Strategic Timing of IPO - A Dynamic Model of Multiple Firms[*]

Cyrus Aghamolla[†]

Graduate School of Business

Columbia University

Ilan Guttman[‡]

Stern School of Business

New York University

November 2015

Abstract

We study a dynamic timing game between multiple firms, who decide when to sell the firm (IPO) or a project. A firm's IPO pricing is a function of its privately observed idiosyncratic type and the period's realization of a factor common to all the firms. The common factor follows a stochastic mean-reverting process, and the market learns about its realization in a given period if there is at least one IPO in that period. Firms consider the trade-off between the direct costs of delaying the IPO and the value of the real option from potentially learning the common factor. We characterize the unique symmetric threshold equilibrium and find that: higher-type firms go public earlier; following successful IPO(s) in the first period we should expect more (clustering of) IPOs in the second period; in more concentrated industries fewer firms go public early, but these are met with more intense clustering (IPO waves). The results also relate delay and clustering of IPOs to changes in the initial market uncertainty.

1 Introduction

In 2014, U.S. public equity markets saw more initial public offerings (IPOs) than in any year since the 2000 dot-com boom. The recent wave of IPOs has been especially interesting given the initial difficulty the market had in evaluating firms in new industries, particularly social media and cloud computing. As one commentator noted during the 100% price increase on the initial day of trading for LinkedIn: "New internet companies based on new and innovative technologies are more difficult to value."[1] In new industries with uncertain fundamentals, firms that had received higher than expected valuations led to further, more immediate public offerings by other firms within the same industry, whereas firms who received less favorable valuations led to delay in the IPO plans of other similar firms. For example, consider the pioneer firm to go public in the new social media industry, Facebook. The price fall that ensued Facebook's IPO allegedly pushed back the offering of Twitter for several months. Twitter went public only when the market was better able to assess Facebook's value, in a very favorable way, which resulted in a tremendous price increase around the IPO. Indeed, the ability to observe the market sentiment before going public provide firms an advantage in choosing the timing of their IPO.[2] The strategic timing of IPOs has also been well-documented in the news (e.g., the case of Virtu who delayed its IPO due to dissatisfaction over flash-trading[3]) and in the empirical literature (e.g. Lowry and Schwert (2002), Brau and Fawcett (2006)).

We seek to study this phenomenon in a strategic game of disclosure/IPO by multiple firms, in which each firm chooses when to disclose its private information and go public (or sell a project). Unlike existing models of IPO timing, which largely feature either a single firm, multiple firms whose actions are independent or irrelevant to one another, or multiple firms who act in an exogenously determined order, we consider many firms whose endogenous IPO timing decisions are *interdependent*. We are thus able to show the endogenous emergence of pioneer firms in the face of free-riding, as we all as capture interesting properties of the

---

[1] "Wall Street 'mispriced' LinkedIn's IPO." *Financial Times*, March 30, 2011.

[2] Several other firms, such as Kayak, have been reported to delay their IPO dates specifically because of the market reaction to the Facebook IPO. See "Did IPO damage Facebook brand?", *CBS Money Watch,* June 6, 2012.

[3] "For Virtu IPO, Book Prompts a Delay." *The Wall Street Journal*, April 3, 2014. The timing is a serious concern for firms: "Analysts said Virtu had little choice but to postpone the offering. 'The timing couldn't be worse,' said Pat Healy, CEO of Issuer Advisory Group LLC, which advises companies on going public."

timing and clustering of IPOs which have hitherto not been characterized in the literature.[4]

We study the following three-period multi-firm/entrepreneur setting. The value of each firm/project is determined by two components: an idiosyncratic component, which we refer to as the firm's type, and a common component which affects all firms in the industry/economy who consider an IPO. The first ingredient of our model is that, given all else equal, each firm's manager/entrepreneur prefers to sell the firm/project as early as possible. This assumption could reflect that delaying the IPO leads to, for example: forgoing profitable investment and expansion opportunities, potential loss of market power relative to competitors and hence reduced payoff, the costs of debt that is used to finance projects or operations, or even the tendency of a firm's idiosyncratic component to mean-revert. To capture this time preference, we assume that firms/managers discount the future payoff from selling the firm/project. The second ingredient of our model pertains to the common factor (or "state of nature"), which can capture an industry-specific valuation discovered during an IPO process, the state of the economy/industry, or market sentiment.[5] The state of nature is assumed to follow a mean-reverting stochastic process.[6] Bessembinder et al. (1995) found that all the markets they examined are characterized by mean-reversion, where there is substantial variation across industries in terms of the reversal rates. The state variable can also be thought of as reversal of macroeconomic shocks, as evidenced by Bloom (2009) and Bloom et al. (2014).

As part of an IPO process, the market learns and forms an opinion about the new technology or the market conditions (captured by the state of nature/common factor) and reveals this information through the pricing of the IPO. The fewer firms go public in a given period the less the market learns about the state of nature. Had Facebook not gone public in May 2012, there would have been a much greater uncertainty about the market's perception of the value and the potential of the social media industry. To capture this key aspect of IPOs in the simplest, most tractable, way we assume in the base model that a period's state

---

[4]Several studies, such as Hoffman-Burchardi (2002) and Benveniste et al. (2003), have noted the puzzling emergence of pioneer firms. Particularly, Benveniste et al. (2003) comment, "What then prevents the market from collapsing around the incentive for potential issuers to free-ride one another?" (p. 577).

[5]There is evidence that firms in different industries have different market sentiments, and that IPOs within an industry share similar one-day returns and similar average returns. For example, technology IPOs performed very well in 2014, whereas bank IPOs often failed to meet their price range. See "Bank IPO Falls Short of Target Price Range," *The Wall Street Journal*, September 24, 2014. For empirical evidence, see Maur and Senbet (1992).

[6]The key trade-off that we identify does not rely on mean reversion, but rather exists for any serial correlation structure in the state of nature.

of nature is observed as long as at least one of the firms goes public in this period. We later relax this assumption and show that the results hold in the more realistic setting, in which the precision of the beliefs about the period's state of nature increases in the number of firms that go public (see Section 5.2). It is easy to see that the main results of the base model hold under the exogenous assumption that the precision of the information about the common factor increases in the number of IPOs. We then endogenize the link between the precision of the information about the state of nature and the number of IPOs by assuming that following an IPO, the market observes the pricing of the IPO but cannot perfectly disentangle the firm's idiosyncratic component from the common factor. In such a setting, the precision of the inference about the common factor is increasing in the number of IPOs. The higher the precision of the beliefs about the common factor following the IPOs, the higher the expected value of the real option from delaying the IPO, and hence the higher the incentive to delay the IPO.

The notion of a common factor in an IPO setting has been suggested previously in empirical studies, notably Lowry and Schwert (2002) and Benveniste et al. (2003). Indeed, Lowry and Schwert (2002) note that "initial returns [of recent IPOs] contain valuable information for private companies considering an IPO" (p. 1183).[7]

The mean reverting nature of the common factor gives rise to a real option from delaying the IPO in the first period. In case a firm delays its IPO and another firm goes public, the state of nature in the first period is revealed. If the realization of the state of nature in the first period is sufficiently low, the firm is better off delaying its IPO until the third period. After a poor IPO, the state is expected to be low in the second and third periods as well, however, the mean-reverting property implies an expected improvement in market conditions from the second to the third period. Likewise, if the realization of the state of nature in the first period is sufficiently high, a firm that did not IPO in the first period finds it more profitable to go public in the second period than delaying its IPO. When deciding whether to IPO in the first period, the firm considers the trade-off between the direct costs of delaying the IPO and the benefit from the expected value of the real option from delaying the IPO. The firm considers the probability that the other firms will disclose and IPO in the

---

[7]Lowry and Schwert (2002) also note that "We find that more companies file IPOs following periods of high initial returns because the high returns are related to positive information learned during the registration periods of those offerings, suggesting that companies can raise more money in an IPO than they previously though" (p. 1173).

4

first period, as if no other firm goes public in the first period, the state of nature will not be revealed and the option value will not be realized. This introduces strategic interaction between firms, as the IPO strategy of one firm affects the payoff and the optimal strategy of the other firms.

We analyze the above setting and show that there exists a unique symmetric equilibrium in which firms follow a threshold strategy in each period. In particular, each firm goes public in the first period if and only if the realization of its idiosyncratic component is sufficiently high. If there was no IPO by any firm in the first period, then the first-period state of nature is not revealed, and hence, all firms go public in the second period (as the game ends in the third period). If at least one firm went public in the first period, then a firm that did not IPO in the first period goes public in the second period only if the realization of the first-period state was sufficiently high. The threshold realization of the first-period state of nature following which a firm will delay its IPO is decreasing in the firm's type, i.e. a lower realization of the state is needed for high type firms to take advantage of the real option. Low-type firms are thus comparatively more inclined to delay their IPO not only in the first period, but in the second period as well. The reason is two-fold: (i) the cost of delay due to the discount is comparatively lower for low-type firms, and (ii) the value of the real option from delaying the IPO in the first period is decreasing in a firm's type.

The results of the model are in line with several empirical regularities. Our model predicts the clustering of IPOs following successful IPOs, which has been documented by Ibbotson and Jaffe (1975), Ritter (1984), Ibbotson, Sindelar, and Ritter (1988, 1994), and Hoffman-Burchardi (2001), among others. Moreover, the results are in line with the findings of Lougran, Ritter, and Rydqvist (1994), Lerner (1994), Pagano, Panetta, and Zingales (1998), Lowry and Schwert (2002) and Benveniste et al. (2003), which document the strategic timing of IPOs. We formally capture the hypothesis of Lowry and Schwert (2002), who interpret their findings of increased IPO volume following high initial returns of recent IPOs as due to observational learning by private firms, as well as Benveniste et al. (2003) who interpret similar findings as due to the presence of a common valuation factor. The results of our model also imply that clustering should be composed primarily of firms within a specific industry and that high returns of recent IPOs should induce more clustering, as documented by Ritter (1984) and Lowry and Schwert (2002).

Furthermore, several interesting insights and empirical predictions emerge from this

analysis. We see that there is always a positive amount of delay of going public in equilibrium, where sufficiently high type firms do not delay. In general, the model predicts that the higher a firm's value (idiosyncratic component), the earlier it will go public, as higher type firms exhibit a lower value of the real option from delaying the IPO and a greater discounting costs. Hence, IPO timing is determined by firm value, and pioneer IPOs are issued by the higher value firms. The results also predict that the extent of clustering and delay depends on the concentration of firms in the industry. Specifically, industries composed of comparatively numerous private firms will experience greater delay. Moreover, because a higher industry concentration induces some high-type firms to delay, the likelihood of clustering following an IPO is greater with a higher concentration, thus giving rise to more intense IPO "waves." The results of the model also imply that there is greater delay when there is more initial uncertainty about the state of nature. This is especially applicable for new industries without a close counterpart, who thus potentially face a great deal of uncertainty. The effect of the level of serial correlation in the state of nature on the incentive to delay the IPO is non-monotone. In particular, the disclosure threshold, as a function of the degree of mean-reversion (in both the first and second period), exhibits an inverse U-shape.

1.1 Related Literature

The extant theoretical literature on IPO timing largely consists of models which include either a single firm (or multiple firms embedded in a single-firm setting), or multiple firms that move in an exogenously determined order. We first discuss the latter models and how they differ from our model.

Benveniste, Busaba, and Wilhelm (2002) examine a two-firm model that, similar to our model, includes both an idiosyncratic component and an unknown common factor. The timing and IPO order, however, is assumed to be fixed, such that one firm is designated to move first and the other firm follows. Benveniste, Busaba, and Wilhelm argue that potential free-riding can be solved by the investment bank "bundling" IPOs together so that the costs of information discovery are shared across firms. However, this bundling crucially depends on the investment bank's monopoly power over issuing firms. For example, the results are less applicable in a setting where issuing firms may choose one of several underwriting banks. Moreover, as evidenced by Lowry and Schwert (2002), there is a significant positive relation between initial returns of recent IPOs and subsequent IPO filings. Hence, their

model does not account for firms who do not even begin the preliminary *filing* for an issue until after observing the market reaction of recent IPOs, which substantially contribute to IPO clustering/waves. In contrast, we allow the firms' timing of the IPO to be endogenously determined, and show that pioneer firms endogenously emerge and bear the (implicit) cost of information production (in terms of giving up their real option).

Persons and Warther (1997) develop a model of financial innovation among several firms who may move sequentially. Each firm observes the noisy cash flow returns of firms who have already adopted the innovation and based on this information decides whether to adopt the innovation. They generate "booms" in the adoption of the new technology, as each additional firm that adopts the innovation may lead to another firm's subsequent adoption. However, a fundamental assumption in their model is that it is common knowledge which firms benefit the most from the adoption of the technological innovation, and, correspondingly, the firms adopt the technology in a predetermined order, beginning with the firm that benefits the most. This would be equivalent to the model here where each firms' idiosyncratic component was commonly known, the state of nature does not follow a mean-reversion process, and adoption of the innovation increases the precision of the beliefs about the profitability of the innovation.

Likewise, Alti (2005) develops a model of information spillover in an IPO setting, where information asymmetry decreases following an IPO, which consequently lowers the cost of going public for the other firms. The cost of going public is due to adverse pricing by the market in a second price auction in the presence of an informed trader. The common component among firms is the cash flow generated in the period of IPO, which is assumed to be identical to all firms (and not mean-reverting). The support of per-period cash flow, however, is assumed to be binary and unchanging. Maksimovic and Pichler (2001) consider a timing game where firms may delay their IPO plans, however, firms designated as "pioneering" move first while those designated as "potential entrants" move second. The present setting does not make such designations.

Pastor and Veronesi (2003) model the strategic timing of an IPO as an inventor who faces a problem analogous to an American call option. The inventor can exercise the option to capitalize on abnormal profits, but sacrifices the possibility that market conditions may worsen to cover the initial investment. Our model varies in that we incorporate strategic interaction between firms that affects the timing of IPOs. A number of other papers look

at the strategic timing of IPOs in a single-firm setting. He (2007) considers a game between investment banks and investors to generate high first day returns during periods of high IPO volume. Chemmanur and Fulghieri (1999) models IPO timing as a trade-off between selling the firm to a risk-averse venture capitalist at a discount or through the loss in informational advantage from going public. Benninga, Helmantel, and Sarig (2005) model the decision to go public as a trade-off between diversification and the private benefits of control. They generate IPO waves during periods when expected cash flows are high. Our model differs from these three as they are all single-firm models, whereas we are principally interested in the strategic interaction between firms and the resulting clustering effects.

Our model varies from the literature on dynamic voluntary disclosure (e.g., Dye and Sridhar (1995), Acharya DeMarzo, and Kremer (2011), Guttman, Kremer, and Skrzypacz (2014), Aghamolla and An (2015)) in three ways. In our setting (i) the manager receives information with probability one and disclosure is costless, (ii) the entrepreneur is only concerned with the firm's value in the period of disclosure and IPO, and (iii) there are multiple firms/entrepreneurs whose decisions are interrelated.[8]

The following section presents the setting of the model and section three analyzes the equilibrium. Section four examines comparative statics and offers empirical predictions. Section five studies extensions of the model in which the state is imperfectly observed, firms can IPO without disclosure, and where the support of the firms' type is bounded. The final section concludes. Proofs are relegated to the Appendix, unless otherwise stated.

2 Model Setup

We study a setting with three periods, $t \in \{1, 2, 3\}$, and $N \geq 2$ firms. A firm's value is a function of its idiosyncratic component and the value of a common factor. Prior to $t = 1$, each firm's manager/entrepreneur privately observes the idiosyncratic component of her firm's value or project, θ_i, which is the realization of a random variable $\tilde{\theta}$ with a cumulative density $G(\theta)$ and probability density function $g(\theta)$. We will often refer to θ_i as the type of firm i. The support of θ is $[0, \infty)$ and $g(\theta)$ is positive over the entire support of θ.[9] For all $i \neq j$ the idiosyncratic components, θ_i and θ_j, are independent. We constrain

---

[8]The latter feature is present in Dye and Sridhar (1995).

[9]We later study the case in which the support of θ is bounded from above, i.e., $\theta \in [0, \bar{\theta}]$ and show that the symmetric equilibrium that we characterize in the current section still holds.

θ to be non–negative since this simplifies the analysis, however, the results would not be qualitatively affected with negative firm values.[10] Firms' managers/owners are assumed to be risk-neutral.

Every firm manager must IPO the firm (or sell the project) in one of the periods, while as part of the IPO the manager discloses the private type, θ_i. Disclosure of the type is credible and costless. The managers are assumed to maximize the firm's market price at the time of IPO. For example, the manager/owner may want to IPO the firm and needs to make a disclosure at the time of the IPO. In section 5.2, we examine the model without disclosure, where firms only observe the prices of other firms that went public. Our results are qualitatively insensitive to this alternative specification.

The firm's price at the time of the IPO depends on investors' beliefs about both the idiosyncratic component, θ_i, as well as on the state of nature at the time of the IPO, which is denoted by s_t. The market price at time τ of firm i that discloses θ_i at $t = \tau$ equals investors' expectation of $\theta_i + s_\tau$ given all the available information at $t = \tau$, which we denote by Ω_τ. Every firm's manager has a time preference (discount) which is denote by r, such that the expected utility of the owner/manager of firm i from going public and disclosing θ_i at $t = \tau$ is given by:

$$u_{i,\tau} = \frac{E\left(\theta_i + s_\tau | \Omega_\tau\right)}{(1+r)^{\tau-1}}.$$

Discounting is meant to capture the costs associated with delaying the sale of a project or shares. Such cost could be due, for example: costs of debt, the cost from forgoing investmenting, operating and acquisition opportunities due to lack of financing, and the decrease in profitability due to increase in competition.

The state of nature in each period, s_t, is ex-ante unobserved, however, upon IPO by at least one of the firms, all firms learn s_t at the end of the period in which an IPO took place.[11] We assume that the state of nature follows a mean-reverting $AR(1)$ process of the form:

$$s_t = \gamma s_{t-1} + \varepsilon_t,$$

---

[10]With negative values, firms would be compelled to delay disclosure since discounting works to improve the firm's payoff. We eliminate this case so as not to confound the results.

[11]In Section 5.2 we analyze the more realistic setting in which precision of the beliefs about the period's state of nature increases in the number of firms that go public. All the qualitative results of the base model are robust to the extended setting.

where $\gamma \in (0,1)$ and $\varepsilon_t \sim N(0, \sigma^2)$ with a cumulative distribution function $F(\cdot)$ and density function $f(\cdot)$.[12] The initial state is given by $s_0 = 0$, and so the first period's state is given by $s_1 = \tilde{\varepsilon}_1$. Hence, the state of the economy in the first period is simply a mean-zero error term.[13]

The mean-reversion property of the state of nature, which is one of the central assumptions in our model, is taken exogenously. However, both the empirical and theoretical literature provide ample support for mean reversion of both specific stock returns (e.g., Fama and French (1988) and Poterba and Summers (1988)) and of macroeconomic measures, such as stock market indices (e.g., Richards (1997)). Mean reversion can be motivated by fully rational settings (e.g., Cecchetti, Lam, and Nelson 1990) and high-order beliefs in an overlapping generation (as in Allen, Morris, and Shin (2006))[14], or by behavioral explanations such as investors sentiment and limits to arbitrage (e.g., Baker and Wurgler (2006)). Mean reversion of the state of nature in our setting can also be motivated by dynamic competition in the market that affects the common factor. For example, when the state of nature, which may represent the perceived profitability of the relevant technology, is high in the first period, firms have an incentive to increase their activity in this market/technology, which in return will decrease the profitability in this market. A symmetric argument applies to a low state of nature.

The sequence of events in the game is as follows: Prior to $t = 1$ all managers/firms privately observe the idiosyncratic component of their firm value, θ_i. In $t = 1$, each firm decides weather to IPO in this period. In each period, firms make their decisions simultaneously. If at $t = 1$ at least one firm made an IPO the state of nature at $t = 1$, s_1, is publicly observed and firms that disclosed and IPO receive their market valuation. Those firm managers receive their corresponding payoff and the remainder of the game is irrelevant for them. At period $t = 2$, all firms that did not IPO at $t = 1$ decide whether to IPO or delay the IPO to

---

[12] Alternatively, we could have the variance of the error decreasing in each period to reflect the market's ability to better evaluate the firm in later periods. This would not affect the results since firms are assumed to be risk neutral and only the variance level in the first period, which affects the value of the real option from delaying disclosure, is consequential.

[13] We assume normality of ε_t primarily for consistency with the literature and for tractibility of some of the comparative statics. However, as we later show, the main results hold for any distribution of ε_t as long as mean-reversion of s_t is preserved.

[14] Mean reversion due to high-order beliefs in an Allen, Morris and Shin setting is as follows. Since in the first period the private signals are underweighted in the price formation it gives rise to a biased investors beliefs about the intrinsic value. As time goes by, on expectation, this bias decreases and the price converges to the unbiased mean.

$t = 3$. If at least one firm IPO at $t = 2$ the realization of the state of nature, s_2 is publicly revealed. The market valuation of firms that IPO at $t = 2$ is determined and managers of those firms receive their payoff. Finally, at $t = 3$, which is the last period of the game, all firms that have not yet gone through IPO must do so and those firm's managers obtain their payoff. The timeline of a generic period is given in Figure 1.

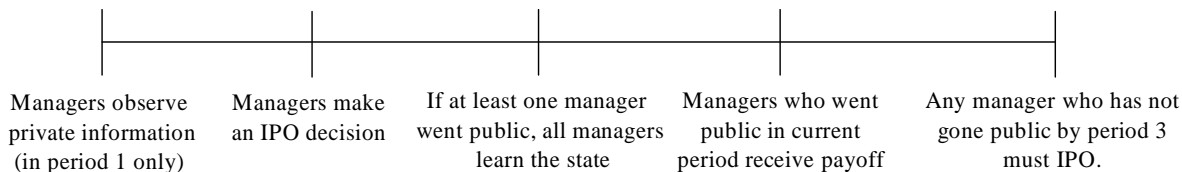| Managers observe private information (in period 1 only) | Managers make an IPO decision | If at least one manager went public, all managers learn the state | Managers who went public in current period receive payoff | Any manager who has not gone public by period 3 must IPO. |

Figure 1 – Sequence of the stage game.

We assume that all firms are ex-ante homogeneous, that is, all firms have the same distribution of idiosyncratic component of value, θ_i, the same discount rate, r, and that the common factor, s_t, affects every firm's market value in the same way. The following section analyzes the equilibrium of the above reporting game.

3   Equilibrium

Before we derive the equilibrium of our setting, note that in a two-period (rather than three-period) version of our model, all firms IPO at the beginning of the game. The reason is that in the first period, none of the managers have any information about s_1, and hence, the expected value of s_2 (which in this case is the last period) is zero. As such, the expected payoff from delaying the IPO is $\frac{\theta_i}{1+r}$, which is lower than the expected payoff from IPO at $t = 1$ (which is θ_i).

We conjecture a symmetric threshold equilibrium, in which each firm IPO in the first period if and only if its type, θ_i, is greater than a threshold θ_1^*, which is a function of all the parameters of the model (the number of firms, the distributions of the types, the distribution of the state of nature, the degree of mean-reversion and the discount factors). At $t = 2$, if at least one firm $j \neq i$ went public at $t = 1$ and the state of nature s_1 was revealed, firm i IPO if and only if $\theta_i > \theta_2^*(s_1)$. Note that if there were no IPOs at $t = 1$, then this reduces to the two-period setting mentioned above, and hence, all firms IPO in $t = 2$. Given that there is positive probability of IPO by at least one other firm in the first period, firm i has

11

a real option from delaying the IPO at $t = 1$, hoping to observe s_1 at the end of period 1. Upon observing the state of nature, s_1, for sufficiently negative realizations of the state of the economy, the firm rather delay the IPO until $t = 3$, as the state of nature follows a mean-reverting process, such that the state of nature is expected to increase towards zero at $t = 3$.

In light of the above behavior in period 2, firms at $t = 1$ have to take into consideration the trade-off between the benefit from the above real option and the cost of delaying the IPO. The cost of delaying, due to the discount factor r, increases in the firm's type, θ_i. Moreover, as we show below, the value of the real option from delaying the IPO at $t = 1$ is decreasing in the firm's type, θ_i. As such, both of the above effects work in the same direction. That is, any firm follows a threshold strategy at $t = 1$ such that, for realizations of θ_i that are sufficiently high, the manager prefers to IPO at $t = 1$, whereas for lower realizations the manager is better off delaying the IPO at $t = 1$. We solve for the unique symmetric threshold equilibrium. We start by deriving the IPO policy in the second period and then analyze the first period's decision.

3.1 Period 2

As indicated above, if no firm went public at $t = 1$, all firms IPO at $t = 2$.

Given an IPO by at least one firm at $t = 1$ and the realization of s_1, firm i of type θ_i is indifferent between going public and delaying the IPO at $t = 2$ if and only if the following indifference condition holds:

$$\frac{\theta_i + E\left(s_2|s_1\right)}{1 + r} = \frac{\theta_i + E\left(s_3|s_1\right)}{\left(1 + r\right)^2}.$$

The above has a unique solution. The unique optimal strategy in $t = 2$, which we denote by $\theta_2^*\left(s_1\right)$, is as follows.

Lemma 1 *In any equilibrium, the strategy of firm i that did not IPO at $t = 1$ is as follows. If no firm went public at $t = 1$, firm i goes public at $t = 2$. If at least one firm went public at $t = 1$ (and hence s_1 is observed) firm i follows a threshold strategy at $t = 2$ such that it*

goes public if and only if [15]

$$\theta_i \geq \theta_2^*(s_1) \equiv -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right). \tag{1}$$

Having observed the market condition in the first period, s_1, firms will delay the IPO only for sufficiently negative values of s_1. Note that for all $s_1 \geq 0$, all managers that did not IPO at $t = 1$ will IPO at $t = 2$, as both effects (discounting and the reversal of the state of nature) work in the same direction - not to delay IPO. When the realization of s_1 is negative (or in general lower than the mean of s) the mean-reversion property of s implies that s_3 is expected to be higher than both s_1 and s_2, which provides an incentive to delay the IPO to $t = 3$. However, delaying the IPO is costly due to discounting, and hence, the manager's IPO threshold at $t = 2$ resolves the trade-off between these two effects.

To further the intuition for the threshold at $t = 2$, it is useful to consider extreme parameter values. For $\gamma = 1$, such that the state of nature follows a random walk, the manager goes public at $t = 2$ if and only if $\theta + s_1 > 0$. On the contrary, when $\gamma = 0$, such that s_1 and s_2 are independent, the manager goes public immediately. For extreme values of the discount rate it is easy to see that for $r = 0$ firms IPO at $t = 2$ if and only if $\gamma s_1 > 0$, or equivalently $s_1 > 0$, as the only effect in place is the reversal of the state of nature. As the discount rate goes to infinity, all firms would have gone public at $t = 1$ (and if did not IPO at $t = 1$ IPO at $t = 2$ if and only if $\theta + s_1 > 0$). We investigate the comparative statics in section 4.

Next, we analyze the equilibrium behavior at $t = 1$.

3.2   Period 1 and the option value from delayed disclosure

We conjecture a threshold strategy at $t = 1$ such that firm i goes public in the first period if and only if $\theta_i \geq \theta_1^*$. Recall that if the manager of firm i goes public in $t = 1$, her expected payoff is $\theta_i + E[s_1] = \theta_i$. If manager i does not IPO at $t = 1$, then her payoff depends on whether at least one other firm goes public at $t = 1$. If there were no IPOs at $t = 1$, firm i (as well as all other firms) will IPO at $t = 2$ and will obtain an expected payoff of $\frac{E(\theta_i + s_2)}{1+r} = \frac{\theta_i}{1+r}$.

---

[15] An alternative way to think about the disclosure strategy is to take θ_i as given and to specify the realizations of s_1 for which the firm will and will not disclose at $t = 2$. This approach yields that for a given θ_i firm i discloses at $t = 2$ if and only if $s_1 < s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$.

If at least one firm went through IPO at $t = 1$, then firm i will IPO at $t = 2$ if and only if $\theta_i > \theta_2^*(s_1)$, or equivalently, if and only if $s_1 > s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}$, where $s_1^*(\theta_i)$ is the realization of s_1 for which a firm θ_i is indifferent between IPO at $t = 2$ or delaying the IPO to $t = 3$. So, conditional on the realization of s_1 being sufficiently high to induce an IPO of firm i at $t = 1$, the expected payoff of the firm is $\frac{E(\theta_i + s_2 | s_1 > s_1^*(\theta_i))}{1+r}$. If the realization of s_1 is sufficiently low, i.e., $s_1 < s_1^*(\theta_i)$ firm i will delay the IPO to $t = 3$, in which case its expected payoff is $\frac{E(\theta_i + s_3 | s_1 < s_1^*(\theta_i))}{(1+r)^2}$. In summary, the expected payoff of manager i from delaying the IPO at $t = 1$ is:

$$\Pr\left(ND_{j \neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) \tag{2}$$

$$+ \left(1 - \Pr\left(ND_{j \neq i}^1\right)\right)\left(\begin{array}{c} \Pr\left(D_i^2\right) E\left[\text{payoff at } t = 2 | \theta_i,\, D_i^2\right] \\ + \Pr\left(ND_i^2\right) E\left[\text{payoff at } t = 3 | \theta_i,\, ND_i^2\right] \end{array}\right),$$

where $\Pr\left(ND_{j \neq i}^1\right)$ is the probability that no IPO is made by any other firm at $t = 1$, D_i^2 (ND_i^2) indicates that firm i goes public (does not IPO) at $t = 2$, and $\Pr\left(D_i^2\right)$ $(\Pr\left(ND_i^2\right))$ is the probability that firm i, which did not IPO at $t = 1$, will IPO (not IPO) at $t = 2$.

We analyze a symmetric equilibrium of $N \geq 2$ firms whose types θ_j are independent, so the ex-ante probability of IPO is identical to all firms. Consequently, the probability that no IPO is made at $t = 1$ by any other firm is $\Pr\left(ND_{j \neq i}^1\right) = \left[G\left(\theta_1^*\right)\right]^{N-1}$. The probability that firm i with type θ_i that did not IPO at $t = 1$ will IPO at $t = 2$, given that s_1 was revealed, is the probability that the realization of s_1 will be sufficiently high, such that (1) holds. That is, for any given θ_i the firm will IPO at $t = 2$ if and only if $s_1 > s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}$. The probability of such an event is $F\left(\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}\right)$. Substituting the above into the expected payoff of the manager of firm i from not going public at $t = 1$, given in (2), yields:

$$\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_1^*}{1+r}\right) \tag{3}$$

$$+ \left(1 - \left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left(\begin{array}{c} F\left(\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}\right) E\left[\text{payoff at } t = 2 | \theta_i,\, D_i^2\right] \\ + \left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}\right)\right) E\left[\text{payoff at } t = 3 | \theta_i,\, ND_i^2\right] \end{array}\right).$$

Note that unlike the threshold in $t = 2$, which depends on the manager's type and the realization of s_1, the IPO threshold of the first period, θ_1^*, depends only on the firm's type,

θ_i (and all the other parameters of the model).

In order to derive and analyze the equilibrium, it is useful to define and characterize the properties of the manager's real option from delaying IPO at $t = 1$. The option value arises from the manager's opportunity to determine his IPO decision at $t = 2$ based on the observed value of s_1 (whenever at least one other manager IPO at $t = 1$). As Lemma 1 prescribes, the manger prefers to take advantage of the real option and to delay IPO at $t = 2$ only for sufficiently low values of θ and s_1. To capture the option value that stems from not going public at $t = 1$ we first express the expected payoff of a type θ_i manager who is not strategic and always IPO at $t = 2$. We denote the expected payoff of such non-strategic manager by $NS(\theta_i)$, which is given by:

$$NS(\theta_i) \equiv E\,[\text{Payoff if IPO at } t = 2] = E\left[\frac{\theta_i + s_2}{1 + r}\right] = \frac{\theta_i}{1 + r}.$$

The expected payoff of a type θ_i manager that never goes public at $t = 1$ but is strategic at $t = 2$, which we denote by $S(\theta_i)$ (where S stands for strategic), is given by:

$$S(\theta_i) \equiv E\,[\text{Payoff if follows IPO strategy } \theta_2^* \text{ at } t = 2].$$

Finally, we define the option value as the increase in the expected payoff of a manager who does not IPO in $t = 1$ from being strategic in $t = 2$, relative to always IPO in $t = 2$. The option value, which we denote by $V_2(\theta_i)$ is given by:

$$
\begin{aligned}
V_2(\theta_i) &\equiv S(\theta_i) - NS(\theta_i) \\
&= \Pr\left(s_1 < s_1^*(\theta_i)\right) E\left[\frac{\theta_i + s_3}{(1 + r)^2} - \frac{\theta_i + s_2}{1 + r}\Big| s_1 < s_1^*(\theta_i)\right].
\end{aligned}
$$

The following Lemma describes a fairly intuitive property of the option value, which is very useful in showing existence and uniqueness of the symmetric threshold equilibrium.

Lemma 2 *The option value is decreasing in θ_i, i.e.,*

$$\frac{\partial V_2(\theta_i)}{\partial \theta_i} < 0.$$

Intuitively, the option value is decreasing in θ_i due to two effects. The first is that the discounting is comparatively more punitive for higher type firms, and hence, delaying

disclosure is relatively more costly for high type firms. The second, and more salient effect, is that the likelihood of taking advantage of the real option in period 2 is decreasing in θ_i. The reason for this can be seen from Lemma 1; the manager at time $t = 2$ only delays the IPO for sufficiently negative realizations of s_1. Moreover, higher θ firms require even lower realizations of s_1 in order to find it profitable to delay the IPO until $t = 3$. As such, the likelihood of obtaining a sufficiently low realization of s_1 such that the manager take advantage of the real option and delay the IPO at $t = 2$ is decreasing in his type, θ. So both of the above effects point at a decreasing real option as a function of the firm's type, θ. The proof of the Lemma provides a full and formal analysis.

Having established that the option value from delaying the IPO is decreasing in θ, and given that the cost of delaying the IPO (due to discounting) is increasing in θ for any given strategy of the other firms, we can conclude that the optimal strategy in any equilibrium can be characterized by a disclosure/IPO threshold.

Corollary 1 *In any equilibrium, any firm's optimal strategy is characterized by an IPO threshold in both $t = 1$ and $t = 2$.*

We next solve for and analyze the symmetric equilibrium, in which all firms follow the same strategy. We show that there is a unique symmetric equilibrium. While our main focus is the symmetric equilibrium in the setting with an unbounded support, we study in section 5 an extension of the model in which the support of firm's type is bounded from above, i.e., $\theta \in [0, \bar{\theta}]$. For this setting, the symmetric equilibrium still always exists, however, for sufficiently low discount factors (and sufficiently low $\bar{\theta}$) we show the existence of another equilibrium, in which one firm always goes public at $t = 1$ and all the other firms never IPO at $t = 1$. Such an equilibrium does not exist in our main setting in which the support of the firm's type is unbounded above.

In a symmetric equilibrium, each manager's best response to all other managers' strategies, who play a threshold strategy θ_1^*, is consequently given by θ_1^*. The $t = 1$ threshold level of all firms is such that each manager of the threshold type, θ_1^*, is indifferent between going public and not going public at $t = 1$. Therefore, the threshold level is the type for which θ_1^* equals the expected payoff from not going public at $t = 1$, given in equation (3).

Lemma 3 *The threshold at $t = 1$ is given by the solution to the following indifference condition of the manager at $t = 1$:*

$$\theta_1^* = [G(\theta_1^*)]^{N-1}\left(\frac{\theta_1^*}{1+r}\right) \tag{4}$$

$$+ \left(1 - [G(\theta_1^*)]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right].$$

3.3 Unique Symmetric Equilibrium

In this part we establish that there exists a unique equilibrium in which all firms follow the same threshold strategy. We refer to this equilibrium as the symmetric equilibrium. Using Lemmas $1-3$, we show existence and uniqueness of a symmetric threshold equilibrium. Lemmas 1 and 3 tie down the IPO thresholds in a symmetric equilibrium. We use Lemma 2 to show that this equilibrium exists – any firm whose value is above the threshold indeed finds it optimal to go public at $t = 1$, given the discounting costs and since the option value is decreasing in θ. Moreover, we show that the threshold characterized by Lemma 1 and Lemma 3 is the unique threshold level in the symmetric equilibrium.

Theorem 1 *There exists a unique symmetric equilibrium in which firm i, $i \in \{1, 2, ..N\}$, uses the following IPO threshold strategy:*
(i) Firm i goes public at $t = 1$ if and only if $\theta_i \geq \theta_1^*$, where θ_1^* is given by the solution to (4);
(ii) If there was at least one IPO at $t = 1$, firm i goes public at $t = 2$ if and only if $\theta_i \geq \theta_2^*(s_1) \equiv -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)$, when firm i did not IPO at $t = 1$;
(iii) If no IPO was made by any firm at $t = 1$, firm i goes public at $t = 2$ for all θ_i.

Proof. Given that the IPO strategy of firm i at $t = 2$ does not depend on beliefs about θ_j, the IPO strategy at $t = 2$ is given by (ii) and (iii) (note that if no other firm went public at $t = 1$ we are back to a two-period setting, in which all firms IPO immediately as they can). Under the assumption of existence of a threshold equilibrium, any IPO threshold at $t = 1$ should satisfy the first period's indifference condition in (4).

At $t = 2$ the firm will IPO if an only if the expected payoff from IPO is higher than if it delays the IPO, i.e., it will IPO if $\frac{\theta_i+E(s_2|s_1)}{1+r} \geq \frac{\theta_i+E(s_3|s_1)}{(1+r)^2}$, which holds for all $\theta_i > \theta_2^*(s_1) = -s_1((1+r)-\gamma)\left(\frac{\gamma}{r}\right)$. Therefore, no type has an incentive to deviate at $t = 2$.

Next, we show that no type has an incentive to deviate at $t = 1$. Assume that type $\theta_i = \theta_1^*$ is indifferent between going public and delaying the IPO at $t = 1$. To show that all types higher (lower) than θ_1^* strictly prefer to IPO (not to IPO) note that the marginal loss in delaying the IPO for higher (lower) θ_i is greater (smaller) due to discounting, i.e. discounting is more pronounced for higher θ_i's. In addition, the marginal benefit from delaying the IPO (captured by the option value) is lower (higher) for higher θ_i, as shown in Lemma 2. Hence, no type has an incentive to deviate at $t = 1$.

Next, we show uniqueness of a symmetric IPO threshold. Assume by contradiction that there are two values of $\theta_1^* : \theta_L$ and θ_H, where $\theta_H > \theta_L$, that are consistent with a symmetric equilibrium. If all firms move from θ_L to θ_H, the probability that the other managers will IPO decreases, which in turn increases any manager's incentive to IPO. That is, it decreases the best response IPO threshold. However, this contradicts the assumption of the existence of a higher threshold θ_H. A similar argument follows for a lower IPO threshold. More formally, the manager's indifference condition at $t = 1$ is given by:

$$
\begin{aligned}
\theta_i \;=\; & \Pr\left(ND_{j\neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) \\
& + \left(1 - \Pr\left(ND_{j\neq i}^1\right)\right)\left[
\begin{array}{c}
\Pr\left(D_i^2\right) E\left[\text{payoff at } t = 2 | \theta_i, \text{ and IPO at } t = 2\right] \\
+ \Pr\left(ND_i^2\right) E\left[\text{payoff at } t = 3 | \theta_i, \text{ and delay IPO at } t = 2\right]
\end{array}
\right] \\
=\; & \Pr\left(ND_{j\neq i}^1\right)\left(\frac{\theta_i}{1+r}\right) + \left(1 - \Pr\left(ND_{j\neq i}^1\right)\right)\left(\Pr\left(D_i^2\right)\frac{\theta_i}{1+r} + \Pr\left(ND_i^2\right)\left(\frac{\theta_i}{1+r} + V_2\left(\theta_i\right)\right)\right) \\
=\; & \frac{\theta_i}{1+r} + \left(1 - \Pr\left(ND_{j\neq i}^1\right)\right)\Pr\left(ND_i^2\right) V_2\left(\theta_i\right).
\end{aligned}
$$

If the IPO threshold of firm $j \neq i$ increases to θ_H, it has no effect on the option value (conditional on getting to $t = 2$ when firm j went public at $t = 1$ and firm i's type is $\theta_i > \theta_2^*$), however the probability of this event decreases as the threshold of firm i increases. As such, the right hand side of the above indifference condition decreases, which implies that, in order for firm i to be indifferent at $t = 1$, the IPO threshold of firm i at $t = 1$ must decrease as well – in contradiction to the assumption of the increased IPO threshold. ∎

Note that in obtaining the above results we imposed no restriction regarding the distributions of θ, and for tractability we assumed that ε_t is normally distributed (one can show that the above Theorem holds for other distributions of the noise term, including distributions with bounded support such as a uniform distribution).

As is typical in games with multiple players and a continuous type space, asymmetric equilibria may also exist. Under certain conditions of the distribution of θ, $G(\theta)$, we are can shed some light on the potential asymmetric equilibria as well. For the two-agent setting, there exist at most three equilibria – the symmetric equilibrium and two asymmetric equilibria – when $G(\theta)$ has a nondecreasing hazard rate. This implies that the best response function for firm i is convex and thus there can be at most three intersections of the two agents' best response functions. We haven't been able to confirm or preclude the existence of these two asymmetric equilibria, however, we can preclude the existence of any other equilibrium. The following Claim formalizes this result.[16]

Claim 1 *When $N = 2$, $g'(\theta) < 0$, and $G(\theta)$ has a nondecreasing hazard rate, there are at most three equilibria: the symmetric equilibrium and two analogous asymmetric equilibria.*

The condition that the density function is strictly decreasing and that $G(\theta)$ has a nondecreasing hazard rate hold for a wide variety of distributions, such as the exponential and certain parameterizations of the Chi-squared and generalized Weibull distributions. Claim 1 implies that the two possible asymmetric equilibria are analogous in the sense that the equilibrium first period threshold pairs mirror one another, i.e. $\left(\theta_{1,i}^*, \theta_{1,j}^*\right) = \left(\theta_{1,j}^*, \theta_{1,i}^*\right)$. Hence, in any asymmetric equilibrium, one firm discloses more often with a low threshold while the other keeps quiet more often with a high threshold. We explore asymmetric equilibria for bounded distributions in section 5.

In the next section, we provide comparative statics and empirical predictions that arise from the symmetric equilibrium.

4   Comparative Statics and Empirical Predictions

We now analyze how the equilibrium is affected by the various parameters. In particular, we generate empirical predictions with respect to changes in the following parameters of the model: the discount factor, the rate of mean reversion, and the variance of the error term. We start by studying the effect of the parameters on the disclosure threshold in the second period and then study the effect on the first period's disclosure threshold.

---

[16]We can show similar result also for uniform distributions of $\tilde{\theta}$, where the difference is that the best response strategy is concave rather than convex.

4.1 Comparative Statics for θ_2^*

We begin the analysis with the second period's equilibrium threshold, $\theta_2^*(s_1)$. Note that the threshold of the second period, which is the unique best response at $t = 2$, is independent of the other firm's characteristics. So the analysis of this part is independent of whether the firms are homogeneous or not and the specific characteristics of all the other firms.

Recall that the IPO threshold at $t = 2$, given that there was at least one IPO at $t = 1$ (and hence s_1 is observed) is given by:

$$\theta_2^*(s_1) = -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right).$$

We will keep everything constant (including s_1) and see how the threshold at $t = 2$ is affected by changes in: (i) manager i's discount factor, r; (ii) the extent of persistence in the state of nature, γ (where lower γ implies higher mean-reversion); and (iii) the variance of the shock to the state of nature, σ_ε.

Taking the derivative of the threshold with respect to the discount factor, r, yields:

$$\frac{\partial}{\partial r}\theta_2^*(s_1) = \frac{\partial}{\partial r}\left(-s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right) = -\frac{1}{r^2}\gamma s_1(\gamma - 1) < 0.$$

Note that $\frac{\partial}{\partial r}\theta_2^*(s_1) < 0$ since $\gamma \in (0,1)$ and at the threshold we have $s_1 < 0$. The fact that the threshold level at $t = 2$ is decreasing in r is very intuitive. To see this, recall that at the IPO threshold $\theta_2^*(s_1)$, at which the manager is indifferent between going public and delaying the IPO, it must be that $\theta + s_1 > 0$ (otherwise the manager would strictly prefer to delay the IPO to $t = 3$). Since the expected payoff of the threshold type is positive, an increase in the discount factor increases the cost from delaying the IPO, and hence, decreases the IPO threshold (equivalently, for a given θ, the threshold level of s_1 is lower).

Next we analyze the effect of the extent of mean-reversion of the state of nature, γ, on the IPO threshold at $t = 2$. While the mathematical derivation of this effect is straight forward, the intuition for the result is a little more complex.

Taking the derivative of the second period's threshold with respect to γ, yields:

$$\frac{\partial}{\partial \gamma} \theta_2^* (s_1) = \frac{\partial}{\partial \gamma} \left(-s_1 \left((1+r) - \gamma \right) \left(\frac{\gamma}{r} \right) \right)$$

$$= -\frac{1}{r} s_1 \left(r - 2\gamma + 1 \right) = \left\{ \begin{array}{cc} > 0 & \text{for } \gamma < \frac{r+1}{2} \\ 0 & \text{for } \gamma = \frac{r+1}{2} \\ < 0 & otherwise \end{array} \right\}.$$

The direction of the effect of changes in γ on the threshold $\theta_2^* (s_1)$ varies with the level of γ. To illustrate the effect of γ on $\theta_2^* (s_1)$, the figure below plots $\frac{\partial}{\partial \gamma} \theta_2^* (s_1)$ as a function of γ using parameter values $r = 0.1$ and $s = -\frac{1}{2}$.
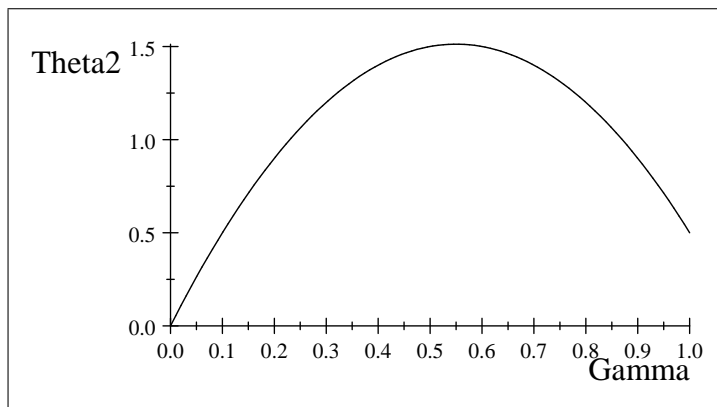

Figure 2: The effect of γ on $\theta_2^* (s_1)$, for $r = 0.1$ and $s = -\frac{1}{2}$

To get better intuition for the above result, it might be useful to consider separately the effect of the idiosyncratic component, θ, and the common factor, s_1, on the incentive to IPO or delay the IPO at $t = 2$. Since $\theta_i \geq 0$ and is constant over time, it always provides an incentive not to delay the IPO due to discounting. This incentive increases in θ. The incentive due to the state of nature, which is more complex, is determined by two effects: (i) the mean-reverting feature of the state of nature (characterized by γ) which provides incentive to delay the IPO for low realizations of s_1; and (ii) the discount factor. For $\gamma = 0$ the realizations of the state of nature are iid and s_2 and s_3 are independent of s_1. Hence, since $E (s_2 | s_1) = 0$ there is no benefit from delaying the IPO. As such, for $\gamma = 0$ all managers IPO at $t = 2$ (if they did not IPO already at $t = 1$). As γ increases from $\gamma = 0$ and the mean-reversion effect is no longer perfect, s_1 becomes more informative about s_2 and s_3. Hence for negative values of s_1 the value from delaying the IPO increases in γ. However, there is a second, mitigating, effect that stems from the fact that the mean-reversion of s_3 decreases as

γ increases - which decreases the benefit from delaying the IPO at $t = 2$ for a given $\{\theta_i, s_1\}$. For sufficiently low γ the former effect dominates and the option value increases in γ, and hence, the IPO threshold increases in γ. As γ further increases, the second effect becomes relatively more pronounce, such that from one point and on the option value decreases in γ. As γ approaches one, the process of the state converges to a random-walk and there is no mean-reversal. Hence, the part of the option value that stems from mean-reversion of low realizations of s_1 disappears, and the only reason the option is still valuable is that when the expected firm value $(\theta + s)$ is negative, there is a benefit from delaying a negative payoff.

Finally, $\theta_2^*(s_1)$ is independent of the variance in the noise of the state of nature, σ^2, and independent for the distribution of θ (recall that θ is assumed to have a positive support), conditional on the state of nature s_1 being revealed in $t = 1$. The threshold level in $t = 2$ is consequently unaffected by changes in σ^2.

4.2   Comparative Statics for θ_1^*

The comparative statics for the first period threshold level, θ_1^*, are slightly less intuitive, however, the analysis of $\theta_2^*(s_1)$ serves as a useful guide. We start with the effect of γ on θ_1^*:

Proposition 1 *The effect of the rate of mean-reversion, γ, on the IPO threshold at $t = 1$, θ_1^*, is similar to its effect on the second period's threshold, $\theta_2^*(s_1)$. Specifically,*

$$\frac{\partial \theta_1^*}{\partial \gamma} = \left\{ \begin{array}{ll} 0 & for\ \gamma = \frac{r+1}{2} \\ > 0 & for\ \gamma < \frac{r+1}{2} \\ < 0 & otherwise \end{array} \right\}.$$

Recall that $\frac{\partial \theta_2^*}{\partial \gamma} = \left\{ \begin{array}{ll} 0 & \text{for } \gamma = \frac{r+1}{2} \\ > 0 & \text{for } \gamma < \frac{r+1}{2} \\ < 0 & otherwise \end{array} \right\}$. Let's assume by contradiction that $\frac{\partial \theta_1^*}{\partial \gamma} < 0$

for $\gamma < \frac{r+1}{2}$. An increase in γ affects the expected option value from not going public at $t = 1$ in several ways. First, conditional on another firm going public at $t = 1$, the threshold at $t = 2$ increases in γ, which consequently increases the expected value of the option. Moreover, under the contradictory assumption, the probability that the other firm IPO at $t = 1$ increases in γ, and hence the probability of taking advantage of the option value at $t = 2$ also increases in γ. Overall, the expected option value increases. The manager thus

has a stronger incentive not to IPO at $t = 1$, which contradicts the assumption that θ_1^* is decreasing in γ. A symmetric argument applies for the case of $\gamma > \frac{r+1}{2}$. A more formal proof is included in the Appendix. The intuition for the non-monotonicity of the disclosure threshold in γ follows similar arguments to our discussion in the analysis of the comparative statics for the second period's IPO threshold.

Next we analyze the effect of the discount factor, r, on the first period's threshold. Similar to the second period's threshold, the first period threshold is also decreasing in the discount rate:

$$\frac{\partial \theta_1^*}{\partial r} < 0.$$

From the comparative statics for θ_2^*, we know that $\frac{\partial \theta_2^*(s_1)}{\partial r} < 0$, i.e., for a given level of θ_i the manager is more likely to IPO in the second period for higher values of r, and hence, is less likely to take advantage of the real option. In addition, a higher r increases the manager's cost from delaying the IPO. Both effects lead to a stronger incentive to IPO at $t = 1$. This results in a lower IPO threshold at $t = 1$ for higher values of r.

Next, we consider the effect of the variance of the periodic innovation in the state of nature, σ, on the first period threshold.

Proposition 2 *The first period IPO threshold is increasing in the variance of the state of nature, σ_ε^2, i.e., a higher variance induces less IPO in the first period:*

$$\frac{\partial \theta_1^*}{\partial \sigma_\varepsilon^2} > 0.$$

The intuition for this result is that an increase in volatility increases the value of the option, and hence, induces less IPO in the first period. This implies that the threshold of the first period is increasing in the variance, σ_ε^2. While this is intuitive, the proof (which is relegated to the appendix) requires a few steps. Finally, we examine the effect of an increase in the number of firms on the $t = 1$ threshold.

Proposition 3 *The first period threshold is increasing in the number of firms, i.e.*

$$\frac{\partial \theta_1^*}{\partial N} > 0.$$

We see that the threshold level in the first period rises as more private firms enter the industry. This occurs since the likelihood of disclosure by at least one firm rises in N, thus

leading to a greater incentive to delay the IPO. The following section discusses the empirical implications of these results.

4.3   Empirical implications and predictions

The results help explain several documented results as well as offer numerous avenues for future research in terms of empirical predictions. The model gives implications for the timing–when firms decide to IPO–and the clustering of IPOs. By (information-based) *clustering*, we mean that at least two firms disclose and go public within two consecutive periods, primarily with respect to the first two periods. The results of the model imply that the clustering of IPOs is driven by the realization of a common valuation factor, which is consistent with the hypotheses of Lowry and Schwert (2002) and Benveniste et al. (2003). Moreover, the results imply that IPO clustering should disproportionately feature firms within a specific industry, and that clustering emerges following high initial returns of recent IPOs, which has been documented by Ritter (1984) and Lowry and Schwert (2002). The model also implies dispersion of IPOs under weak market conditions, which helps to explain bust patterns of IPOs, as documented in Ibbotson and Ritter (1995).

The results are in contrast to the predictions of Benveniste, Busaba, and Wilhelm (2002), who suggest that IPO clustering emerges due to bundling by investment banks. When the timing decision is endogenized, we see that there is always a positive amount of delay in the IPO times of some firms, but that other firms find it profitable to go public without delay and forgo potential informational rents. Hence, the first immediate prediction of the model is that firms with higher market valuations (e.g. in terms of historical earnings) go public earlier than firms with comparatively lower pre-IPO valuations. Another implication is that following a "successful" IPO in the first period, in which the state of nature is revealed to be relatively high, we expect clustering of IPOs. Our particular and stylized setting assumes that the distribution of the innovation in the state of nature is symmetric, which implies that all firms will IPO following a state of nature that is above the mean. However, under a more general distribution of the innovation in the state of nature, higher realizations of the state of nature in the first period increase the expected number of firms that will go public in the second period. The following corollary summarizes these immediate predictions of the model:

Corollary 2 *In the unique symmetric equilibrium:*

- *The higher a firm's type, the earlier it will disclose and go public.*

- *The expected number of IPOs in the second period, following an IPO in the first period, is increasing in the realization of the state of nature in the first period.*

The results also imply several other predictions which are less intuitive. With respect to the timing, Proposition 2 implies that there are fewer "early" IPOs in nascent industries who face greater uncertainty over their common valuation factor or market conditions. Thus, we expect more delay in the IPOs, perhaps inefficiently so, with more uncertainty. This is expected, however, the model also implies that the likelihood of clustering and the intensity of IPOs waves is also lower following initial IPOs in industries with greater uncertainty in their pricing prospects. As shown in Proposition 2, fewer firms go public in the first period when there is greater uncertainty. Moreover, since the likelihood of drawing a low state, s_1, is higher with a greater variance, σ_ε^2, there is a higher probability that firms delay their IPOs until the last period, after having observed an IPO in the first period. Hence, we expect more delay and less clustering in the IPOs of firms in less mature industries or when there is greater market or macroeconomic uncertainty. This prediction stated in the following corollary:

Corollary 3 *More firms delay their IPO issues as σ_ε^2 increases, and the likelihood of clustering is decreasing in σ_ε^2.*

The results of the model also imply that there should be more delay and fewer early IPOs in more heavily populated industries where firms are predominately private. However, the likelihood and size of clustering, conditional on at least one firm going public in the first period, should be higher in industries with a comparatively larger number of (private) firms. The first prediction is implied from Proposition 3–the more concentrated the industry, the fewer firms that go public in the first period. Furthermore, as more high-type firms delay their IPO time, the likelihood of clustering increases as these high-type firms are also more likely to disclose in the second period. This occurs since a lower state, s_1, is necessary to induce delay in the second period for high-type firms (as shown by Lemma 1). Hence, the more populous an (nascent) industry is with private firms, the less timely the IPOs, but the more likely clustering ensues and with a more intense IPO wave:

Corollary 4 *With a greater number of firms, overall delay increases and there is a greater likelihood of clustering. Clustering which emerges is more intense; more firms go public in the second period following an IPO.*

Lastly, Proposition 1 implies that there should be less delay with high persistence of the common valuation or the market/macroeconomic conditions. Similarly, the likelihood of clustering is also greater with high persistence.

5 Extensions and Robustness

This section offers several extensions of the base model. We start with the, almost trivial, extension in which following an IPO the market's information about the common factor is not perfect (Section 5.1). This is then used in our main extension in which the precision of market's beliefs about the common factor increases in the number of firms that go public (Section 5.2). We start with the simpler variation in which we exogenously assume that the precision of the information about the common factor increases in the number of IPOs. We then discuss the case in which the relation between the precision of beliefs about the common factor and the number of IPOs evolves endogenously. In particular, at the end of each period, the market gets to observe the prices of all the IPOs that took place, however, the market cannot perfectly disentangle each firm's idiosyncratic component (θ_i - which is not perfectly disclosed in this setting) and the common factor. As such, the more IPOs take place in a given period the more the market learns about the common factor. In all of the above extensions, the main results of the basic setting still hold. Finally, we study a setting in which the firms' type belong to a bounded support and show that while the symmetric equilibrium always exists, there may be an additional type of equilibrium in which one firm always IPO in the first period where all the other firms always delay their IPO in the first period.

5.1 Noisy Signal of the Common Factor

In the baseline model, we assumed that all firms observe the realization of the common factor at $t = 1$, s_1, if there was at least one IPO in the first period. We now relax this feature and instead assume that firms receive an imperfect signal of s_1 upon an IPO by at

least one firm at time $t = 1$. Denote this signal by $q_1 = s_1 + \delta$, where $\delta \sim N(0, \sigma_\delta^2)$. The second period threshold stated in Lemma 1 is qualitatively identical under this alternative framework, where the quantitative difference is only due to the difference in the precision of beliefs about the common factor. The unique optimal threshold in $t = 2$, which we denote by $\theta_2^*(q_1)$, is given by:

$$\theta_2^*(q_1) = -q_1 \cdot \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\delta^2} ((1+r) - \gamma) \left(\frac{\gamma}{r}\right).$$

For negative realizations of q_1, the threshold is decreasing in the variance of the noise in the signal about the common factor, σ_δ^2, and hence disclosure is more likely in the second period with a less informative signal. The first period threshold is also analogously derived from the baseline setting. Denote the posterior distribution of s_1 by $H(\cdot)$ and the conditional variance of s_1 after observing q_1 by σ_c^2. The equilibrium first period threshold is given as:

$$
\begin{aligned}
\theta_i^* &= [G(\theta_1^*)]^{N-1} \left(\frac{\theta_i^*}{1+r}\right) \\
&+ \left(1 - [G(\theta_1^*)]^{N-1}\right) \left[\begin{array}{l} H\left(\frac{\theta_i^*\left(\sigma_\varepsilon^2 + \sigma_\delta^2\right)}{\sigma_\varepsilon^2((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \frac{\theta_i^*}{1+r} + \frac{1}{1+r}\gamma\sigma_c^2 \cdot h\left(-\frac{\theta_i^*\left(\sigma_\varepsilon^2 + \sigma_\delta^2\right)}{\sigma_\varepsilon^2((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - H\left(\frac{\theta_i^*\left(\sigma_\varepsilon^2 + \sigma_\delta^2\right)}{\sigma_\varepsilon^2((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right) \frac{\theta_i^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma\sigma_c^2 \cdot h\left(-\frac{\theta_i^*\left(\sigma_\varepsilon^2 + \sigma_\delta^2\right)}{\sigma_\varepsilon^2((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array} \right].
\end{aligned}
$$

The qualitative properties of the first period threshold are unchanged. Moreover, Lemma 2 and Theorem 1 continue to hold in this setting as well. The effect of the precision of the signal q_1 is presented in the following corollary:

Corollary 5 *Both θ_2^* and θ_1^* decrease in σ_δ^2.*

The first part of the result follows immediately from the expression of θ_2^*. Intuitively, when the signal becomes less informative, the value of learning about the state of nature decreases, and hence, firms have an incentive to go public for lower values of θ_i. Likewise, the benefit of delaying the IPO in the first period decreases as σ_δ^2 increases, as the value of the real option declines. Hence, firms are induced to disclose more frequently, thus resulting in a lower disclosure threshold in the first period.

5.2 Increased Learning in the Number of IPOs

In this extension we relax the assumption that the precision of the market beliefs about the common factor is independent of the number of IPO. In particular, we study a setting in which the more IPOs that occur in a given period, the higher the precision the market has about the common factor. The simplest way of introducing this feature is by extending the setting in Section 5.1, in which firms observe a noisy signal of s_1, by adding the assumption that the precision of the signal increases with the number of IPOs at $t = 1$. That is, assuming that $q_1 = s_1 + \delta$, where $\delta \sim N\left(0, \sigma_\delta^2\right)$ and σ_δ^2 is decreasing in the number of first period IPOs. As will be apparent by the end of this section, all the results of the previous sections are robust to this specification of the model.

Another way to capture this feature, in an endogenous way, is by assuming that at the end of each period the market observes the pricing of every IPO. The pricing, as before, is determined by the firm's idiosyncratic component and the common factor, however in this setting the firm does not disclose its type θ_i, so the market cannot perfectly disentangle the firm's idiosyncratic component from the common factor. In such a setting, the precision of the inference about the state of nature, s_1, is increasing in the number of IPOs in the first period.

The baseline model assumes for simplicity that the firm must make a truthful disclosure of its idiosyncratic component, θ_i, prior to going public. This is meant to capture the information that is shared in its prospectus and SEC disclosures. However, it may be the case that remaining firms and the market are unable to perfectly learn the firm's type, and hence unable to perfectly disentangle the two components of the price for a first period IPO, $P_1^i = \theta_i + s_1$, and rather just observe the vector of prices for the n firms who went public at time 1, $\mathbf{P}_1 = (P_1^1, ..., P_1^n)$. We show that our model is largely insensitive to this framework, though it poses additional algebraic intensity which may shift attention away from the fundamental tension we study.

Similar to the imperfect observability case in section 5.1, the second period threshold is given by:

$$\theta_2^*\left(\mathbf{P}_1\right) = -E\left(s_1 | \mathbf{P}_1\right)\left[(1 + r) - \gamma\right]\frac{\gamma}{r}, \tag{5}$$

which is an increasing function of $E\left(s_1 | \mathbf{P}_1\right)$, given that $E\left(s_1 | \mathbf{P}_1\right)$ is monotonically increasing in $\mathbf{P}_1$. Since s_1 is normally distributed, monotonicity of $E\left(s_1 | \mathbf{P}_1\right)$ holds for a wide range of

distributions for θ_i, such as normal, uniform, exponential, gamma, and Pareto distributions. For simplicity, we assume that $\theta_i \sim N\left(\mu_\theta, \sigma_\theta^2\right)$. This assumption implies that the distribution of s_1 conditional on observing an IPO price, P_1^i, is a truncated normal distribution. As such, the qualitative results concerning the second period threshold are not affected.

Similarly, the first period threshold level is also qualitatively unchanged. The primary results rely on the fact that the option value is decreasing in θ_i. When the agent indirectly observes s_1 after an IPO, the option value continues to decrease in θ_i. The decreasing option value is a result of the more severe beliefs about the state s_1 necessary for higher types to exercise the option in the second period. This property continues to hold in the modified second period threshold in equation (5). Specifically, higher θ_i values require a lower $E\left(s_1 | \mathbf{P}_1\right)$ in order to go public in the second period, thus lowering the probability of going public in the second period. This implies, as in the baseline setting, that the option value is decreasing in θ_i.

The IPO threshold in $t = 1$ is similar to the baseline case and the setting in Section 5.1, with the difference that now each firm must take account for all the permutations of the number of IPOs by other firms and the corresponding value of the real option from delaying the IPO. Since the posterior distribution of s_1, in particular the precision of the beliefs about s_1, varies with the number of IPOs, the number of IPOs by other firms affects the value of a firm's the real option from delaying its IPO.

We first rearrange equation (5) to be in terms of the minimum expected value of the common factor for which a firm with type θ_i will go public in $t = 2$. We denote it by $x^*\left(\theta_i\right)$ where

$$x^*\left(\theta_i\right) \equiv E^*\left(s_1 | \mathbf{P}_1\right) = -\frac{\theta_i}{\left((1+r) - \gamma\right)\frac{\gamma}{r}}.$$

That is, for any given θ_i, firm i will IPO at $t = 2$ if and only if $E\left(s_1 | \mathbf{P}_1\right) > x^*$. The probability of such an event is $C_n\left(\frac{\theta_i}{\left((1+r) - \gamma\right)\frac{\gamma}{r}}\right)$ where $C_n\left(\cdot\right)$ is the joint CDF of $s_1 + \theta_{-i}$ given that n firms went public at time 1. Note that we have a shifting support, as any $P_1^i < \theta_1^*$ implies that $s_1 < 0$. Moreover, because the distribution of IPO firms is bounded below by the first period threshold, θ_1^*, we can always determine an upper bound for the support of the posterior distribution of s_1 (e.g. when $P_1^j = 20$ and $\theta_1^* = 10$, then s_1 is at most 10). Thus, the conditional CDF C_n will be the convolution of a normal and a truncated distribution(s).

The setting here is a bit more involved than in the baseline case, as the precision of the

beliefs about s_1 depends on the number of firms who went public in period 1. Hence, the manager's strategy must account for the different potential number of IPOs by other firms. This makes the derivation of the optimal threshold algebraically more cumbersome, however the economic forces driving the result are maintained. For ease of exposition, we assume $N = 3$. The symmetric $t = 1$ indifference condition is given by:

$$
\begin{aligned}
\theta_1^* = {} & G\left(\theta_1^*\right)^2 \left(\frac{\theta_1^*}{1+r}\right) \\[2mm]
& + 2\left(1 - G\left(\theta_1^*\right)\right) G\left(\theta_1^*\right) \left(
\begin{array}{l}
C_1\left(\frac{\theta_1^*}{((1+r)-\gamma)\frac{\gamma}{r}}\right) E\left[\frac{\theta_1^*+s_2}{1+r} \Big| x^* > -\frac{\theta_1}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right] \\[3mm]
+ \left(1 - C_1\left(\frac{\theta_i}{((1+r)-\gamma)\frac{\gamma}{r}}\right)\right) E\left[\frac{\theta_1^*+s_3}{(1+r)^2} \Big| x^* \leq -\frac{\theta_1}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right]
\end{array}
\right) \\[4mm]
& + \left(1 - G\left(\theta_1^*\right)\right)^2 \left(
\begin{array}{l}
C_2\left(\frac{\theta_i}{((1+r)-\gamma)\frac{\gamma}{r}}\right) E\left[\frac{\theta_1^*+s_2}{1+r} \Big| x^* > -\frac{\theta_1}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right] \\[3mm]
+ \left(1 - C_2\left(\frac{\theta_1^*}{((1+r)-\gamma)\frac{\gamma}{r}}\right)\right) E\left[\frac{\theta_1^*+s_3}{(1+r)^2} \Big| x^* \leq -\frac{\theta_1}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right]
\end{array}
\right)
\end{aligned}
\tag{6}
$$

The first term on the RHS in equation (6) is manager i's expected payoff following no IPOs in the first period. The second term on the RHS is the expected payoff following an IPO by one of the other two firms, where the probability of such an event is $2\left(1 - G\left(\theta_1^*\right)\right) G\left(\theta_1^*\right)$. The third term is the expected payoff if both of the other firms go public, where the distribution of this even is $\left(1 - G\left(\theta_1^*\right)\right)^2$. We see that this is analogous to the baseline setting, except for the fact that we now have a different distribution of ε_t. Note that the proof of Lemma 2 was shown for general distributions of ε_t. Hence, the fact that the option value is decreasing in θ_i holds under this setting. The existence argument of Theorem 1 continues to hold under this setting as well.

One concern in this setting is if multiple symmetric equilibria can exist. Recall that the symmetric threshold θ_1^* exactly offsets the cost of waiting with the benefit of waiting. Correspondingly, multiple symmetric equilibria occur only if a different symmetric threshold for θ_i results in a different posterior about s_1 that lowers the option value of waiting. However, the informativeness of $\mathbf{P}_1$ does not depend on the threshold level θ_1^*, but only on the number of firms going public. Hence, the monotone properties of the option as related to θ_1^* continue to hold (the option value is lower when the threshold is higher, and the option value is higher when the threshold is lower), thus preserving uniqueness.

This alternative specification heavily complicates the algebra and the subsequent comparative statics analysis, without adding significant qualitative insights. As such, we do not use it as our baseline model, but rather present it as an extension to reinforce the results of our simpler baseline model.

5.3 Bounded Support - Symmetric and Non-Symmetric Equilibria

In this subsection we show that, when the support of θ is bounded from above, i.e. $\theta \in [\underline{\theta}, \bar{\theta}]$, and the discount rate is sufficiently low, there exists, in addition to the symmetric equilibrium which we characterized in Theorem 1, equilibria in which only one firm always discloses at $t = 1$ and the others always delay. We define this special asymmetric threshold equilibrium as the "asymmetric" equilibrium:

Definition 1 *Define the asymmetric equilibrium as one where the first period threshold for player $j \neq i$ is $\theta_{1,j}^* = \underline{\theta}$, and the first period threshold for all other players is $\theta_{1,-j}^* = \bar{\theta}$.*

We further divide the support of the discount rate, r, into three regions. $\left(0, r^L\right)$, $\left(r^L, r^H\right)$ and $\left(r^H, \infty\right)$, where:

Definition 2 r^H *is such that, given disclosure by at least one other firm at $t = 1$ with probability 1 (so that s_1 is revealed for sure), a firm with the lowest type, $\theta = 0$, is indifferent between disclosing and not disclosing at $t = 1$.*
r^L *is such that, given disclosure by at least one other firm at $t = 1$ with probability 1 (so that s_1 is revealed for sure), a firm with the highest type, $\theta = \bar{\theta}$, is indifferent between disclosing and not disclosing at $t = 1$.*

We know show the existence of the discount rate thresholds that define the set of equilibria in the given regions of r:

Proposition 4 *The set of equilibria for each of the above regions of the discount factor are as follows:*

1. *For $r \in \left(r^H, \infty\right)$ the unique equilibrium is the symmetric equilibrium in which all firms disclose at $t = 1$, i.e., $\theta_1^* = \underline{\theta}$.*

31

2. *For $r \in \left(r^L, r^H \right)$ the unique equilibrium is the symmetric equilibrium defined in Theorem 1, in which all firms disclose at $t = 1$ if and only if their type is greater than the interior disclosure threshold, θ_1^*.*

3. *For $r \in \left(0, r^L \right)$: there exist both the symmetric equilibrium with interior disclosure threshold as well as N asymmetric equilibria.*

The intuition for the proof is relatively straightforward. For $r \in \left(r^H, \infty \right)$ any firm always prefers to disclose at $t = 1$, as even if the lowest type, $\theta = 0$, knows for certainty that s_1 will be revealed, the discounting is too severe to justify delay of disclosure. For $r \in \left(0, r^L \right)$ any firm that believes that at least one other firm will disclose is better off not disclosing over disclosing at $t = 1$. To show the existence of the *asymmetric equilibrium* assume that one firm, firm i, always discloses at $t = 1$. The best response of all other firms is not to disclose at $t = 1$. Now, given that the probability that any other firm will disclose at $t = 1$ is zero, it is optimal for firm i to disclose at $t = 1$. So for $r \in \left(0, r^L \right)$ there exist N *asymmetric* equilibria such that in each one of them a single firm always discloses at $t = 1$ and all the other firms do not disclose at $t = 1$. Finally, for $r \in \left(r^L, r^H \right)$ there are sufficiently high types that will disclose at $t = 1$ even if they are certain that s_1 will be observed. Hence, there is always a positive probability that at least one firm will disclose at $t = 1$. Let's assume by contradiction that there exists an *asymmetric equilibrium* in which firm i always discloses. Then, there exists a disclosure threshold, such that any other firm discloses if and only if its type is lower than this threshold. This, however implies that there is a positive probability that a firm other than firm i will disclose at $t = 1$. As such, if the realized type of firm i is sufficiently low, the discount effect can be arbitrarily low and the value of the real option is strictly positive. Therefore, firm i will disclose for sufficiently low types - in contradiction to the assumption that firm i does not disclose.

6   Conclusion

In this study we have developed a model to help shed light on the strategic interaction between firms who decide to disclose information and sell shares or a project. We have shown that the unique equilibrium is in threshold strategies where all players follow identical strategies. The primary implication of this result is that, in the presence of other firms and

common uncertainty, there is always a positive amount of delay of IPOs in equilibrium.

Several extensions can be considered for future work. We have considered only cases in which the disclosure of the firm's value if verifiable and non-manipulable. A possibly interesting study would be to relax this assumption, in which case firm managers can engage in costly manipulation of the firm's value. We have also assumed that the firm's type (idiosyncratic component) is constant over time. A potentially interesting research question is to investigate a model where the firm's value also follows a stochastic process. Lastly, our model can be extended to a continuous time setting with finite number of firms. We conjecture that in a continuous time setting there exists an equilibrium in which each firm's delay of the IPO is decreasing in the firm's type and the more negative the revealed state of nature is, the more firms delay their IPOs. As such, the continuous time setting seems to share the main characteristics of our discrete time model.

7 Appendix

Proof of Lemma 1. By the second period indifference condition, we have:

$$\frac{\theta_i + E\left(s_2|s_1\right)}{1+r} = \frac{\theta_i + E\left(s_3|s_1\right)}{\left(1+r\right)^2}$$

$$\frac{\theta_i + \gamma s_1}{1+r} = \frac{\theta_i + \gamma^2 s_1}{\left(1+r\right)^2}$$

$$\theta_i \left(\frac{r}{1+r}\right) = \frac{\gamma^2 s_1}{1+r} - \gamma s_1 = s_1 \left(\frac{\gamma}{1+r} - 1\right) \gamma$$

$$\theta_2^*\left(s_1\right) = -s_1 \left(\left(1+r\right) - \gamma\right) \left(\frac{\gamma}{r}\right).$$

∎

Proof of Lemma 2. The option value is equal to the likelihood that the firm which did not disclose at $t = 1$ chooses not to disclose at $t = 2$ times the increase in expected payoff due to the delay in the disclosure, which is

$$V_2\left(\theta_i\right) = S\left(\theta_i\right) - NS\left(\theta_i\right) = \Pr\left(S < s_1^*\left(\theta_i\right)\right) E\left[\frac{\theta_i + s_3}{\left(1+r\right)^2} - \frac{\theta_i + s_2}{1+r} | s_1 < s_1^*\left(\theta_i\right)\right], \quad (7)$$

where $s_1^*\left(\theta_i\right)$ is the value of s_1 such that the agent is indifferent between disclosing and not disclosing at $t = 2$. From equation (1), we have:

$$s_1^*\left(\theta_i\right) = -\frac{\theta_i}{\left(\left(1+r\right) - \gamma\right)\left(\frac{\gamma}{r}\right)}$$

Note that

$$\frac{\partial s_1^*\left(\theta_i\right)}{\partial \theta_i} < 0,$$

which implies that also

$$\frac{\partial \Pr\left(S < s_1^*\left(\theta_i\right)\right)}{\partial \theta_i} < 0.$$

The derivative of the option value with respect to θ_i is:

$$\frac{\partial}{\partial \theta_i} V_2\left(\theta_i\right) = \frac{\partial}{\partial \theta_i}\left[\Pr\left(S < s_1^*\left(\theta_i\right)\right) E\left[\frac{\theta_i + s_3}{\left(1+r\right)^2} - \frac{\theta_i + s_2}{1+r} | s_1 < s_1^*\left(\theta_i\right)\right]\right]$$

$$= \frac{\partial}{\partial \theta_i}\left[F\left(s_1^*\left(\theta_i\right)\right) \cdot \left(\frac{1}{F\left(s_1^*\left(\theta_i\right)\right)} \int_{-\infty}^{s_1^*\left(\theta_i\right)} \left(\frac{\theta_i + E\left(s_3|s_1\right)}{\left(1+r\right)^2} - \frac{\theta_i + E\left(s_2|s_1\right)}{1+r}\right) f\left(s_1\right) ds_1\right)\right]$$

Plugging in $E(s_2|s_1) = \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2$ and
$E(s_3|s_1) = \int_{-\infty}^{\infty} \left(\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3\right) f(\varepsilon_3) d\varepsilon_3$, yields:

$$
\begin{aligned}
\frac{\partial}{\partial \theta_i} V_2(\theta_i) &= \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \int_{-\infty}^{\infty} \left(\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3\right) f(\varepsilon_3) d\varepsilon_3}{(1+r)^2} \right. \\
&\qquad\qquad\qquad\qquad \left. - \frac{\theta_i + \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2}{1+r} \right] f(s_1) ds_1 \\
&= \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1
\end{aligned}
$$

Recall that $s_1^*(\theta_i)$ is the value of s_1 such that a firm of type θ_i is indifferent between disclosing in $t = 2$ or $t = 3$ upon the realization of s_1 in the beginning of $t = 2$. Hence, by definition, we have that $\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} > 0$ for all $s < s_1^*(\theta_i)$ (i.e. it is more profitable to wait until $t = 3$ for even worse/more negative realizations of s_1. A marginal increase in θ_i thus has two effects. First, we see immediately that $\frac{\partial}{\partial \theta_i} \left(\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right) = \frac{1}{1+r} \left(\frac{1}{1+r} - 1 \right) < 0$ since $r > 0$. Moreover, $s_1^*(\theta_i)$ is decreasing in θ_i (i.e. the s_1 required for a higher θ_i to be indifferent must be even more negative), and thus the interval over which we integrate is truncated as θ_i increases. Hence, the integral $\int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1$ is decreasing in θ_i.

This can also be explicitly shown. Using Leibniz's rule, we have

$$
\begin{aligned}
&\frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 \\
&= \int_{-\infty}^{s_1^*(\theta_i)} \frac{\partial}{\partial \theta_i} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial \theta_i} \left[\frac{\theta_i + \gamma^2 s_2^*(\theta_1)}{(1+r)^2} - \frac{\theta_i + \gamma s_1^*(\theta_i)}{1+r} \right] f(s_2^*) \\
&= \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r} \right] f(s_1) ds_1 \\
&\quad + \left[\frac{\partial \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{\partial \theta_i} \right] \left[\frac{\theta_i + \gamma^2 \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{(1+r)^2} - \frac{\theta_i + \gamma \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{1+r} \right] f(s_2^*) \\
&= \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r} \right] f(s_1) ds_1 - \left[\frac{r}{\gamma(r-\gamma+1)} \right] [0] f(s_2^*) \\
&= \int_{-\infty}^{s_1^*(\theta_i)} \frac{-r}{(1+r)^2} f(s_1) ds_1 \\
&< 0.
\end{aligned}
$$

Note that $s_1 = \varepsilon_1$ and we define the integral in terms of s_1 rather than ε_1 for presentational

ease. Also note that the proof does not rely on a specific distribution of ε, as such the result that the option value decreases in θ_i holds for any distribution of ε. ∎

Proof of Lemma 3.   Starting from (2), given our disclosure threshold in $t = 2$, (2) becomes:

$$[G(\theta_1^*)]^{N-1}\left(\frac{\theta_i}{1+r}\right) + \left(1 - [G(\theta_1^*)]^{N-1}\right)$$

$$\cdot \left[\begin{array}{l} \Pr\left[\theta_i > -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] E\left[\frac{\theta_i + s_2}{1+r}|\theta_i > -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] \\ + \Pr\left[\theta_i \le -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] E\left[\frac{\theta_i + s_3}{(1+r)^2}|\theta_i \le -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] \end{array} \right]. \quad (8)$$

Note that in any point in time, the agent knows the value of her θ. Next, we calculate each of the terms above:

$$\Pr\left[\theta_i > -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] = \Pr\left[s_1 > -\frac{\theta_i}{((1+r) - \gamma)\left(\frac{\gamma}{r}\right)}\right] = F\left(\frac{\theta_i}{((1+r) - \gamma)\left(\frac{\gamma}{r}\right)}\right).$$

And:

$$E\left[\frac{\theta_i + s_2}{1+r}|\theta_i > -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] = E\left[\frac{\theta_i + s_2}{1+r}|s_1 > -\frac{\theta_i}{((1+r) - \gamma)\left(\frac{\gamma}{r}\right)}\right].$$

Which becomes:

$$\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)} \int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty} \frac{\theta_i + E(s_2|s_1)}{1+r} f(s_1) \, ds_1$$

$$= \frac{\theta_i}{1+r} + \frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)} \int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty} [E(s_2|s_1)] f(s_1) \, ds_1$$

$$= \frac{\theta_i}{1+r} + \frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)} \int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty} \left[\int_{-\infty}^{\infty}(\gamma s_1 + \varepsilon_2) f(\varepsilon_2) \, d\varepsilon_2\right] f(s_1) \, ds_1$$

$$= \frac{\theta_i}{1+r} + \frac{1}{1+r}\frac{1}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)} \int_{-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}}^{\infty} \gamma s_1 f(s_1) \, ds_1$$

$$= \frac{\theta_i}{1+r} + \frac{1}{1+r}\gamma E\left[s_1|s_1 > -\frac{\theta_i}{((1+r) - \gamma)\left(\frac{\gamma}{r}\right)}\right].$$

Recall that the formula for the expectation of the truncated normal distribution where $x \sim N\left(\mu_x, \sigma^2\right)$ is[17]:

$$E\left(x \mid x \in [a,b]\right) = \mu_x - \sigma^2 \frac{f(b) - f(a)}{F(b) - F(a)}.$$

Using the above formula, we have:

$$
\begin{aligned}
E\left[\frac{\theta_i + s_2}{1+r} \mid \theta_i > -s_1\left((1+r) - \gamma\right)\left(\frac{\gamma}{r}\right)\right] &= \frac{\theta_i}{1+r} + \frac{1}{1+r}\gamma\left(0 - \sigma_\varepsilon^2 \frac{-f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{1 - F\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\right) \\
&= \frac{\theta_i}{1+r} + \frac{1}{1+r}\gamma\left(\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\right).
\end{aligned}
$$

Finally:

$$
\begin{aligned}
E\left[\frac{\theta_i + s_3}{(1+r)^2} \mid \theta_i \leq -s_1\left((1+r)-\gamma\right)\left(\frac{\gamma}{r}\right)\right] &= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2}\gamma^2 E\left[s_1 \mid s_1 < -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right] \\
&= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) - 0}{F\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) - 0}\right) \\
&= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{\left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)}\right).
\end{aligned}
$$

Plugging this back to (2):

$$
\left[G\left(\theta_1^*\right)\right]^{N-1}\left(\frac{\theta_i}{1+r}\right) \tag{9}
$$
$$
+ \left(1 - \left[G\left(\theta_1^*\right)\right]^{N-1}\right)\left[\begin{array}{l} F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\left(\frac{\theta_i}{1+r} + \frac{1}{1+r}\gamma\left(\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}\right)\right) \\ + \left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\left(\frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2}\gamma^2\left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)}{\left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)}\right)\right) \end{array}\right]
$$

---

[17]For $a = -\infty$ we have

$$E\left(x \mid x < b\right) = \mu_x - \sigma^2 \frac{f(b)}{F(b)}$$

$$= [G(\theta_1^*)]^{N-1}\left(\frac{\theta_i}{1+r}\right) \tag{10}$$

$$+ \left(1 - [G(\theta_1^*)]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_i}{1+r} + \frac{1}{1+r}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - F\left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_i}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right]$$

The disclosure threshold for $t = 1$, θ_1^*, is such that the agent is indifferent between disclosing at $t = 1$ and obtaining $\theta_1^* + E[s_1] = \theta_1^*$ and the expected payoff from not disclosing at $t = 1$, given in (10). So the candidate for a disclosure threshold is the solution to:

$$\theta_1^* = [G(\theta_1^*)]^{N-1}\left(\frac{\theta_1^*}{1+r}\right)$$

$$+ \left(1 - [G(\theta_1^*)]^{N-1}\right)\left[\begin{array}{c} F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - F\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right]$$

∎

Proof of Proposition 1. Recall that $\theta \sim G(\theta)$. To simplify notation, we let the first period threshold for player i, $\theta_{1,i}^*$, be denoted as θ_i^*, e.g. firm 2's first period threshold is given by θ_2^*. Firm 1's best response function is defined by the indifference condition found in Lemma 3:

$$\theta_1^* = G(\theta_2^*)\left(\frac{\theta_1^*}{1+r}\right) \tag{11}$$

$$+ (1 - G(\theta_2^*))\left[\begin{array}{c} F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - F_\varepsilon\left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon\left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right]$$

$$= G(\theta_2^*)\left(\frac{\theta_1}{1+r}\right) + (1 - G(\theta_2^*))\left[NS(\theta_1^*) + V_2(\theta_1^*)\right] \tag{12}$$

Let $K(\theta_1, \theta_2)$ be defined as:

$$K(\theta_1, \theta_2) = G(\theta_2)\left(\frac{\theta_1}{1+r}\right) + (1 - G(\theta_2))\left[NS(\theta_1) + V_2(\theta_1)\right] - \theta_1$$

And hence,

$$
\begin{aligned}
\frac{d\theta_1}{d\theta_2} &= -\frac{\frac{\partial K}{\partial \theta_2}}{\frac{\partial K}{\partial \theta_1}} = -\frac{g\left(\theta_2\right)\left(\frac{\theta_1}{1+r}\right) - g\left(\theta_2\right)\left[NS\left(\theta_1\right) + V_2\left(\theta_1\right)\right]}{G\left(\theta_2\right)\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\frac{\partial\left[NS\left(\theta_1\right)+V_2\left(\theta_1\right)\right]}{\partial \theta_1} - 1} \\
&= -\frac{g\left(\theta_2\right)\left(\frac{\theta_1}{1+r}\right) - g\left(\theta_2\right)\left[NS\left(\theta_1\right) + V_2\left(\theta_1\right)\right]}{G\left(\theta_2\right)\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{1}{1+r} + \int_{-\infty}^{s_1^*(\theta_i)}\frac{-r}{(1+r)^2}f\left(s_1\right)ds_1\right] - 1} \\
&= -\frac{-g\left(\theta_2\right)V_2\left(\theta_1\right)}{G\left(\theta_2\right)\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{1}{1+r} + \frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right] - 1} \\
&= -\frac{-g\left(\theta_2\right)V_2\left(\theta_1\right)}{\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right] - 1} \\
&< 0
\end{aligned}
$$

Note that $\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right) < 0$ and hence the numerator is negative when $f\left(\cdot\right) > 0$ for it's support. This is expected as an increase in θ_2 results in a decrease in the probability that agent 1 observes s_1, which results in a lower threshold θ_1 necessary to satisfy indifference.

Next, we take the second derivative:

$$
\frac{d^2\theta_1}{(d\theta_2^*)^2} = \frac{d^2g\left(\theta_1, \theta_2^*\right)}{(d\theta_2^*)^2} = \frac{d}{d\theta_2^*}\left(\frac{g\left(\theta_2\right)V_2\left(\theta_1\right)}{\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right] - 1}\right)
$$

which is:

$$
\frac{d^2\theta_1}{(d\theta_2^*)^2} = V_2\left(\theta_1\right)\frac{g'\left(\theta_2\right)\left(\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right] - 1\right) + \left(g\left(\theta_2\right)\right)^2\left[\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right]}{\left(\left(\frac{1}{1+r}\right) + \left(1 - G\left(\theta_2\right)\right)\left[\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right] - 1\right)^2}
$$

(13)

Note that the denominator in equation (13) is always positive. Let $a = \frac{r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)$. Examining the numerator, we have:

$$
\begin{aligned}
g' \cdot V_2\left(\theta_1\right)\left(\left(\frac{1}{1+r}\right) + \left(1 - G\right)\left[-a\right] - 1\right) + g^2\left[-a\right]V_2\left(\theta_1\right) &> 0 \\
g'\left(\left(\frac{1}{1+r}\right) + \left(1 - G\right)\left[-a\right] - 1\right) + g^2\left(-a\right) &> 0 \\
g'\left(\left(\frac{-r}{1+r}\right) + \left(1 - G\right)\left[-a\right]\right) + g^2\left(-a\right) &> 0,
\end{aligned}
$$

which holds if

$$g'\left(\left(\frac{-r}{1+r}\right) + (1-G)\left[-a\right]\right) - g' \cdot (1-G)(-a) > 0$$

$$\left(\left(\frac{-r}{1+r}\right) + (1-G)\left[-a\right]\right) - (1-G)(-a) < 0$$

$$-\left(\left(\frac{-r}{1+r}\right) + (1-G)\left[-a\right]\right) + (1-G)(-a) > 0$$

$$\frac{r}{1+r} - (1-G)\left[-a\right] + (1-G)(-a) > 0$$

$$\frac{r}{1+r} > 0.$$

Which always holds since $r > 0$. Note this uses the fact that $g^2 \geq -g' \cdot (1-G)$, which can be seen by the nondecreasing hazard rate:

$$h'(\theta) = \frac{g' \cdot (1-G) + g^2}{(1-G)^2} \geq 0,$$

which implies

$$g' \cdot (1-G) + g^2 \geq 0$$
$$g' \cdot (1-G) \geq -g^2$$
$$-g' \cdot (1-G) \leq g^2$$

Therefore

$$\frac{d^2\theta_1^*}{(d\theta_2^*)^2} > 0$$

Which implies that the best response is a decreasing convex function. Since the two best response functions are symmetric, there can only be 3 possible intersections. ∎

Proof of Proposition 1. From Lemma 2 we know that

$$V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r}\right] f(s_1)\, ds_1.$$

Since the discount rate is held constant, the first period threshold changes in γ according to the change in the option value and the change in θ_2^*. Taking the derivative of $V_2(\theta_i)$ with

respect to γ and substituting $s_1^*(\theta_i) = -\frac{\theta_i r}{\gamma(1+r)-\gamma^2}$ we get

$$\frac{\partial}{\partial\gamma}\int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}-\frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)\,ds_1$$

$$= \int_{-\infty}^{s_1^*(\theta_i)}\frac{\partial}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}-\frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)\,ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1^*(\theta_i)}{(1+r)^2}-\frac{\theta_i+\gamma s_1^*(\theta_i)}{1+r}\right]$$

$$= \int_{-\infty}^{s_1^*(\theta_i)}\frac{\partial}{\partial\gamma}\left[\frac{\theta_i+\gamma^2 s_1}{(1+r)^2}-\frac{\theta_i+\gamma s_1}{1+r}\right]f(s_1)\,ds_1 + \frac{\partial s_1^*(\theta_i)}{\partial\gamma}\left[\frac{\theta_i-\frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2}-\frac{\theta_i-\frac{\theta_i r}{(1+r)-\gamma}}{1+r}\right]$$

$$\frac{\partial}{\partial\gamma}V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2}-\frac{s_1}{1+r}\right]f(s_1)\,ds_1$$

$$+ \left[\theta_i r\left(\gamma(1+r)-\gamma^2\right)^{-2}(1+r-2\gamma)\right]\left[\frac{\theta_i-\frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2}-\frac{\theta_i-\frac{\theta_i r}{(1+r)-\gamma}}{1+r}\right]$$

$$= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2}-\frac{s_1}{1+r}\right]f(s_1)\,ds_1 + \left[\theta_i r\left(\gamma(1+r)-\gamma^2\right)^{-2}(1+r-2\gamma)\right][0]$$

$$= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2}-\frac{s_1}{1+r}\right]f(s_1)\,ds_1$$

Next we show how the sign of $\frac{\partial\theta_1^*}{\partial\gamma}$ depends on the value of γ.

First note that for $\gamma = \frac{r+1}{2}$,

$$\frac{\partial}{\partial\gamma}V_2(\theta_i) = \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{2\frac{r+1}{2}s_1}{(1+r)^2}-\frac{s_1}{1+r}\right]f(s_1)\,ds_1$$

$$= \int_{-\infty}^{s_1^*(\theta_i)}\left[\frac{s_1}{(1+r)}-\frac{s_1}{1+r}\right]f(s_1)\,ds_1 = 0.$$

For $\gamma > \frac{r+1}{2}$, we have that:

$$\int_{-\infty}^{s_2^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2}-\frac{s_1}{(1+r)}\right]f(s_1)\,ds_1 < 0.$$

And finally, for $\gamma < \frac{r+1}{2}$:

$$\int_{-\infty}^{s_2^*(\theta_i)}\left[\frac{2\gamma s_1}{(1+r)^2}-\frac{s_1}{(1+r)}\right]f(s_1)\,ds_1 > 0.$$

Since θ_2^* follows the same direction as the change in the option value, the behavior of θ_1^* can

41

be characterized by the above. For example, for $\gamma < \frac{r+1}{2}$, since $\frac{\partial \theta_2^*}{\partial \gamma} > 0$ and $\frac{\partial}{\partial \gamma} V_2(\theta_i) > 0$, then $\frac{\partial \theta_1^*}{\partial \gamma}$. I.e. since the option value increases in $\gamma < \frac{r+1}{2}$, the period 1 threshold will increase since it waiting becomes more valuable, while the cost of waiting, r, remains the same. Likewise, since the second period threshold increases in $\gamma < \frac{r+1}{2}$, the likelihood of taking advantage of the real option is increasing for fixed s_1, thus making the real option more valuable, resulting in an increased period one threshold for fixed r. Both of these effects work in the same direction and hence the θ_1^* is increasing in $\gamma < \frac{r+1}{2}$. A similar argument applies for $\gamma > \frac{r+1}{2}$ and $\gamma = \frac{r+1}{2}$. ■

Proof of Proposition 2.   Recall that the disclosure threshold in the second period, $\theta_2^*(s_1)$, is independent of σ. In addition, for any θ the manager will disclose for any $s_1 > \mu_s = 0$. So, the manager will take advantage of the real option only for sufficiently low realizations of s_1, which are all lower than the mean of s_1.

An increase in σ, increases the probability that a manager that does not disclose at $t = 1$ will take advantage of the real option (and delay disclosure to $t = 3$). This however, is not sufficient to increase the incentive to delay disclosure at $t = 1$. A sufficient argument for the comparative static is to keep the threshold at $t = 1$ constant and to show that following an increase in σ the manager is no longer indifferent between disclosing and not disclosing for $\theta = \theta_1^*$ but rather strictly prefers not to disclose at $t = 1$.

A type θ_1^* will disclose at $t = 2$ either if the other manager did not disclose at $t = 1$ or if s_1 is lower than a threshold $s_1^*(\theta_i) = -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$. So the value from delaying disclosure comes only from realizations $s_1 < s_1^*(\theta_i) < 0$. First, note that following an increase in σ the probability of a realization of $s_1 < s_1^*(\theta_i)$ increases, i.e., $\frac{\partial \Pr\left(s_1 < s_1^*(\theta_i)\right)}{\partial \sigma} > 0$. Second, the expected value from delaying disclosure decreases in s_1.

There exists a value s' such that for all $s_1 < s'$ the probability of such an s_1 increases in σ. If $s' > s_1^*(\theta_i)$ that completes the proof. For $s' < s_1^*(\theta_i)$, following an increase in σ the probability of $s_1 < s'$ increase where $\Pr(s_1 \in (s', s_1^*(\theta_i)))$ decreases. It can be shown that we can "shift" mass from realization $s_1 < s'$ to realization $(s_1 \in (s', s_1^*(\theta_i)))$ under the high variance distribution such that pdf for all $(s_1 \in (s', s_1^*(\theta_i)))$ will be identical to the distribution with the low variance. Note that any such shift decreases the expected value from delaying disclosure at $t = 1$. Since the cumulative distribution for $s_1 < s_1^*(\theta_i)$ is higher under the high variance distribution, following this "shifting procedure" for any $s_1 < s'$ the pdf under the new distribution is still higher than under the low variance distribution (since

the overall mass for $s_1 < s_1^* (\theta_i)$ is higher for the high variance distribution). This implies that the option value under the high variance distribution is strictly higher than under the low variance distribution. ■

Proof of Proposition 3. Recall that the equilibrium first period threshold is:

$$
\theta_1^* = G(\theta_1^*)^{N-1} \left(\frac{\theta_1^*}{1+r} \right) \tag{14}
$$

$$
+ \left(1 - G(\theta_1^*)^{N-1} \right) \left[\begin{array}{c} F_\varepsilon \left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f_\varepsilon \left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \\ + \left(1 - F_\varepsilon \left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \right) \frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f_\varepsilon \left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \end{array} \right]
$$

$$
= G(\theta_1^*)^{N-1} \left(\frac{\theta_1}{1+r} \right) + \left(1 - G(\theta_1^*)^{N-1} \right) [NS(\theta_1^*) + V_2(\theta_1^*)] \tag{15}
$$

Let $K(\theta_1^*, N)$ be defined as:

$$
K(\theta_1^*, N) = G(\theta_1^*)^{N-1} \left(\frac{\theta_1}{1+r} \right) + \left(1 - G(\theta_1^*)^{N-1} \right) [NS(\theta_1^*) + V_2(\theta_1^*)] - \theta_1^*
$$

And hence,

$$
\frac{d\theta_1^*}{dN} = -\frac{\frac{\partial K}{\partial N}}{\frac{\partial K}{\partial \theta_1^*}}
$$

$$
= -\frac{G(\theta_1^*)^{N-1} \ln(G(\theta_1^*)) \left(\frac{\theta_1^*}{1+r} \right) - G(\theta_1^*)^{N-1} \ln(G(\theta_1^*)) [NS(\theta_1^*) + V_2(\theta_1^*)]}{\left[\begin{array}{c} (N-1) G(\theta_1^*)^{N-2} g(\theta_1^*) \left(\frac{\theta_1^*}{1+r} \right) + G(\theta_1^*)^{N-1} \left(\frac{1}{1+r} \right) \\ - (N-1) G(\theta_1^*)^{N-2} g(\theta_1^*) [NS(\theta_1^*) + V_2(\theta_1^*)] + \left(1 - G(\theta_1^*)^{N-1} \right) \cdot \frac{\partial[NS(\theta_1^*)+V_2(\theta_1^*)]}{\partial \theta_1} - 1 \end{array} \right]}
$$

This becomes

$$
= -\frac{G(\theta_1^*)^{N-1} \ln(G(\theta_1^*)) \left(\frac{\theta_1^*}{1+r} \right) - G(\theta_1^*)^{N-1} \ln(G(\theta_1^*)) [NS(\theta_1^*) + V_2(\theta_1^*)]}{\left[\begin{array}{c} (N-1) G(\theta_1^*)^{N-2} g(\theta_1^*) \left(\frac{\theta_1^*}{1+r} \right) + G(\theta_1^*)^{N-1} \left(\frac{1}{1+r} \right) \\ - (N-1) G(\theta_1^*)^{N-2} g(\theta_1^*) [NS(\theta_1^*) + V_2(\theta_1^*)] \\ + \left(1 - G(\theta_1^*)^{N-1} \right) \cdot \left[\frac{1}{1+r} + \int_{-\infty}^{s_1^*(\theta_i)} \frac{-r}{(1+r)^2} f(s_1)\, ds_1 \right] - 1 \end{array} \right]}
$$

which is

$$
= -\frac{-G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)}{\left[\begin{array}{c}-\left(N-1\right)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)V_2\left(\theta_1^*\right)+G\left(\theta_1^*\right)^{N-1}\left(\frac{1}{1+r}\right)\\+\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{1}{1+r}+\frac{-r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right]-1\end{array}\right]}
$$

$$
= \frac{G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)}{-\left(N-1\right)G\left(\theta_1^*\right)^{N-2}g\left(\theta_1^*\right)V_2\left(\theta_1^*\right)-\left(1-G\left(\theta_1^*\right)^{N-1}\right)\cdot\left[\frac{r}{(1+r)^2}F\left(s_1^*\left(\theta_i\right)\right)\right]-\frac{r}{1+r}}
$$

$$
> 0
$$

We have that $G\left(\theta_1^*\right)^{N-1}\ln\left(G\left(\theta_1^*\right)\right)V_2\left(\theta_1^*\right)<0$ since $\ln\left(G\left(\theta_1^*\right)\right)<0$, and we can easily see that the denominator is negative. Thus $\frac{d\theta_1^*}{dN}>0$. ∎

Proof of Proposition 4. Assume that in the case of indifference, the firm discloses. Note that when $r=0$, we have no interior solution. The only equilibria are asymmetric equilibria. It is easy to show that these are equilibria and that no interior equilibrium exists–in any equilibrium in which firm j discloses with positive probability, type $\bar{\theta}_i$ is better off waiting with probability 1, as this gives her strictly higher expected utility over disclosing when $r=0$. Note that there always exists an $r>0$ in which we have the asymmetric equilibria.

Setting $G\left(\theta_{1,j}^*\right) = 0$, we have from Lemma 3 that, as $r \to 0$,

$$\lim_{r \to 0} G\left(\theta_j^*\right)\left(\frac{\theta_i^*}{1+r}\right)$$

$$+ \left(1 - G\left(\theta_j^*\right)\right)\left[\begin{array}{c} F\left(\frac{\theta_i^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\frac{\theta_1^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f\left(-\frac{\theta_i^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \\ + \left(1 - F\left(\frac{\theta_i^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right)\right)\frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f\left(-\frac{\theta_i^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}\right) \end{array}\right]$$

$$= F\left(0\right)\frac{\theta_i^*}{1+r} + \frac{1}{1+r}\gamma\sigma^2 f\left(-\frac{r}{\gamma}\cdot\frac{\theta_i^*}{((1+r)-\gamma)}\right)$$

$$+ \left(1 - F\left(0\right)\right)\frac{\theta_i^*}{(1+r)^2} - \frac{1}{(1+r)^2}\gamma^2\sigma^2 f\left(-\frac{r}{\gamma}\cdot\frac{\theta_i^*}{((1+r)-\gamma)}\right).$$

$$= F\left(0\right)\theta_i^* + \gamma\sigma^2 f\left(0\right) + \left(1 - F\left(0\right)\right)\theta_i^* - \gamma^2\sigma^2 f\left(0\right)$$

$$= \theta_i^* + \gamma\sigma^2 f\left(0\right) - \gamma^2\sigma^2 f\left(0\right)$$

$$= \theta_i^* + \sigma^2 f\left(0\right)\left(\gamma - \gamma^2\right)$$

$$= \theta_i^* + \sigma^2 \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\mu}{2\sigma^2}}\gamma\left(1 - \gamma\right)$$

$$= \theta_i^* + \frac{\sigma}{e\sqrt{2\pi}}\gamma\left(1 - \gamma\right).$$

Since $\gamma \in (0,1)$ and $\sigma > 0$, the benefit of waiting in the limit is strictly positive. Hence, for all $\sigma > 0$ and $\gamma \in (0,1)$, we can find r sufficiently close to zero such that an asymmetric equilibrium can be supported when $G\left(\theta_{1,j}^*\right) = 0$. Recall that the upper bound of the asymmetric equilibria is denoted by r^L. Now for any $r > r^L$, type $\bar{\theta}_{-j}$ still finds disclosure profitable even when $G\left(\theta_{1,j}^*\right) = 0$, and hence the asymmetric equilibria do not exist for $r > r^L$. Finally, as $r \to \infty$, the payoff from waiting to disclose goes to zero. For θ with bounded support, we can find an $r < \infty$ such that $\theta_i^* \leq \bar{\theta}$ when $G\left(\theta_j^*\right) = 0$. Denote the maximum r that supports this equilibrium as r^H:

$$r^H = \max_r \left\{ \begin{array}{c} \Pr\left(s_1 > \frac{r}{\gamma}\cdot\frac{-\theta_i}{((1+r)-\gamma)}\right) E\left[\frac{\theta_i+s_2}{1+r}\Big|s_1 > \frac{r}{\gamma}\cdot\frac{-\theta_i}{((1+r)-\gamma)}\right] \\ + \left(\Pr\left(s_1 < \frac{r}{\gamma}\cdot\frac{-\theta_i}{((1+r)-\gamma)}\right)\right) E\left[\frac{\theta_i+s_3}{(1+r)^2}\Big|s_1 \leq \frac{r}{\gamma}\cdot\frac{-\theta_i}{((1+r)-\gamma)}\right] \end{array} < \bar{\theta} \right\}.$$

Which we know exists by Theorem 1. ∎

References

[1] Acharya, Viral V., Peter DeMarzo, and Ilan Kremer. 2011. "Endogenous Information Flows and the Clustering of Announcements." *American Economic Review* 101 (7): 2955–79.

[2] Aghamolla, C., and Byeong-Je An. 2015. "Voluntary Disclosure with Evolving News." Working paper.

[3] Allen, Franklin, Stephen Morris, and Hyun Song Shin. 2006. "Beauty Contests and Iterated Expectations in Asset Markets." *Review of Financial Studies* 19 (3): 719-752.

[4] Alti, Aydogan. 2005. "IPO Market Timing." *Review of Financial Studies* 18 (3): 1105-1138.

[5] Baker, Malcom, and Jeffrey Wurgler. 2006. "Investor Sentiment and the Cross-Section of Stock Returns." *Journal of Finance* 61 (4): 1645-1680.

[6] Benninga, Simon, Mark Helmantel, and Oded Sarig. 2005. "The Timing of Initial Public Offerings." *Journal of Financial Economics* 75: 115-32.

[7] Benveniste, Lawrence M., Walid Busba, and William J. Wilhelm. 2002. "Information Externalities and the Role of Underwriters in Primary Equity Markets." *Journal of Financial Intermediation* 11: 61-86.

[8] Benveniste, Lawrence M., Alexander Ljungqvist, William J. Wilhelm, and Xiaoyun Yu. "Evidence of Information Spillovers in the Production of Investment Banking Services." *Journal of Finance* 58 (2): 577-608.

[9] Bessembinder, Hendrik, Jay F. Coughenour, Paul J. Seguin, and Margaret Monroe Smoller. 1995. "Mean Reversion in Equilibrium Asset Prices: Evidence From the Futures Term Structure." *Journal of Finance* 5 (1): 361-375.

[10] Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77 (3): 623-685.

[11] Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta, and Stephen Terry. 2014. "Really Uncertain Business Cycles." Working paper.

[12] Brau, James C., and Stanley E. Fawcett. 2006. "Initial Public Offerings: An Analysis of Theory and Practice." *Journal of Finance* 61 (1): 399-436.

[13] Cecchetti, Stephen G., Pok-Sang Lam, and Nelson C. MarkSource. "Mean Reversion in Equilibrium Asset Prices." *American Economic Review* 80 (3): 398-418.

[14] Chemmanur, Thomas J., and Paolo Fulghieri. 1999. "A Theory of the Going-Public Decision." *Review of Financial Studies* 12 (2): 249-79.

[15] Dembosky, April. "Wall Street 'mispriced' LinkedIn's IPO." *Financial Times*, March 30, 2011.

[16] Demos, Telis, and Bradley Hope. "For Virtu IPO, Book Prompts A Delay." *The Wall Street Journal*, April 3, 2014.

[17] Driebusch, Corrie, and Daniel Huang. "Bank IPO Falls Short of Targeted Price Range." *The Wall Street Journal*, September 14, 2014.

[18] Dye, Ronald A., and Sri S. Sridhar. 1995. "Industry-Wide Disclosure Dynamics." *Journal of Accounting Research* 33(1): 157–74.

[19] Fama, Eugene F., and Kenneth R. French. 1988. "Permanent and Temporary Components of Stock Prices." *Journal of Political Economy* 96 (2): 246-73.

[20] Guttman, Ilan, Ilan Kremer, and Andrzej Skrzypacz. 2014. "Not Only What but Also When: A Theory of Dynamic Voluntary Disclosure." *American Economic Review* 104 (8): 2400-2420

[21] He, Ping. 2007. "A Theory of IPO Waves." *Review of Financial Studies* 20 (4): 983-1020.

[22] Helwege, Jean, and Nellie Liang. 2004. "Initial Public Offerings in Hot and Cold Markets." *Journal of Financial and Quantitative Analysis* 39 (3): 541-69.

[23] Hoffman-Burchadi, Ulrike. 2001. "Clustering of Initial Public Offerings, Information Revelation, and Underpricing." *European Economic Review* 45: 353-383.

[24] Ibbotson, Roger G. and Jeffrey F. Jaffe. 1975. "'Hot issue' markets." *Journal of Finance* 30: 1027-1042.

[25] Ibbotson, Roger, and Jay R. Ritter. 1995. Initial public offerings, in Robert A. Jarrow, Vojislav Maksimovic, and William T. Ziemba, eds.: *Handbooks of Operations Research and Management Science* (Amsterdam: North-Holland).

[26] Ibbotson, Roger G., Jody L. Sindelar, and Jay R. Ritter. 1988. "Initial Public Offerings." *Journal of Applied Corporate Finance* 1: 37-45.

[27] Ibbotson, Roger G., Jody L. Sindelar, and Jay R. Ritter. 1994. "The Market's Problems with the Pricing of Initial Public Offerings." *Journal of Applied Corporate Finance* 7: 66-74.

[28] Jung, Woon-Oh, and Young K. Kwon. 1988. "Disclosure When the Market Is Unsure of Information Endowment of Managers." *Journal of Accounting Research* 26 (1): 146–53.

[29] Lerner, Joshua. 1994. "Venture capitalists and the decision to go public." *Journal of Financial Economics* 35: 293-316.

[30] Loughran, Tim, Jay R. Ritter, and Kristen Rydqvist. 1994. "Initial public offerings: International insights." *Pacific-Basin Finance Journal* 2: 166-99.

[31] Lowry, Michelle, and G. William Schwert. 2002. IPO Market Cycles: Bubbles or Sequential Learning?" *Journal of Finance* 57 (3): 1171-1198.

[32] Maskimovic, Vojislav, and Pegaret Pichler. 2001. "Technological Innovation and Initial Public Offerings." *Review of Financial Studies* 14 (2): 459-494.

[33] Maur, David C., and Lemma W. Senbet. 1992. "The Effect of the Secondary Market on the Pricing of Initial Public Offerings: Theory and Evidence." *Journal of Financial and Quantitative Analysis* 27 (1): 55-79.

[34] Pagano, Marco, Fabio Panetta, and Luigi Zingales. 1998. "Why Do Companies Go Public? An Empirical Analysis." *Journal of Finance* 53 (1): 27-64.

[35] Pastor, Lubos, and Pietro Veronesi. 2005. "Rational IPO Waves." *Journal of Finance* 60 (4): 1713-57.

[36] Persons, John C., and Vincent A. Warther. 1997. "Boom and Bust Patterns in the Adoption of Financial Innovations." *Review of Financial Studies* 10 (4): 939-967.

[37] Poterba, James M., and Lawrence H. Summers. 1988. "Mean Reversion in Stock Prices." *Journal of Financial Economics* 22: 27-59.

[38] Richards, Anthony J. 1997. "Winner-Loser Reversals in National Stock Market Idices: Can they be explained?" *Journal of Finance* 52: 2129-44.

[39] Ritter, Jay R. 1984. "The 'hot issue' market of 1980." *Journal of Business* 57: 215-240.

[40] Tobak, Steve. "Did IPO damage Facebook brand?" *CBS Money Watch,* June 6, 2012.

[41] Winkler, Rolfe, Matt Jarzemsky, and Evelyn Rusli. "Drop in Tech Stocks Hits Startup Funding." *The Wall Street Journal*, April 16, 2014.