public profile P_t to period t wages. Each agent's strategy maps the entire history H_t to his period t effort choice. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Among the large set of PBE's, we choose the one that is best for the principal subject to collusion-proofness. To be collusion-proof, there can be no other PBE that has only the agents changing their strategies and provides each agent with a higher payoff (in a Pareto sense) than their equilibrium payoff.

Because the production technology is stationary and that the principal always induces high efforts in each period (a stationary effort policy), we know from Theorem 2 in Levin (2003) that the same contract will be offered in each period. That is, the principal can do no better than to offer the same $\mathbf{w}$ and $\underline{w}$ in each period unless she has reneged on the promised wages in some previous period.[6]

To motivate the principal to honor her implicit contract with the agents, we consider the following trigger strategy played by the agents: both agents behave as if the principal will honor the implicit contract until the principal reneges, after which the agents punish the principal by choosing (*shirk, shirk*) in all future periods. Given the bonus floor, this punishment is the severest punishment the agents can impose on the principal. The principal will not renege if:

$$\frac{2\left[q_1 H - \pi(1,1;\mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n}\{w_{mn} + w_{nm} - \underline{w}\}. \qquad \text{(Principal's IC)}$$

[6]The basic idea is that the agent's incentives come from two sources: performance measure contingent payments in the current period and a (possible) change in continuation payoffs that depend on the current period's performance measures. Because of the limited liability constraints, even the lowest of these continuation payoffs must be nonnegative. Since all parties are risk neutral and have the same discount rate, any credible promise the principal makes to condition continuation payoffs on current period performance measures can be converted into a change in current payments that replicates the variation in continuation payoffs without affecting the principal's incentives to renege on the promise or violating the limited liability constraints on payments. The new continuation payoffs are functions of future actions and outcomes only, removing the history dependence. The new contract is a short-term one. If instead the effort level to be motivated is not stationary, then, of course, non-stationary contracts can be optimal.

This constraint assures the principal will abide by the promised wage scheme rather than payout the minimum joint bonus. The left hand side is the cost of reneging, which is the present value of the production loss in all future periods net of wage payment.[7] The agents choosing (*shirk, shirk*) and the principal paying zero to each agent is a stage game equilibrium. Therefore, the agents' threat is credible. The right hand side of this constraint is the principal's benefit of paying out only the minimum joint bonus. If instead a bonus floor is not allowed, the right hand of the principal's incentive constraint becomes $\max\{w_{mn} + w_{nm} - 0\}$, i.e., the principal can always breach her promise and pay zero to both agents.

3 Implicit Contracting between the Agents

The fact that agents observe each other's effort choice, together with their multi-period relationship, gives rise to the possibility that they use implicit contracts to motivate each other to work (mutual monitoring) as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). Consider the following trigger strategy used to enforce (*work, work*): both agents play work until one agent i deviates by choosing shirk; thereafter, the agents play (*shirk, shirk*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w})-1\right] \geq \pi(0,1;\mathbf{w}) + \frac{1}{r}\pi(0,0;\mathbf{w}). \qquad \text{(Mutual Monitoring)}$$

Such mutual monitoring requires two conditions. First, each agent's expected payoff from playing (*work, work*) must be at least as high as from playing the punishment strategy (*shirk, shirk*). In other words, (*work, work*) must Pareto dominate the punishment strategy from the agents' point of view in the stage game because, otherwise, (*shirk, shirk*) will not be a punishment at all:

$$\pi(1,1;\mathbf{w}) - 1 \geq \pi(0,0;\mathbf{w}). \qquad \text{(Pareto Dominance)}$$

[7] If she reneges on her promise, the principal knows the agents will retaliate with *(shirk, shirk)* in all future periods. That is, the agents revert to the stage game equilibrium. As in Baker, Gibbons, and Murphy (1994) and Kvaløy and Olsen (2006), we assume that, once the relational contract has been violated (by an observable action), the employment relationship reverts to spot employment–the agents shirk in all future periods, and the principal offers them a fixed salary of zero.

Second, the punishment (*shirk, shirk*) must be self-enforcing. That is, the punishment strategy (*shirk, shirk*) is a stage game Nash equilibrium:

$$\pi(0,0;\mathbf{w}) \geq \pi(1,0;\mathbf{w}) - 1. \qquad \text{(Self-Enforcing Shirk)}$$

When motivating mutual monitoring is not optimal for the principal, the principal must instead ensure that (*work, work*) is a stage game Nash equilibrium.

$$\pi(1,1;\mathbf{w}) - 1 \geq \pi(0,1;\mathbf{w}). \qquad \text{(Static NE)}$$

However, the Nash constraint may not be sufficient to motivate the agents to act as the principal intends. The agents may find other implicit contracting desirable–that is, they may tacitly collude against the principal. A collusion strategy is an implicit contract between agents that is harmful for the principal. In the case of collusion, the trigger strategy to support it will be of the form: each agent sticks to the collusion strategy until any agent deviates. After any deviation, the agents revert to (*work, work*) in all future periods. This threat is credible, as (*work, work*) is a Nash equilibrium of the stage game. The punishment is also without loss of generality because, as the following lemma shows, playing *(work,work)* is the harshest punishment the agents can impose on each other. Lemma 1 (and Lemma 2) are borrowed from Baldenius, Glover, and Xue (2016) but are repeated here for completeness.

Lemma 1 *Suppose (Static NE) holds as a strict inequality for some symmetric grand contract. Then the harshest off-equilibrium punishment is for the agents to play $(work, work)$ forever.*

Proof. All proofs are provided in an appendix. ■

The above lemma, which implies $\pi(1,1;\mathbf{w}) - 1 \leq \pi(0,0;\mathbf{w})$, does not contradict the Pareto Dominance constraint because the two constraints are invoked under mutually exclusive scenarios. In particular, the Pareto Dominance constraint is imposed if and only if the principal intends to us mutual monitoring between the agents.[8]

[8]Mathematically, we formulate an integer program to solve for the optimal solution. *Pareto Dominance* is imposed when the principal intends to utilize mutual monitoring between the two agents, i.e., the integer indicator $T = 1$; while $\pi(1,1) - 1 \leq \pi(0,0)$ is used for $T = 0$.

Given the infinitely repeated relationship, the space of potential collusions between the two agents is very rich. As it turns out, we can confine attention to two specific intuitive collusive strategies when constructing collusion-proof contracts. If we prevent these two types of collusion, all other possible collusion strategies are also upset.

First, the contract has to satisfy the following condition to prevent collusion on (*shirk, shirk*) in all periods:

$$\pi(1,0;\mathbf{w}) - 1 + \frac{\pi(1,1;\mathbf{w}) - 1}{r} \geq \frac{1+r}{r}\pi(0,0;\mathbf{w}). \qquad \text{(No Joint Shirking)}$$

The left-hand side is the agent's expected payoff from unilaterally deviating from (*shirk, shirk*), or "*Joint Shirking*," for one period by unilaterally choosing work and then being punished indefinitely by the other agent by playing the stage game equilibrium (*work, work*) in all future periods, while the right-hand side is his expected payoff from sticking to *Joint Shirking*.

Second, the following condition is needed to prevent agents from colluding by "*Cycling*," i.e., alternating between (*shirk, work*) and (*work, shirk*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w}) - 1\right] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;\mathbf{w}) + \frac{(1+r)}{r(2+r)}\left[\pi(1,0;\mathbf{w}) - 1\right]. \qquad \text{(No Cycling)}$$

The left hand side is the agent's expected payoff if he unilaterally deviates by choosing work when he is supposed to shirk and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff if the agent instead sticks to the *Cycling* strategy.

Lemma 2 *A contract is collusion-proof if it satisfies No Joint Shirking and No Cycling conditions.*

The intuition for Lemma 2 is that all other potential collusive strategies can only provide some period t' shirker with a higher continuation payoff than under *Joint Shirking* or *Cycling* if some other period t'' shirker has a lower continuation payoff than under *Joint Shirking* or *Cycling*. Hence, if the contract motivates all potential shirkers under *Joint Shirking* and *Cycling* to instead deviate to work, then so will the period t'' shirker under the alternative strategy.

It is helpful to distinguish three classes of contracts and point out how they are related to the two collusion-proof conditions above. The wage contract creates a *strategic complementarity* if $\pi(1,1) - \pi(0,1) > \pi(1,0) - \pi(0,0)$, which is equivalent to a payment complementarity $w_{HH} - w_{LH} > w_{HL} - w_{LL}$. Similarly, the contract creates a *strategic substitutability* if $\pi(1,1) - \pi(0,1) < \pi(1,0) - \pi(0,0)$, or equivalently $w_{HH} - w_{LH} < w_{HL} - w_{LL}$. The contract creates *strategic independence* if $\pi(1,1) - \pi(0,1) = \pi(1,0) - \pi(0,0)$, or $w_{HH} - w_{LH} = w_{HL} - w_{LL}$. The reason a payoff and payment complementarity (substitutability) are equivalent is that the performance measures are uncorrelated. As noted in the following observation, this classification has implications for which collusion strategy is the most costly for the principal to upset.

Observation

(1) *No Joint Shirking* implies *No Cycling* if the contract creates a strategic complementarity in the agents' payoffs.

(2) *No Cycling* implies *No Joint Shirking* if the contract creates a strategic substitutability in the agents' payoffs.

(3) *No Cycling* and *No Joint Shirking* are equivalent if the contract creates a strategic independence in the agents' payoffs.

Whether a contract exhibits a strategic complementarity or a strategic substitutability is endogenous and has an important effect on the nature of the agent-agent collusion strategy. Investigating when and why the principal purposely designs the contract to exhibit strategic complementarity, substitutability, or independence is a key element of the analysis of the remainder of the paper.

Although we do not confine attention to particular wage schemes, some (but not all) of the wage schemes that emerge as optimal in our model have been studied before. A wage scheme exhibits individual performance evaluation (*IPE*) if $w_{HH} = w_{HL}$ and $w_{LH} = w_{LL}$, relative performance evaluation (*RPE*) if $w_{HL} \geq w_{HH}$ and $w_{LL} \geq w_{LH}$ (with one $>$), and joint performance evaluation (*JPE*) if $w_{HH} \geq w_{HL}$ and $w_{LH} \geq w_{LL}$ (with one $>$). *JPE* contracts create strategic complementarity in the agents' payoffs, while *RPE* contracts create strategic substitutability and *IPE* contracts create strategic independence. We will say that the contract has a bonus pool (*BP*)-type feature if it specifies a positive total bonus floor, i.e., $\underline{w} > 0$. Our *BP*-type contracts can be thought of as discretionary bonus pools that allow the principal to pay the agents more than the contractually agreed upon minimum $\underline{w}$. There will always be some room for such discretion in our model, since the model is a dynamic one.

4 The Principal's Problem

The principal designs an *explicit* contract that specifies only a joint bonus floor $\underline{w}$ (i.e., a minimum total payment) and an *implicit* contract that specifies self-enforcing promises $\mathbf{w} = \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ that ensure (*work, work*) in every period is the equilibrium-path behavior of a collusion-proof equilibrium. If the principal breaches the explicit contract to pay a total bonus of at least $\underline{w}$, the agents can take the principal to court to insist that she honor the explicit contract. The implicit promises $\mathbf{w}$ must satisfy the *Principal's IC* constraint in order to make the promise self-enforcing, since the courts cannot be used to enforce the implicit contract. When designing the optimal contract, the principal can choose to motivate mutual monitoring between agents if it is worthwhile. Alternatively, she can implement a static Nash equilibrium subject to collusion proof constraints. The following integer program summarizes the principal's problem.

$$\textbf{Program P:} \quad \min_{T\in\{0,1\},\underline{w}\geq 0,w_{mn}\geq 0} \pi(1,1)$$

$$\begin{aligned} & s.t. \\ & \text{No Cycling} \\ & \text{Principal's IC} \\ & T \times \text{Mutual Monitoring} \\ & T \times \text{Pareto Dominance} \\ & T \times \text{Self-enforcing Shirk} \\ & (1-T) \times \text{Static NE} \\ & (1-T) \times \text{No Joint Shirking} \end{aligned} \tag{1}$$

The variable T (short for team incentives) takes a value of either zero or one. $T = 1$ means the principal designs the contract to induce team incentives by motivating mutual monitoring between the agents. When $T = 1$, the agents will not collude by jointly shirking because (*shirk, shirk*) is Pareto dominated by (*work, work*) from the agents' perspective. Therefore, we know from Lemma 2 that the contract in this case is collusion-proof as long as it satisfies the *No Cycling* constraint.

The following lemma links the explicit joint bonus floor $\underline{w}$ and the implicit promises $\{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$.

Lemma 3 *It is optimal to set* $\underline{w} = \min_{m,n\in H,L}\{w_{m,n} + w_{n,m}\}$.

That is, the principal optimally sets the contractible joint bonus floor equal to the minimum total compensation specified by the principal's implicit contract with the agents. The result is intuitive. If the bonus floor is set lower than the minimum promised total compensation, then the principal can relax *IC* by increasing the bonus floor. If the bonus floor is greater than the minimum promised compensation, then the agents will use the courts to enforce the bonus floor. That is, the actual minimum compensation will not be the promised minimum but instead the bonus floor. The same equilibrium payments can be achieved by revising the principal's implicit contract so that the minimum promised compensation is equal to the bonus floor.

Lemma 3 allows us to rewrite the (Principal's IC) constraint as follows:

$$\frac{2\left[q_1 H - \pi(1,1;\mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n,m',n' \in \{H,L\}} \{w_{mn} + w_{nm} - (w_{m',n'} + w_{n',m'})\}.$$

As the reformulated Principal's IC constraint suggests, in our model, an explicit bonus floor is equivalent to an explicit contract that specifies the range of possible payments and asks the principal to *self-report* the non-verifiable performance measures. The courts would then be used to enforce payments that are consistent with some realizations of the performance measures but not verify the actual realizations, since they are non-verifiable by assumption.

We solve Program **P** by the method of enumeration and complete the analysis in two steps. In Sections 4.1 and 4.2, we solve Program P while setting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T in Section 4.3.

4.1 Team Incentives

We are now ready to characterize the optimal solution, taking team incentives ($T = 1$) as given.

Proposition 1 *Given $T = 1$, the solution to Program P is one of the following (with $w_{LH} = 0$ in all cases):*

(i) JPE1: $w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$, $w_{HL} = w_{LL} = \underline{w} = 0$ if and only if $r \in (0, r^A]$;

(ii) BPC1: $w_{HH} > 0, w_{HL} = 2w_{LL} = \underline{w} > 0, w_{HH} > w_{HL} - w_{LL}$ if and only if $r \in (r^A, \min\{r^L, r^C\}]$;

(iii) JPE2: $w_{HH} > w_{HL} > 0, w_{LL} = \underline{w} = 0$ if and only if $r \in (\max\{r^L, r^A\}, r^B]$;

(iv) BPC2: $w_{HH} > 0, w_{HL} > 2w_{LL} = \underline{w} > 0, w_{HH} > w_{HL} - w_{LL}$ if and only if $r \in (\max\{r^L, r^B\}, r^C]$;

(v) infeasible otherwise,

where closed-form expressions for BPC1, JPE2, and BPC2 are given in the appendix and $r^L = \frac{q_1+q_0-1}{(1-q_1)^2}$, *and* r^A, r^B, *and* r^C *are increasing functions of* H *and are also specified in the appendix.*

It is easy to check that all the solutions create a strategic payoff complementarity (denoted by the C in the *BP*-type contracts).[9] When the discount rate r is small ($r \in (0, r^A]$), the contract offered is the same as the "Joint Performance Evaluation" (*JPE1*) contract studied in Che and Yoo (2001), i.e., the agents are rewarded only if the outcomes from both agents are high. *JPE1* is optimal because it is the optimal means of tying the agents together, so that they have both the incentive to mutually monitor each other and the means of punishing each other for free-riding.

Starting from *JPE1*, agents become less patient as r increases and, therefore, the principal must increase w_{HH} to incentivize the agents to mutually monitor each other. Because the principal is also less patient as r increases, she will eventually loose credibility to promise *JPE1* to the agents. To see this, note that the benefit for the principal to renege ($2\,w_{HH}$) increases in r while the cost of reneging (in the reduction of production in all future periods) is perceived as less costly. In particular, (Principal's IC) starts to bind at $r = r^A$.

To make her promise credible, the principal will have to choose between *JPE2* and *BPC1*. *JPE2* has the principal substitute individual for team incentives by setting $w_{HL} > 0$ and holding w_{LL} at 0, and *BPC1* has the principal setting $w_{LL} > 0$ in order to increase w_{HH} and keep the focus on team incentives. It is tempting to think that the principal should only start using *BPC1* when its alternative *JPE2* is infeasible, but this is not correct.

Corollary 1a Given $T = 1$, *BPC1* is sometimes optimal when *JPE2* is also feasible

When will this happen? Intuitively, *BPC1* is optimal when the agents' credibility to mutually monitor each other is *stronger* than the principal's credibility to honor her promises at $r = r^A$. The reason is that *BPC1* keeps the focus on team incentives, which is valuable when

[9] Since $w_{LH} = 0$, strategic payoff complementarity is reduced to $w_{HH} > w_{HL} - w_{LL}$.

the agents' credibility is strong, even at the cost of paying for joint bad performance ($\underline{w} > 0$). Technically, we know from Proposition 1 that a necessary condition for *BPC1* to be optimal is $r^A < r^L$. This condition is equivalent to restricting the high output of the project $H < H^*$ for some constant H^* because r^A increases in H while r^L is independent of H. To gain some intuition for the requirement $H < H^*$, notice that the principal's ability to commit to honoring the *JPE1* contract is enhanced by a high value of H, as the punishment the agents can bring to bear on the principal is more severe. Therefore, the region over which the principal can commit to the *JPE1* contract becomes larger (i.e., a bigger r^A) as H increases. For sufficiently large H ($H > H^*$), r^A is so large that by the time the principal looses her credibility to honor the *JPE1* contract, the agents have already lost their ability to mutually monitor each other. Without the benefit of mutual monitoring, the positive bonus floor is not justified and *JPE2* is instead optimal. In other words, *JPE2* can be thought of as a mix of *JPE1* and *IPE*.

A feature of Proposition 1 is that no feasible solution exists when r is large enough. There is a conflict between principal's desire to exploit the agents' mutual monitoring and her ability to make credible promises. Once r is sufficiently large, the intersection between the Mutual Monitoring constraint and the Principal's IC constraint is an empty set.

Before turning to collusion (T=0), compare Proposition 1 to the same setting without a contractual commitment to a bonus floor. In this case, the principal can always renege on her promise and so setting $w_{LL} = 0$ is optimal as in Kvaløy and Olsen (2006).

Corollary 1b Given $T = 1$ and no commitment to a bonus floor, the solution to Program P is one of the following (with $w_{LL} = w_{LH} = 0$ in all cases):

(i) *JPE1* if and only if $r \in (0, r^A]$;

(ii) *JPE2* if and only if $r \in (r^A, r^B]$;

(iii) infeasible otherwise.

4.2 Collusion

We are now ready to characterize the solution to Program P, taking individual incentives ($T = 0$) as given.

Proposition 2 *Given $T = 0$, the solution to the Program P is one of the following ($w_{LH} = 0$ in all cases):*

(i) IPE: $w_{HH} = w_{HL} = \frac{1}{q_1 - q_0}, w_{LL} = \underline{w} = 0$ if and only if $r \in (0, r^B]$;

(ii) BPI: $w_{HH} > 0$, $2w_{LL} = \underline{w} > 0, w_{HL} = w_{HH} + w_{LL}$ if and only if $r \in (r^B, r^H]$;

(iii) RPE: $w_{HL} > w_{HH} > 0$, $w_{LL} = \underline{w} = 0$ if and only if $r \in (\max\{r^B, r^H\}, r^D]$;

(iv) BPS: $w_{HH} > 0$, $w_{HL} = 2w_{HH} > 2w_{LL} = \underline{w} > 0$ if and only if $r > \max\{r^H, r^D\}$;

where closed-form expressions for BPI, RPE, and BPS are given in the appendix and $r^H = \frac{2q_1 - 1}{(1-q_1)^2}$ and r^B, r^D are increasing functions of H, which are also specified in the appendix.

We use "S", and "I" to denote a strategic (payoff) substitutability and strategic independence, respectively. In Proposition 2, individual performance evaluation (*IPE*) is optimal if both parties are patient enough ($r \leq r^B$).

As r increases, *IPE* is no longer feasible because the impatient principal has incentive to renege when the output pair is (H, H). To see this, note that the benefit of reneging the *IPE* contract is a constant ($2 * w_{HH} = \frac{2}{q_1 - q_0}$), while the cost of reneging (in lowering production in all future periods) is lower as r increases. As the principal becomes less patient, eventually she has incentives to renege when the output pair is (H, H). In particular, the Principal's IC constraint starts binding at $r = r^B$. As r increases further, the gap between w_{HH} and w_{LL} must be decreased in order to prevent the principal from reneging.

The principal has two methods of decreasing the gap between w_{HH} and w_{LL}. First, she can decrease w_{HH} and, therefore, increase w_{HL} to provide incentives – i.e., increasing her reliance on *RPE*. Second, the principal can increase w_{LL}, corresponding to *BPI* in Proposition 2. *BPI* makes the two collusion constraints equally costly to deal with by making the agents' payoffs strategically independent. It is tempting to think that one would only start using *BPI* when *RPE* is no longer feasible because paying out $w_{LL} > 0$ in *BPI* does not provide any incentives to the agents. However, the corollary below shows that this intuitive thinking is incorrect.

Corollary 2a Given $T = 0$, BPI is sometimes optimal when RPE is also feasible.

To understand the corollary, note that RPE creates a strategic substitutability in the agents' payoffs, which means that each agent's high effort level has a negative externality on the other agent's incentive to choose high effort. Invoking the observation made in Section 3, we know that the *Cycling* collusion more difficult (more expensive) to upset under RPE. In contrast, by increasing $w_{LL} > 0$, BPI creates a payoff strategic independence, making the two collusion constraints equally costly to upset. Because of the desirable feature of BPI in combating the agent–agent collusion, we would expect BPI to be optimal when the agents' credibility to collude is relatively stronger than the principal's credibility to honor her promises at $r = r^B$. More precisely, from Proposition 2, we know BPI is optimal if and only if $r^B < r^H$, which as argued in Subsection 4.1, means that H is too large as a large as higher H strengthens the principal's credibility. If in contrast the principal has a relative stronger credibility (due to for sufficiently large H), by the time the principal's limited commitment becomes a binding constraint, the agents are so impatient that mutual monitoring is of little value. Because the only reason that BPI is optimal is its efficiency in combatting agent-agent collusion on the Cycling strategy, the RPE contract instead of BPI will be optimal if H is large (and thus $r^B > r^H$).

The role of BPS is to expand the feasible region. The commitment to a joint bonus floor allows the principal to construct an incentive compatible and collusion proof scheme for any parameters. As pointed out by Levin (2003), "the variation in contingent payments is limited by the future gains from the relationship." The variation of wage payment is extremely limited under BPS, because both parties are sufficiently impatient ($r > \max\{r^H, r^D\}$). As a result, the principal has to set $w_{HL} = 2w_{HH}$ and also increase $w_{LL} = \underline{w}/2 > 0$ to make the contract self-enforcing. As the discount rate becomes arbitrarily large, the optimal incentive scheme converges to a bonus pool with a fixed total payment to the agents. This coincides with the traditional view that bonus pools will eventually come into play because they are the only self-enforcing compensation form in such cases.

Let us now compare Proposition 2 to the same setting without a contractual commitment to a bonus floor. Again, the principal can always renege on her promise, and so setting w_{LL} $= 0$ is optimal.

Corollary 2b Given $T = 0$ and no commitment to a bonus floor, the solution to Program P is one of the following (with $w_{LL} = w_{LH} = 0$ in all cases):

(i) *IPE* if and only if $r \in (0, r^B]$;

(ii) *RPE* if and only if $r \in (r^B, r^D]$;

(iii) infeasible otherwise.

4.3 Overall optimal contract

The following proposition endogenizes the principal's choice of $T \in \{0, 1\}$ and characterizes the overall optimal contract.

Proposition 3 *The overall optimal contract is:*

(i) JPE1 if and only if $r \in (0, r^A]$;

(ii) BPC1 if and only if $r \in (r^A, \min\{r^L, r^C\}]$;

(iii) JPE2 if and only if $r \in (\max\{r^L, r^A\}, r^B]$;

(iv) BPIif and only if $r \in (r^B, r^H]$;

(v) RPE if and only if $r \in (\max\{r^B, r^H\}, r^D]$;

(vi) BPS if and only if $r > \max\{r^H, r^D\}$.

Relative to Propositions 1 and 2 in which the team vs. individual incentive choice was made exogenously, once we allow for an optimal choice, neither $BPC2$ nor IPE are ever (uniquely) optimal. If *JPE1*, *BPC1*, or *JPE2* are optimal when team incentives are exogenously imposed, they remain optimal when the choice between team and individual incentives is endogenous. **Corollary 3a** When the principal cannot commit to a joint bonus floor, i.e., $\underline{w} = 0$, the overall optimal contract is:

(i) *JPE1* if and only if $r \in (0, r^A]$;

(ii) *JPE2* if and only if $r \in (r^A, r^B]$;

(iii) RPE if and only if $r \in (r^B, r^D]$;

(iv) infeasible otherwise.

As the Corollary above shows, in the absence of a bonus floor, there is no feasible solution once the discount rate is above r^D. The temptation for the principal to renege in the current period is then larger than any future punishment the agents can impose on her. At the singular discount rate r^B, $JPE2$ becomes IPE, and so IPE is also optimal. In our model (with a bonus floor), IPE may or may not be optimal at r^B. IPE and our BPI contract can be thought of as two forms of a broader class of incentive arrangements that create strategic payoff independence. Unlike IPE, which is optimal at most at a singular discount rate, BPI is optimal for a range of discount rates.

Since the joint bonus floor relaxes both the team incentive and individual incentive programs, it is unclear whether such a commitment results in more or less team incentives once team incentives is a choice variable. The following corollary states that the region in which team incentives are optimal expands once the bonus floor is introduced.

Corollary 3b

(i) When the principal *cannot* commit to a joint bonus floor, team incentives are optimal if and only if $r < r^B$.

(ii) When the principal *can* commit to a joint bonus floor, team incentives are optimal if and only if (1) $r < r^B$ or (2) $r < \min\{r^L, r^C\}$ if $r^L \geq r^B$.

The following discussion and examples are intended to provide an intuitive interpretation of the conditions given in Proposition 3 and Corollary 3b. Fix $q_0 = 0.53$ and $q_1 = 0.75$ for all the examples to ease comparison. In the first example, set $H = 200$. The principal's ability to commit is high because the expected production she forgoes after reneging is large. In this case, the agents' ability to commit is limited relative to that of the principal, so collusion is never the driving determinant of the form of the optimal compensation arrangement. As the discount becomes large enough that *JPE1* is not feasible, the agents ability to make commitments is also relatively limited, making mutual monitoring relatively costly to motivate and *JPE2* (which partially substitutes individual for team incentives) optimal. As the discount rate is increased even more, *RPE* is used immediately after *JPE2*–again, because collusion is not costly to deal with.

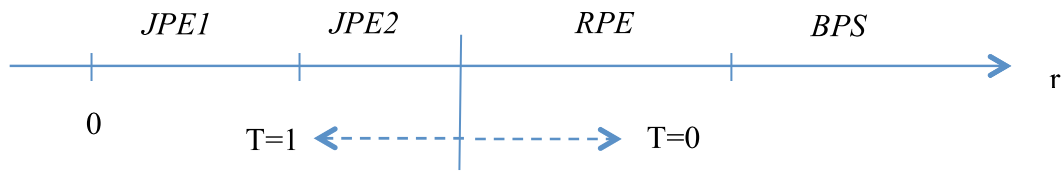

Figure 1: Numerical Example ($H = 200$)

In the second example, set $H = 170$. The principal's ability to commit is still high relative to the agents', but the relative comparison is not as extreme. As the discount rate is increased, we move from $JPE2$ to BPI, since collusion is costly to prevent. As the discount rate is increased even more, we move from *BPI* to *RPE*, since the principal still has enough ability to commit to make *RPE* feasible after the discount rate is so large that collusion is not the key determinant of the form of the compensation contract.

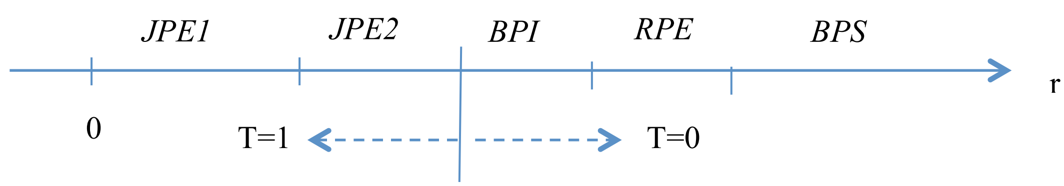

Figure 2: Numerical Example ($H = 170$)

If we further lower the value of H to $H = 100$, the principal's ability to commit becomes low relative to the agents'. As the discount becomes large enough that *JPE1* is no longer feasible, the agents' ability to make commitments is still strong, making mutual monitoring highly valuable and *BPC1* preferred to *JPE2*. As the discount rate continues to increase to the point that individual incentives are optimal, collusion is costly to upset, making *BPI* optimal. By the time the discount rate is large enough that the agents' collusion is not the key determinant of the form of the compensation contract, the principal's ability to commit is so limited that *BPS* is the only feasible solution.

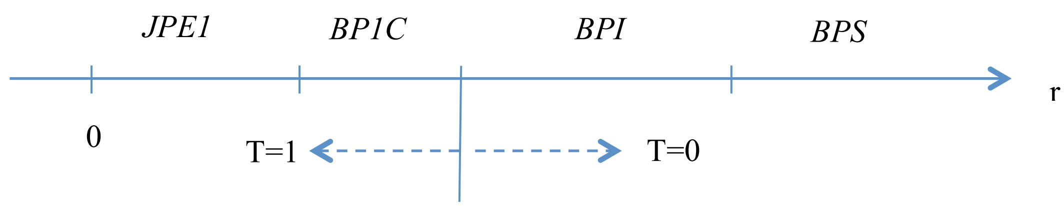

Figure 3: Numerical Example ($H = 100$)

To illustrate the last case (Figure 3), we present the optimal contract and the payoff matrix of the stage game, as a function of the discount rate r. For a low discount $r = 3$, *JPE1* is optimal with $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\} = (0, 0, 0, 5.15)$. This is the optimal contract in Che and Yoo (2001). The stage game payoff matrix follows.

JPE1 ($r = 3$)

A/B	0	1
0	(1.45,1.45)	(2.05,1.05)
1	(1.05,2.05)	(1.90,1.90)

The benefit of free-riding of 2.05 - 1.90 = 0.15 is exactly equal to the punishment imposed by reverting to the stage game Nash equilibrium of (1.90 - 1.45)/3 = 0.15.

If we increase r to $r = 4$, $JPE2$ with $\mathbf{w} = (0, 0, 3.84, 4.66)$ is feasible but is not optimal. Instead, *BPC1* is the optimal wage scheme with $\mathbf{w} = (0.68, 0, 1.36, 5.35)$. The payoff matrix follows.

BPC1 ($r = 4$)

A/B	0	1
0	(1.99,1.99)	(2.39,1.69)
1	(1.69,2.39)	(2.31,2.31)

Again, the benefit of free-riding of 2.39 - 2.31 = 0.08 is exactly equal to the punishment of reverting to the stage game Nash equilibrium of (2.31 - 1.99)/4 = 0.08.

For $r = 5$, team incentives are no longer optimal. The optimal means of preventing collusion is *BPI* with $\mathbf{w} = (1.09, 0, 5.82, 4.73)$. The payoff matrix follows.

BPI ($r = 5$)

A/B	0	1
0	(3.02,3.02)	(2.78,3.06)
1	(3.06,2.78)	(2.82,2.82)

The principal provides any shirking agent with a benefit of 0.04 for instead working and upsetting collusion on either *Joint Shirking* or *Cycling*. It is easy to verify that the shirking agent's continuation payoff under *Joint Shirking* of $\frac{3.02}{5} = 0.604$ is the same as his continuation payoff under *Cycling*. Under the equilibrium strategy (*work,work*), each agent's continuation payoff is $\frac{2.82}{5} = 0.564$. Therefore, the difference in continuation payoffs of $0.604 - 0.564 = 0.04$ is exactly equal to the benefit an agent receives for upsetting collusion by playing *work* instead of *shirk* in the current period. That is, both (No Joint Shirking) and (No Cycling) hold as equality – a feature of the contract that creates payoff strategic independence.

For $r = 10$, *BPS* is optimal with $\mathbf{w} = (2.71, 0, 8.94, 4.47)$. The payoff matrix follows.

BPS ($r = 10$)

A/B	0	1
0	(4.08,4.08)	(3.28,4.25)
1	(4.25,3.28)	(3.36,3.36)

As r continues to increase, BPS becomes more and more like a proper bonus pool under which the total payments are constant.

5 Stationary Collusion Strategies

In addition to allowing for a joint bonus floor, another difference between our paper and Kvaløy and Olsen's (2006) is that they restrict the agents to playing stationary strategies, while we do not. To incorporate the stationary equivalent of our *Cycle* strategy, they allow the agents to play correlated strategies. If we restrict attention to stationary strategies (while keeping the ability to commit to a bonus floor), the (No Cycling) becomes:

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w})-1\right] \geq \pi(0,1;\mathbf{w}) + \frac{[\pi(1,0;\mathbf{w})-1+\pi(0,1;\mathbf{w})]/2}{r}.$$

The second term of the right hand side of the equation is the continuation payoff for the shirking agent from indefinitely playing either $(1,0)$ or $(0,1)$ with equal probability in each period. A similar argument to our Lemma 2 shows that, if we restrict the agents collusive strategy to be stationary as in Kvaløy and Olsen (2006),[10] the contract is collision proof if it satisfies both the (No Joint Shirking) and the modified (No Cycling) constraint above. The corollary below shows that the stationary *No Cycle* constraint does not qualitatively change the nature of our results, except that strategic payoff independence no longer plays a critical in combatting collusion.

Corollary 4 If we restrict attention to stationary strategies,

(i) Given $T = 1$, the wage contract and the cutoffs are same as in Proposition 1.

(ii) Given $T = 0$, the wage contract and the cutoffs are characterized by the same binding constraints as those in Proposition 2. The expected wage payment, $\pi(1,1)$, is weakly lower than in Proposition 2.

(iii) Strategic payoff independence is never optimal.

Part (i) shows that the stationary *No Cycle* constraint does not affect our results when it is optimal to motivate mutual monitoring. Mutual monitoring endogenously requires the wage contract to satisfy strategic complements, and, in this case, the *No Cycle* constraint does not bind. Part (ii) of Corollary 4 implies that the stationary *No Cycling* constraint is not without loss of generality–the non-stationary constraint is strictly more demanding/tighter. Part (iii) states that strategic payoff independence does not emerge as optimal in response to stationary

[10]Stationary (symmetric) collusive strategies are characterized in Kvaløy and Olsen (2006) as probabilities $(a, b, b, 1-a-2b)$ on effort combinations $(1,1), (1,0), (0,1)$, and $(0,0)$, respectively.

collusive strategies. This is because it is only under the non-stationary formulation of the collusion constraints that *No Cycle* and *No Joint Shirk* become equally costly under strategic payoff independence.

6 Conclusion

Stepping back from the principal's commitment problem, the general theme of team (mutual monitoring) vs. individual incentives seems to be under-explored. For example, the models of team-based incentives typically assume the agents are symmetric (e.g., Che and Yoo, 2001; Arya, Fellingham, and Glover, 1997). With agents that have different roles (e.g., a CEO and a CFO), static models predict the agents would be offered qualitatively different compensation contracts. Yet, in practice, the compensation of CEOs and CFOs are qualitatively similar, which seems to be consistent with a team-based model of dynamic incentives (with a low discount rate/long expected tenure).

If we apply the team-based model to thinking about screening, then we might expect to see compensation contracts that screen agents for their potential productive complementarity with agents already in the firm's employ (or in a team of agents being hired at the same time), since productive complementarities reduce the cost of motivating mutual monitoring because the benefit to free-riding is small. A productive substitutability (e.g., hiring an agent similar to existing ones when there are overall decreasing returns to effort) is particularly unattractive, since the substitutability makes it appealing for the agents to tacitly collude on taking turns working. We might also expect to see agents screened for their discount rates. Patient agents would be more attractive, since they are the ones best equipped to provide and receive mutual monitoring incentives. Are existing incentive arrangements such as employee stock options with time-based rather than performance-based vesting conditions designed, in part, to achieve such screening?

Even if screening is not necessary because productive complementarities or substitutabilities are driven by observable characteristics of agents (e.g., their education or work experience), optimal team composition is an interesting problem. For example, is it better to have one team with a large productive complementarity and another with a small substitutability or to have two teams each with a small productive complementarity?

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Appendix

Proof of Lemma 1. We first argue that the off-diagonal action profiles (L, H) and (H, L) cannot be punishment strategies harsher than (H, H). We illustrate the argument for (L, H); similar logic applies to (H, L):

		Agent B	
		L	H
Agent A	L	U_{LL}, U_{LL}	U_{LH}, U_{HL}
	H	U_{HL}, U_{LH}	U_{HH}, U_{HH}

If playing (L, H) were a harsher punishment for Agent A (i.e., $U_{LH} < U_{HH}$), he would be able to profitably deviate to (H, H), implying that (L, H) is not a stage-game equilibrium and thus cannot be used as a punishment strategy. If (L, H) were a harsher punishment for Agent B (i.e., $U_{HL} < U_{HH}$), we would need $U_{HL} \geq U_{LL}$ to prevent Agent B from deviating from (L, H) to (L, L). However, $U_{HL} < U_{HH}$, $U_{HL} \geq U_{LL}$, and (StageNE) together imply $U_{HH} \geq \max\{U_{HL}, U_{LL}, U_{LH}\}$, which means there is no scope for collusion because at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy.

To establish that (H, H) is the (weakly) harshest punishment, it remains to show that $U_{HH} \leq U_{LL}$. Suppose, by contradiction, $U_{HH} > U_{LL}$. If the wage scheme $\mathbf{w} \equiv \{w_{mn}^x\}$, $x = \{0, 1\}$, $m, n = \{L, H\}$, creates strategic payoff substitutability, i.e., $U_{HL} - U_{LL} > U_{HH} - U_{LH}$, (StageNE) again implies that (L, L) is not a stage-game equilibrium and thus cannot be used as a punishment strategy in the first place. If instead $\mathbf{w}$ creates (weak) strategic payoff complementarity, i.e., $U_{HH} - U_{LH} \geq U_{HL} - U_{LL}$, we have $U_{HL} + U_{LH} \leq U_{HH} + U_{LL} < 2U_{HH}$, where the last inequality is due to the assumption $U_{HH} > U_{LL}$. But $U_{LL} < U_{HH}$ and $U_{HL} + U_{LH} < 2U_{HH}$ together mean that at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy, meaning there is no scope for any collusion. ■

Proof of Lemma 2. Let

$$V_t^i(\sigma) \equiv \sum_{\tau=1}^{\infty} \frac{U_{t+\tau}^i(a_\tau^i, a_\tau^j)}{(1+r)^\tau}$$

be agent i's continuation payoff from $t+1$ and forwards, discounted to period t, from playing $\{a_{t+\tau}^A, a_{t+\tau}^B\}_{\tau=1}^\infty$ specified in an action profile $\sigma = \{a_t^A, a_t^B\}_{t=0}^\infty$, for $a_t^A, a_t^B \in \{0,1\}$. We allow any σ satisfying the following condition to be a potential collusive strategy:

$$\sum_{t=0}^{\infty} \frac{U_t^i(a_t^i, a_t^j)}{(1+r)^t} \geq \frac{1+r}{r} U_{1,1}, \ \forall i \in \{A, B\}, \tag{2}$$

where $U_t^i(a_t^i, a_t^j)$ is Agent i's stage-game payoff at t given the action pair (a_t^i, a_t^j) specified in σ, and $\frac{1+r}{r}U_{1,1}$ is the agent's payoff from always working.

The outline of the proof is as follows:

Step 1: Any collusive strategy that contains only $(1,1)$, $(1,0)$ and $(0,1)$ (i.e., without $(0,0)$ in any period) is easier for the principal to upset than *Cycle*.

Step 2: Any collusive strategy that ever contains $(0,0)$ at some period t would be easier for the principal to upset than either *Shirk* or *Cycle*.

Step 1: The basic idea here is that, compared with *Cycle*, any reshuffling of $(0,1)$ and $(1,0)$ effort pairs across periods and/or introducing $(1,1)$, can only leave some shirking agent better off in some period if it also leaves another shirking agent worse off in another period, in terms of their respective continuation payoffs.

In order for the agents to be better-off under the collusive strategy σ that contains only $(1,1)$, $(1,0)$ and $(0,1)$ than under jointly work $(1,1)^\infty$, condition (2) requires $U_{1,0}+U_{0,1} > 2U_{1,1}$. Therefore, we know

$$\overline{V}^{CYC} + \underline{V}^{CYC} \geq V_t^i(\sigma) + V_t^j(\sigma), \ \forall t, \tag{3}$$

where $\overline{V}^{CYC} = \sum\limits_{t=1,3,5,\ldots} \frac{U_{1,0}}{(1+r)^t} + \sum\limits_{t=2,4,6,\ldots} \frac{U_{0,1}}{(1+r)^t}$ and $\underline{V}^{CYC} = \sum\limits_{t=1,3,5,\ldots} \frac{U_{0,1}}{(1+r)^t} + \sum\limits_{t=2,4,6,\ldots} \frac{U_{1,0}}{(1+r)^t}$ are the continuation payoffs (under *Cycle*) of the shirking agent and the working agent, respectively. We drop the time index in $\overline{V}^{CYC}$ and $\underline{V}^{CYC}$ because they're time independent. Since *StageNE* and $U_{1,0} + U_{0,1} > 2U_{1,1}$ together imply $U_{1,0} > U_{1,1} \geq U_{0,1}$, simple algebra shows

$$\overline{V}^{CYC} > \max\{\underline{V}^{CYC}, V^*\}, \tag{4}$$

where $V^* \doteq \frac{U_{1,1}}{r}$ is the continuation payoff from playing $(1,1)^\infty$.

To prove the claim that the collusive strategy σ is easier for the principal to upset than *Cycle*, it is sufficient to show the following:

$$\exists\, t|\{(a_t^i = 0, a_t^j = 1) \wedge V_t^i(\sigma) \leq \overline{V}^{CYC}\}. \tag{5}$$

That is, there will be some period when the agents i is supposed to be the *only* "shirker" in that period faces a weakly lower continuation payoff (hence stronger incentives to deviate from shirking) under the collusive strategy σ than under *Cycle*. Suppose by contradiction that (5) fails. That is,

$$V_t^i(\sigma) > \overline{V}^{CYC}, \forall\, t|\{(a_t^i = 0, a_t^j = 1)\}. \tag{6}$$

We know from (3) that (6) implies the following for the other agent j:

$$V_t^j(\sigma) < \underline{V}^{CYC}, \forall\, t|\{(a_t^i = 0, a_t^j = 1)\}. \tag{7}$$

Since one agent always playing 0 is not a sub-game perfect equilibrium, there must be a switch from $(0,1)$ to $(1,0)$, possibly sandwiched by one or more $(1,1)$, in any strategy σ. We pick any such block in σ, and denote τ and $\tau+1+n$ ($n \in \{0,1,2,3,...\}$) as the time $(0,1)$ and $(1,0)$ are sandwiched by n period(s) of $(1,1)$.

We show below that, for all n, (6) leads to a contradiction, which then verifies (5) and proves the claim. We name the first agent as Agent A throughout the analysis.

- If $n = 0$, i.e., $(0,1)$ is followed immediately by a $(1,0)$ at $\tau+1$. One can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,0} + V_{\tau+1}^A(\sigma)}{1+r} \\ &< \frac{U_{1,0} + \underline{V}^{CYC}}{1+r} \\ &= \overline{V}^{CYC}, \end{aligned}$$

which contradicts (6). The inequality above applies (7) to $t = \tau + 1$.

- If n is an even number ($n = 2, 4, 6, ...$), i.e., there are even numbers of $(1, 1)$ sandwiched between $(0, 1)$ and $(1, 0)$. We prove the case for $n = 2$ and same argument applies for all even n.

	...	$t = \tau$	$t = \tau + 1$	$t = \tau + 2$	$t = \tau + 3$	...
σ	...	$(0, 1)$	$(1, 1)$	$(1, 1)$	$(1, 0)$	...

We can show the following for Agent A's continuation payoff at τ:

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + V_{\tau+3}^A(\sigma)}{(1+r)^3} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &< \frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &= \overline{V}^{CYC}, \end{aligned}$$

which again contradicts (6). The first inequality applies (7) to $t = \tau + 3$. The second inequality applies (4) and the fact that $\overline{V}^{CYC} > V^*$ if and only if $\frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} > \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2}$.

- If n is an odd number ($n = 1, 3, 5, ...$). Consider, without loss of generality, the following case for $n = 1$, i.e., there is one $(1, 1)$ sandwiched between $(0, 1)$ and $(1, 0)$.

$$\begin{array}{cccccc} & ... & t=\tau & t=\tau+1 & t=\tau+2 & ... \\ \sigma & ... & (0,1) & (1,1) & (1,0) & ... \end{array}$$

We can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,0}+V_{\tau+2}^A(\sigma)}{(1+r)^2} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,0}+\underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{U_{1,0}+\underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{\overline{V}^{CYC}}{1+r} \\ &< \overline{V}^{CYC}, \end{aligned}$$

which contradicts (6). The first inequality applies (7) to $t=\tau+2$, the second equalities is from the definition of $V^*=\frac{U_{1,1}}{r}$, and the last equality is by the definition of $\underline{V}^{CYC}$ and $\overline{V}^{CYC}$ so that $\overline{V}^{CYC}=\frac{U_{1,0}+\underline{V}^{CYC}}{1+r}$.

Step 2: Given any contract offered by the principal, one of the following must be true:

$$\max\{2U_{1,1}, 2U_{0,0}, U_{1,0}+U_{LH}\} = \begin{cases} 2U_{1,1} & \text{case 1} \\ U_{1,0}+U_{0,1} & \text{case 2} \\ 2U_{0,0} & \text{case 3.} \end{cases}$$

Case 1 is trivially collusion proof as no other action profile Pareto-dominates the equilibrium strategy $\{1,1\}_{t=0}^{\infty}$.

Case 2 has two sub-cases: sub-case 2.1 where $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{1,1} \geq U_{0,0}$ and sub-case 2.2 in which $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$. In sub-case 2.1, first note that we can ignore collusive strategy σ that contains $(0,0)$ at some period without loss of generality. The reason is that we can construct a new strategy σ' by replacing $(0,0)$ in σ by $(1,1)$, and σ' is more difficult to upset than σ because (a) both agents' continuation payoffs are higher under σ' and (b) σ' does not have the possibility of upsetting the collusive strategy at $(0,0)$. After ruling out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (NoCycle) constraint.

In sub-case 2.2 (i.e., $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$), we argue that any collusive strategy σ that contains $(0,0)$ at some point is easier to be upset than (thus implied by) the *Cycle* strategy. The reason is clear by comparing the action profiles between σ and *Cycle* from any $\tilde{t}$ such that $a_{\tilde{t}}(\sigma) = (0,0)$:

	$t = \tilde{t}$	$t = \tilde{t}+1, \tilde{t}+2, \cdots$
σ	$(0,0)$	$\{a^A(\sigma), a^B(\sigma)\}_{\tilde{t}+1}^{\infty}$
Cycle	$(0,1)$	$\{Cycle\}_{\tilde{t}+1}^{\infty}$

$\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$ implies $U_{1,0} - U_{0,0} > U_{1,1} - U_{0,1}$. That is, the benefit for either Agent A or B to unilaterally deviate from (L,L) at $\tilde{t}$ is higher than the benefit for A to deviate from (L,H) to (H,H) at $\tilde{t}$ under the *Cycle* strategy. In addition, we know $V_{\tilde{t}}^A(CYC) + V_{\tilde{t}}^B(CYC) \geq V_{\tilde{t}}^A(\sigma) + V_{\tilde{t}}^B(\sigma)$ holds under Case 2, and therefore either $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(CYC)$ or $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(CYC)$. If $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(CYC)$ holds, then A has stronger incentive to deviate at $\tilde{t}$ under σ than he would have under *Cycle*. If it is $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(CYC)$, we make use of the observation that $V_{\tilde{t}}^B(CYC) < V_{\tilde{t}}^A(CYC)$ to conclude $V_{\tilde{t}}^B(\sigma) < V_{\tilde{t}}^A(CYC)$, which means that B has stronger incentive to deviate at $\tilde{t}$ under σ than A would have under *Cycle* at $\tilde{t}$. Again, once we rule out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (NoCycle) constraint.

Case 3 implies $V_t^A(SHK) + V_t^B(SHK) = \max_\sigma V_t^A(\sigma) + V_t^B(\sigma)$, $\forall t$. If a collusive strategy σ contains $a_{t'}^A(\sigma) = a_{t'}^B(\sigma) = 0$ at some period t', then either $V_{t'}^A(\sigma) \leq V_{t'}^A(SHK)$ or $V_{t'}^B(\sigma) \leq V_{t'}^B(SHK)$, which means at least one of the agents who is supposed to (jointly) shirk at t' will have a weakly stronger incentive to deviate than he would have under *Shirk* strategy. If the collusive strategy does not contain $a_t^A = a_t^B = 0$ in any period, we can refer to Step 1 and show that any such collusive strategy is deterred by the (NoCycle) constraint. ■

Proof of the Observation. Rewrite the *Joint Shirking constraint* and the *No Cycling* *constraint* as $f^{SHK} \leq 0$ and $f^{CYC} \leq 0$, respectively. It is straightforward to verify that

$$a \times f^{SHK} - b \times f^{CYC} = [\pi(1,1) - 1 - \pi(0,1)] - [\pi(1,0) - 1 - \pi(0,0)], \tag{8}$$

where $a = \frac{r}{1+r}$ and $b = \frac{r(2+r)}{(1+r)^2}$ are two positive constant.

If the contract creates strategic complementarity (the RHS of (8) is positive), we know $f^{SHK} > \frac{b}{a} f^{CYC}$. Therefore, $f^{SHK} \leq 0$ implies $f^{CYC} \leq 0$. That is, *No Joint Shirking* implies *No Cycling* if the contract creates strategic complementarity.

A similar argument shows that if the contract creates strategic substitutability (the RHS of (8) is negative), then $f^{CYC} \leq 0$ implies $f^{SHK} \leq 0$. ■

Proof of Lemma 3. Given any wage scheme $\mathbf{w}$, denote $M = \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$. The lemma claims $\underline{w} = M$. Suppose $\underline{w} < M$ in a wage scheme $\mathbf{w}$. We can construct a new wage scheme $\mathbf{w}'$ by increasing $\underline{w}$ (while keep the payment $w_{m,n}$ unchanged). It is easy to see that (i) $\mathbf{w}'$ and $\mathbf{w}$ yield the same objective value, and (ii)$\mathbf{w}'$ is feasible as long as $\mathbf{w}$ is. Since Principal's IC is more relaxed in $\mathbf{w}'$ than in $\mathbf{w}$, it is optimal to set $\underline{w} \geq M$.

We now show $\underline{w} \leq M$. Suppose by contradiction that $\underline{w} > M$, which means $\underline{w} > w_{m,n} + w_{n,m}$ for some outcome pair (m, n). Since the court will enforce $\underline{w}$, it will allocate the difference $\Delta = \underline{\mathrm{w}} - (w_{m,n} + w_{n,m})$ between the two agents according to a pre-determined allocation rule. The principal can directly give the agents the same payment as what they would have received through court enforcement by increasing the payments so that $(w'_{m,n} + w'_{n,m}) = \underline{w}$ and allocating it appropriately. Label the new wage scheme as $\mathbf{w}'$. It is easy to see that (i) $\mathbf{w}'$ costs the same as $\mathbf{w}$, and (ii) $\mathbf{w}'$ is feasible as long as the original $\mathbf{w}$ is because the agents' ex-post payment (after taking into account the court's enforcement) is same across the two scheme. Since the principal can optimize over the ways of setting $(w'_{m,n} + w'_{n,m}) = \underline{w}$ (as opposed to replicating the court's allocation rule), $\mathbf{w}'$ is at least weakly better than $\mathbf{w}$. ■

The following parameters will be useful in the remaining proofs.

$$
\begin{aligned}
r^A &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2 q_1 H - 1 - q_1^2 + \\ \sqrt{((q_0-q_1)^2 q_1 H - 1 - q_1^2)^2 + 4(q_0-q_1)^2(q_0+q_1)H - 4q_1^2} \end{array}\right], \\
r^B &= (q_0-q_1)^2 H - q_1, \\
r^C &= (q_0-q_1)^2(q_0+q_1)H - q_0 - q_1^2, \\
r^D &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2 + \\ \sqrt{((q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2)^2 + 4(q_1-2)q_1 - 8(q_0-q_1)^2(q_1-1)H} \end{array}\right], \\
r^L &= \frac{q_1+q_0-1}{(1-q_1)^2}, \\
r^H &= \frac{2q_1-1}{(1-q_1)^2}.
\end{aligned}
$$

Note that r^A,r^B,r^C,and r^D increase in H, and we assume throughout the paper that H is larger enough to rank term by comparing the coefficient of the linear term of H. In particular, we obtain (i) $r^A < r^B < r^D$, $r^A < r^C$, and (ii) $r^B < r^C$ if and only if $q_0 + q_1 > 1$. The agent's effort is assumed to be valuable enough ($q_0 - q_1$ is not too small) such that $r^A > \sqrt{2}$ and $(q_0-q_1)^2 H \geqslant \max\{1 + \frac{1}{q_1-q_0}, \frac{q_1^2}{2q_1-1}, \frac{q_1(2-q_1)}{2(1-q_1)}, \frac{q_0-(1-q_1)q_1}{q_0+q_1-1}\}$.

Proof of Proposition 1. The program can be written as follows.

$\min (1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$

$s.t$

$(2-q_0-q_1)w_{LL} + (q_0+q_1-1)w_{LH} + (q_0+q_1-1)w_{HL} - (q_0+q_1)w_{HH} \leq \frac{-1}{q_1-q_0}$ (Pareto Dominance, λ_{Pareto})

$(2-q_0-q_1+r(1-q_1))w_{LL} + (q_0+q_1+q_1 r-1)w_{LH} + (q_0+(q_1-1)(1+r))w_{HL} - (q_0+q_1+q_1 r)w_{HH} \leqslant \frac{-(1+r)}{q_1-q_0}$ (Mutual Monitoring, $\lambda_{Monitor}$)

$(1-q_1)(2+r)w_{LL} + (-1+q_1(2+r))w_{LH} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \leqslant -\frac{1+r}{q_1-q_0}$ (No Cycling, λ_{CYC})

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max_{m,n,m',n'}\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn \succ m'n'}$)

$-w_{LL} \leqslant 0\,(\mu_{LL})$; $\quad -w_{HL} \leqslant 0(\lambda_{11})$; $\quad -w_{HH} \leqslant 0(\mu_{HH})$; $\quad -w_{LH} \leqslant 0(\mu_{LH})$;

$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \leqslant \frac{1}{q_1-q_0}$ (Self-Enforcing Shirk)

We first solve a relaxed program without the "Self-Enforcing Shirk" constraint and then verify that solutions of the relaxed program satisfy the "Self-Enforcing Shirk" constraint.

Claim: Setting $w_{LH} = 0$ is optimal in the *relaxed program* (without Self-Enforcing Shirk constraint.)

Proof of the Claim: Suppose the optimal solution is $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Consider the solution $w' = \{w'_{HH}, w'_{HL}, w'_{LH}, w'_{LL}\}$, where $w'_{LH} = 0$, $w'_{HL} = w_{HL} + w_{LH}$, $w'_{LL} = w_{LL}$ and $w'_{HH} = w_{HH}$. It is easy to see that w and w' generate the same objective function value. We show below that the constructed w' satisfies all the $ICP_{mn \succ m'n'}$ constraints and further relaxes the rest of the constraints (compared to the original contract w). Since w_{LH} and w_{HL} have the same coefficient in all the $ICP_{mn \succ m'n'}$ constraints, w' satisfies these constraints as long as w does. Denote the coefficient on w_{LH} as C_{LH} and the coefficient on w_{HL} as C_{HL} for the "*Pareto Dominate*", "*Mutual Monitoring*", and "*No Cycling*" constraints. We can show that $C_{LH} - C_{HL} \geq 0$ holds for each of the three constraints. Given $C_{LH} \geq C_{HL}$, it is easy to show that w' will relax the three constraints compared to the solution w, which complete the proof of Lemma.

The Lagrangian for the problem is

$$L(w, \lambda, \mu) = f_0(w) - \sum_i \lambda_i f_i(w) - \sum_s \mu_s w_{a_i a_j}.$$

A contract w is optimal if and only if one can find a pair (w, λ, μ) that satisfies the following four conditions: (i) Primal feasible, i.e., $f_i(w) \geq 0$ and $w_{a_i a_j} \geq 0$, (ii) Dual feasible, i.e., vector $\lambda \geq 0$ and $\mu \geq 0$, (iii) Stationary condition, i.e., $\nabla_w L(w, \lambda, \mu) = 0$, and (iv) Complementary slackness, i.e. $\lambda_i f_i(w) = \mu_s w_{a_i a_j} = 0$. The proof lists the optimal contract, in particular the pair (w, λ, μ), as a function of the discount rate r.

For $r < r^A$, the solution, denoted as as *JPE1*, is:

$w_{LL} = 0, w_{HL} = 0, w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$;

$\lambda_{Pareto} = q_1, \lambda_{Monitor} = \frac{q_1^2}{q_0+q_1+q_1 r}, \lambda_{CYC} = 0, \lambda_{HH \succ LL} = 0,$

$\lambda_{HL \succ LL} = 0, \lambda_{HH \succ HL} = 0, \lambda_{HH \succ HH} = 0, \lambda_{LL \succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = \frac{q_0 - 2q_0q_1 + q_1(1+r-q_1r)}{q_0+q_1+q_1r}, \mu_{HL} = \frac{q_0q_1}{q_0+q_1+q_1r}, \mu_{HH} = 0.$

The $ICP_{HH\succ LL}$ constraint yields the upper bound on r under *JPE1.*

For $r^A < r \leqslant \min(r^L, r^C)$, the solution, denoted as *BPC1*, is:

$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)}, w_{HL} = 2 * w_{LL},$

$w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1+(-1+q_1)r)H-(1+r)(-1+q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)};$

$\lambda_{Pareto} = 0, \lambda_{Monitor} = \frac{-r}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_{CYC} = 0, \lambda_{HH\succ LL} = \frac{q_0-(q_1-2)((q_1-q)r-1)}{q_0+q_1+q_1r-q_1^2r-r^2},$

$\lambda_{HL\succ LL} = 0, \lambda_{HH\succ HL} = \frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_{HH\succ HH} = 0, \lambda_{LL\succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = 0, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *BPC1* , the non-negativity of w_{LL} and w_{HL} requires $r > r^A$, while the *Pareto Dominant* constraint and $\lambda_{HH\succ HL} \geqslant 0$ impose upper bounds r^C and r^L respectively.

For $\min\{r^L, r^A\} < r \leqslant r^B$, the solution, denoted as *JPE2*, is:

$w_{LL} = 0, w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0} - (q_1-q_0)(q_0+q_1+q_1r)H}{(1-q_1)r(1+r)-q_0(q_1+r)},$

$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0+(-1+q_1)(q+r))H}{(q_1-q_0)((-1+q_1)r(1+r)+q_0(q_1+r))};$

$\lambda_{Pareto} = 0, \lambda_{Monitor} = \frac{(q_1-1)q_1r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{CYC} = 0, \lambda_{HH\succ LL} = \frac{-q_0q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$

$\lambda_{HL\succ LL} = 0, \lambda_{HH\succ HL} = 0, \lambda_{HH\succ HH} = 0, \lambda_{LL\succ HH} = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = \frac{r(q_0-(q_1-1)((q_1-q)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *JPE2*, the non-negativity of w_{HL} requires $r > r^A$ and $\mu_{LL} \geqslant 0$ yields another lower bound r^L on r. The non-negativity of w_{HH} and w_{HL} also requires $r > s' \equiv \frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}$. In addition, both the Pareto Dominance and No Cycle constraints require $r < r^B$. We claim $(\max\{s', r^A, r^L\}, r^B] = (\max\{r^A, r^L\}, r^B]$. The claim is trivial if $s' \leqslant r^A$ and therefore consider the case where $s' > r^A$. Since r^A increases in H while s' is independent of H, one can show $s' > r^A$ is equivalent to $H < H'$ for a unique positive H'. Meanwhile, algebra shows that $r^B < r^A$ for $H < H'$. Therefore $s' > r^A$ implies $r^B < r^A$, in which case $(\max\{s', r^A, r^L\}, r^B] = (\max\{r^A, r^L\}, r^B] = \varnothing$.

For $\max\{r^L, r^B\} < r \leqslant r^C$ and $q_1 + q_0 \geqslant 1$, the optimal solution, denoted as *BPC2*, is:

$w_{LL} = \frac{q_0(q_1-q_0)^2H-q_0(q_1+r)}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}, w_{HL} = \frac{(q_1-q_0)^2+r-2q_0r-(q_1-q_0)^3H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))},$

$w_{HH} = \frac{(q_0+q_1)(q_1-1)+q_0r+(q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))};$

$\lambda_{Pareto} = \frac{-q_0+(-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_{Monitor} = \frac{q_0+q_1-1}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_{CYC} = 0, \lambda_{HH \succ LL} = \frac{q_0(1-q_1)}{(q_1-1)r+q_0(-1+q_1+r)},$

$\lambda_{HL \succ LL} = 0, \lambda_{HH \succ HL} = 0, \lambda_{HH \succ HH} = 0, \lambda_{LL \succ HH} = 0,$

$\lambda_{LL \succ HL} = 0, \mu_{LL} = 0, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *BPC2* , the non-negativity of λ_{Pareto} requires $q_1 + q_0 \geqslant 1$. Given $q_1 + q_0 \geqslant 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > r^B$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0+q_1-1}$. The other lower bound r^L on r is generated by intersecting requirements for $\lambda \geqslant 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leqslant r^C$. We claim $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$. Subtracting q_1 from both sides of $r^B \leqslant r^C$ and collecting terms, one obtains $s'' \leqslant r^B$ which means $r^B \leqslant r^C$ if and only if $s'' \leqslant r^B$. Therefore $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$ is verified.

As r becomes even larger, the problem $T = 1$ becomes infeasible because the intersection of the Mutual Monitoring constraint and the Principal's IC constraint(s) is an empty set. Finally, tedious algebra verifies that the solutions characterized above satisfy the "Self-Enforcing Shirk" constraint that we left out in solving the problem. Therefore adding this constraint back does not affect the optimal objective value. ■

Proof of Corollary 1a and 1b. The corollary follows directly from Proposition 1. ■

Proof of Proposition 2. Similar argument as in the Proof of Proposition 1 shows that setting $w_{LH} = 0$ is optimal. Given $w_{LH} = 0$, one can rewrite the program as follows.

$\min (1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$

s.t

$(1-q_1)w_{LL} - (1-q_1)w_{HL} - q_1 w_{HH} \leqslant \frac{-1}{q_1-q_0}$ (*Stage NE*, λ_{SNE})

$((2-q_1-q_0)+(1-q_0)r)w_{LL} + ((q_0-1)(1+r)+q_1)w_{HL} - (rq_0+q_0+q_1)w_{HH} \leqslant \frac{-(1+r)}{q_1-q_0}$ *No Joint Shirking*, (λ_{SHK})

$(1-q_1)(2+r)w_{LL} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \leqslant -\frac{1+r}{q_1-q_0}$ (*No Cycling*, (λ_{CYC})

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn \succ m'n'}$)

$-w_{LL} \leqslant 0(\mu_{LL})$; $\quad -w_{HL} \leqslant 0(\mu_{HL})$; $\quad -w_{HH} \leqslant 0(\mu_{HH})$.

For $r \leqslant r^B$, the solution, denoted as *IPE*, is:

$w_{LL} = 0$, $w_{HL} = w_{HH} = \frac{1}{q_1 - q_0}$

$\lambda_{SNE} = q_1$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = 0$, $\lambda_{HH \succ LL} = 0$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = 1 - q_1$, $\mu_{HL} = 0$, $\lambda_{12} = 0$.

Under IPE, the $ICP_{HH \succ LL}$ constraint imposes the upper bound r^B on r.

For $r^B < r < r^H$, the solution, denoted as *BPI*, is:

$w_{LL} = \frac{(q_1-q_0)^2(1+r)H-(1+r)(q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))}$, $w_{HL} = w_{HH} + w_{LL}$,

$w_{HH} = \frac{(q_1-q_0)^2 H-(1+r)(-1+q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))}$;

$\lambda_{SNE} = 0$, $\lambda_{SHK} = \frac{r(1+r+q_1^2 r-2q_1(q+r))}{(q_1-q_0)(1+r)(-1+(-1+q_1)r+r^2)}$, $\lambda_{CYC} = \frac{r(-1+q_0+q_1-r+q_0 r-q_1^2 r)}{(q_0-q_1)(1+r)(-q+(-1+q_1)r+r^2)}$, $\lambda_{HH \succ LL} = \frac{1+r-q_1 r}{-1-(1-q_1)r+r^2}$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = 0$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *BPI*, both the non-negativity of w_{LL} and the Stage NE constraints require $r > r^B$ and $\lambda_{SHK} \geqslant 0$ requires $r < r^H$.

For $\max\{r^H, r^B\} < r \leqslant r^D$, the optimal solution, denoted as *RPE*, is:

$w_{LL} = 0$, $w_{HL} = \frac{(q_1-q_0)q_1(2+r)H - \frac{(1+r)(q_1^2+r)}{q_1-q_0}}{q_1^2-r(1+r)+q_1 r(2+r)}$,

$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(-1-r+q_1 r(2+r))H}{(q_1-q_0)(q_1^2-r(1+r)+q_1 r(2+r))}$,

$\lambda_{SNE} = 0$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = \frac{(1-q_1)q_1 r}{r(1+r)-q_1^2-q_1 r(2+r)}$, $\lambda_{HH \succ LL} = \frac{q_1^2}{r(1+r)-q_1^2-q_1 r(2+r)}$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = \frac{r(1+r+q_1^2 r-2q_1(1+r))}{r(1+r)-q_1^2-q_1 r(2+r)}$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *RPE*, the Stage NE constraint and $\mu_{LL} \geqslant 0$ yields two lower bounds r^B and r^H on r. $ICP_{HL \succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leqslant r^D$. $w_{HH} \geqslant 0$ also requires $r > s \equiv \frac{2q_1-1+\sqrt{(2q_1-1)^2+4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, r^B, r^H\}, r^D] = (\max\{r^B, r^H\}, r^D]$. Consider the case where $s > r^B$ (as the claim is trivial if instead $s \leqslant r^B$). Since r^B increases in H while s is independent of H, one can show $s > r^B$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $r^D < r^B$ for $H < H^*$. Therefore $s > r^B$ implies $r^D < r^B$, in which case both $(\max\{s, r^B, r^H\}, r^D]$ and $(\max\{r^B, r^H\}, r^D]$ are empty sets.

For $r > \max\{r^D, r^H\}$, the optimal solution, denoted as *BPS*, is:

$w_{LL} = \frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}, w_{HL} = 2w_{HH},$

$w_{HH} = \frac{(1+r)(-(1-q_1)^2+r)-(q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))},$

$\lambda_{SNE} = q_1, \lambda_{SHK} = 0, \lambda_{CYC} = \frac{r}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ LL} = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2r-r^2-q_1(2+3r)},$

$\lambda_{HL\succ LL} = \frac{2(1+r+q_1^2r-2q_1(1+r))}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ HL} = 0, \lambda_{HL\succ HH} = 0, \lambda_8 = 0,$

$\lambda_{LL\succ HL} = 0, \mu_{LL} = 1 - q_1, \mu_{HL} = 0, \mu_{HH} = 0.$

Where the two lower bound r^D and r^H on r are derived from the non-negativity constraint of w_{LL} and $\lambda_{HL\succ LL}$. Collecting conditions verifies the proposition. ■

Proof of Corollary 2a and 2b. The corollary follows directly from Proposition 2. ■

Proof of Proposition 3. The proposition is proved by showing a sequence of claims.

Claim 1: $T = 0$ is optimal for $r > \max\{r^B, r^C\}$.

Claim2: *BPC2* of $T = 1$ is never the overall optimal contract.

Claim 3: If *JPE1* is optimal given $T = 1$, it is the overall optimal contract.

Claim 4: If *JPE2* is optimal given $T = 1$, it is the overall optimal contract.

Claim 5: If *BPC1* is optimal given $T = 1$, it is the overall optimal contract.

Claim 6: $\min\{r^L, r^C\} > r^B$ if and only if $r^L > r^B$.

Using Claims 1 - 5, one can verify the following statement: when $\min\{r^L, r^C\} \leqslant r^B$, $T = 1$ is optimal if and only if $r < r^B$; otherwise for $\min\{r^L, r^C\} > r^B$, $T = 1$ is optimal if and only if $r < \min\{r^L, r^C\}$. Claim 6 shows that condition $\min\{r^L, r^C\} > r^B$ is equivalent to $r^L > r^B$ and, thus, is equivalent to the statement in the proposition.

Proof of Claim 1: The claim is trivial as we know from Proposition 2 that $T = 1$ does not have feasible solution on the region.

Proof of Claim 2: Recall that *BPC2* of $T = 1$ is obtained by solving the following three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH\succ LL}$. It is easy to see that $\pi(0, 0; \mathbf{w}) = \pi(0, 1; \mathbf{w})$ when both Mutual Monitoring constraint and the Pareto Dominance constraint are binding, in which case the Mutual Monitoring constraint can be re-written as follows:

$$\frac{1+r}{r}[\pi(1,1;\mathbf{w})-1] \geqslant \pi(0,1;\mathbf{w})+\frac{1}{r}\pi(0,1;\mathbf{w}).$$

Note this is same as the "Stage NE" constraint in $T=0$ and therefore all the constraints in $T=0$ are implied by those in $T=1$ under the *BPC2* solution. In this case, $T=1$ has a smaller feasible set, so it can never do strictly better than $T=0$.

Proof of Claim 3: We know from Proposition 2 that *JPE1* is the optimal solution of $T=1$ for $r \in (0, r^A]$, over which the optimal solution of $T=0$ is *IPE* (Proposition 1). Substituting the corresponding solution into the principal's objective function, we obtain $obj_{JPE1} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1 r)}$ and $obj_{IPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{IPE} - obj_{JPE1} = \frac{q_0 q_1}{(q_1-q_0)(q_0+q_1+q_1 r)} > 0$, which verifies the claim.

Proof of Claim 4: *JPE2* is the solution of $T=1$ for $r \in (\max\{r^L, r^A\}, r^B]$, over which *IPE* is the corresponding solution of $T=0$. Algebra shows that $obj_{JPE2} = \frac{(q_1-1)q_1 r(1+r)+q_0(q_0-q_1)^2 q_1 H}{(q_0-q_1)((q_1-1)r(1+r)+q_0(q_1+r))}$, $obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE2} - obj_{IPE} \leqslant 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)} \leqslant r \leqslant r^B$ (with equality on the boundary). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)}, r^L, r^A\} \leqslant r \leqslant r^B$, which was shown in the proof of Proposition 2 to be equivalent to $r \in (\max\{r^L, r^A\}, r^B]$. Therefore, $JPE2$ is the overall optimal contract whenever it is feasible.

Proof of Claim 5: We know that *BPC1* is the solution of $T=1$ if $r \in (r^A, \min\{r^L, r^C\}]$. In this region, *IPE* and *BPI* are potential solutions in $T=0$ because the other two solutions (*RPE* and *BPS*) require $r \geqslant r^H > r^L$. Let us compare first *BPC1* of $T=1$ and *BPI* of $T=0$. It is easy to show $obj_{BPC1} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1 r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1 r-r^2)}$ and $obj_{BPI} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_0-q_1)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BPC1} < obj_{BPI}$ for $r^B < r \leqslant \min\{r^L, r^C\}$ where both solutions are feasible.

Showing *BPC1* is always more cost efficient than the *IPE* solution is more involved and is presented in two steps. We first derive the sufficient condition for this to be true and then show that the sufficient condition holds whenever both solutions are optimal in their corresponding program, namely $r^A < r \leqslant \min\{r^L, r^B, r^C\}$. Given obj_{BPC1} and obj_{IPE} defined above, one can show that $obj_{BPC1} < obj_{IPE} \Leftrightarrow r < \delta$, where

$$\delta = \frac{1}{2(1-q_1)}[((q_1-q_0)^2H - q_1)q_1(1-q_1) - 1 + \sqrt{(((q_1-q_0)^2H - q_1)q_1(1-q_1) - 1)^2 + 4(1-q_1)((q_1-q_0)^2H - q_1)(q_1+q_0)}].$$

Note that if $\delta \geqslant r^B$, then $r < \delta$ (thus $obj_{BPC1} < obj_{IPE}$) is satisfied trivially for $r^A < r \leqslant \min\{r^L, r^B, r^C\}$. Consider the opposite case in which $\delta < r^B$. For $q_0 \in [0, q_1)$, one can show that $\delta < r^B$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. Since the latter condition contradicts the maintained assumption that $(q_1-q_0)^2H > q_1$, we consider $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ only. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$, one can show $\delta > r^L$ for any $q_0 \in [0, q_1)$. Therefore, under the maintained assumption $(q_1-q_0)^2H > q_1$, $\delta < r^B$ implies $r^L < \delta$. If the choice is between *BPC1* and *IPE*, $r \leqslant \min\{r^L, r^B, r^C\}$. Then $r^L < \delta$ implies $r < \delta$. $r < \delta$ implies $obj_{BPC1} < obj_{IPE}$ whenever both are feasible (which is in the region $r^A < r \leqslant \min\{r^L, r^B, r^C\}$).

Proof of Claim 6: The "only if" direction is trivial. To show the "if" direction, note that if $r^L > r^B$, we know $q_1 + q_0 > 1$ as otherwise $r^L < 0 < r^B$. Under the maintained assumption on H, $q_1 + q_0 > 1$ implies $r^C > r^B$. Therefore, $\min\{r^L, r^C\} > r^B$ if and only if $r^L > r^B$. ■

Proof of Corollary 3a and 3b. The corollary follows directly from Proposition 3. ■

Proof of Corollary 4. For $T = 1$, recall from Proposition 1 that the (old) *No Cycling* constraint does not bind and that all solutions in Proposition 1 satisfy strategic complements, i.e., $U(1,1) - U(0,1) > U(1,0) - U(0,0)$. It is easy to show that strategic complements and the *Pareto Dominance* constraint together imply $2U(1,1) > U(0,1) + U(1,0)$. Therefore, if $T = 1$, agents will not collude on playing any strategies that involve only *work* and *shirk*. This proves the part (i).

For $T = 0$, it is straightforward to plug in the new cycling constraint, investigate the Lagrangian, and verify the first part of Corollary 4 - (ii). The second part of (ii) follows from the observation that the (old) *No Cycling* constraint is more restrictive than the new cycling constraint. Note that the two agents are willing to collude on playing the off-diagonal *work* and *shirk* only if their *joint* payoff satisfies $U(1,0) + U(0,1) > 2U(1,1)$. Given $T = 0$, $U(1,0) + U(0,1) > 2U(1,1)$ implies $U(1,0) - U(1,1) > U(1,1) - U(0,1) \geq 0$, where the last inequality follows from the *Static NE* constraint that must hold if $T = 0$. The observation $U(1,0) - U(1,1) > U(1,1) - U(0,1) \geq 0$ further implies $U(1,0) > U(1,1) > U(0,1)$, which together with the time value argument, suggests that our *No Cycling* constraint provides strictly higher continuation payoff for the shirking agent in (0,1) (hence more costly for the principal to break) than any collusive strategy having agents randomizes between *work* and *shirk*.

Part (iii) follows directly form Proposition 3 and parts (i) and (ii) Corollary 4. ■