Influence activities, coalitions, and uniform policies

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Abstract

This paper examines the effects of endogenous coalition formation in a setting where agents lobby a policy-maker. For example, financial firms lobby for weaker regulation to facilitate privately beneficial activities that impose welfare-decreasing externalities (e.g., systemic risk) on the financial system. Policy uniformity (e.g., one-size-fits-all rules) causes agents to free ride on each other's lobbying and gives them an incentive to form lobbying coalitions. We show that the coalitions may be formed by similar or dissimilar agents. Additionally, endogenous coalition formation causes the effects of policy uniformity and lobbying costs on aggregate lobbying activity and policy strength to be non-monotonic. Finally, our model suggests that increased competition in the market for coalition-facilitating lobbyists leads to less lobbying.

Keywords: lobbies, coalitions, one-size-fits-all, regulation

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1 Introduction

Regulatory policies are usually set with a degree of uniformity, applying similar treatments to diverse agents. These agents, in turn, often have the ability to influence the policies that they are exposed to and can organize into coalitions or lobbies that coordinate their influence activities. In this paper, we explore the agents' choices of how to organize into lobbies, and how agents' ability to organize endogenously influences their lobbying activities and a policy-maker's choices.

In our model, a regulator (or enforcement agency) chooses the policies that apply to a set of heterogeneous agents. Each agent can take an action that is privately beneficial but socially harmful, and stronger regulatory policies reduce the probability that the agents succeed at taking their socially inefficient actions. The regulator is interested in minimizing the welfare losses, but is subject to influence activities undertaken by the agents, who lobby for weaker policies. As a motivating example, we have in mind the regulation of financial institutions, particularly those that may pose risks to the entire financial system. Regulators such as the Federal Reserve write capital rules, run stress tests, and perform on-site examination reviews of banks to help ensure the stability of the financial system. These regulatory actions reduce financial institutions' opportunities to pursue strategies that impose systemic risk on the economy (e.g., excess leverage and risk-taking, insufficient capital buffers, failure to account for counterparty risk, exposure to short-term financing runs, levered exposures in exchange traded funds).

Although the agents in our model differ in the socially-harmful but privately-beneficial actions they can take, the regulator is constrained to treat the agents similarly (though not necessarily equivalently). In the U.S., banks and other financial institutions face similar regulatory policies and oversight, i.e., through federal laws such as Dodd-Frank. However, different types of financial institutions are overseen by different regulators, reflecting regulatory heterogeneity. For example, the Office of the Comptroller of the Currency (OCC) regulates nationally chartered banks, while the Office of Thrift Supervision (OTS) oversees savings and loan associations, and the Federal Reserve, Federal Deposit Insurance Corporation (FDIC), and state-level banking departments oversee state-chartered banks. Finally, the Federal Reserve and Financial Stability Oversight Council (FSOC) regulate bank holding companies and Systemically Important Financial Institutions (SIFIs), which can include large insurers and asset managers. The US Congress, through laws that set up different regulators for different sets of financial institutions and overlapping oversight responsibilities, has enacted a degree of partial uniformity in the regulation of financial firms in the US.

With lobbying, exposing different agents to similar policies (e.g., rules from the same oversight body) causes an externality of one agent's lobbying on other agents' policies results, which, in turn, results in a lobbying-related free rider problem among agents (Friedman and Heinle, 2016). To overcome the free-rider problem and, thus, to more effectively reduce the extent of regulation or enforcement, agents can organize into coalitions (or lobbies). Among financial institutions, public lobbying filings list several multi-firm lobbying organizations, including the American Banking Association, the Credit Union National Association, Independent Community Bankers of America, and the Financial Services Roundtable. Aside from forming their own lobbying organizations, different financial institutions can also retain the services of the same lobbyist to facilitate coordination. For example, Citigroup, Goldman Sachs, Hartford Financial Services Agents, and other financial institutions retained the services of Subject Matter, a lobbying services firm.[1] Facing potential designation as SIFIs, large asset managers including BlackRock and Fidelity engaged in concerted pushback, and "held numerous meetings with congressional staff" (see Ackerman and Tracy, 2014). Within a coalition in our model, agents choose their lobbying efforts to maximize the joint utility of all agents in the coalition. That is, agents eliminate the free rider problem because they internalize the effect of their lobbying on the other agents in the coalition. We endogenize the formation of lobbies and analyze the characteristics of the coalitions that form.

The model consists of three agents, which can represent specific financial institutions (e.g.,

[1] Data from https://www.opensecrets.org/lobby/issuesum.php?id=BAN.

Citigroup, Goldman Sachs, and Wells Fargo) or broader sectors (e.g., commercial banks, insurers, and asset managers). We assume that there is a cost of forming a coalition, which increases in the size of the coalition. In essence, this cost represents the cost to overcome the free-rider problem. Absent the cost, the grand coalition (i.e., the lobby including all agents) is always optimal from the agents' perspective, due to the positive externalities that agents enjoy from each other's lobbying. Furthermore, we assume that each coalition needs a specialist (or lobbyist) to enable within-coalition coordination. That is, we assume that lobbyists do not reduce lobbying costs but, instead, allow agents to overcome the free rider problem. While lobbyists cannot force agents to join coalitions, we assume that they extract a fraction of the net coalition gains from coalition formation. This fraction captures the intensity of competition in the unmodeled lobbyist sector.

We focus on the implications of endogenously-formed coalitions. To define an equilibrium coalition, we introduce the notion of offer-stable coalitions. When a set of coalition members receive an offer to deviate, offer-stable coalitions can prevent these members from leaving by making a successful counteroffer. As such, offer-stability is closely related to notions based on the core and bargaining sets (e.g., Von Neumann and Morgenstern, 1944; Aumann and Maschler, 1964; Ray and Vohra, 1999). However, in our setting of heterogeneous agents, existing externalities, and not necessarily superadditivity, offer-stability predicts at least one stable coalition structure for any feasible set of parameters. When offer-stability predicts multiple stable coalition structures, we allow the lobbyist to "break the tie" and form the coalition that provides her with the greatest benefit.[2] Interestingly, we find that this coalition formation mechanism can support the grand coalition, a coalition of the two agents with the largest potential for private benefits, or a coalition between the agents with the smallest and the largest potential for private benefits. The only two-member coalition that is never stable is the one that includes the two agents with the smallest potential for private benefits. In other words, offer stable coalitions can be between either similar agents (i.e., medium and

[2]The unique coalition predicted by offer-stability and the lobbyist's preferences is in the core when the core is non-empty and there exists a unique equilibrium coalition even when the core is empty.

large), between dissimilar agents (small and large), or between all agents.

We show that regulatory uniformity has non-monotonic effects on aggregate lobbying and policy strength when coalitions form endogenously. This occurs because the primary effect of uniformity–promoting free-riding between lobbying agents–also encourages them to form coalitions. That is, uniformity tends to decrease lobbying and strengthen regulatory policies, except around thresholds that cause agents to coalesce into new or larger lobbying coalitions. These coalitions cause discrete jumps in the agents' ability to overcome free-rider problems, leading to increased lobbying and, in turn, weaker policies.[3]

Additional comparative statics show similar non-monotonicities related to the costs of lobbying and the costs of coalition formation. Higher lobbying costs lead directly to less lobbying and less distorted policy, which in turn reduces the free-riding-on-lobbying problem that motivates coalition formation. However, in our setting, one agent benefits when the two other agents form a coalition, because the coalition-members increase their lobbying and this tends to weaken the regulation imposed on the non-coalition agent. Typically, higher lobbying costs lead to smaller coalitions, but there are sets of parameters for which higher lobbying costs can cause the coalition structure to transition from two-agent to three-agent, through the non-proportional influence on coalition-member and non-coalition agents. Changes in coalition costs, through similar mechanisms, can have similar non-monotonic effects on coalition structures, and, through their influence on the coalition structure, have non-monotonic effects on lobbying and policy strength. In contrast, an increase in the lobbyists' share of the gains from coalition formation, i.e., a decrease in competition in the unmodeled lobbying sector, always causes a shift towards larger coalitions. This tends to increase total lobbying and decrease average regulatory strength. Restricting the supply of lobbyists (e.g., through laws against revolving doors), can thus have a *negative* effect on regulatory strength and can *increase* the degree to which policies are influenced by lobbying.

[3] This result should also hold in a setting with a continuous mass of firms when the number of coalitions is discrete and can change in the degree of uniformity. If the number of coalitions tends to infinity, then the mass of a given coalition tends to zero, eliminating the benefit of the coalition in influencing policy.

Beyond the regulation of systemically important financial institutions, the central tensions in our model carry over to several settings. These key tensions arise from a policy maker that is constrained in some way to set similar policies for a set of heterogeneous agents, and agents who differ in their abilities to take actions that are socially harmful (e.g., through negative externalities, lower consumer surplus, or tax burdens imposed on unrelated parties). For example, the agents in our model could be lobbying for fewer restrictions on investments that impose negative externalities, e.g., involving pollution or medical testing. Agents could also be lobbying for barriers to entry, such as government granted monopolies related to patents, taxi medallions, or flight routes. Similarly, with slight modification, the agents in our model can be interpreted as different industries that seek trade protection or a set of countries that seek to influence product standards set out in a trade deal (e.g., Charlemagne, 2016).[4] The importance of coalitions is highlighted by recent popular press articles, noting that "Corporate America can't seem to get enough of the ad hoc coalitions that are formed to put muscle and money behind a lobbying push, and hardly a week passes in Washington without a new group appearing on the scene." (Bogardus, 2013).[5]

The foundation for our study is the literature on regulatory choice in economics (e.g., Arrow, 1950), which helps explain observed choices by highlighting how lobbying and regulatory capture cause regulators to choose non-welfare-maximizing rules and transfers (e.g., Stigler, 1971; Grossman and Helpman, 1996). Our model is most closely related to Friedman and Heinle (2016), who present a similar two-agent model involving a regulator who can probabilistically prevent a privately costly but socially wasteful action through regulation but is subject to regulatory capture via lobbying pressure. However, the model in Friedman and Heinle (2016) precludes the formation of lobbying coalitions, leaving agents no way to

[4] As a related example, Rodrik (1986) focuses on trade-offs between industry-wide tariffs and firm-specific subsidies in a setting in which industry-wide tariffs promote free-riding on firms' tariff-seeking. Damania and Fredriksson (2000) explore coalition formation and lobbying in a model focused on environmental protections.

[5] Bogardus (2013) and Ho (2015) provide examples of: FedEx, Nike, and Verizon joining coalitions lobbying for tax reform; Intel, Microsoft, Qualcomm, and Texas Instruments lobbying through a coalition for immigration reform; and 3M, Caterpillar, and GE lobbying through the Coalition for 21st Century Patent Reform for revisions to laws covering patents.

overcome the free-riding problem generated by uniform policies. Similarly, Bebchuk and Neeman (2010) investigate a model in which different groups lobby the regulator over the level of investor protection in a perfectly uniform regulatory regime. Although this is a regime in which coalitions would be most valuable (which we show in our model), lobbying coalitions are assumed impossible. In Bertomeu and Magee (2014), regulatory outcomes are chosen by a combination of a majoritarian vote by firms and the standard setter's bliss point. In their model, as in most that feature voting as the policy selection tool, voter collusion (e.g., via trading or selling votes) is excluded by assumption.

Related studies in the trade literature also explore lobby formation.[6] Mitra (1999), for example, allows firms to form lobbies to coordinate their efforts on lobbying for trade protections or subsidies. Lobbies pay an exogenous cost of organizing, but only firms within the same industry can organize with each other. In this way, the bounds of a lobby are essentially exogenous, as firms are assigned to industries and the regulator has to treat all firms within an industry homogeneously. Furthermore, lobbies in different industries compete for trade subsidies, generating negative externalities, in contrast to the lobbies in our model which have positive externalities on each other. Magee (2002), building on Pecorino (1998), models a single industry with homogeneous firms and a focus on the interplay between industry concentration and the ease of overcoming free-riding between firms. Drazen et al. (2007) examine the effect of limits on political spending, finding potentially unintended consequences because upper limits can improve the per-dollar efficiency of lobbying. In a model with endogenous lobby formation, this increase in efficiency can cause greater entry of lobbyists, leading to greater equilibrium lobbying and increasing the consequent policy distortions.

[6] Beyond lobbying, endogenous coalitions are important in theories explaining the formation and actions of clubs (e.g., Ellickson et al., 1999 or Nordhaus, 2015).

2 Model setup

In the model, there are five risk neutral actors: three agents, a policy-setter/regulator, and a lobbyist.[7] Agents are indexed by $i \in \{s, m, \ell\}$ for small, medium, and large, respectively. Each agent can take a privately-beneficial action, for which she gains D_i, where $0 < D_s < D_m < D_\ell$. The privately-beneficial action associated with D_i imposes a social cost of $A_i = (1 + \lambda) D_i$, where $\lambda > 0$.[8] The privately-beneficial action is therefore socially inefficient and imposes a net welfare loss of $D_i\lambda > 0$. Our assumption of $D_i > 0$ implies that each agent always prefers to take the personally beneficial action. For ease of exposition, we refer to agents with higher D_i as larger and agents with lower D_i as smaller. This nomenclature relates to the impact of the action, and should not be interpreted as directly reflecting, say, firm size. Relating back to our setting of the regulation of systemically-important financial institutions, we could interpret agent ℓ as large banks, agent m as large insurers, and agent s as large asset managers. Large banks are plausibly the most systemically important, while insurers and asset managers potentially impose smaller risks on the financial system, on average. Alternatively, taking all agents as banks, ℓ could be JPMorgan Chase & Co. (~$2.5 trillion in assets), m could be US Bancorp (~$450 in assets) and s could be Suntrust (~$200 billion in assets).[9]

Regulation limits each agent's opportunity to take the privately-beneficial action. Specifically, we model the intensity of regulation governing each agent i as the probability, π_i, that an agent is unable to take the action. The action therefore is taken with probability $(1 - \pi_i)$. Finally, before the regulator specifies the regulatory intensities, each agent can exert effort B_i to lobby the regulator to relax the regulatory intensity she faces.

Agents benefit only from the privately-beneficial action. Each agent incurs a personal cost of lobbying the regulator, $\frac{c}{2}B_i^2$. The parameter $c > 0$ captures the ability of agents to

[7] Because of the similarity of the lobbying game, we borrow large parts of the model description from Friedman and Heinle (2016).

[8] Agents in our model are heterogeneous in the impact of their actions, although they are homogeneous in the proportional costs of their actions, $1 + \lambda$.

[9] Amounts based on annual reports dated December 31, 2016.

effectively lobby. A higher value of c reflects a less severe problem related to the lobbying that facilitates inefficient regulatory policies. Each agent's expected utility is given by

$$U_i = (1 - \pi_i) D_i - \frac{c}{2} B_i^2. \tag{1}$$

With probability $(1 - \pi_i)$, the agent is able to take the privately beneficial action and consume D_i. Agents always bear the cost of lobbying because they lobby the regulator before the action takes place. We assume that agents cannot commit to "share the spoils" with the regulator and do not use resources gained from their privately beneficial actions to extract regulatory concessions.

When the regulator decides on the regulatory intensity, the costs of lobbying, $\frac{c}{2}B_i^2$, are sunk. Therefore, the aggregate utility that can be influenced by the regulator is given by the expected losses from the socially-inefficient action:

$$L(\pi, D, \lambda) = -\lambda \sum_{i \in \{s,m,\ell\}} D_i (1 - \pi_i). \tag{2}$$

The welfare-interested regulator is only concerned about the actions because of the welfare loss, $\lambda \bar{D} = \lambda \sum_{i \in \{s,m,\ell\}} D_i$, that the actions impose on society. This welfare loss occurs with probability $(1 - \pi_i)$, for each agent i. The regulator wants to minimize this welfare loss subject to the costs of regulation.[10]

Regulation is costly for three reasons. First, regulation is costly in and of itself, with a convex cost of regulation, $\frac{1}{2}\pi_i^2$ for regulation covering each agent. More stringent regulation and enforcement, e.g., more frequent bank inspections, naturally require larger staffs and potentially more costly training. Second, each agent can influence the regulator through lobbying activity B_i, which increases the cost of regulatory intensity by $B_i \pi_i$. Third, defining

[10] We assume that the regulator cares only about the welfare loss. Glaeser et al. (2001) model judges and regulators as alternative enforcement mechanisms and assume that regulators, while easier to motivate, can also be overzealous and prefer excessively-strict policies. In such a setting, lobbying can be beneficial as a counterweight to regulator's overzealousness. See Friedman and Heinle (2016) for further discussion of interpretations of c, λ, and π_i.

a different regulatory intensity for different agents imposes additional costs, which we model as $\frac{k}{2}\sum_i \left(\pi_i - \bar{\pi}_{\setminus i}\right)^2$, where $\bar{\pi}_{\setminus i} = \frac{1}{2}\sum_{i' \neq i} \pi_{i'}$. We interpret the parameter k as the degree of regulatory uniformity; when $k = 0$, the regulator is free to choose individualized regulation (IR) without incurring any penalty whereas the regulator sets the same regulatory intensity for all agents as $k \to \infty$, enacting a one-size-fits-all uniform regime (UR). Crucial for our results is that the regulator is committed to apply somewhat similar regulatory policies to different agents for any $k > 0$. Instead of modeling this commitment through a cost term, we could assume that the regulator receives a benefit from more similar regulation.

Alternatively, we could assume that the regulator maximizes utility subject to some constraints that force regulatory similarity, such as $\pi_j - \frac{1}{\kappa} \leq \pi_i \leq \pi_j + \frac{1}{\kappa}$, with the commitment to uniformity captured by $\kappa > 0$. Here, the regulator would be committed to choose π_i not too far away from π_j, with higher κ signifying a commitment to greater uniformity. One of the main economic consequences of uniformity is the free rider problem induced by agent i's lobbying affecting π_j. In a setting where the commitment comes from similarity bounds given by κ rather than a cost of dissimilarity given by k, agent i's lobbying efforts will affect π_j as long as some κ-based constraints bind.[11]

Given that heterogeneous regulation plausibly requires greater care in drafting and increased expenditures in enforcement (e.g., staff costs), k can be interpreted as a technical constraint on the regulator. Alternatively, k could be an institutional commitment (for example, a mission statement) to regulate different firms in a similar fashion. In the setting of the regulation of financial institutions, higher k, might reflect a single regulator covering a broader set of firms (e.g., the Federal Reserve supervising systemically important financial institutions whether they are banks, insurers, or asset managers), while lower k would reflect a regulatory environment in which there are several regulators, each with a different domain

[11] In essence, the similarity-bounds impose a convex cost on the regulator where small differences in regulatory policies are free but large differences are infinitely costly. Indeed, the similarity-bounds and cost-constraint can be linked if the regulator can, at an earlier stage, pay a k-based cost to reduce κ. The regulator would then compare the shadow price from the constrained maximization with the k-based cost to relax the constraint.

(i.e., the OCC, OTS, and FDIC covering different types of banks). In this example, the Federal Reserve would tend to set more similar policies for the three different types of economic agents it oversees, relative to the policies set by each of the three smaller regulators each focused on their own domain. We capture this intuition by using a higher k to represent the former case of three types of agents overseen by the Fed, and a lower k to represent the latter case. We use a continuous k rather than a binary IR vs. UR indicator to allow for smoother transitions across regimes and comparative statics analysis with finer regime gradations. While we take k as an exogenous parameter, Friedman and Heinle (2016) examine how a system designer, such as a legislature, would optimally set k in a setting with two agents who cannot form a coalition.

The total cost of regulation is given by

$$C(\pi, B, k) = \sum_{i \in \{s,m,\ell\}} \left(\frac{\pi_i^2}{2} + B_i \pi_i + \frac{k}{2} \left(\pi_i - \bar{\pi}_{\setminus i} \right)^2 \right), \tag{3}$$

and the regulator's expected utility is

$$U_R = L(\pi, D, \lambda) - C(\pi, B, k). \tag{4}$$

The regulator chooses the regulatory intensity for all agents to minimize the expected welfare loss subject to the cost of regulation.

Agents, in turn, can form coalitions to coordinate their lobbying efforts. A coalition is defined as a set of agents, denoted by $l_j \equiv \{i : i \text{ is a member of coalition } j\}$, where j indicates a particular coalition. Each coalition also includes a lobbyist (described below). We impose an increasing cost to forming larger lobbies, defined as $\chi_{|l_j|}$, where $|l_j|$ is the number of agents in the lobby, with $\chi_1 = 0 < \chi_2 < \chi_3$. These costs prevent the payoff structure from generally being superadditive. Aumann and Dreze (1974) discuss how difficulties associated with collaboration, such as transaction costs of side payments or within-coalition monitoring, would give rise to such size-based coalition costs.

There are five possible coalition structures, as shown in Table 1.

Structure name	Notation	Coalition structure
Independent agents	I	$\{\{s\},\{m\},\{\ell\}\}$
Small-medium lobby	sm	$\{\{s,m\},\{\ell\}\}$
Small-large lobby	$s\ell$	$\{\{s,\ell\},\{m\}\}$
Medium-large lobby	$m\ell$	$\{\{s\},\{m,\ell\}\}$
Grand lobby	G	$\{\{s,m,\ell\}\}$

Table 1
Coalition Structures

Let U_{i,l_j} be the expected utility, before coalition costs, $\chi_{|l_j|}$, generated by agent i in coalition structure $l_j \in \{I, sm, s\ell, m\ell, G\}$. For example, $U_{i,I}$ is the expected utility generated by agent i if all agents lobby independently, $U_{s,m\ell}$ is the expected utility generated by agent s when agents m and ℓ form a coalition together, and $U_{m,G}$ is the expected utility generated by agent m in the grand coalition, G. We define the (potentially negative) net gains from coalition formation as

$$\Delta U_{l_j} = \sum_{i \in l_j} \left(U_{i,l_j} - U_{i,I}\right) - \chi_{|l_j|}.$$

The gain from coalition formation reflects the additional expected utility for the agents in the coalition net of the coalition costs, prior to splitting any net gains with the lobbyist.

The lobbyist, who is necessary for coalition formation, can extract an exogenous fraction of ΔU_{l_j}. We assume that the agents keep a fraction α of these gains, the lobbyist keeps $(1-\alpha)$, and $\alpha \in [0,1]$. The share that the lobbyist retains captures the competitiveness or specialization in the lobbying sector, as, in our model, only one coalition at most will form. The lobbyist's utility is given by

$$U_L\left(l_j\right) = (1-\alpha)\left(\sum_{i \in l_j}\left(U_{i,l_j} - U_{i,I}\right) - \chi_{|l_j|}\right). \qquad (5)$$

This assumption reflects the idea that the lobbyist has in her possession a technology necessary for coalition formation.[12] We interpret $\chi_{|l_j|}$ as a monitoring cost of ensuring compliance

[12]Essentially, only the lobbyist possess the technology to facilitate collusion on lobbying between insiders.

with the coalition's strategy, rather than a fee paid to the lobbyist. Essentially, the lobbyist is necessary for the coalition to exist (in a binary sense), and her fee is a fraction of the gains from coalition formation net of within-coalition monitoring costs. In some sense, the lobbyist is a monitoring device that helps ensure that the coalition members pursue strategies that maximize the coalition's net gain.

Within a lobby, utility is transferable. This is a convenience assumption, as our interest is in the effects of coalitions on lobbying activity and policy choice, rather than within-coalition allocations of utility. When agents choose their lobbying activities, whether individually or jointly, the coalition costs, $\chi_{|l_j|}$, are sunk. So, the agents who are joined in a particular lobby choose lobbying efforts to maximize $\sum_{i \in l_j} U_{i,l_j}$, which maximizes the total amount split between the agents and the lobbyist.

We solve the model by backward induction, and Table 2 shows the timeline.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
Coalition formation	Agents choose influence activities	Regulator chooses regulatory intensities	Privately-beneficial actions may occur
l_j	B_i	π_i	$Payoffs$

Table 2
Timeline

We leave the issue of coalition formation open for now and return to it after we derive the regulatory policies and lobbying strategies in Section 3, conditional on coalition structures. Throughout the analysis, we use the terms lobby and coalition interchangeably. We solve for the Perfect Bayesian Nash Equilibrium using backward induction.

Because there can be no more than one multiple-firm coalition in our three-agent setting, we assume that there is only one lobbyist. This abstracts from potential competition between lobby-forming specialists, but substantially simplifies the problem. Ray and Vohra (1999), focusing on endogenous coalition formation, label some players as "proposers", who, by proposing coalition structures and within-coalition transfers, play essentially the same role as the lobbyist here. Bloch (2002) similarly presents a sequential coalition formation game featuring proposers.

3 Regulation and influence activities

Given lobbying efforts, the regulator chooses regulatory intensities in period $t = 2$ as $\underset{\pi_i, i\in\{s,m,\ell\}}{\arg\min} \sum_i \left[\lambda\left(1-\pi_i\right)D_i + \frac{\pi_i^2}{2} + B_i\pi_i + \frac{k}{2}\left(\pi_i - \bar{\pi}_{\setminus i}\right)^2\right]$. The first-order-conditions (FOC) are a set of three equations that imply,

$$\hat{\pi}_i = \frac{4\left(\lambda D_i - B_i\right) + 9k\left(\lambda\frac{\bar{D}}{3} - \frac{\bar{B}}{3}\right)}{4+9k}. \tag{6}$$

In what follows, we derive the optimal lobbying efforts, conditional on the coalition structure. We assume throughout that the exogenous parameters are such that regulation is defined by (6) and $\hat{\pi}_i \in (0,1)\,\forall i$.

In each setting, a coalition (including a one-agent coalition) chooses lobbying to maximize the expected utility of its members, which is $\sum_{i\in l_j} U_i|_{\pi_i=\hat{\pi}_i}$, where U_i is defined in (1) as $U_i = \left(1-\pi_i\right)D_i - \frac{c}{2}B_i^2$, and $\hat{\pi}_i$ is defined in (6). We do not specify transfers within lobbies, restricting them only to be feasible (i.e., the sum of the individual utilities of the lobby members equals the total utility of the members of the lobby).

To develop the optimal lobbying, regulatory strengths, and expected agent and lobbyist utilities for each of the coalition structures, we introduce additional notation. Specifically, to represent agents within coalition structures, we use the subscripts $\{g,h,j,i\}$, where $g,h,j,i \in \{s,m,\ell\}$ and where $g \neq h \neq j$. Therefore, g, h, j, and i are each agents and g, h, and j are different agents. This allows us to introduce flexible terms, like $\bar{D}_{gh} = D_g + D_h$ instead of separately introducing $\bar{D}_{sm}$, $\bar{D}_{s\ell}$, and $\bar{D}_{m\ell}$. Additionally, let $\overline{D_{gh}^2} = D_g^2 + D_h^2$, and $\overline{D^2} = \sum_i D_i^2$.

Independent agents. When the three agents act independently, lobbying, regulatory strengths, and agents' utility are given by

$$\hat{B}_{i,I} = \frac{4+3k}{c\,(4+9k)} D_i, \tag{7}$$

$$\hat{\pi}_{i,I} = \frac{\lambda c\,(4+9k) - (4+3k)}{c\,(4+9k)^2} \left(4D_i + 3k\bar{D}\right), \text{ and} \tag{8}$$

$$\hat{U}_{i,I} = \left(1 - \hat{\pi}_{i,I}\right) D_i - \frac{1}{2c} \left(\frac{4+3k}{4+9k}\right)^2 D_i^2. \tag{9}$$

Two-agent lobbies. We present the results for each of the three potential two-agent lobbies in Appendix A. Generally, in the two-agent setting, if agents g and h form a lobby or coalition, leaving agent j out, we have the following: $\hat{B}_{j,gh} = \hat{B}_{j,I} = D_j \frac{4+3k}{c(4+9k)}$; $\hat{B}_{g,gh} = \hat{B}_{g,I} + \frac{3kD_h}{c(4+9k)} = \frac{4D_g+3k\bar{D}_{gh}}{c(4+9k)}$; and $\bar{B}_{gh} = \hat{B}_I + \frac{3k\bar{D}_{gh}}{c(4+9k)}$. Regulatory strengths are: $\hat{\pi}_{j,gh} = \hat{\pi}_{j,I} - \frac{9k^2\bar{D}_{gh}}{c(4+9k)^2}$ and $\hat{\pi}_{g,gh} = \hat{\pi}_{g,I} - \frac{3k\left(4D_h+3k\bar{D}_{gh}\right)}{c(4+9k)^2}$. The coalitions' expected utilities are: $\hat{U}_{gh,gh} = \hat{U}_{g,I} + \hat{U}_{h,I} + \frac{9k^2\overline{D^2_{gh}}}{2c(4+9k)^2}$ and $\hat{U}_{j,gh} = \hat{U}_{j,I} + \frac{9k^2 D_j \bar{D}_{gh}}{c(4+9k)^2}$, where and $\hat{U}_{gh,gh}$ is the total utility of the agents and the lobbyist in the gh coalition. Note that $\hat{U}_{j,gh} \geq \hat{U}_{j,I}$, implying that the non-coalition member benefits from the other agents forming a coalition.

Grand coalition/three-agent lobby. When all agents join together in the three-agent lobby (the grand coalition), lobbying is $\hat{B}_{i,G} = \frac{4D_i+3k\bar{D}}{c(4+9k)}$ for each agent, which implies that total lobbying is given by $\bar{B}_G = \frac{\bar{D}}{c}$. Regulatory strengths are:

$$\hat{\pi}_{i,G} = \frac{4D_i\left(\lambda - \frac{4}{c(4+9k)}\right) + 3k\bar{D}\left(\lambda - \frac{1}{c} - \frac{4}{c(4+9k)}\right)}{4+9k} \tag{10}$$

and the total expected utility of the grand coalition is

$$\hat{U}_G = \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2\left(\overline{D^2} + D_sD_m + D_sD_\ell + D_mD_\ell\right)}{c\,(4+9k)^2}. \tag{11}$$

Whenever the regulator is at least somewhat constrained to enact uniform regulation, i.e., whenever $k > 0$, the free-riding problem between agents provides an opportunity for gains from forming coalitions. Whenever agents form coalitions, they lobby more, because they internalize the externality effect of their lobbying on the other agents(s) in the coalition. This drives regulatory strength down for all agents. Although lobbying efforts are greater, each agents' expected utility is also greater, and not just for the members of the coalition. Regulatory uniformity means that the non-coalition member (e.g., agent ℓ in the presence of the sm coalition) faces weaker regulation due only to the coalition members increasing their lobbying.

First, note that absent coalition costs, the combined expected utility of all coalitions is greatest for the three-agent coalition, and lowest when each agent lobbies independently, as

$$\hat{U}_G > \hat{U}_{j,gh} + \hat{U}_{gh,gh} > \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I}. \tag{12}$$

Furthermore, while the agent that is not part of a two-agent coalition lobbies as if all agents were independent, this agent gains from the existence of the lobby, i.e., $\hat{B}_{j,gh} = \hat{B}_{j,I}$ and $\hat{U}_{j,gh} > \hat{U}_{j,I}$. Finally, the gains from forming coalitions are increasing in the degree of regulatory uniformity, k, as $\frac{d}{dk}\left(\frac{9k^2}{c(4+9k)^2}\right) = \frac{72k}{c(9k+4)^3} > 0$.

In the next section, we investigate the equilibrium coalition structure. This introduces substantial complication into the model, which primarily relates to issues that arise in any model of endogenous coalition formation, i.e., combinatorial problems of cooperative game theory. Two important features of our model limit our ability to use well-known cooperative game theory solution techniques or to directly apply the results of earlier studies.

First, the externalities of lobbying in a policy regime with any degree of uniformity imply that the value of a given coalition to its members depends on the overall coalition structure. Therefore, we cannot write a function for the value of a coalition that depends only the characteristics of that coalition and ignores the overall coalition structure (i.e., a characteristic function). As such, solutions based on characteristic functions are not applicable (e.g., those based on the core and Shapley value; see Myerson (2013)). Thrall and Lucas (1963) and Myerson (1978) analyze games in partition function form where the value to a coalition depends on the entire coalition structure. Thrall and Lucas (1963) present results primarily for 2- and 3-player games, and allow for transfers across coalitions, which we prohibit. Myerson (1978) explores the role of commitments to threat strategies, although these threats may not be sequentially rational *ex post*. Kóczy (2007), Ray and Vohra (1997, 1999), and Yi (1997) also explore games with externalities in which, by definition, a player's utility is influenced by the coalition structure as long as the coalition structure influences other players' equilibrium actions. Yi (1997) develops rules for stable coalition structures in the presence of positive externalities, but assumes (Yi's condition P.2) that per-member payoffs are decreasing in coalition size. In our model, the payoff structure emerges as a function of the regulatory environment, and is not in general characterized by per-member payoffs that decrease in coalition size. Instead, absent coalition costs, larger coalitions yield larger per-member payoffs because coalitions are able to eliminate the free-rider problem between all members.

Second, by the nature of our focus on the importance of regulatory uniformity, it is crucial for the players, i.e., the agents in our model, to be heterogeneous. In a model with homogeneous agents, restricting the regulator to any degree of regulatory uniformity is free, as the regulator optimally desires to set homogeneous regulation across the cross-section of agents. The lack of homogeneity means that we care about which agents are members of which lobbies or coalitions, and cannot simply use coalition size as an outcome variable of interest, as in the model of Bloch (2002). A benefit of allowing for heterogeneity is that we can derive predictions on which agents find it optimal to associate with each other, and which associations can be sustained in equilibrium (as in Baccara and Yariv, 2016).

4 Coalition formation

We adopt the following convention for coalition formation. First, if $\alpha = 0$, then the lobbyist extracts all the net gains from coalition formation, leaving agents indifferent across coalition structures. In this case, the lobbyist will choose the coalition structure to maximize the net coalition gain, i.e., $\hat{l}_j \in \arg\max_{l_j} \Delta U_{l_j}$. For $\alpha > 0$, agents are no longer indifferent across coalition structures. Therefore, for $\alpha > 0$, we allow agents the option of forming a coalition, then hiring the lobbyist via the exogenous α-based sharing rule described above. In some cases, as shown below, each agent would prefer to be the outsider and for the other two agents to form a coalition, i.e., each agent $j \in \{s, m, \ell\}$ prefers structure gh to either jg or jh. This arises due to the positive externalities that coalition formation have on the non-coalition member, as shown by $\hat{U}_{j,gh} = \hat{U}_{j,I} + \frac{9k^2 D_j \bar{D}_{gh}}{c(4+9k)^2}$. When each agent prefers to be the outsider, the agents are in a sort of 3-way prisoners' dilemma, in that each player prefers to be the outsider in the presence of a two-agent coalition, but is worse off if no coalitions are formed at all.[13] In such a scenario, we allow the lobbyist to choose the coalition structure, breaking the stalemate.

In the remainder of this section, we derive the optimal coalition structures. We start with the case of $\alpha = 0$, as this case illustrates several of the tradeoffs absent some issues that arise when agents retain some of the gains from coalition formation.

4.1 Coalitions formation benchmark

As a benchmark, we assume $\alpha = 0$ in this subsection, which implies that agents are indifferent across coalition structures, as the lobbyist extracts the full net gain from coalition formation. Due to agent indifference, the lobbyist can determine the coalition structure.

The lobbyist, maximizing (5), always prefers to form the $m\ell$ lobby over either the $s\ell$ or the sm lobby. This implies that the lobbyist will only form either the grand or the $m\ell$ lobby. To facilitate the discussion, we introduce "effective coalition formation costs", $\chi_2^E = \chi_2 \frac{2c(4+9k)^2}{9k^2}$ and $\chi_3^E = \chi_3 \frac{2c(4+9k)^2}{9k^2}$. These effective coalition formation costs absorb the parameters associated with the net costs and benefits of lobbying, c and k, that affect the net

[13] An alternative analogy is a 3-way Mexican standoff, wherein each player wants someone else to act (shoot) first. Laver and Shepsle (1990) discuss the issue of the Mexican standoff in the context of coalition governments.

gains from coalition formation. Additionally, let $\Xi_G = 2\left(\overline{D^2} + D_s D_m + D_s D_\ell + D_m D_\ell\right)$, such that there is a net benefit to the formation of the grand coalition if $\Xi_G - \chi_3^E > 0$ and a net benefit to the formation of the $m\ell$ coalition if $\overline{D_{m\ell}^2} - \chi_2^E > 0$. Additionally, let $\Delta\chi^E = \chi_3^E - \chi_2^E$ be the extra effective cost of a three-agent coalition relative to a two-agent coalition. The equilibrium lobbies are given by the following theorem.

Theorem 1 (Lobbyist-based coalitions) *When lobbyists form coalitions:*

1. *If $\chi_2 > \overline{D_{m\ell}^2}$ and $\chi_3 > \Xi_G$, the lobbyist will not form a coalition.*
2. *If $\chi_2 > \overline{D_{m\ell}^2}$ and $\chi_3 < \Xi_G$, the lobbyist will optimally form the grand lobby.*
3. *If $\chi_2 < \overline{D_{m\ell}^2}$ and $\Delta\chi > \Xi_G - \overline{D_{m\ell}^2}$, the lobbyist will optimally form the $m\ell$ coalition.*
4. *If $\chi_2 < \overline{D_{m\ell}^2}$ and $\Delta\chi < \Xi_G - \overline{D_{m\ell}^2}$, the lobbyist will optimally form the grand lobby.*

4.2 Offer-stability

Necessarily, coalitions that emerge must be stable in that no set of agents (or individual agent) prefers to deviate and form a new coalition (or go off on her own). We formalize this requirement and introduce "offer-stability" as our equilibrium concept below. For each region of the parameter space we find at least one offer-stable coalition, which, combined with our assumptions about the lobbyist choosing when there are multiple offer-stable coalitions, allows for clean empirical predictions. We begin by defining feasible allocations of utility across agents for a given structure, X, where the possible coalition structures are listed in Table 2.

Definition 1 (Allocation) *An allocation A is a vector of utilities u_i for each agent $i \in \{s, m, \ell\}$. An allocation is feasible if $\sum_{i \in l_j} u_i = \sum_{i \in l_j} U_{i,I} + \alpha \Delta U_{l_j}$ for each i given the coalition structure, X.*

A feasible allocation allows utility transfers within a coalition, but not across coalitions. Transfers across coalitions imply a measure of cooperation across coalitions, which is inconsistent with our assumption that there are costs to forming coalitions that are borne by the coalitions themselves. Implicitly, then, cross-coalition transfers would allow all three agents to form a two-agent lobby while still coordinating among three agents. Then, agents should be able to save all coalition forming costs and coordinate for free. Instead, we require that the cost of coordinating three agents (two agents) is χ_3 (χ_2).

Traditional notions of the core (e.g., Von Neumann and Morgenstern, 1944) and extensions into cooperative games with externalities (e.g., Kóczy, 2007) rely on the concept of blocking (Ray and Vohra, 1997, 2014). An allocation-structure pair (A, X) is blocked if there is an alternative allocation-structure pair (A', X') that makes a subset of players better off, assuming that the subset of players can cause a deviation from (A, X) to (A', X'). An allocation-structure pair is said to be in the core if it is not blocked by any achievable alternative. Whether (A', X') makes the deviating subset of players better off depends on how the other players, referred to as the residual players, react to the deviation to (A', X'). Various definitions of cores have been put forth, with different assumptions about how the residual players behave (i.e., whether the residual players are punitive towards the deviators, supportive towards the deviators, or optimizers, as well as whether and how the residual players rearrange themselves; see Kóczy (2007) for a concise discussion).

A well-known problem with cores is that they can be empty or non-unique. Both an empty core and a core that includes more than one coalition structure imply a lack of clarity on predictions about the equilibrium coalition structure. Furthermore, in games with superadditive payoff structures and fully transferable utility (i.e., in which larger coalitions are always better), there is no loss in only considering how the spoils from the grand coalition are allocated. Absent superadditivity, determining the coalition structures that are in the core is a non-trivial problem. Our setting features non-superadditive payoffs and admits an empty core in some cases (see the example in Appendix B). To avoid these core-related problems, we introduce offer-stability as a means of determining agent-formed coalition structures.

Definition 2 (Offer-stability) *A feasible allocation-structure pair (A, X) is blocked if there is a nonempty coalition in which each member is made weakly better off and one member is made strictly better off in an alternative feasible allocation-structure pair (A', X'). A struc-*

ture is offer-stable if, for every alternative feasible allocation-structure pair, (A', X')*, with* $X' \neq X$*, there is an allocation-structure pair* (A'', X) *that is not blocked by* (A', X')*.*

We use the term offer-stability because it is based on the idea that coalitions can use counteroffers to prevent being blocked. In the definition, A'' is a counteroffer that prevents X from being blocked by an allocation-structure pair that includes an alternative structure, X'. This is closely related to objections and counterobjections used to define stability in the presence of blocking related to bargaining sets (Aumann and Maschler, 1964). While we use the terms "stable" and "offer-stable" interchangeably, note that our concept of offer-stable coalitions is somewhat different from coalition stability as defined in Von Neumann and Morgenstern (1944) and Aumann and Maschler (1964) (see also Ray and Vohra, 2014). We contrast offer-stability with concepts of stability based on the core and bargaining sets in Appendix B, where we also define core-stability and provide a description of stability based on bargaining sets.

In the analysis below, we show that there is at least one offer-stable coalition structure for any set of parameters. For a large set of the parameter-space, this coalition structure is unique. When it is not unique, we rely on the lobbyist as a means of selecting a particular offer-stable coalition structure. Importantly, when the structure is chosen by the lobbyist, no agent or set of agents has an incentive to deviate. Given that offer-stability allows for weaker blocking conditions than core-stability, it is clear that any core-stable coalition structure must be an offer-stable coalition structure as well. We use offer-stability rather than more traditional notions of the core because, as Appendix B shows, there may be no core-stable coalition structure for some parameter values. Our use of offer-stability avoids this problem, allowing for relatively straightforward comparative statics analysis throughout the parameter-space.

Stepping back, our notion of offer-stability is based on the idea that if agent i considers leaving a given coalition, then the remaining agents in the coalition could offer agent i any amount up to the amount that those agents gain from being members of the coalition including i. The logic extends naturally if we replace agent i with a set of agents. Offer-stability is similar in spirit to several fundamental types of coalition stability described in Ray and Vohra (2014), based on whether various coalition structures are blocked by alternatives.[14] Fundamentally, offer-stability relies on credible counteroffers to individual (sets of) potentially-deviating coalition members.

To illustrate the notion of offer-stable coalitions, consider the grand coalition. If $\Xi_G > \chi_3^E$, then the gain from forming the grand coalition, relative to not forming any coalition, outweighs the costs. The grand coalition is then preferred to each agent lobbying independently and the grand coalition is partially stable because agents j and k can offer up to $\frac{9k^2\left(\Xi_G - \chi_3^E\right)}{2c(4+9k)^2}$ to agent i to prevent her from leaving. However, when χ_2 is not too large, two members of the grand coalition might prefer to form a two-agent coalition over the grand coalition. That is, it may be that the extra costs of having a three-agent coalition (relative to a two-agent coalition) outweigh the benefit. For example, with $\chi_2 = 0$ and $\chi_3^E > \Xi_G$, a two-agent coalition may be optimal. Clearly, there are gains to be had from a two-agent coalition, but we have not yet shown whether any two-agent coalition is offer stable.

4.3 Coalition formation when agents retain some coalition gains

An offer-stable two-agent coalition, as defined, cannot be broken up by the agent who is not a member of the coalition. That is, the most that the agent is willing to offer either of the coalition members is less than either coalition member would offer the other coalition member to leave the coalition that would form if the independent agent's initial offer was successful. Offer-stability implies that the outsider's utility in the presence of a two-agent coalition is relevant for determining the surviving coalition. In the following theorem, we develop the offer stable coalition structures when agents keep a sufficiently high fraction of

[14] Our offer-stability differs from the stability notion used in Ray and Vohra (1997), in which subsets of l_j can deviate, but deviations always make the coalition structure finer, as the deviants are precluded from joining with insiders who were not initially in l_j. In contrast, we allow for the deviating insiders to form coalitions with new partners.

the coalition gains , $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$.

Theorem 2 *When* $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$*, the following coalition structures are offer-stable:*

1. *If* $\chi_2^E > \overline{D_{m\ell}^2}$ *and*
   (a) $\chi_3^E > \Xi_G$*, then no coalitions will form and all agents will lobby individually;*
   (b) $\chi_3^E < \Xi_G$*, then the offer-stable coalition structure consists of the grand lobby.*

2. *If* $\chi_2^E \in \left(\overline{D_{s\ell}^2}, \overline{D_{m\ell}^2}\right)$ *and*
   (a) $2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E < \chi_3^E$*, then the offer-stable coalition structure consists of the* $m\ell$ *lobby.*
   (b) $2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E \geq \chi_3^E$*, then the offer-stable coalition structure consists of the grand lobby.*

3. *If* $\chi_2^E < \overline{D_{s\ell}^2}$ *and*
   (a) $2\overline{D^2} - (D_s - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell} + \chi_2^E < \chi_3^E$*, then the offer-stable coalition structure consists of the* $s\ell$ *lobby;*
   (b) $2\overline{D^2} - (D_s - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell} + \chi_2^E \geq \chi_3^E$*, then the offer-stable coalition structure consists of the grand lobby.*

Note that when the costs to forming a two-agent coalition are sufficiently low, $\chi_2^E < \overline{D_{s\ell}^2}$, and the extra cost of forming the grand coalition is sufficiently large, then the only offer-stable coalition is the $s\ell$ lobby. This is in stark contrast to the setting where a lobbyist retains the full gains from coalition formation. When agents capture enough of the gains from coalition formation, i.e., when $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, the equilibrium coalition has to be stable against an outside offer. In the setting of Theorem 2, part 3(a), the $m\ell$ lobby is not stable because the m agent's gain from the $s\ell$ lobby is relatively high. For that reason, the s agent's gain from joining the lobby exceeds the m agent's loss from leaving the lobby and the $s\ell$ lobby persists. However, if $\chi_2^E \in \left(\overline{D_{s\ell}^2}, \overline{D_{m\ell}^2}\right)$, then it is no longer profitable for the s and ℓ agents to form a coalition together, as their net coalition gains are negative. For χ_2^E in this region, the net coalition gains to the $m\ell$ coalition are positive, implying that this coalition is offer-stable when the negative coalition gains to $s\ell$ make that coalition infeasible.

When α is not sufficiently high, i.e., $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, agents do not capture sufficient gains from coalition formation to make them prefer to be inside the coalition rather than outside. Recall that coalition members' net gain is increasing in α, while the benefit to the non-coalition participant, i.e., agent s in the presence of $m\ell$, has a gain that is independent of α. Similarly, the gain to the grand coalition is increasing in α, meaning that a lower α makes it less desirable for the 2-agent coalition outsider to join the three-agent coalition.

Considering only two-agent coalitions, when α is too low, each agent prefers for the other two agents to form a coalition. Even though each agent would gain from joining a coalition, each agent would prefer to be the outsider in the presence of the other agents forming a coalition. If we are in such a standstill, there are three possible stable 2-agent coalition structures, one for each 2-agent coalition. Once a two-agent coalition forms, it will be stable, as the outsider will not wish to make a deviation-offer. However, prior to a two-agent coalition forming, no agent will willingly join a coalition as long as the option to wait for a coalition that does not include them to form remains. We exploit the lobbyist as the equilibrium-selection mechanism here. If $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, then the lobbyist, who is necessary for a coalition to form, will simply choose the coalition that is in her best interests. As the coalition gain from the $m\ell$ coalition is the largest, the lobbyist will choose this coalition. Each agent knows that the lobbyist will chose this coalition, but both the m and ℓ agents prefer to form the $m\ell$ coalition than to form a two-agent coalition with s in this scenario. Theorem 3 follows.

Theorem 3 *When* $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, *the following coalition structures are offer-stable:*

1. *If* $\chi_2^E > \overline{D_{m\ell}^2}$ *and*
   - *(a)* $\chi_3^E > \Xi_G$, *then no coalitions will form and all agents will lobby individually;*
   - *(b)* $\chi_3^E < \Xi_G$, *then the offer-stable coalition structure consists of the grand lobby.*
2. *If* $\chi_2^E < \overline{D_{m\ell}^2}$ *and*
   - *(a)* $2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E < \chi_3^E$, *then the offer-stable coalition structure consists of the* $m\ell$ *lobby.*

(b) $2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E \geq \chi_3^E$*, then the offer-stable coalition structure consists of the grand lobby.*

Essentially, inaction among agents with respect to forming two-agent coalitions eliminates the possibility for the $s\ell$ coalition to form. Even though the lobbyist chooses the coalition structure, the coalitions that do form (or the parameters that govern particular coalition formation) are slightly different from those that form when the agents are completely indifferent. For $\chi_2^E < \overline{D_{m\ell}^2}$, in which case the two-agent $m\ell$ coalition provides a net coalition gain and is feasible, the difference is driven by the fact that the s agent gains from the $m\ell$ coalition forming when $\alpha > 0$, but does not when $\alpha = 0$. The s agent's gain in the $m\ell$ structure relative to the G structure is sensitive to α, because the s agent would have to share her gain with the lobbyist if she joined G, but does not have to share her gain from the positive externality of lobbying in the presence of the $m\ell$ coalition, of which she is not a member.

5 Analysis

5.1 Coalitions

As the analysis above shows, when the lobbyist captures a sufficient amount of the net gains from coalition formation, i.e., for $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, the two possible equilibrium coalitions are $m\ell$ and G. In either coalition, agents group by similarity. That is, either no agents, the higher types, or all agents form a coalition. This result is similar to much of the prior literature on endogenous lobbying (e.g., Mitra (1999)), in which only the most similar agents organize into coalitions. In contrast, when the agents capture a sufficient amount of the net gains from coalition formation, i.e., with $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, the $s\ell$ coalition is also possible. This resulting coalition structure contrasts with much of the prior literature on endogenous lobbying, in which only the most similar agents organize into coalitions, but is similar in spirit to the results of Baccara and Yariv (2016), who find potential for polarization or similarity of membership in peer-selected groups organized to produce public goods. Our result is summarized in the following proposition.

Proposition 1 ***Lobbyist power and lobby size***
If agents capture more of the net gains from coalition formation, i.e., α is higher, then coalitions are more likely to be polarized, featuring dissimilar agents. In contrast, if lobbyists capture more of the net gains from coalition formation, then only coalitions of similar agents are possible.

Next, we turn to the influence of coalition costs, χ, the cost to agents of lobbying efforts, c, and the degree of regulatory uniformity, k, on coalition formation, lobbying, B, and regulatory strength, π. To facilitate the discussion, we continue to use "effective coalition formation costs", $\chi_2^E = \chi_2 \frac{2c(4+9k)^2}{9k^2}$ and $\chi_3^E = \chi_3 \frac{2c(4+9k)^2}{9k^2}$. This is useful because it subsumes lobbying costs and uniformity into effective coalition formation costs.

Figures 1 and 2 illustrate the topography of coalitions for $\alpha > 0$. The figures maps out regions of coalitions as functions of effective coalition formation costs, χ_2^E and χ_3^E. The area in the bottom-right is not feasible, as it is defined by $\chi_2^E < \chi_3^E \Leftrightarrow \chi_2 < \chi_3$. The remaining area, in the upper-left of the figure, is divided into regions in which different types of coalitions will exist in equilibrium: either no lobbies, the grand lobby, or the $m\ell$ lobby. These regions correspond to the regions described in Theorem 3. Note that the areas of the regions can change, but the shapes defining the regions hold generally.

In addition to these regions, each figure features two rays that each start at a gray dot and proceed up and to the right. These rays are useful for thinking about how the coalition structure changes when k, c, or χ change. Specifically, each ray traces out the nexus of points defined by $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$ for $\chi_2^E \geq 18c\chi_2$. Overall, the χ's determine the slope of the ray, while c determines the point closest to the origin, and c and k jointly determine the relevant point on the ray that defines the equilibrium coalition structure, i.e., where we fall on the plot. The slope is given by the proportional relation between χ_2 and χ_3. For a given c, the lowest and left-most point on the ray, at the gray dot, is defined by the point $\left(\chi_2^E, \chi_3^E\right) = (18c\chi_2, 18c\chi_3)$, because $\lim_{k\to\infty} \frac{2c(4+9k)^2}{9k^2} = 18c$. If c is very small, the gray dot is

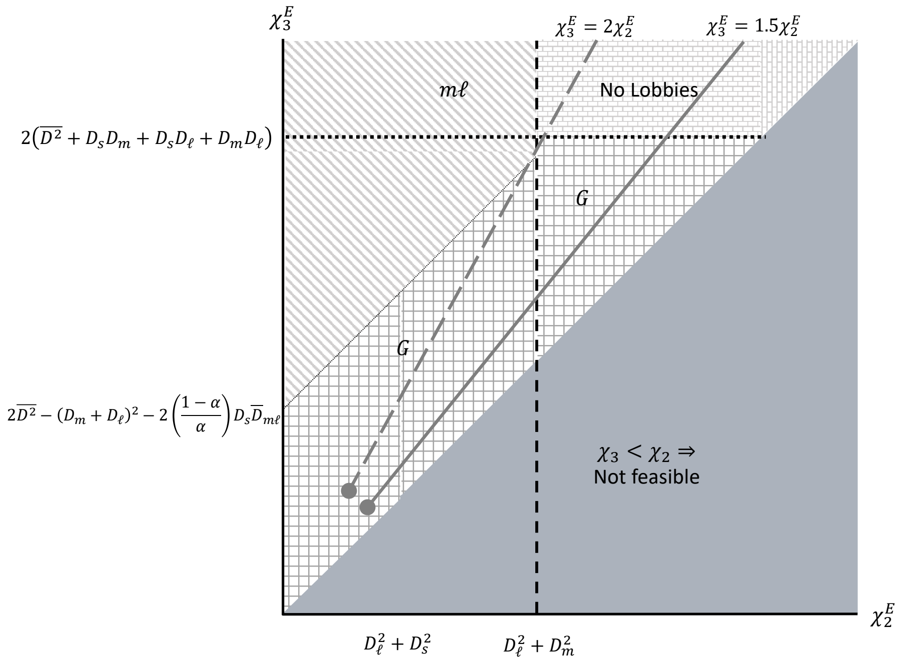

Figure 1

Coalition structures as functions of effective coalition-formation costs, χ_2^E and χ_3^E, when $0 < \alpha < \frac{D_\ell(D_s + D_m)}{D_m^2 + D_\ell^2}$. The grey line segment in each subfigure indicates a nexus of points such that $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$. The grey dot at the lower-left end of each line segment is at $(\chi_2^E, \chi_3^E) = (18c\chi_2, 18c\chi_3)$. $\chi_{|l_j|}^E = \chi_{|l_j|} \frac{2c(4+9k)^2}{9k^2}$.

close to the origin, but if c is large, the gray dot is far. Large values for the cost of lobbying, c, imply that effective coalition costs are large. The reason is that for large lobbying costs, agents do not lobby much, such that the benefit of forming coalitions is small. In general, if the ray were extended down and to the left, it would intersect the origin for any χ_2 and χ_3, i.e., for any slope. Via the $\frac{2c(4+9k)^2}{9k^2}$ term in the χ^E's, c and k can be thought of as determining the location on the ray. Increasing k or decreasing c causes a shift down and to the left. Decreasing k or increasing c, in contrast, cause shifts in the other direction.

Several of the comparative statics below rely on transitions driven by changes in parameters. These transitions can be understood graphically from Figures 1 and 2. Additionally, in Appendix A, Section 8.3, we characterize the conditions under which parameter changes lead to transitions across coalition structures.

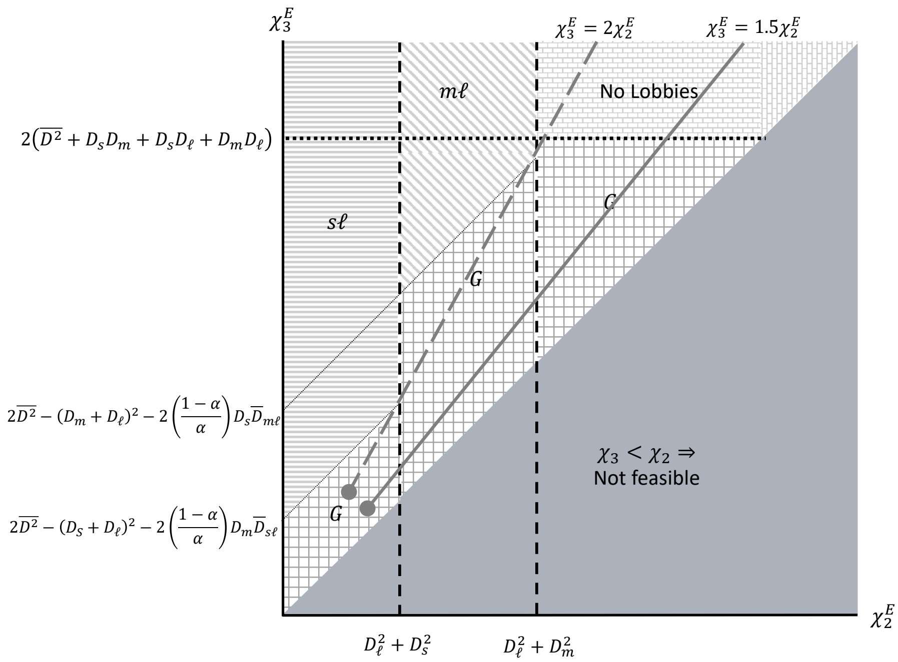

Figure 2

Coalition structures as functions of effective coalition-formation costs, χ_2^E and χ_3^E, when $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$. The grey line segment in each subfigure indicates a nexus of points such that $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$. The grey dot at the lower-left end of the line segment is at $(\chi_2^E, \chi_3^E) = (18c\chi_2, 18c\chi_3)$. $\chi_{|l_j|}^E = \chi_{|l_j|} \frac{2c(4+9k)^2}{9k^2}$.

Proposition 2 ***Coalition costs and lobby size***

1. *An increase in χ_3 causes weakly smaller lobbies.*
2. *An increase in χ_2 can cause lobbies to grow or to disband.*
3. *Concurrent proportional increases in χ_2 and χ_3 can lead to larger or smaller lobbies.*

In Figures 1 and 2, an increase in χ_3 corresponds to an upward shift or a steepening of the gray rays, which can cause a transition from the grand coalition to either the $s\ell$ lobby, the $m\ell$ lobby, or to no lobby. Not surprisingly, higher costs can lead to smaller lobbies, as is always the case with the cost of the three-agent coalition, χ_3. An increase in χ_2 corresponds to a rightward shift or a flattening of the gray rays in Figures 1 and 2 (e.g., a transition from the dashed gray ray to the solid gray ray). Holding c and k constant, this can cause the coalition shifts described in part 2 of Proposition 2. The $m\ell$ coalition can become unstable with an increase in χ_2, causing a transition either to the grand coalition or to no coalitions as χ_2^E moves from below to above $D_\ell^2 + D_m^2$. Finally, increasing χ_2 and χ_3 proportionately at the same time causes a shift up and to the right along the gray rays. As in Proposition 2, part 1, this can cause a transition from G to $m\ell$, from G to I, or from $m\ell$ to I, similar to the effect of increasing c at intermediate levels.

Graphically, a proportional increase in both χ_2 and χ_3 can be interpreted as a shift up and to the right along any of the four gray rays in Figures 1 and 2. For example, in Figure 2, a shift up and to the right along the gray dashed line can cause transitions from G to $s\ell$, from $s\ell$ to G, from G to $m\ell$, from $m\ell$ to G, or from G to I. Much of this non-monotonicity occurs because the non-coalition member's utility is important for determining the offer-stable coalition. Specifically, when the equilibrium moves from the grand coalition to a two-agent coalition, the utility of the non-coalition member plays a role. Because the coalition members are able to overcome the free-rider problem and each agent's lobbying decreases all agents' regulatory strength, the non-coalition member gains from the other agents forming a coalition (relative to the non-coalition case). This externality of the two-agent lobby can make it profitable for one of the agents to leave the grand coalition because the two remaining members are not able to make a successful offer to stay in the grand coalition.

Proposition 3 ***Lobbying costs, uniformity, and lobby size***
Increases in lobbying costs, c, and decreases in regulatory uniformity, k, have similar effects as proportional increases in both coalition costs, χ_2 and χ_3, and can lead to larger or smaller lobbies.

Note that $\frac{\chi_3^E}{\chi_3} = \frac{\chi_2^E}{\chi_2} = \frac{2c(4+9k)^2}{9k^2}$, and that lobbying costs, c, and regulatory uniformity, k, only influence coalition structures through their influence on χ_2^E and χ_3^E. As such, any change in c or k causes a proportional change in both χ_2^E and χ_3^E, just as a proportional change in both χ_2 and χ_3 would.

Graphically, lobbying costs, c, and regulatory uniformity, k, determine the relevant location on a given $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$ ray drawn in Figures 1 and 2; the particular gray lines in the figure are examples. First, higher lobbying costs decrease the benefit of lobbying, and thereby decrease the benefits of forming a coalition that helps overcome the free-rider problem on lobbying effort. At one extreme, as $c \to \infty$, agents have no reason to form lobbies. At the other extreme, as $c \to 0$, agents exert significant lobbying effort, making the grand coalition highly desirable. At intermediate levels of c, increasing c causes a shift up and to the right along a given gray ray, which can cause various transitions between coalition structures.

Increases in regulatory uniformity, k, exacerbate the free-rider problem on lobbying and, thus, tend to promote the formation of lobbies. When k is very low, the free-rider problem is insignificant, giving agents little incentive to bear the costs of forming coalitions. Note that $\lim_{k \to 0} \chi_j^E \to \infty$ which corresponds to locations on the gray rays in Figures 1 and 2 that are in the "No Lobbies" region. Increasing k causes a shift down and to the left along a given ray. When k is sufficiently high, agents can benefit from coordinating their lobbying efforts, which tends to favor larger lobbies, but can locally cause transitions to smaller lobbies as well. Even with high k, though, lobbying can be prohibitively costly to facilitate beneficial coalitions.

An increase in c reduces the net benefit of lobbying. A decrease in k reduces the externalities of lobbying, thus lowering the net benefit of forming a coalition. Graphically, either of these causes a shift up and to the right along the gray rays in Figures 1 and 2. The overall trend is towards smaller or no lobbies, but switches from G to $s\ell$, G to $m\ell$, and back yield non-monotonicities. As noted above, in Figure 2, an increase in lobbying costs or a decrease in regulatory uniformity can cause a shift up and to the right along the gray dashed line leading to transitions from G to $s\ell$, from $s\ell$ to G, from G to $m\ell$, from $m\ell$ to G, and from G to I. As for proportional changes in χ_2 and χ_3, the non-monotonicity is due to the non-coalition member's utility contributing to the determination of the offer-stable coalition.

5.2 Lobbying activity and regulatory strength

We turn to the effects of parameter changes on lobbying efforts and regulatory strength.

Corollary 1 ***Lobbyist power, lobbying, and policy strength***
An increase in the fraction of the net gains from coalition formation that are retained by the agents, i.e., higher α, causes weakly lower lobbying, B, higher regulatory strength, π, and lower expected welfare losses from agents' privately beneficial activities.

As described in Proposition 1, and illustrated in comparisons between Figures 1 and 2, higher α makes the $s\ell$ coalition feasible in regions of parameter-space that would otherwise be characterized by either the $m\ell$ coalition or the grand coalition. Higher α in these regions thus leads to smaller coalitions that pursue less total lobbying, which lessens the agents' influence on equilibrium regulatory strength, allowing the regulator to set stronger policies, i.e., higher π. This in turn deters the privately beneficial action more frequently, lowering the expected welfare losses.

Similar to α, changes in χ_2 and χ_3 only affect lobbying efforts and regulatory strengths through their effects on the equilibrium coalition structure. These are discussed next. Changes in c and k, discussed below, affect lobbying and regulatory strength both directly (through agents' choice of lobbying and the regulator's choice of regulatory strength conditional on coalition structure) and indirectly (through their effects on the endogenous coalition structures).

Corollary 2 ***Coalition costs, lobbying, and policy strength***

1. *An increase in χ_3, all else equal, causes weakly lower lobbying, B, higher regulatory strength, π, and lower expected welfare losses from agents' privately beneficial activities.*
2. *An increase in χ_2, all else equal, can cause either weakly lower lobbying, B, higher regulatory strength, π, and lower expected losses from diversion; or greater lobbying, B, lower regulatory strength, π, and greater expected welfare losses from agents' privately beneficial activities.*

The combination of an increase in lobbying from larger coalitions and the results in Proposition 2 explains Corollary 2. More lobbying in turn weakens regulation and increases agents' chances to inefficiently take the privately beneficial actions. Coalition formation costs influence lobbying and regulatory strength indirectly, that is, only through their influence on the coalition structure. In a regression of total lobbying on formation costs and observed coalition structures, for instance, formation costs should have no explanatory power because their explanatory power is completely subsumed by the coalition structures that emerge.

As noted above, unlike coalition formation costs, both lobbying costs and regulatory uniformity have direct effects on equilibrium lobbying behavior and regulatory strengths. In fact, as shown in Friedman and Heinle (2016), absent lobbying coalitions, k and c have monotonic effects on lobbying and regulatory strength. Absent coalitions, more regulatory uniformity and higher lobbying costs both lead to less lobbying and stronger regulatory policies, all else equal. In the presence of endogenously-formed coalitions, both regulatory uniformity and lobbying costs influence whether and which agents organize into lobbying coalitions, causing the effects of c and k to be non-monotonic.

Corollary 3 ***Lobbying costs, uniformity, lobbying, and policy strength***
An increase in lobbying costs, c, or regulatory uniformity, k, all else equal, can cause either weakly lower lobbying, B, and higher regulatory strength, π; or greater lobbying, B, and lower regulatory strength, π.

Corollary 3 combines the results of Proposition 3 with the monotonic effects of lobbying costs and regulatory uniformity shown in Friedman and Heinle (2016). The results described in Corollary 3 are illustrated in Figure 3, which plots total lobbying, $\bar{B}$, and average regulatory strength, $\pi^{ave} = \frac{1}{3}\sum \pi_i$, as functions of regulatory uniformity, k, for two sets of parameters that differ only in χ_2. The solid gray curves have $\chi_2 = 1$, while the dashed black curves have $\chi_2 = 2$. In both cases, $\chi_3 = 2.65$.[15]

When $\chi_2 = 2$, coalition formation costs are concave, corresponding to a relatively flat $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ solid ray in Figure 2 and the solid plots in Figure 3. Starting from $k = 0$, increasing k tends to reduce lobbying and increase regulatory strength. These continue monotonically in k until we reach a threshold level of k that makes the benefit of forming a three-agent coalition sufficiently large. At this point, as the coalition is formed, we see a discrete jump in total lobbying and a drop in average regulatory strength. Further increases

[15] Other parameters are set as $\alpha = 1$, $D_s = 1$, $D_m = 15$, $D_\ell = 30$, $c = 1$, and $\lambda = 1.2$.

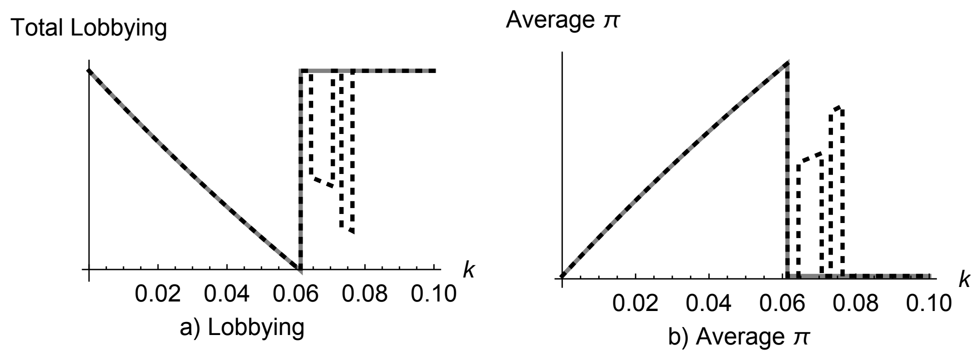

Figure 3
Total lobbying and average regulatory strength as functions of regulatory uniformity, k, when $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$. Parameters are set as $\alpha = 1$, $D_s = 1$, $D_m = 15$, $D_\ell = 30$, $\chi_3 = 2.65$, $c = 1$, and $\lambda = 1.2$. For the solid gray line, $\chi_2 = 1$. For the dotted black line, $\chi_2 = 2$.

in regulatory uniformity have no more effects, as k is moot in the presence of the grand coalition and increases in k maintain the dominance of the grand coalition.

When $\chi_2 = 1$, coalition costs are convex, corresponding to a steeper $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ dashed ray in Figure 2 and the dashed plots in Figure 3. Total lobbying and average regulatory strength mostly behave as they do when coalition costs are concave. There is, however, a key difference in the intermediate range of $k \in (0.06, 0.08)$. In this region, as we increase k from 0.06, we first see a transition from the grand coalition to the $m\ell$ coalition around $k = 0.061$. This first transition occurs when the $m\ell$ coalition becomes feasible, as χ_2^E drops below $D_\ell^2 + D_m^2$. As k continues to increase, the $m\ell$ coalition remains optimal, but total lobbying decreases and average regulatory strength increase, as the free-rider problem between the $m\ell$ coalition and agent s worsen. As k increases past about 0.07, χ_3^E becomes less than $\chi_2^E + 2\overline{D^2} - \overline{D_{m\ell}^2} + D_m D_\ell$, and, as the gain from the grand coalition starts to dominate the gain from the $m\ell$ coalition, we see a shift back to the high lobbying and low regulatory strength associated with the grand coalition. (Recall that when costs are convex, χ_3^E will decrease faster in k than χ_2^E will.) Next, as k increases further, the $s\ell$ coalition becomes feasible with χ_2^E dipping below $D_\ell^2 + D_s^2$. At $k \approx 0.074$, the $s\ell$ coalition is preferable to the grand coalition, but at $k \approx 0.076$, the grand coalition again becomes preferable, as χ_3^E drops below $\chi_2^E + 2\overline{D^2} - \overline{D_{s\ell}^2} + D_s D_\ell$. With k between 0.074 and 0.076, increases in regulatory uniformity again cause decreases in lobbying and increases in regulatory strength, as the free-rider problem between the $s\ell$ coalition and agent m gets worse. As k increases beyond 0.076, the grand coalition is again optimal, and further increases in regulatory uniformity cease to play a significant role.

Results, though not plotted, are similar for changes in lobbying costs, c, as increases in c have similar effects as decreases in k as described in Proposition 3 and Corollary 3 and illustrated graphically with shifts along the gray rays in Figures 1 and 2. Figure 3 shows that increases in k have non-monotonic effects on lobbying and regulatory strength when agents can organize into lobbies to overcome free-rider problems on lobbying. Almost everywhere, the effects of k on lobbying (regulatory strength) are locally negative (positive), but these effects can be significantly outweighed by discrete jumps or drops as changes in k cause agents to change how they organize into coalitions.

6 Discussion and implications

In this section we relate our results to our motivating example of systemically important financial institutions. We consider several institutional changes that may have affected the values of the primitives in our model related to competition between lobbyists, the costs of regulatory influence, and the degree of regulatory uniformity.

In 2006, the Lobbying Disclosure Act of 1995 was amended to require that lobbyists register with congressional officials. Arguably, this registration requirement imposes a cost on becoming a lobbyist and, thus, creates a barrier to entry that restricts the supply of lobbyists. We expect that the 2006 amendment of the Lobbying Disclosure Act has increased the bargaining power of the remaining lobbyists. In our model, this translates into a reduction in α. Our model (Proposition 1 and Corollary 1) predicts that following the 2006 amendment, lobbying coalitions are more likely to be formed by larger financial institutions and that total lobbying should increases as a result. Consistent with this prediction, in the 1998 to 2016 period, the number of unique registered lobbyists who have actively lobbied on behalf of the Finance, Insurance & Real Estate sector peaked in 2007 and has steadily decreased every year since, with the exception of a slight uptick in 2013. During the same period, the total lobbying spending by firms in the Finance, Insurance & Real Estate sector increased each year from 2008 to 2016, with the exception of a slight reduction in 2015.[16]

The 2008 to 2016 period has also seen an increase in the prevalence of lobbying coalitions, as highlighted in the popular press (e.g., Bogardus, 2013; Ho, 2015), and the dollar amount of political contributions, particularly soft money contributions, made by the financial sector.[17] Below, we discuss how these trends are consistent with the potential effects of changes in lobbying costs and regulatory uniformity brought about by recent changes in the institutional environment introduced by the Supreme Court and Congress.

In 2010, the US Supreme Court ruled in *Citizens United v. FEC* (Federal Election Commission) that the government cannot restrict political expenditures made by organizations including for-profit corporations. In the context of our model, we interpret this relaxation of the political expenditure restrictions as a reduction in the cost of lobbying, as *Citizens United* made it easier for corporations, managers, and wealthy individuals (e.g., managers and directors of systemically important financial institutions) to use personal or corporate resources to influence elections and, in turn, politicians. More specifically, we interpret *Citizens United* as a decrease in our marginal lobbying cost, c. While our model predicts non-monotonic results for decreases in c, the general trend is that a decrease in lobbying costs increases lobbying and increases the incentives to form coalitions. (See the discussion to Proposition 3). Therefore, our model would predict an increase in lobbying and larger

[16] Data from https://www.opensecrets.org/lobby/indus_lobs.php?id=F&year=2016 and https://www.opensecrets.org/lobby/indus.php?id=F&year=2016.

[17] Data for this trend comes from https://www.opensecrets.org/industries/totals.php?cycle=2016&ind=F.

coalitions since 2010.

Turning to regulatory uniformity, Title 1 of Dodd-Frank established the Financial Stability Oversight Council as a coordinator of the oversight and regulation of systemically important financial institutions. Additionally, Section 312 of the Dodd-Frank Act specified that the Office of Thrift Supervision be closed down and its oversight responsibilities reallocated to other financial regulators including the Federal Reserve, FDIC, and OCC. Both changes tend towards regulatory policies that treat financial institutions more uniformly. Similar to changes in c, our model predicts non-monotonic results for changes in k. However, the overall trend in our model is that the increased uniformity provides greater incentives for firms to lobby and to form coalitions.

7 Conclusion

In this study we investigate the propensity of agents to form coalitions that lobby against stricter policies. For example, financial firms can take privately-beneficial actions that impose systemic risks on the financial system and lobby against policies that reduce their ability to take such actions. In our model, agents form coalitions because regulation is, at least partly, uniform across agents. This uniformity, in turn, implies that one agent's lobbying has effects on all other agent's regulation, such that a free-rider problem on lobbying arises among agents. The benefit to forming coalitions arises because we assume that a coalition is able to overcome the free-rider problem among the agents in the coalition, which increases the lobbying efforts of all agents in the coalition. As the free-rider problem is caused by uniformity, a more uniform regulatory policy increases the benefits to forming a lobbying coalition.

We assume that a lobbyist is necessary to form a coalition, and allow the lobbyist's bargaining power, i.e., the fraction of the net coalition benefit she extracts, to vary. In this setting, the degree of lobbyist power influences the potential lobbies that can arise in equilibrium, with dissimilar agents potentially joining together only when agents retain a high fraction of the net coalition gains. We further find that endogenous coalition formation causes the effects of regulatory uniformity and lobbying costs on total lobbying and average policy strength to be non-monotonic. Increasing the degree of uniformity both: (i) increases the free-rider problem, which decreases lobbying and increase average policy strength; and (ii), increases the benefit of forming coalitions, which increases lobbying and decreases average policy strength. In an environment with a fixed coalition structure, only the first effect is present. However, since the coalition structure reacts to changes in policy uniformity, we find non-monotonic effects of increasing the extent of uniformity. In our three-agent setting, these manifest as jumps. Similarly, increasing the cost agents bear personally for lobbying has a direct effect that decreases lobbying but has an indirect effect on the benefit of forming a lobby. Surprisingly, we find that lobbying decreases (and regulatory quality increases) when competition in the lobbyist market increases. The reason is that when agents retain a higher share of the benefits to forming coalitions, they tend to form smaller coalitions which results in less lobbying. As a result, increasing barriers to entry in the lobbyist market, e.g., through anti-revolving-door policies, can lead to more lobbying for policies that benefit private interests.

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8 Appendix A

8.1 Section 3: Two-agent coalition results

Small-medium agent coalition: When the small (s) and medium (m) agents form a lobby together, lobbying is as follows:

$$\begin{aligned}
\hat{B}_{s,sm} &= \hat{B}_{s,I} + \frac{3kD_m}{c(4+9k)} = \frac{4D_s + 3k\bar{D}_{sm}}{c(4+9k)} \\
\hat{B}_{m,sm} &= \hat{B}_{m,I} + \frac{3kD_s}{c(4+9k)} = \frac{4D_m + 3k\bar{D}_{sm}}{c(4+9k)} \\
\hat{B}_{\ell,sm} &= \hat{B}_{\ell,I} = D_\ell \frac{4+3k}{c(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{sm}}{c(4+9k)}
\end{aligned}$$

where $\bar{D}_{sm} = D_m + D_s$. Regulatory strengths are:

$$\begin{aligned}
\hat{\pi}_{s,sm} &= \frac{4\left(\lambda D_s - \hat{B}_{s,I} - \frac{3kD_m}{c(4+9k)}\right) + 3k\left(\lambda\bar{D} - \hat{B}_I - \frac{3k\bar{D}_{sm}}{c(4+9k)}\right)}{4+9k} \\
&= \hat{\pi}_{s,I} - \frac{3k\left(4D_m + 3k\bar{D}_{sm}\right)}{c(4+9k)^2}, \\
\hat{\pi}_{m,sm} &= \hat{\pi}_{m,I} - \frac{3k\left(4D_s + 3k\bar{D}_{sm}\right)}{c(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,sm} &= \hat{\pi}_{\ell,I} - \frac{9k^2\bar{D}_{sm}}{c(4+9k)^2}.
\end{aligned}$$

The utilities are: .

$$\begin{aligned}
\hat{U}_{sm,sm} &= \frac{c(9k+4)\left((9k+4)\bar{D}_{sm} - \lambda\left(3k\bar{D}_{sm}\bar{D} + 4\overline{D^2_{sm}}\right)\right) + 3k\bar{D}_{sm}\bar{D}(3k+4) + 8\overline{D^2_{sm}}}{c(9k+4)^2} \\
&= \hat{U}_{s,I} + \hat{U}_{m,I} + \frac{9k^2\overline{D^2_{sm}}}{2c(4+9k)^2}, \text{ and} \\
U_{\ell,sm} &= \frac{D_\ell\left(4D_\ell\lambda - 4 - 2c(9k+4)(3k(\lambda\bar{D} - 3)) + D_\ell(3k+4)^2 + 12k(3k+2)\bar{D}_{sm}\right)}{2c(9k+4)^2} \\
&= \hat{U}_{\ell,I} + \frac{9k^2 D_\ell \bar{D}_{sm}}{c(4+9k)^2}
\end{aligned}$$

where $\overline{D^2_{sm}} = D_s^2 + D_m^2$.

Small-large agent coalition: When the small (s) and large (ℓ) agents form a lobby together, lobbying is as follows:

$$\begin{aligned}
\hat{B}_{s,s\ell} &= \hat{B}_{s,I} + \frac{3kD_\ell}{c\,(4+9k)} = \frac{4D_s + 3k\bar{D}_{s\ell}}{c\,(4+9k)} \\
\hat{B}_{m,s\ell} &= \hat{B}_{m,I} = D_m \frac{4+3k}{c\,(4+9k)} \\
\hat{B}_{\ell,s\ell} &= \hat{B}_{\ell,I} + \frac{3kD_s}{c\,(4+9k)} = \frac{4D_\ell + 3k\bar{D}_{s\ell}}{c\,(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{s\ell}}{c\,(4+9k)}
\end{aligned}$$

where $\bar{D}_{s\ell} = D_s + D_\ell$. Regulatory strengths are:

$$\begin{aligned}
\hat{\pi}_{s,s\ell} &= \hat{\pi}_{s,I} - \frac{3k\left(4D_\ell + 3k\bar{D}_{s\ell}\right)}{c\,(4+9k)^2}, \\
\hat{\pi}_{m,s\ell} &= \hat{\pi}_{m,I} - \frac{9k^2\bar{D}_{s\ell}}{c\,(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,s\ell} &= \hat{\pi}_{\ell,I} - \frac{3k\left(4D_s + 3k\bar{D}_{s\ell}\right)}{c\,(4+9k)^2}.
\end{aligned}$$

and utilities are:

$$\begin{aligned}
\hat{U}_{s\ell,s\ell} &= \frac{c(9k+4)\left((9k+4)\,\bar{D}_{s\ell} - \lambda\left(4\overline{D^2_{s\ell}} + 3k\bar{D}_{s\ell}\bar{D}\right)\right) + 8\overline{D^2_{s\ell}} + 3k\bar{D}_{s\ell}\bar{D}\,(3k+4)}{c(9k+4)^2} \\
&= \hat{U}_{s,I} + \hat{U}_{\ell,I} + \frac{9k^2\overline{D^2_{s\ell}}}{2c\,(4+9k)^2}, \text{ and} \\
\hat{U}_{m,s\ell} &= \frac{D_m\left(-2c(9k+4)(3k(\lambda\bar{D}-3) + 4D_m\lambda - 4) + 12k(3k+2)\bar{D}_{s\ell} + D_m(3k+4)^2\right)}{2c(9k+4)^2} \\
&= \hat{U}_{m,I} + \frac{9k^2 D_m\bar{D}_{s\ell}}{c\,(4+9k)^2}
\end{aligned}$$

where $\overline{D^2_{s\ell}} = D_s^2 + D_\ell^2$.

Medium-large agent coalition: When the medium (m) and large (ℓ) agents form a lobby together, lobbying is as follows:

$$
\begin{aligned}
\hat{B}_{s,m\ell} &= \hat{B}_{s,I} = D_s \frac{4+3k}{c\,(4+9k)} \\
\hat{B}_{m,m\ell} &= \hat{B}_{m,I} + \frac{3kD_\ell}{c\,(4+9k)} = \frac{4D_m + 3k\bar{D}_{m\ell}}{c\,(4+9k)} \\
\hat{B}_{\ell,m\ell} &= \hat{B}_{\ell,I} + \frac{3kD_m}{c\,(4+9k)} = \frac{4D_\ell + 3k\bar{D}_{m\ell}}{c\,(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{m\ell}}{c\,(4+9k)},
\end{aligned}
$$

where $\bar{D}_{m\ell} = D_m + D_\ell$. Regulatory strengths are:

$$
\begin{aligned}
\hat{\pi}_{s,m\ell} &= \hat{\pi}_{s,I} - \frac{9k^2\bar{D}_{m\ell}}{c\,(4+9k)^2}, \\
\hat{\pi}_{m,m\ell} &= \hat{\pi}_{m,I} - \frac{3k\left(4D_\ell + 3k\bar{D}_{m\ell}\right)}{c\,(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,m\ell} &= \hat{\pi}_{\ell,I} - \frac{3k\left(4D_m + 3k\bar{D}_{m\ell}\right)}{c\,(4+9k)^2}.
\end{aligned}
$$

and utilities are:

$$
\begin{aligned}
\hat{U}_{m\ell,m\ell} &= \frac{c(9k+4)\left((9k+4)\,\bar{D}_{m\ell} - \lambda\left(4\overline{D^2_{m\ell}} + 3k\bar{D}_{m\ell}\bar{D}\right)\right) + 8\overline{D^2_{m\ell}} + 3k\bar{D}_{m\ell}\bar{D}\,(3k+4)}{c(9k+4)^2} \\
&= \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2\overline{D^2_{m\ell}}}{2c\,(4+9k)^2}, \text{ and} \\
\hat{U}_{s,m\ell} &= \frac{D_s\left(-2c(9k+4)(3k(\lambda\bar{D}-3) + 4D_s\lambda - 4) + 12k(3k+2)\bar{D}_{m\ell} + D_s(3k+4)^2\right)}{2c(9k+4)^2} \\
&= \hat{U}_{s,I} + \frac{9k^2D_s\bar{D}_{m\ell}}{c\,(4+9k)^2},
\end{aligned}
$$

where $\overline{D^2_{m\ell}} = D_m^2 + D_\ell^2$.

8.2 Proof of Theorems 2 and 3: Offer-stable coalitions

To help develop coalition stability among two-agent coalitions, we define a maximum deviation offer.

Definition 3 (Two-agent maximum deviation offer) *If agents g and h are members of the gh coalition, the maximum deviation offer from agent j to agent g, $MDO^{gh}_{j,g}$ is the greatest amount that agent j would offer agent g to leave the gh coalition and join the new jg coalition.*

We can calculate the maximum deviation offers that a given agent is willing to make as

$$\begin{aligned} MDO^{gh}_{j,g} &= \hat{U}_{j,jg} + \hat{U}_{g,jg} - \hat{U}_{L,jg} - \hat{U}_{j,gh}, & (13) \\ &= \hat{U}_{j,I} + \hat{U}_{g,I} + \alpha \left(\frac{9k^2 \overline{D^2_{jg}}}{2c\,(4+9k)^2} - \chi_2 \right) - \left(\hat{U}_{j,I} + \frac{9k^2 D_j \bar{D}_{gh}}{c\,(4+9k)^2} \right), & (14) \\ &= \hat{U}_{g,I} + \frac{9k^2 \left(\alpha \left(\overline{D^2_{jg}} - \chi_2 \right) - D_j \bar{D}_{gh} \right)}{2c\,(4+9k)^2}, & (15) \end{aligned}$$

which is the utility that the agents in the jg coalition achieve minus the utility that agent j expects to achieve in the presence of the gh coalition. The $MDO^{gh}_{j,g}$ is the maximum offer, because agent j in coalition jg faces a budget constraint (in terms of transferable utility) of $\hat{U}_{j,jg} + \hat{U}_{g,jg} - \hat{U}_{L,jg}$. Furthermore, agent j's outside option is to remain independent in a structure featuring the gh coalition, meaning that she should be willing to offer agent g no more than the gain achieved from g's deviation from gh to jg. For example,

$$\begin{aligned} MDO^{m\ell}_{s,\ell} &= \hat{U}_{\ell,I} + \frac{9k^2}{2c\,(4+9k)^2} \left(\alpha \left(\overline{D^2_{s\ell}} - \chi_2 \right) - 2 D_s \bar{D}_{m\ell} \right), \text{ and} & (16) \\ MDO^{s\ell}_{m,\ell} &= \hat{U}_{\ell,I} + \frac{9k^2}{2c\,(4+9k)^2} \left(\alpha \left(\overline{D^2_{m\ell}} - \chi_2 \right) - 2 D_m \bar{D}_{s\ell} \right). & (17) \end{aligned}$$

The following definition illustrates the offer-stability concept within the set of two-agent coalitions.

Lemma 1 (Offer stability for a two-agent coalition) *A two-agent coalition gh is offer-stable relative to other two-agent coalitions if $MDO^{gh}_{j,g} < MDO^{gj}_{h,g}$ and $MDO^{gh}_{j,h} < MDO^{hj}_{g,h}$,* *i.e., if: 1) agent j's maximum deviation offer to g conditional on coalition gh is lower than agent h's maximum deviation offer to g conditional on coalition jg; and 2) agent j's maximum deviation offer to h conditional on coalition gh is lower than agent g's maximum deviation offer to h conditional on coalition jh. A coalition structure containing a stable two-agent coalition is two-agent stable.*

Lemma 1 allows us to prove Theorems 2 and 3. First, $MDO^{sm}_{s,\ell} > MDO^{s\ell}_{m,\ell}$:

$$\begin{aligned} MDO^{m\ell}_{s,\ell} &> MDO^{s\ell}_{m,\ell} \\ \alpha D_s^2 + \alpha D_\ell^2 - 2D_sD_m - 2D_sD_\ell &> \alpha D_m^2 + \alpha D_\ell^2 - 2D_mD_s - 2D_mD_\ell \\ 2D_mD_\ell - 2D_sD_\ell &> \alpha D_m^2 - \alpha D_s^2 \\ 2D_\ell\left(D_m - D_s\right) &> \alpha\left(D_m + D_s\right)\left(D_m - D_s\right) \\ 2D_\ell &> \alpha\left(D_m + D_s\right), \end{aligned}$$

which implies that agent s can make a credible offer to agent ℓ to leave the $m\ell$ coalition. Note that the $\alpha\chi_2$ terms on both sides cancel. Second, $MDO^{sm}_{\ell,s} > MDO^{s\ell}_{m,s}$:

$$\begin{aligned} MDO^{sm}_{\ell,s} &> MDO^{s\ell}_{m,s} \\ \hat{U}_{s,I} + \frac{9k^2\left(\alpha\overline{D^2_{\ell s}} - 2D_\ell\bar{D}_{ms}\right)}{2c\left(4+9k\right)^2} &> \hat{U}_{s,I} + \frac{9k^2\left(\alpha\overline{D^2_{ms}} - 2D_m\bar{D}_{\ell s}\right)}{2c\left(4+9k\right)^2} \\ \alpha D_\ell^2 + \alpha D_s^2 - 2D_\ell\left(D_m + D_s\right) &> \alpha D_s^2 + \alpha D_m^2 - 2D_m\left(D_\ell + D_s\right) \\ \alpha\left(D_\ell + D_m\right) &> 2D_s, \end{aligned}$$

which is true for $\alpha > \frac{2D_s}{D_\ell + D_m}$, and false for $\alpha < \frac{2D_s}{D_\ell + D_m}$. When $\alpha < \frac{2D_s}{D_\ell + D_m}$, $MDO^{sm}_{\ell,s} < MDO^{s\ell}_{m,s}$, implying that agent m can make a credible offer to agent s to leave the $s\ell$ coalition. Finally,

$$\begin{aligned} MDO^{sm}_{\ell,m} &> MDO^{m\ell}_{s,m} \Leftrightarrow \\ \hat{U}_{m,I} + \frac{9k^2\left(\alpha\overline{D^2_{m\ell}} - 2D_\ell\bar{D}_{ms}\right)}{2c\left(4+9k\right)^2} &> \hat{U}_{m,I} + \frac{9k^2\left(\alpha\overline{D^2_{sm}} - 2D_s\bar{D}_{m\ell}\right)}{2c\left(4+9k\right)^2} \Leftrightarrow \\ \alpha\left(D_\ell + D_s\right) &> 2D_m. \end{aligned}$$

So, $\alpha > \frac{2D_m}{D_\ell+D_s} \Rightarrow MDO^{sm}_{\ell,m} > MDO^{m\ell}_{s,m}$, implying that ℓ can successfully motivate m to leave the sm coalition. But $\alpha < \frac{2D_m}{D_\ell+D_m} \Rightarrow MDO^{sm}_{\ell,m} < MDO^{m\ell}_{s,m}$, implying that s can successfully motivate m to leave the $m\ell$ coalition.

Comparing the thresholds, we have

$$\begin{aligned}
\frac{2D_s}{D_\ell + D_m} &< \frac{2D_m}{D_\ell + D_s} \\
2D_s\left(D_\ell + D_s\right) &< 2D_m\left(D_\ell + D_m\right) \\
2D_sD_\ell + 2D_s^2 &< 2D_mD_\ell + 2D_m^2.
\end{aligned}$$

If $\alpha > \frac{2D_m}{D_\ell+D_s}$, then $\alpha > \frac{2D_s(D_m+D_\ell)}{D_\ell^2+D_s^2}$, as

$$\begin{aligned}
\frac{2D_m}{D_\ell + D_s} &> \frac{2D_s\left(D_m + D_\ell\right)}{D_\ell^2 + D_s^2} \\
2D_mD_\ell^2 + 2D_mD_s^2 + 4D_mD_\ell D_s &> 2D_s^2D_\ell + 2D_mD_s^2 + 2D_sD_\ell^2 + 2D_mD_sD_\ell \\
D_\ell^2\left(D_m - D_s\right) + D_\ell D_s\left(D_m - D_s\right) &> 0.
\end{aligned}$$

If $\alpha > \frac{2D_m}{D_\ell+D_s}$, then $\alpha > \frac{2D_m(D_s+D_\ell)}{D_m^2+D_\ell^2}$ may or may not be true, as

$$\begin{aligned}
\frac{2D_m}{D_\ell + D_s} &> \frac{2D_m\left(D_s + D_\ell\right)}{D_m^2 + D_\ell^2} \\
2D_m\left(D_m^2 + D_\ell^2\right) &> 2D_m\left(D_s + D_\ell\right)^2 \\
D_m^2 + D_\ell^2 &> D_s^2 + D_\ell^2 + 2D_sD_\ell \\
\frac{D_m + D_s}{D_s + D_s}\left(D_m - D_s\right) &> D_\ell.
\end{aligned}$$

Overall, we have the following for regions of parameter-space defined by α:

1. $\alpha > \frac{2D_m}{D_\ell+D_s} \Rightarrow MDO^{m\ell}_{s,\ell} > MDO^{s\ell}_{m,\ell}$, $MDO^{sm}_{\ell,s} > MDO^{s\ell}_{m,s}$, and $MDO^{sm}_{\ell,m} > MDO^{m\ell}_{s,m}$. $s\ell$ is offer-stable.

2. $\frac{2D_s}{D_\ell+D_m} < \alpha < \frac{2D_m}{D_\ell+D_s} \Rightarrow MDO^{m\ell}_{s,\ell} > MDO^{s\ell}_{m,\ell}$, $MDO^{sm}_{\ell,s} > MDO^{s\ell}_{m,s}$, and $MDO^{sm}_{\ell,m} < MDO^{m\ell}_{s,m}$. $s\ell$ is offer-stable.

3. $\alpha < \frac{2D_s}{D_\ell + D_m} \Rightarrow MDO^{m\ell}_{s,\ell} > MDO^{s\ell}_{m,\ell}$, $MDO^{sm}_{\ell,s} < MDO^{s\ell}_{m,s}$, and $MDO^{sm}_{\ell,m} < MDO^{m\ell}_{s,m}$. sm is hypothetically offer-stable.

In the following subsections we derive coalition structures in each region of the parameter-space.

8.2.1 $\chi_2 < \overline{D^2_{s\ell}}$

If $MDO^{m\ell}_{s,\ell} > 0$, then the s agent is better-off in a pairing with ℓ than as the outsider when $m\ell$ form. To be able to form a pairing with ℓ, s must be able to offer ℓ more than what ℓ would gain from being the outsider in the presence of the sm lobby, meaning $\alpha \left(\overline{D^2_{\ell s}} - \chi_2\right) - 2D_\ell \bar{D}_{sm} > 0 \Leftrightarrow \alpha > \frac{2D_\ell(D_s+D_m)}{D^2_\ell + D^2_s - \chi_2}$, which in turn implies

$$\begin{aligned}
\alpha &> \frac{2D_\ell\left(D_s + D_m\right)}{D^2_\ell + D^2_s - \chi_2} \\
&\Rightarrow \alpha\left(D^2_\ell + D^2_s + 2D_sD_\ell\right) - \alpha\chi_2 > 2D_\ell D_s\left(1+\alpha\right) + 2D_\ell D_m \\
&\Rightarrow \alpha\left(D_\ell + D_s\right)\left(D_\ell + D_s\right) - \alpha\chi_2 > +2D_\ell D_m + 2D_m D_s + 2D_\ell D_s\left(1+\alpha\right) - 2D_m D_s \\
&\Rightarrow \alpha > \frac{2D_m}{D_\ell + D_s} + \frac{2D_s\left(D_\ell - D_m\right) + 2\alpha D_\ell D_s + \alpha\chi_2}{\left(D_\ell + D_s\right)^2} > \frac{2D_m}{D_\ell + D_s} \\
&\Rightarrow \alpha > \frac{2D_m}{D_\ell + D_s}.
\end{aligned}$$

It is possible, that for $\alpha < \frac{2D_\ell(D_s+D_m)}{D^2_\ell + D^2_s - \chi_2}$, the ℓ agent will turn down the s-agent's offer to form a coalition, preferring to wait for the sm coalition to form or willing to form an $m\ell$ coalition. The ℓ agent will prefer to form a coalition with m rather than waiting for the sm coalition to potentially form if

$$\alpha > \frac{2D_\ell\left(D_s + D_m\right)}{D^2_m + D^2_\ell - \chi_2}.$$

So, if $\alpha \in \left(\frac{2D_\ell(D_s+D_m)}{D^2_m + D^2_\ell - \chi_2}, \frac{2D_\ell(D_s+D_m)}{D^2_\ell + D^2_s - \chi_2}\right)$, then the ℓ agent will be willing to form an $m\ell$ coalition but unwilling to form an $s\ell$ coalition. In this range, the m agent will be willing to form the coalition with ℓ rather than wait as long as

$$\alpha > \frac{2D_m(D_s + D_\ell)}{D_m^2 + D_\ell^2 - \chi_2},$$

which is implied by $\alpha > \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}$ as $\frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2} > \frac{2D_m(D_s+D_\ell)}{D_m^2+D_\ell^2-\chi_2}$. For $\alpha \in \left(\frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}, \frac{2D_\ell(D_s+D_m)}{D_\ell^2+D_s^2-\chi_2}\right)$, $\alpha > \frac{2D_s}{D_\ell+D_m}$ is true and implies that $s\ell$ is offer-stable. So, s can make a successful deviation offer to ℓ to leave the $m\ell$ coalition. The problem is that ℓ would rather be the outsider in the presence of the sm coalition than be in the $s\ell$ coalition, but would rather be in the $m\ell$ coalition than be the outsider. This means that the following hold:

$$\begin{aligned} MDO_{\ell,s}^{sm} &< 0, \\ MDO_{\ell,m}^{sm} &> 0, \text{ and} \\ MDO_{s,\ell}^{m\ell} &> MDO_{m,\ell}^{s\ell} > 0. \end{aligned}$$

So, this arrangement should result in the $s\ell$ lobby forming as well, via the following sequence: 1) m forms a coalition with ℓ; 2) s makes a successful deviation-offer to ℓ.

If $\alpha < \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}$ (i.e., $MDO_{\ell,m}^{sm}$), then the ℓ agent will prefer to wait rather than join a coalition with s or m. In this case, the sm coalition will emerge if s and m both prefer joining the coalition to waiting, i.e., if

$$\alpha > \frac{2D_s(D_m + D_\ell)}{D_s^2 + D_m^2 - \chi_2} \text{ and } \alpha > \frac{2D_m(D_s + D_\ell)}{D_s^2 + D_m^2 - \chi_2}$$

Now, $\alpha > \frac{2D_m(D_s+D_\ell)}{D_s^2+D_m^2-\chi_2} \Rightarrow \alpha > \frac{2D_s(D_m+D_\ell)}{D_s^2+D_m^2-\chi_2}$, so we can focus only on the $\alpha > \frac{2D_m(D_s+D_\ell)}{D_s^2+D_m^2-\chi_2}$ condition. It is not possible to have $\alpha \in \left(\frac{2D_m(D_s+D_\ell)}{D_s^2+D_m^2-\chi_2}, \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}\right)$, i.e., this range is empty, as $\frac{2D_m(D_s+D_\ell)}{D_s^2+D_m^2-\chi_2} > \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}$. To show this, let $D_m = D_s + d_m$ and $D_\ell = D_s + d_m + d_\ell$ with

$d_\ell > 0$ and $d_m > 0$. Then $\frac{2D_m(D_s+D_\ell)}{D_s^2+D_m^2-\chi_2} < \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}$ implies

$$
\begin{aligned}
& 2\left(D_s+d_m\right)\left(D_s+D_s+d_\ell\right)\left(\left(D_s+d_m\right)^2+\left(D_s+d_\ell\right)^2-\chi_2\right) \\
< \;& 2\left(D_s+d_\ell\right)\left(D_s+D_s+d_m\right)\left(D_s^2+\left(D_s+d_m\right)^2-\chi_2\right) \\
\Rightarrow 0 > \;& 2\left(D_s+d_m\right)\left(D_s+D_s+d_m+d_\ell\right)\left(\left(D_s+d_m\right)^2+\left(D_s+d_m+d_\ell\right)^2-\chi_2\right) \\
& -2\left(D_s+d_m+d_\ell\right)\left(D_s+D_s+d_m\right)\left(D_s^2+\left(D_s+d_m\right)^2-\chi_2\right) \\
\Rightarrow 0 > \;& 2\begin{pmatrix} 5D_sd_m^3+4D_s^3d_m+D_sd_\ell^3+2D_s^3d_\ell+d_md_\ell^3+3d_m^3d_\ell+d_m^4+8D_s^2d_m^2 \\ +4D_s^2d_\ell^2+3d_m^2d_\ell^2+7D_sd_md_\ell^2+10D_sd_m^2d_\ell+10D_s^2d_md_\ell+\chi_2X_sd_\ell \end{pmatrix}.
\end{aligned}
$$

Therefore, the m agent will prefer to wait rather than join with s. So, for $\alpha < \frac{2D_\ell(D_s+D_m)}{D_m^2+D_\ell^2-\chi_2}$, neither the m nor ℓ agents are willing to join a coalition and we have a standstill. Overall, for $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, we should see the offer-stable $s\ell$ coalition. For $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, the lobbyist will choose the $m\ell$ coalition.

When $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ and $\chi_2 < \overline{D_{s\ell}^2}$, the agents will rationally form a 3-agent lobby if

$$
\begin{aligned}
\hat{U}_G-\chi_3 &\geq \hat{U}_{s\ell,s\ell}-\chi_2+\hat{U}_{m,s\ell} \\
\alpha\left(\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c\left(4+9k\right)^2}-\chi_3\right) &\geq \alpha\left(\frac{9k^2\overline{D_{s\ell}^2}}{2c\left(4+9k\right)^2}-\chi_2\right)+\frac{9k^2D_m\bar{D}_{s\ell}}{c\left(4+9k\right)^2} \\
2\overline{D^2}-\left(D_s-D_\ell\right)^2-2\left(\frac{1-\alpha}{\alpha}\right)D_m\bar{D}_{s\ell} &\geq \frac{2c\left(4+9k\right)^2}{9k^2}\left(\chi_3-\chi_2\right) \\
2\overline{D^2}-\left(D_s-D_\ell\right)^2-2\left(\frac{1-\alpha}{\alpha}\right)D_m\bar{D}_{s\ell}+\chi_2^E &\geq \chi_3^E.
\end{aligned}
$$

When $\alpha < \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ and $\chi_2 < \overline{D_{s\ell}^2}$, the agents will rationally form a 3-agent lobby if

$$
\begin{aligned}
\hat{U}_G-\chi_3 &\geq \hat{U}_{m\ell,m\ell}-\chi_2+\hat{U}_{s,m\ell} \\
\alpha\left(\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c\left(4+9k\right)^2}-\chi_3\right) &\geq \alpha\left(\frac{9k^2\overline{D_{m\ell}^2}}{2c\left(4+9k\right)^2}-\chi_2\right)+\frac{9k^2D_s\bar{D}_{m\ell}}{c\left(4+9k\right)^2} \\
2\overline{D^2}-\left(D_m-D_\ell\right)^2-2\left(\frac{1-\alpha}{\alpha}\right)D_s\bar{D}_{m\ell} &\geq \chi_3^E-\chi_2^E.
\end{aligned}
$$

8.2.2 $\chi_2 \in \left(\overline{D^2_{s\ell}}, \overline{D^2_{m\ell}}\right)$

If $\chi_2^E > \overline{D^2_{s\ell}}$, then the s and ℓ agents will choose to disband the lobby, even though it is stable relative to other two-agent coalitions. If $\chi_2 \in \left(\overline{D^2_{s\ell}}, \overline{D^2_{m\ell}}\right)$, then both the $s\ell$ and sm lobbies would each impose a net cost on the its member agents, but the $m\ell$ lobby would result in a net gain to its members. Therefore, the $m\ell$ lobby is the only two-agent coalition that would feasibly form, regardless of the value of α, as $\overline{D^2_{s\ell}} - \chi_2 > 0 \Rightarrow \alpha\left(\overline{D^2_{s\ell}} - \chi_2\right) \geq 0 \forall \alpha \in [0,1]$. Suppose the $m\ell$ coalition has formed. Then, if agent ℓ defects and joins the $s\ell$ coalition, we know that the instant later, that coalition will fall apart. However, if $\chi_2^E < \overline{D^2_{m\ell}}$, then agent m can offer a portion of the positive net coalition gain to agent ℓ, which is better than what agent ℓ would obtain from defecting to the doomed $s\ell$ coalition.

The agents will rationally form a 3-agent lobby if

$$\begin{aligned}
\hat{U}_G - \chi_3 &\geq \hat{U}_{m\ell,m\ell} - \chi_2 + \hat{U}_{s,m\ell} \\
\alpha\left(\frac{9k^2\left(\overline{D^2} + D_sD_m + D_sD_\ell + D_mD_\ell\right)}{c\,(4+9k)^2} - \chi_3\right) &\geq \alpha\left(\frac{9k^2\overline{D^2_{m\ell}}}{2c\,(4+9k)^2} - \chi_2\right) + \frac{9k^2 D_s\bar{D}_{m\ell}}{c\,(4+9k)^2} \\
2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s\bar{D}_{m\ell} &\geq \chi_3^E - \chi_2^E
\end{aligned}$$

8.2.3 $\chi_2 > \overline{D^2_{m\ell}}$

If $\chi_2 > \overline{D^2_{m\ell}}$, then no two-agent lobby benefits its members, so none will form. A 3-agent lobby will form when $\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c(4+9k)^2} - \chi_3 > 0$, or $\overline{D^2} + D_sD_m + D_sD_\ell + D_mD_\ell > \chi_3^E$.

8.3 Boundaries between coalition structures

Below is a list of the potential parameter changes that lead to transitions from one coalition structure to another. Each parameter change is provided only for a one-directional transition. Reversing the parameter change yields a transition in the opposite direction.

1. The following lead to transitions *to* the $s\ell$ coalition. These changes all lead to either

smaller coalitions or coalitions with smaller welfare impacts. Reversing them lead to the $s\ell$ coalition becoming less feasible or less likely, and tend towards larger coalitions or coalitions with larger welfare impacts:

(a) With $\chi_2 < \overline{D_{s\ell}^2}$ and $\chi_3^E > 2\overline{D^2} - (D_m + D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E$, α increasing from below $\frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ to above $\frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ yields a transition from $m\ell$ to $s\ell$

(b) With $\chi_2 < \overline{D_{s\ell}^2}$ and

$$\begin{aligned} & 2\overline{D^2} - (D_s - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell} + \chi_2^E \\ < \quad & \chi_3^E < 2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E, \end{aligned}$$

α increasing from below $\frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ to above $\frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ yields a transition from G to $s\ell$

(c) With $\chi_3^E > 2\overline{D^2} - (D_m + D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E$ and $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, χ_2^E decreasing from above $\overline{D_{s\ell}^2}$ to below $\overline{D_{s\ell}^2}$ yields a transition from $m\ell$ to $s\ell$

(d) With

$$\begin{aligned} & 2\overline{D^2} - (D_s - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell} + \chi_2^E \\ < \quad & \chi_3^E < 2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E, \end{aligned}$$

and $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$, χ_2^E decreasing from above $\overline{D_{s\ell}^2}$ to below $\overline{D_{s\ell}^2}$ yields a transition from G to $s\ell$

(e) With $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ and χ_2^E below $\overline{D_{s\ell}^2}$, increasing χ_3^E from below to above $2\overline{D^2} - (D_s - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell} + \chi_2^E$ yields a transition from G to $s\ell$.

(f) With $\alpha > \frac{D_\ell(D_s+D_m)}{D_m^2+D_\ell^2}$ and χ_2^E below $\overline{D_{s\ell}^2}$, decreasing χ_2^E from above to below $\chi_3^E - 2\overline{D^2} + (D_s - D_\ell)^2 + 2\left(\frac{1-\alpha}{\alpha}\right) D_m \bar{D}_{s\ell}$ yields a transition from G to $s\ell$.

2. The following lead to shifts from G to $m\ell$, implying smaller coalitions with smaller welfare impacts. Reversing them lead to shifts from $m\ell$ to G, implying larger coalitions or coalitions with larger welfare impacts:

   (a) When $\chi_2^E \in \left(\overline{D_{s\ell}^2}, \overline{D_{m\ell}^2}\right)$, increasing χ_3^E from below to above $2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + \chi_2^E$ yields a shift from G to $m\ell$.

   (b) When $\chi_2^E \in \left(\overline{D_{s\ell}^2}, \overline{D_{m\ell}^2}\right)$, decreasing χ_2^E from below to above $\chi_3^E - 2\overline{D^2} + (D_m - D_\ell)^2 + 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell}$ yields a shift from G to $m\ell$.

   (c) When

$$
\begin{aligned}
& 2\left(\overline{D^2} + D_s D_m + D_s D_\ell + D_m D_\ell\right) \\
> \; & \chi_3^E \\
> \; & 2\overline{D^2} - (D_m - D_\ell)^2 - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell} + D_\ell^2 + D_m^2 \\
= \; & 2\overline{D^2} + 2D_m D_\ell - 2\left(\frac{1-\alpha}{\alpha}\right) D_s \bar{D}_{m\ell},
\end{aligned}
$$

   decreasing χ_2^E from above $\overline{D_{m\ell}^2}$ to below $\overline{D_{m\ell}^2}$ yields a shift from G to $m\ell$.

3. The following lead to shifts from I (no lobbies) to either $m\ell$ or G, implying larger coalitions with larger welfare impacts. Reversing them lead to shifts from $m\ell$ or G to I, implying smaller coalitions or coalitions with smaller welfare impacts:

   (a) When $\chi_2^E > \overline{D_{m\ell}^2}$, decreasing χ_3^E from above to below $2\left(\overline{D^2} + D_s D_m + D_s D_\ell + D_m D_\ell\right)$ yields a shift from I to G.

   (b) When $\chi_3^E > 2\left(\overline{D^2} + D_s D_m + D_s D_\ell + D_m D_\ell\right)$, decreasing χ_2^E from above to below $\overline{D_{m\ell}^2}$ yields a shift from I to $m\ell$.

9 Appendix B

In this Appendix we contrast offer-stability with notions of stability based on the core and bargaining sets. Addressing the core first, we start with a definition of core-stability.

Definition 4 (Core stability) *A feasible allocation-structure pair (A, X) is blocked if there is a nonempty coalition in which each member is made weakly better off and one member is made strictly better off in an alternative feasible allocation-structure pair (A', X'). An allocation-structure pair (A, X) is in the core and the structure X is core-stable if (A, X) is not blocked by any alternatives.*

Under core stability, a given allocation has to be robust to all possible alternatives. That is, the same allocation has to prevent any possible deviation if that allocation-structure pair is in the core. Under offer-stability, the structure must be consistent, but we allow the allocation within a structure to adjust to prevent blocking by different alternatives. For example, suppose we are in the $m\ell$ structure, with allocation $A = \{u_s, u_m, u_\ell\} = \{1, 2, 2\}$. Now, if the sm coalition formed, assume that any allocation $A' = \{u'_s, 3.5 - u'_s, 0.6\}$ is feasible, implying that sm have a budget of 3.5 to split between them. In this example, allocation A is blocked by $A' = \{1.1, 2.4, 0.6\}$ because A' makes agents s and m better off.

X	A
G	$\{-\infty, -\infty, -\infty\}$
sm	$\{u'_s, 3.5 - u'_s, 1.2\}$
$s\ell$	$\{u''_s, 1.1, 4 - u''_s\}$
$m\ell$	$\{1, u'_m, 4.5 - u'_m\}$
I	$\{0.9, 1, 1.1\}$

Table 3
Payoff Structure for a sample game

Assume $\chi_3 \to \infty$, so we only consider 2-agent coalitions. Clearly, the allocation provided by I can be dominated by allocations in any of the two-agent coalition structures. In any two-agent coalition, we must have $u_s > 0.9$, $u_m > 1$, and $u_\ell > 1.1$. Now, any allocation in $X = sm$ is blocked, since either $3.5 - u'_s \leq 1 \Rightarrow m$ unilaterally deviates, or $u'_s \leq 2.5$, in which case s and ℓ can deviate to, for example, $(s\ell, \{2.6, 1.1, 1.4\})$. Therefore, sm is not core-stable. An allocation in $s\ell$ must have $u''_s > 0.9$, implying $u_\ell < 4 - 0.9 = 3.1$. But, in this case, m and ℓ can deviate to $(m\ell, \{1, 1.3, 3.2\})$, implying that $s\ell$ is not core-stable. For $m\ell$, $u'_m > 1$ prevents unilateral deviation of m, but m can also get up to 2.5 for deviating with s to sm. Similarly, ℓ can get up to 3 for deviating with s to $s\ell$. It is impossible for an allocation in $m\ell$ to provide both $u'_m \geq 2.5$ and $u_\ell = 4.5 - u'_m \geq 3$ implying that any allocation in $m\ell$ is blocked by either sm or $s\ell$. Therefore, the core to the game above (again, with $\chi_3 \to \infty$) is empty, and there is no core-stable coalition structure, as every allocation is dominated by another allocation with an alternative structure. If, instead, the set of feasible allocations in $m\ell$ were $\{1, u'_m, 5.7 - u'_m\}$, then the core would consist of $(m\ell, \{1, x, 5.7 - x\})$, with $x \in (2.5, 2.7)$.

Despite the lack of a core-stable coalition structure, there is an offer-stable coalition structure, $m\ell$. The $m\ell$ structure is offer-stable because: any deviation from $m\ell$ to $s\ell$ can be prevented by giving ℓ at least 3 and still leaving m better off with at least the 1.1 he would receive under $s\ell$; and any deviation to sm can be prevented by giving m at least 2.5 while leaving ℓ with more than the 1.2 he would receive under sm. Clearly, these cannot both be accomplished, but the can be accomplished as counteroffers to proposals received individually. Our nomenclature of offer-stability comes directly from the idea that a coalition structure is offer-stable if coalition partners can be given counteroffers that prevent deviations. That is, if s proposes an allocation to m that involves a move to the sm structure, we allow ℓ to make a counteroffer to m to keep him in the coalition.

Myerson (2013) provides a concise and intuitive presentation of the bargaining set solution to cooperative games introduced by Aumann and Maschler (1964). The discussion here borrows from Myerson (2013). To set the stage, consider an N-player game in which the value available to a coalition S in structure X is $v(S, X)$. Consider an initial payoff-structure pair (A, X_I), where the initial allocation is A and the initial coalition structure is X_I. The bargaining set is based on objections to allocations and counterobjections to those objections. An objection by player (or set of players) i against another player (or set of players) j is an allocation-coalition-structure triple, (A', S, X_S), such that $A' \in R^N$, $S \subseteq N$, $i \in S$, $j \notin S$, $v(S, B_S) = \sum_{k \in S} a'_k$, and $A' >_S A$, where $>_S$ indicates that the players in S prefer allocation A' to allocation A and a'_k is the payoff to player k in allocation A'. A counterobjection to i's objection (A', S, X_S) against j and A is similarly a triple (A'', T, B_T) such that $A'' \in R^N$, $T \subseteq N$, $j \in T$, $i \notin T$, $T \cap S \neq \varnothing$, $v(T, B_T) = \sum_{k \in S} a''_k$, $A'' >_T A$, and $A'' >_{T \cap S} A'$. In the counterobjection, player j can form a coalition T that takes away some of i's partners in the objection (but not i) and makes them at least as well off as in the objection; thus, j can restore himself/themselves and the other members of T to payoffs at least as good as they had in x. Additionally, all the players in T weakly prefer the allocation under A'' to the allocation under A'. A payoff-structure pair (A, X) is stable if there is a counterobjection to each possible objection.

Offer-stability relies on offers and counteroffers, which are essentially synonymous with objections and counterobjections. The primary difference is that with offer-stability, we weaken the bargaining set to ensure only that player (or set of players) j is at least as well off under the counteroffer or counterobjection as he (they) would be if the original offer or objection were accepted. That is, we replace $A'' >_T A$ with $A'' >_T A'$.