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The use of analyst forecasts as proxies for investors' earnings expectations is commonplace in empirical research. An implicit assumption behind their use is that they reflect analysts' private information in an unbiased manner. As demonstrated here, this assumption is not necessarily valid. There is shown to be a tendency for analysts to release forecasts closer to prior earnings expectations than is appropriate, given their information. Further, analysts exhibit herding behavior, whereby they release forecasts similar to those previously announced by other analysts, even when this is not justified by their information. These results are shown to have interesting empirical implications.

Analyst forecasts have been widely used in empirical research in both accounting and finance to proxy for investors' earnings expectations.[1] Other empirical research has focused on comparing analysts' forecast accuracy to that of both time-series and publicly announced managerial forecasts.[2] An implicit assumption underlying much of this research is that the forecasts publicly released by analysts reflect their private information in an unbiased manner. I demonstrate that this assumption is not necessarily valid.

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[1] An abbreviated reference list includes Brown, Foster, and Noreen (1985), Hughes and Ricks (1987), McNichols (1989), and Waymire (1984).

[2] Representative of such work are Brown et al. (1987), Brown and Rozeff (1978), and O'Brien (1988).

The Review of Financial Studies Spring 1994 Vol. 7, No. 1, pp. 97–124

The analysis yields two principal results. First, under certain circumstances an analyst prefers to release a forecast that is close to prior earnings expectations, *even if issuing a more extreme forecast is justified by his private information.* Such action positively impacts investors' assessment of the analyst's forecasting ability and so enables him to charge a higher fee for his forecasts.[3] Second, the likelihood that the analyst releases a forecast similar to those previously announced by other analysts *is greater than could be justified by his own information.* Such action is a manifestation of herding behavior and, as before, is undertaken in order to favorably affect investors' assessment of the analyst's forecasting ability.[4]

These results yield several empirical implications, discussed in detail in Section 4. The first relates to studies that measure the price impact of earnings announcements and that use analysts' reported forecasts as a proxy for investors' prior earnings expectations. It is predicted that for more extreme earnings surprises, the price reaction will be smaller than would be expected theoretically under the assumption that analysts incorporate their information into their forecasts in an unbiased manner. This is a result of investors recognizing the possibility that an analyst who issues a forecast close to prior expectations actually has information that justifies a more extreme forecast. Consequently, a large difference between realized earnings and the analyst's forecast does not surprise them as much as if they had taken the announced forecast at face value. A second implication is that for those forecasts that represent small deviations from prior earnings expectations, a positive covariance is expected between the ex post forecast error and both the price change at the time of the forecast announcement and the forecast itself. Again, this is a result of the possibility that an analyst releasing a small positive (negative) forecast [which causes a positive (negative) market reaction at the time of release] actually has information that expected earnings are even more positive (negative). Consequently, his ex post forecast error is also expected to be positive (negative). With respect to herding behavior, it is predicted that the forecasts of analysts with greater ability will be less influenced by previous forecasts than will those of weaker analysts. Additionally, given analyst herding, it is shown that calculating a consensus forecast by simply averaging individual

[3] In related research, Trueman (1990) shows that an analyst may also be reluctant to revise a previously issued forecast upon receipt of new information. This is because a forecast revision implies that the analyst's original information was inaccurate and, as a consequence, may result in investors lowering their assessment of the analyst's ability to collect accurate information in a timely manner.

[4] Herding behavior has been studied in other contexts by Bikhchandani, Hirshleifer, and Welch (1992). Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1993), Scharfstein and Stein (1990), and Welch (1992).

analyst forecasts is inappropriate. The order in which the forecasts are released must also be taken into account in the calculation.

The plan of this article is as follows. In Section 1 the economic setting is described. The analyst's optimal forecast disclosure strategy is examined in Section 2 under the assumption that all analysts release their forecasts simultaneously. A similar analysis for the case where analysts release their forecasts sequentially appears in Section 3. Implications of the analysis are discussed in Section 4, and I conclude with some additional remarks.

1. Economic Setting

Consider an economy in which there are many investors, one risky firm, and two security analysts, each of whom is paid by clients to prepare forecasts of the firm's earnings, $\tilde{e}$. At the beginning of each period, the prior expectation of the period's earnings is given by $\bar{e}$, which, for simplicity, is set equal to zero. The realized earnings of the period take one of four possible values, denoted by e_h^+, e_l^+, e_l^-, and e_h^-, where $e_h^+ > e_l^+ > 0$, $e_h^+ = -e_h^-$, and $e_l^+ = -e_l^-$. The + (−) superscript signifies that the earnings realization is greater (less) than the prior expectation for earnings, and the h (l) subscript signifies that the magnitude of earnings is high (low). The realized earnings are disclosed at the end of the period. The prior probability that earnings will equal e_l^+ (as well as the prior probability that they will equal e_l^-) is denoted by t, where $.25 < t < .5$, while the prior probability that earnings will equal e_h^+ (as well as the prior probability that they will equal e_h^-) is given by $.5 - t$. Consequently, the prior distribution of the firm's earnings is symmetric around its expected value of zero, with greater weight assigned to the less extreme earnings levels.

At the beginning of the period each of the two security analysts obtains private information, $\tilde{y}$, about the earnings to be realized. The private information takes one of four possible values, denoted by y_h^+, y_l^+, y_l^-, and y_h^-. The relation between the private information and the realized earnings of the period is given as follows:

$$\text{prob}(y_i^z \mid e_i^z) = k, \qquad i \in \{h, l\}; \quad z \in \{+, -\}, \tag{1}$$

$$\text{prob}(y_j^z \mid e_i^z) = 1 - k, \qquad i, j \in \{h, l\}; \quad i \neq j; \, z \in \{+, -\}, \tag{2}$$

$$\text{prob}(y_j^{z'} \mid e_i^z) = 0, \qquad i, j \in \{h, l\}; \, z, z' \in \{+, -\}; \, z \neq z'. \tag{3}$$

Expression (3) implies that if the analyst's private information is either y_h^+ or y_l^+ (y_h^- or y_l^-), then he can perfectly infer that earnings will be greater (less) than the prior mean.[5] However, as reflected in expressions (1) and (2), he cannot tell with certainty whether the magnitude of those earnings will be high or low.

The parameter k reflects the analyst's ability to predict earnings. For an analyst with strong (weak) ability, k is equal to g (b), where $g > b > .5$.[6] While the analyst knows his ability with certainty, his clients do not.[7] The prior probability that the analyst's predictive ability is strong (weak) is denoted by p_s (p_w). In the subsequent analysis, a strong (weak) analyst is referred to as an analyst of type s (w).

If the analyst of type m, $m \in \{s, w\}$, observes the signal y_i^z, his expectation for the firm's earnings, $E(\tilde{e} \mid y_i^z, m)$, is equal to

$$E(\tilde{e} \mid y_i^z, m) = e_b^z p(e_b^z \mid y_i^z, m) + e_l^z p(e_l^z \mid y_i^z, m), \tag{4}$$

where $p(e_b^z \mid y_i^z, m)$ and $p(e_l^z \mid y_i^z, m)$ are the probabilities that earnings of e_b^z and e_l^z, respectively, will be realized, given that an analyst of type m observes a signal of y_i^z. Note that $E(\tilde{e} \mid y_i^z, m)$ is not equal to e_i^z. Because of the possible error in the analyst's private information, his earnings expectation conditional on the signal y_b^z (y_l^z) is closer to (further from) the prior mean than are the earnings $e_b^z(e_l^z)$.

Using Bayes' rule along with expressions (1) and (2) reveals that

$$p(e_l^z \mid y_b^z, m) = \frac{(1-k)t}{(1-k)t + k(.5-t)}, \tag{5}$$

$$p(e_b^z \mid y_b^z, m) = \frac{k(.5-t)}{(1-k)t + k(.5-t)}, \tag{6}$$

$$p(e_l^z \mid y_l^z, m) = \frac{kt}{kt + (1-k)(.5-t)}, \tag{7}$$

$$p(e_b^z \mid y_l^z, m) = \frac{(1-k)(.5-t)}{kt + (1-k)(.5-t)}. \tag{8}$$

Given the symmetries in the structure of the analyst's private information, it is straightforward to show that $p(e_l^- \mid y_b^-, m) = p(e_l^+ \mid y_b^+, m)$ and $p(e_l^- \mid y_l^-, m) = p(e_l^+ \mid y_l^+, m)$. Consequently, $E(\tilde{e} \mid y_i^-, m)$ is equal to $-E(\tilde{e} \mid y_i^+, m)$.

[5] Allowing for a small, but positive, probability that the analyst is incorrect in his prediction of the sign of the period's earnings does not affect the nature of the equilibrium in this economy. See Appendix C for further details.

[6] The restriction that g and b are both greater than .5 ensures that observing a signal of y_i^z not only reveals the sign of earnings, z, but also increases the probability, conditional on z, that earnings of e_i^z will be realized. It also guarantees that the strict monotone likelihood ratio property, $\text{prob}(y_b^z \mid e_b^z)/\text{prob}(y_b^z \mid e_l^z) > \text{prob}(y_l^z \mid e_b^z)/\text{prob}(y_l^z \mid e_l^z)$, is satisfied. [See Milgrom (1981).]

[7] The impact on the analyst's equilibrium actions of varying the degree of uncertainty clients have over his ability is the subject of Proposition 3(b).

After observing his signal at the beginning of the period and forming his posterior expectation for the firm's earnings, the analyst releases a forecast of expected earnings to his clients, after which his forecast is made public.[8,9] While his posterior expectation for earnings is given by (4) conditional on observing signal y_i^z, it is not clear that this will be his announced forecast. In deciding on the forecast to disclose, I assume that the analyst's goal is to maximize his expectation of his clients' end-of-period assessment of the probability that his ability is strong. As shown, this assessment is based on the forecast that the analyst releases and the earnings that are disclosed at the end of the period.[10]

The analyst's objective arises naturally if the fee that he can charge next period for his forecasts is directly related to his clients' assessment of his predictive ability at the end of this period. It also arises naturally if the analyst is employed by a financial services firm, such as an investment bank, whether or not the firm or the analyst is paid directly by clients for the analyst's forecasts. The reason is that the revenues that the firm can generate from trading commissions are likely to depend, to a great extent, on the perceived forecasting ability of its analysts.

A recent *Wall Street Journal* article[11] confirms the importance that investment bankers place on their analysts' forecasting ability. It quotes the director of research at Dean Witter as stating that "If your performance is bad and you generate a lot of business, you've done a terrible job," since clients will remember the analyst's poor performance and be reluctant to do business with the analyst's firm in the future. He continues by saying that "If your performance is good and you generate a lot of business, you've done a terrific job." In summarizing the opinions of the director of research at Shearson Lehman Brothers the article states that "More important, he [the director of research] says, are whether the companies covered think the analysts are expert, and how the analysts' recommendations perform." These

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[8] The results of the subsequent analysis would not change if it were assumed that the analyst announced either y_l^z or y_h^z rather than an earnings expectation.

[9] It is a common practice for investment banks or brokerage houses to publicly release the earnings forecasts of their analysts after they are privately disclosed to clients.

[10] A multiperiod generalization of this objective function would involve the analyst choosing a forecast disclosure this period that maximizes his expectation of a weighted average of this period's *and* future periods' posterior assessments of the probability that he is strong. Such a generalization is not expected to affect the nature of the results derived here, because any action that the analyst takes this period affects future periods only through its effect on this period's posterior probability assessment, since this assessment is equal to next period's prior. Consequently, even in a multiperiod context, the analyst's focus remains on his clients' end-of-period posterior assessment of the probability that he is strong.

[11] "Analysts Devote More Time to Selling as Firms Keep Scorecard on Performance," *Wall Street Journal,* October 29, 1991, p. C1.

remarks suggest that, while an analyst's compensation may be formally based on several factors, among the most fundamental, if not the most fundamental, is the analyst's perceived forecasting ability. This conclusion is reinforced by an analyst at Ferris, Baker Watts, who is quoted as stating that "If your estimates aren't accurate, nobody's going to buy your stocks. So it doesn't matter" how your compensation is determined.[12]

2. Equilibrium in the Case of Simultaneous Forecast Release

In this section each analyst's equilibrium reporting strategy is derived under the assumption that the two analysts release their forecasts simultaneously. This equilibrium provides useful insights for understanding the nature of equilibrium in the case of sequential release, which is considered in Section 3.

Before proceeding with the analysis, some further notation and definitions must be introduced. Let $\mathbf{X}$ represent the set of possible earnings forecast disclosures, $\{E(\tilde{e} \mid y', m'); y' \in \{y_h^+, y_h^-, y_l^+, y_l^-\}, m' \in \{s, w\}\}$.[13] Denote by $\boldsymbol{\alpha}_{y,m} \equiv \{\alpha_{y,m}(y', m'); y' \in \{y_h^+, y_h^-, y_l^+, y_l^-\}, m' \in \{s, w\}\}$ a probability distribution over $\mathbf{X}$ for an analyst of type $m \in \{s, w\}$ who observes the forecast $y \in \{y_h^+, y_h^-, y_l^+, y_l^-\}$, where $\alpha_{y,m}(y', m')$ represents the probability of the analyst reporting the forecast $E(\tilde{e} \mid y', m')$. Finally, define $p(s \mid E(\tilde{e} \mid y', m'), e_j^z)$ as the clients' posterior probability that an analyst is strong, given a reported forecast of $E(\tilde{e} \mid y', m')$ and realized earnings of e_j^z, $j \in \{h, l\}$, $z \in \{+,-\}$. An equilibrium is then defined as a set of strategies for each analyst type m and observed signal y, $\boldsymbol{\alpha}^*_{y,m} \equiv \{\alpha^*_{y,m}(y', m')\}$, such that

$$\text{(i)} \qquad \boldsymbol{\alpha}^*_{y,m} \in \operatorname*{argmax}_{\boldsymbol{\alpha}_{y,m}} E[p(s \mid E(\tilde{e} \mid \tilde{y}', \tilde{m}'), \tilde{e}) \mid \boldsymbol{\alpha}_{y,m}, y, m],$$

where the outer expectation is taken over all possible values of y', m', and earnings, and (ii) $p(s \mid E(\tilde{e} \mid y', m'), e_j^z)$ is calculated by clients using Bayes' rule, where applicable, under the conjecture that each analyst's equilibrium strategy is given by $\boldsymbol{\alpha}^*_{y,m}$.

[12] An alternative to the objective of maximizing perceived ability is that of maximizing perceived added value relative to the forecasts of other analysts. In the setting of this article, these two objectives are identical and would lead the analysts to take identical actions. There are other settings, though, in which the two objectives would lead to different actions. Note, however, that the maximization of added value can only arise as an objective to the extent that clients obtain investment advice about a given security from more than one analyst. This is not expected to occur frequently. Whether or not it does, the preceding discussion and quotations suggest that the maximization of absolute accuracy remains as an analyst's basic objective.

[13] In principle, the set of possible earnings forecast announcements could be expanded to include forecasts inconsistent with any combination of signal and analyst type. However, as shown in the proof to Proposition 1, it would not be optimal for an analyst to issue such a forecast.

In the subsequent analysis an equilibrium will be shown to exist in which $\alpha^*_{y,m}$ is of the following form:

a. $\alpha^*_{y,s}(y', m') = 1$ if $m' = s$ and $y' = y$; (S1)

b. $\alpha^*_{y,s}(y', m') = 0$ otherwise; (S2)

c. $\alpha^*_{y,w}(y', m') = 1$ if $m' = s$ and $(y, y') \in \{(y_l^+, y_l^+), (y_l^-, y_l^-)\}$; (S3)

d. $\alpha^*_{y,w}(y', m') = \alpha \in [0, 1)$ if $m' = s$ and $(y, y') \in \{(y_h^+, y_l^+), (y_h^-, y_l^-)\}$; (S4)

e. $\alpha^*_{y,w}(y', m') = 1 - \alpha$ if $m' = s$ and $(y, y') \in \{(y_h^+, y_h^+), (y_h^-, y_h^-)\}$; (S5)

f. $\alpha^*_{y,w}(y', m') = 0$ otherwise. (S6)

Under this set of strategies, the strong analyst always releases a truthful forecast, one that is consistent with his private information. In contrast, the weak analyst deviates from issuing a truthful forecast in two respects. First, he mimics the forecasts of a strong analyst by releasing $E(\tilde{e} \mid y_l^z, s)$ rather than $E(\tilde{e} \mid y_l^z, w)$ when he wants to convey to his clients that he has observed signal y_l^z.[14] If he did not do this, his clients would be able to immediately discern his true ability from the announced forecast [since they know that a strong analyst never releases a forecast of $E(\tilde{e} \mid y_i^z, w)$] and reduce his fee accordingly. Second, the weak analyst sometimes releases a forecast of $E(\tilde{e} \mid y_l^z, s)$ upon observing the signal y_h^z. Doing so represents an attempt by the analyst to convince his clients that he has observed the signal y_l^z instead of the signal y_h^z. Much of the remainder of this section is devoted to showing that these reporting strategies for the strong and weak analysts are optimal.

To streamline the following analysis and discussion, I denote an announced forecast of $E(\tilde{e} \mid y_l^z, s)$ by f_l^z and an announcement of $E(\tilde{e} \mid y_h^z, s)$ by f_h^z. Further, in both this section and Section 3, the superscript z can be dropped without causing confusion.

With clients making correct conjectures for the equilibrium reporting strategies of the two analyst types, their assessment of the probability that an analyst is strong, conditional on a reported forecast of f_i and realized earnings of e_j, $p(s \mid f_i, e_j)$, $i,j \in \{h, l\}$, is given by the following set of expressions (with derivations relegated to Appendix A):

[14] This means that for y_i^z equal to y_h^z (y_l^z) the weak analyst places more weight on the earnings level e_h^z (e_l^z) than is appropriate given his private information.

$$p(s \mid f_l, e_l) = \frac{g}{g + bp + \alpha(1 - b)p}, \tag{9}$$

$$p(s \mid f_l, e_h) = \frac{1 - g}{1 - g + (1 - b)p + \alpha bp}, \tag{10}$$

$$p(s \mid f_h, e_l) = \frac{1 - g}{1 - g + (1 - \alpha)(1 - b)p}, \tag{11}$$

$$p(s \mid f_h, e_h) = \frac{g}{g + (1 - \alpha)bp}, \tag{12}$$

where $p \equiv p_w/p_s$.

Comparing expressions (9) and (10) reveals that, conditional on the analyst announcing a forecast that is low (f_l), clients' assessment of the probability that the analyst is strong is greater when the realized earnings are also low (e_l). Similarly, comparing expressions (11) and (12) reveals that, conditional on the analyst announcing a forecast that is high (f_h), clients' assessment of the probability that the analyst is strong is greater when the realized earnings are also high (e_h). Such outcomes are, therefore, preferred by the analyst.

Note that these posterior probabilities, and, consequently, each analyst's reporting strategy, do not depend on the other analyst's forecast. The reason is that, conditional on any analyst's reported forecast and the realized earnings, the forecast of the other analyst provides no additional information to clients about the first analyst's ability.[15]

Recall that the analyst's forecast disclosure at the beginning of the period is chosen to maximize the expectation (over all possible earnings levels) of his clients' posterior probability that he is strong. For an analyst of ability m who observes signal y_h and releases forecast f_h, this expectation, denoted by $E[p(s \mid f_h, \tilde{e}) \mid y_h, m]$, is given by

$$E[p(s \mid f_h, \tilde{e}) \mid y_h, m] = \frac{(1 - g)p(e_l \mid y_h, m)}{1 - g + (1 - \alpha)(1 - b)p} + \frac{gp(e_h \mid y_h, m)}{g + (1 - \alpha)bp}. \tag{13}$$

For an analyst of ability m who observes signal y_h but reports forecast f_l, the expectation of this posterior probability is

$$E[p(s \mid f_l, \tilde{e}) \mid y_h, m] = \frac{gp(e_l \mid y_h, m)}{g + bp + \alpha(1 - b)p} + \frac{(1 - g)p(e_h \mid y_h, m)}{1 - g + (1 - b)p + \alpha bp}. \tag{14}$$

[15] This is a direct result of the fact that, conditional on the realized earnings, the signals of the two analysts are independent.

An examination of expressions (13) and (14) reveals the following:

Observation 1. (a) $E[p(s \mid f_h, \tilde{e}) \mid y_h, s] > E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$.

(b) $E[p(s \mid f_l, \tilde{e}) \mid y_h, w] > E[p(s \mid f_l, \tilde{e}) \mid y_h, s]$.

Proof. From expressions (9)–(12) is it straightforward to see that $p(s \mid f_l, e_l) > p(s \mid f_l, e_h)$ and $p(s \mid f_h, e_h) > p(s \mid f_h, e_l)$. Further, from expressions (5)–(8), $p(e_i \mid y_i, s) > p(e_i \mid y_i, w)$, $i \in \{h, l\}$. From these relations, the observation follows immediately. Q.E.D.

Observation 1 is a direct result of the fact that clients' assessment of the posterior probability that the analyst is strong is greater when the realized earnings and the analyst's forecast are either both high or both low rather than when one is high and the other low. In part (a) of the observation, the analyst observes signal y_h and releases forecast f_h. In this case, earnings are more likely to also be high if the analyst's ability is strong (since there is a greater probability that his signal is correct and that earnings of e_h will be realized). Consequently, such an analyst has a more positive expectation of his clients' end-of-period assessment of the probability that he is strong.

In part (b) the analyst observes signal y_h but releases forecast f_l. In this case, earnings are more likely to also be low if the analyst's ability is weak (since there is a greater probability that his signal is incorrect and that earnings of e_l will be realized). Consequently, conditional on misreporting his information, the weak analyst has a more positive expectation of his clients' end-of-period assessment of the probability that he is strong.

Directly resulting from Observation 1 is Observation 2.

Observation 2. *Conditional on observing a signal of y_h, a weak analyst expects to gain more (or lose less) than a strong analyst by reporting the forecast f_l instead of f_h. That is,*

$$E[p(s \mid f_l, \tilde{e}) \mid y_h, w] - E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$$
$$> E[p(s \mid f_l, \tilde{e}) \mid y_h, s] - E[p(s \mid f_h, \tilde{e}) \mid y_h, s].$$

Observation 2 provides the basis upon which the main result of this section is derived.

Proposition 1. *An equilibrium exists in which*

(a) $\alpha^*_{y,m}$ *is given by* (S1)–(S6).

(b) *Clients' posterior assessment of the probability that an analyst* *is strong is calculated using Bayes' rule, whenever the reported forecast is a member of* $\{E(\tilde{e} \mid y', s),\ y' \in \{y_b^+, y_b^-, y_l^+, y_l^-\}\}$.

(c) *For all other forecasts, the clients believe that the analyst is weak with probability 1.*[16]

Proof. See Appendix B.

The most interesting aspect of this equilibrium is that, in certain cases, a weak analyst who observes a signal of y_b has an incentive to report a less extreme earnings forecast, f_l, rather than the forecast f_b, that deviates more from prior expectations. Comparing expressions (13) and (14) reveals the set of parameter values under which such action is taken with positive probability (that is, for which α is strictly greater than zero).

Proposition 2. *The equilibrium probability that the weak analyst reports the forecast* f_l *after observing the signal* y_b *is strictly greater than zero if and only if* $b < 2t$.

Conditional on observing signal y_b, the weak analyst releases forecast f_l with positive probability as long as the accuracy of his signal, b, is sufficiently low relative to the prior probability, t, that earnings of e_l will occur. It is straightforward to show that the condition $b < 2t$ is equivalent to the condition that $p(e_l \mid y_b, w) > p(e_b \mid y_b, w)$. If it holds, then there is a greater likelihood of the analyst's announced forecast and the realized earnings both being of the same magnitude (low) if he reports f_l rather than if he reports f_b. Since his clients' assessment of the probability that he is strong is higher if the forecast and earnings are of the same magnitude, the analyst has an incentive to report f_l with positive probability. (This would not be true if the analyst's ability were known to the clients; in that case there would be no incentive for the analyst to deviate from announcing a forecast consistent with his private information.) Alternatively stated, when $b < 2t$, a conjecture that the analyst always reports a forecast of f_b when his private information is y_b cannot be fulfilled by the analyst's actions.

There are two comparative statics results that can be derived in this equilibrium. They are stated in the following proposition.

[16] While this proposition demonstrates the existence of an equilibrium in which the set of reporting strategies is as specified in (S1)–(S6), there may be other sets of equilibrium strategies. However, it can be shown that the set analyzed here is the only one with the property that the posterior probability the analyst is strong is higher when realized earnings and the earnings forecast are either both high or both low than when one is high and the other is low. This desirable and intuitively appearing feature justifies focusing on this equilibrium.

Proposition 3. (a) *The equilibrium level of* α *(weakly) increases with* t. (b) *The equilibrium level of* α *(weakly) increases with* $1/p \equiv p_s/p_w$.

Proof. See Appendix B.

An increase in t means that there is a greater ex ante probability that the earnings level e_l will be realized. This implies that there is also a greater likelihood that an announced forecast of f_h will be contradicted by realized earnings of e_l. Consequently, the weak analyst's gain from reporting f_l rather than f_h, conditional on observing y_h, increases. As a result, the equilibrium level of α (weakly) increases with t.[17]

This result has both time-series and cross-sectional implications. Noting that the variance of realized earnings is inversely related to t, it implies that in time periods when a firm's earnings are of higher variance, and, in any given time period, for those firms whose earnings have a higher variance, it is less likely for a weak analyst to release an earnings forecast that is inconsistent with his private information (that is, to release a forecast of f_l when the signal y_h is observed).

Part (b) of the proposition captures the notion that the weak analyst gains more (or loses less) from reporting a forecast inconsistent with his signal the less certain clients are, ex ante, that his predictive ability is weak. In practice, clients are likely to be less knowledgeable of an analyst's ability if (a) he has not been providing forecasts for a long period of time or (b) he has recently changed the set of firms that he is following, and an ability to forecast earnings for one set of firms does not translate into an ability to forecast earnings for a different set of firms, or (c) his ability changes over time. The presence of one or more of these conditions increases the likelihood of a weak analyst reporting a forecast inconsistent with his private information.

3. Equilibrium in the Case of Sequential Forecast Release

In contrast to the analysis of the previous section, here it is assumed that the two analysts release their forecasts sequentially. It is straightforward to analyze the equilibrium in this setting, given the preceding results for the case of simultaneous disclosure. The reporting strategy of the analyst who discloses first (referred to below simply as the first analyst) is, in fact, the same as that in the previous section. This is true because the forecast of the analyst who reports after him (referred

[17] If $\alpha > 0$, then $\partial\alpha/\partial t$ is strictly greater than zero. If $\alpha = 0$, then $\partial\alpha/\partial t$ is strictly positive if and only if the increase in t causes $E[p(s \mid f_l, \tilde{e}) \mid y_h, w]$ to exceed $E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$ at $\alpha = 0$.

to below as the second analyst) is not used by clients in assessing the first analyst's ability.[18]

In contrast, the reporting strategy of the second analyst is affected by the announced forecast of the first analyst. The reason for this dependence is not that the first analyst's forecast provides clients with information useful in assessing the second analyst's ability; given realized earnings, it does not. Rather, it is because the second analyst uses the first forecast to update the probability of occurrence of each of the two earnings levels. From the viewpoint of the second analyst, a forecast announcement of f_l (f_h) by the first analyst increases the ex ante probability, t ($.5 - t$), that earnings of e_l (e_h) will occur. From Proposition 3, this implies that if the second analyst is weak, his equilibrium α is (weakly) higher when the first analyst releases a forecast of f_l rather than when he discloses f_h. This conclusion immediately leads to the main result of this section.

Proposition 4. *If the second analyst's ability is weak and he observes a signal of* y_h, *then*

(a) *The likelihood of his releasing a forecast of* f_l *is (weakly) higher if the first analyst previously reported a forecast of* f_l *rather than* f_h.
(b) *The likelihood of his releasing a forecast of* f_h *is (weakly) higher if the first analyst previously reported a forecast of* f_h *rather than* f_l.

If the second analyst's ability is weak and he observes a signal of y_l, *or if the second analyst's ability is strong, then his announced forecast is not affected by the forecast released by the first analyst.*

As the proposition makes clear, herding behavior is a characteristic of equilibrium, with the reporting decision of a weak second analyst affected by the forecast announcement of the first analyst. If the second analyst observes a signal of y_h, he is more likely to report a forecast of f_h (f_l) as long as the first analyst also reported f_h (f_l).[19] Alternatively stated, the probability α that the second analyst discloses a forecast inconsistent with his information is higher if the first analyst reported f_l rather than f_h.

It should be emphasized that the positive correlation of the two

[18] Again, this is because the two analysts' forecasts are independent, conditional on realized earnings. A possible extension of this analysis would be to allow for one analyst's report to be useful to clients in assessing the other analyst's ability, even in the presence of realized earnings. While a formal derivation of the equilibrium reporting strategies in such a setting would be difficult, it is reasonable to expect that herding behavior on the part of the second analyst, as described later, will continue to exist.

[19] In an extreme case, if a report of f_h by the first analyst causes the second analyst's prior for e_h to exceed that for e_l, the second analyst might actually find it preferable to report f_h, not only when y_h is observed, but also when y_l is observed.

analysts' forecasts that results from herding behavior is separate from that which arises naturally due to the positive correlation of each analyst's signal with realized earnings. That source of correlation is present whether forecasts are released simultaneously or sequentially and is not related to herding behavior.

It is also important to recognize that despite the presence of herding, the second analyst's forecast does have information content. This is certainly true if the second analyst is strong, since he never engages in herding behavior. It is also true if the analyst is weak, since α is less than unity in equilibrium.[20] That the second analyst's forecast provides information to investors is consistent with the finding of Lys and Sohn (1990) that there is incremental information content in an analyst forecast that closely follows in time a forecast disclosure by another analyst.

This analysis has assumed that the order in which the two analysts disclose their earnings forecasts is determined exogenously; an interesting extension would be to give each analyst the flexibility to himself choose between two dates to release his forecast. It is conjectured that in such a setting an analyst choosing to disclose his forecast at the earlier date (labeled, for convenience, date 1) is more likely to be strong than is an analyst who releases his forecast at the later date (labeled date 2). The basis for this conjecture is Proposition 4, which implies that a weak analyst has an incentive to delay his announcement until date 2, since, when he observes y_b, he conditions his disclosure on any previously released forecast; in contrast, a strong analyst has no such incentive to delay his announcement. In order for the weak analyst to find it profitable to disclose his forecast at date 2 with positive probability, though, there must be some chance that each analyst type will be constrained (for exogenous reasons) to report his forecast at date 2. Otherwise, clients would infer that an analyst releasing his forecast at date 2 is weak, eliminating any incentive for such an analyst to delay his disclosure.

This conjecture leads to an interesting implication for the relative accuracies of earnings forecasts released at different times during the year. Conventional wisdom maintains that later forecasts will be more accurate, in general, than those released earlier since, as the year progresses, analysts have access to more information. However, if, as suggested here, the analysts disclosing forecasts earlier are of higher average ability, then it can no longer be unambiguously stated that later forecasts will be more accurate. This conclusion is consistent with some of the results in O'Brien (1988) that show an insignificant

[20] Only if α were equal to 1, so that the analyst always releases the forecast f_l regardless of his information, would his forecast give no information to investors about the period's earnings.

difference between the accuracy of the most recent earnings forecast made for a firm and that of the mean or median of recent forecasts.[21]

4. Empirical Implications of the Analysis

In this section several empirical implications of the analysis in Sections 2 and 3 are explored. Whenever possible, they are linked to the extant empirical literature. The first implication relates to the observed share-price reaction to earnings surprises. For many years, empirical accounting researchers have been measuring the price impact of earnings announcements by regressing price change on earnings surprise.[22] As should be clear from the preceding analysis, a potential problem arises when researchers measure the earnings surprise by the difference between reported earnings and prior analysts' earnings forecasts. Since these forecasts need not reflect the analysts' private information in an unbiased manner, the "apparent" earnings surprise observed by researchers need not equal the earnings surprise as perceived by investors. Their assessment of earnings surprise is the difference between reported earnings and their expectation for earnings given the analysts' forecasts *and* their knowledge of the analysts' reporting strategy.

To understand clearly the effect of this difference on the relation between earnings surprise and price reaction observed by researchers, assume, for simplicity, that there is only one analyst forecasting earnings for a given firm. The adjustments that investors make to the analyst's forecast in order to derive their expectation of the firm's earnings are of two types. First, if a forecast of f_l^z is released, investors take into account the possibility that the analyst is weak and actually observed a private signal of y_h^z. Second, investors adjust for the fact that a weak analyst mimics the forecasts of a strong analyst by releasing either $f_l^z \equiv E(\tilde{e} \mid y_l^z, s)$ or $f_h^z \equiv E(\tilde{e} \mid y_h^z, s)$ rather than $E(\tilde{e} \mid y_l^z, w)$ or $E(\tilde{e} \mid y_h^z, w)$. (Recall the discussion in Section 2.)

If the forecast f_h^z is released, then investors' expectation for the firm's earnings, denoted by $E(\tilde{e} \mid f_h^z)$, is given by

$$E(\tilde{e} \mid f_h^z) = f_h^z p(s \mid f_h^z) + E(\tilde{e} \mid y_h^z, w) p(w \mid f_h^z), \tag{15}$$

where $p(m \mid f_i^z)$ is investors' assessment of the probability that the analyst releasing forecast f_i^z is of type m, $m \in \{s, w\}$. If, instead, the forecast f_l^z is released, investors' earnings expectation, denoted by $E(\tilde{e} \mid f_l^z)$, is

[21] Another potential explanation for these results is that the mean or median forecast reflects the information of several analysts, while the most recent forecast reflects just the information of an individual analyst.

[22] See Lev (1989) for a summary of this research.

$$E(\tilde{e} \mid f_l^z) = f_l^z p(s \mid f_l^z) + E(\tilde{e} \mid y_l^z, w) p(w, y_l^z \mid f_l^z)$$
$$+ E(\tilde{e} \mid y_b^z, w) p(w, y_b^z \mid f_l^z), \tag{16}$$

where $p(w, y_l^z \mid f_l^z)$ $[p(w, y_b^z \mid f_l^z)]$ is investors' assessment of the probability that the analyst releasing forecast f_l^z is weak and has observed the signal y_l^z (y_b^z).

Straightforward algebra reveals the following relation between the analyst's announced forecast and investors' expectation for the firm's earnings.[23]

Observation 3. (a) $E(\tilde{e} \mid f_l^+) > f_l^+$;

(b) $E(\tilde{e} \mid f_l^-) < f_l^-$;

(c) $E(\tilde{e} \mid f_b^+) < f_b^+$;

(d) $E(\tilde{e} \mid f_b^-) > f_b^-$.

Investors' earnings expectation given a report of f_l^+ (f_l^-) is more positive (negative) than the reported earnings forecast, while their earnings expectation given a report of f_b^+ (f_b^-) is more negative (positive) than the forecast. As a result, the price reaction that researchers observe subsequent to an earnings announcement, which reflects investors' reaction given their actual earnings expectation, will appear inappropriate given the "apparent" earnings surprise as computed by researchers. This conclusion is similar to that of Lev (1989), who claims that the difficulty that empiricists have had in explaining stock returns around the time of earnings announcements, whether they have used analysts' forecasts or time-series models as a proxy for the market's expectations, is attributable, in part, to incorrect measurement of earnings surprise.

With Observation 3, the "apparent" earnings surprises can be partitioned into two types. The first is those for which researchers are expected to see price underreaction, defined as a price response which is inappropriately low in absolute magnitude for the given "apparent" earnings surprise. Assuming that price reactions are positively related to *investors'* assessment of the surprise in reported earnings, price underreaction will be observed when the "apparent" earnings surprise is greater in magnitude than investors' assessment of it. The second type is surprises for which researchers are expected to see price overreaction and occur when the "apparent" earnings surprise is smaller in magnitude than investors' assessment of it. The following table lists all of the possible "apparent" earnings surprises

[23] These results depend on the assumption that the forecast released by the analyst is either f_l^z or f_b^z, and not $E(\tilde{e} \mid f_l^z)$ or $E(\tilde{e} \mid f_b^z)$.

in this economy and whether they result in overreaction or underreaction. (The most positive earnings surprises are listed first and the most negative appear last.)

"Apparent" earnings surprise	Under- or overreaction
$e_h^+ - f_l^+ > 0$	Underreaction
$e_l^- - f_h^- > 0$	Underreaction
$e_h^+ - f_h^+ > 0$	Overrreaction
$e_l^- - f_l^- > 0$	Overrreaction
$e_l^+ - f_l^+ < 0$	Overrreaction
$e_h^- - f_h^- < 0$	Overrreaction
$e_l^+ - f_h^+ < 0$	Underreaction
$e_h^- - f_l^- < 0$	Underreaction

As is clear from the table, researchers are expected to find price underreactions for the most extreme positive and negative earnings surprises and price overreactions for the more moderate surprises. Alternatively stated, when earnings surprise is measured relative to prior analyst forecasts, a graph of market return against earnings surprise is expected to be nonlinear, with a slope (or, earnings response coefficient) that is smaller in magnitude for the more extreme levels of surprise. A nonlinear relation is not predicted in the case where surprise is measured relative to a time-series expectation model.

Empirical evidence concerning nonlinearities in the return earnings surprise relation is found in Freeman and Tse (1990), Cheng, Hopwood, and McKeown (1992), and Das and Lev (1991), who do document a nonlinear relation of the form predicted above. However, they find the nonlinear relation to hold both in the case where surprise is measured relative to prior analyst forecasts *and* in the case where it is measured relative to a time-series expectation model of earnings. An explanation for these findings that is suggested in all three studies, but which is abstracted from in the formal analysis of this article, is that the more extreme earnings surprises are likely to be less persistent than are the smaller earnings surprises. If persistence is a factor contributing to a nonlinear return earnings surprise relation independent of the proxy used to measure prior expectations, then it must be controlled for before testing the prediction made above. Freeman and Tse do this. They partition their sample of firms into deciles according to earnings surprise (with decile 1 containing the most negative surprises and decile 10 the most positive) and, then, within each decile, into those with high, medium, and low levels of past earnings persistence. When the time-series model is used as a proxy for prior expectations, the high-persistence group of firms in each of the 10 deciles has a significantly positive mean earnings response coefficient. However, when the prior analysts' forecasts benchmark is used, only groups 1 through 8 exhibit a significantly positive mean earnings response coefficient. That is, for the high-persistence group of firms, the graph of the return earnings surprise relation is flat for the most extreme positive earnings surprises. Further, for every decile the t-statistic for the difference between the mean earnings response coefficients in the high- and low-persistence groups of firms is lower for the analyst forecast benchmark than for the time-series benchmark. In other words, persistence appears to have a greater effect on the return earnings surprise relation when a time-series expectation model is used. These results suggest that there is an additional factor, such as strategic analyst behavior, that contributes to the nonlinear relation when analyst forecasts are used as a proxy for prior expectations.

Observation 3 can also be used to derive predictions for the sign of the covariance between the *ex post error* in the analyst's forecast and the price change at the time of the forecast release (or, equivalently, the forecast, itself). Such predictions are a natural extension of prior empirical research that has examined the sign of the covariance between an analyst's forecast *revision* and the price change at the time of a previous forecast announcement and the previous revision, itself.[24,25] The following predictions arise in this setting.

Observation 4. *There is a positive covariance between the ex post forecast error and the price change at the time of the forecast release (or, equivalently, the forecast, itself) for earnings forecasts that are close to prior earnings expectations (forecasts of either* f_l^+ *or* f_l^-*). For more extreme forecasts (either* f_h^+ *or* f_h^-*), this covariance is negative.*

Proof. See Appendix B.

Given that $E(\tilde{e} \mid f_l^+) > f_l^+$ and $E(\tilde{e} \mid f_l^-) < f_l^-$, a forecast of f_l^+ (f_l^-) is an underestimate (overestimate) of the firm's earnings. As a result, the forecast f_l^+ (f_l^-) is expected to be followed by a positive (negative) forecast error. This leads directly to the conclusion that such forecasts (and the associated price reactions) covary positively with the ex post forecast errors. In contrast, a forecast of f_h^+ (f_h^-) is an overestimate (underestimate) of the firm's earnings and so is expected to be followed by a negative (positive) forecast error. In this case, the forecast (and the associated price reaction) covaries negatively with the ex post forecast error.

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[24] See, for example, Abarbanell (1991), Brown, Foster, and Noreen (1985), Givoly and Lakonishok (1979), and Lys and Sohn (1990).

[25] It is not possible to use this analysis to make predictions involving forecast revisions since in this setting each analyst releases only one forecast during the period.

Observation 3 can also be used to directly address the issue of bias in analyst forecasts. Many empirical studies have found that analysts tend to issue optimistic forecasts.[26] One explanation given for these findings is that analysts perceive a need to be optimistic in order to retain access to management. Another is that analysts' compensation is partly based on the sales commissions they generate and that optimistic forecasts that are accompanied by buy recommendations result in a greater number of trades than do pessimistic forecasts that are accompanied by either hold or sell recommendations. While these factors have not been formally incorporated into our analysis, predictions of analyst bias arise, nevertheless, as reflected in the following observation.

Observation 5. *In an equilibrium for which α is strictly greater than zero, an analyst forecast which is greater (less) than the prior earnings expectation is biased downward (upward), on average.*

Proof. See Appendix B.

As shown formally in the proof, this observation is driven by the positive probability of a weak analyst issuing the forecast f_l^+ (f_l^-) when observing signal y_h^+ (y_h^-). This causes a downward (upward) bias in forecasts that are greater (less) than prior earnings expectations. This result, in conjunction with the (unmodeled) anecdotal evidence that analysts have a tendency to issue optimistic forecasts, in general, leads to the prediction that the degree of analyst optimism will be greater for forecasts representing a negative change from prior earnings expectations than for those reflecting a positive change.

There are two final implications of this analysis, both related to analyst herding behavior. One, which follows from Proposition 4 and is consistent with recent empirical evidence, is that the likelihood of an analyst to herd decreases with his ability to predict earnings. The evidence supporting this conjecture comes from Stickel (1990), who finds that changes in prior consensus analyst forecasts have less of an effect on the revision of an individual analyst's forecast if he is a member of *Institutional Investor*'s "All-American Research Team" than if he is not a member. Stickel concludes from this that members of the All-American Research Team have less of a tendency to "follow the crowd," that is, to exhibit herding behavior. This, combined with the additional finding of Stickel (1992) that the forecasts of All-American Research Team members are more accurate than those of non-

[26] See Fried and Givoly (1982), O'Brien (1988), Abarbanell (1991), Stickel (1990), and Lys and Sohn (1990).

members, provides support for the prediction that analysts with greater forecasting ability exhibit less herding behavior.

Another implication of herding behavior relates to the calculation of a consensus forecast from individual analyst forecasts. As should be clear from the analysis of the previous section, not only are the individual forecasts, themselves, important in deriving a consensus forecast, but so, too, is the order in which the forecasts are released. Given that analysts exhibit herding behavior, a forecast disclosure of f_h^z followed by one of f_l^z, for example, has a different implication for expected earnings than does a forecast of f_l^z followed by one of f_h^z. The reason is that the reporting strategy of the first analyst differs from that of the second. Specifically, the probability that an analyst who reports f_l^z has actually observed the signal y_l^z is less if he is the first to report than if his report comes on the heels of a report of y_h^z by another analyst. This means that, from investors' point of view, prob$(e_h^z \mid f_l^z, f_h^z)$ is greater than prob$(e_h^z \mid f_h^z, f_l^z)$, so that the earnings expectation $E(\tilde{e} \mid f_l^z, f_h^z)$ is greater in magnitude than $E(\tilde{e} \mid f_h^z, f_l^z)$. Interpreting this result in another way, in calculating the consensus forecast it is inappropriate for the two forecasts to receive equal weighting; in this example, the second analyst's forecast should be weighted more heavily than that of the first. This result arises even though the precision of the private information of a type m analyst, $m \in \{s, w\}$, is the same whether he reports first or second. Of course, if the second analyst is expected to have more precise information (a feature which is not part of this model), there would be an additional reason to weigh his forecast more than that of the first analyst.

5. Summary and Conclusions

As shown here, an implicit assumption behind much of the empirical research involving security analyst earnings forecasts—that the forecasts reflect the analysts' private information in an unbiased manner—is not necessarily valid. In some cases analysts choose to release earnings forecasts that do not differ greatly from prior expectations, even though their private information justifies more extreme earnings forecasts. This result has implications for (i) the observed relation between earnings surprises and price changes, (ii) the sign of the covariance between the price change at the time of a forecast announcement and the ex post forecast error, and (iii) the direction of bias in analyst forecasts. It was also shown that analysts have a tendency to report forecasts similar to those previously released by other analysts; that is, they exhibit herding behavior. This implies that naively calculating a consensus analyst forecast by averaging individual analyst forecasts is inappropriate. The order in which the individual forecasts are made must also be taken into account in deriving the consensus forecast.

Appendix A

In this appendix Equation (9) is derived. (The z superscript is suppressed for convenience.) The derivations of Equations (10)–(12) follow along very similar lines and so are omitted. As a first step in the derivation, note that $p(s \mid f_l, e_l)$ can be written as follows:

$$p(s \mid f_l, e_l) = p(s \mid y_l, f_l, e_l) \cdot p(y_l \mid f_l, e_l) + p(s \mid y_b, f_l, e_l) \cdot p(y_b \mid f_l, e_l). \tag{A1}$$

Using Bayes' rule gives

$$p(s \mid y_l, f_l, e_l) = \frac{g}{g + bp}. \tag{A2}$$

Further, given the clients' conjectures of each analyst's set of reporting strategies, we write

$$p(s \mid y_b, f_l, e_l) = 0. \tag{A3}$$

As a next step, note that

$$p(y_l \mid f_l, e_l) = p(s, y_l \mid f_l, e_l) + p(w, y_l \mid f_l, e_l). \tag{A4}$$

Again, using Bayes' rule, we obtain

$$p(s, y_l \mid f_l, e_l) = \frac{p(e_l \mid s, y_l) \cdot p(s, y_l)}{[p(e_l \mid s, y_l) \cdot p(s, y_l) + p(e_l \mid w, y_l) \cdot p(w, y_l) + \alpha p(e_l \mid w, y_b) \cdot p(w, y_b)]} \tag{A5}$$

$$= \frac{g}{g + bp + \alpha(1 - b)p}. \tag{A6}$$

Similarly,

$$p(w, y_l \mid f_l, e_l) = \frac{bp}{g + bp + \alpha(1 - b)p}. \tag{A7}$$

Therefore,

$$p(y_l \mid f_l, e_l) = \frac{g + bp}{g + bp + \alpha(1 - b)p}. \tag{A8}$$

Substituting (A2), (A3), and (A8) into (A1) gives

$$p(s \mid f_l, e_l) = \frac{g}{g + bp + \alpha(1 - b)p}, \tag{A9}$$

which is Equation (9).

Appendix B

Proof of Proposition 1. Consider, first, the action of a weak analyst upon observing signal y_b. From Equations (13) and (14) it is a simple matter to show that

(a) $E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$ is declining in α.
(b) $E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$ is increasing in α.
(c) $E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$ is less than 1 at $\alpha = 1$.
(d) $E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$ is equal to 1 at $\alpha = 1$.

[Point (d) follows from the observation that when $\alpha = 1$, only the strong analyst ever issues a forecast of f_b.]

Several inferences can be drawn from relations (a) through (d). First, α could not equal 1 in equilibrium because the weak analyst would prefer to release the forecast f_b, conditional on observing the signal y_b, if his clients were to conjecture that $\alpha = 1$. This action is inconsistent with the conjecture. Second, if at $\alpha = 0$, $E[p(s \mid f_b, \tilde{e}) \mid y_b, w] \geq E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$, then a conjecture of $\alpha = 0$ is fulfilled by the weak analyst's actions; it is preferable for him to report his signal truthfully with probability 1. Further, given relations (a) and (b), $\alpha = 0$ would also be the only possible conjecture that is fulfilled in equilibrium. Third, if at $\alpha = 0$, $E[p(s \mid f_b, \tilde{e}) \mid y_b, w] < E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$, then the unique equilibrium value of α, denoted by α^*, is where $E[p(s \mid f_b, \tilde{e}) \mid y_b, w] = E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$. If clients conjecture that the weak analyst reports f_l with probability α^*, then the analyst is indifferent between reporting f_l and reporting f_b, so that announcing f_l with probability α^* is consistent with maximizing his objective function. Clients' conjectures are again fulfilled.

Given that the weak analyst (weakly) prefers to report f_b when y_b is observed, the strong analyst must strictly prefer to do so when he observes y_b. (This follows immediately from Observation 2.) Consequently, the conjecture that the strong analyst reports f_b with probability 1 conditional on observing y_b is fulfilled by his action.

Consider, now, the weak analyst's actions when y_l is observed. The analogous expressions to (13) and (14) in this case are

$$E[p(s \mid f_l, \tilde{e}) \mid y_l, w] = \frac{gp(e_l \mid y_l, w)}{g + bp + \alpha(1 - b)p} + \frac{(1 - g)p(e_b \mid y_l, w)}{1 - g + (1 - b)p + \alpha bp} \tag{B1}$$

and

$$E[p(s \mid f_b, \tilde{e}) \mid y_l, w] = \frac{(1-g)p(e_l \mid y_l, w)}{1-g+(1-\alpha)(1-b)p} + \frac{gp(e_b \mid y_l, w)}{g+(1-\alpha)bp}. \tag{B2}$$

It follows from (13), (14), (B1), and (B2) that

(a) $E[p(s \mid f_l, \tilde{e}) \mid y_l, w] > E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$.
(b) $E[p(s \mid f_b, \tilde{e}) \mid y_l, w] < E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$.
(c) $E[p(s \mid f_l, \tilde{e}) \mid y_l, w] > E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$ at $\alpha = 0$.
(d) $E[p(s \mid f_b, \tilde{e}) \mid y_l, w] < E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$ at $\alpha = 0$.

From relations (a) and (b) it follows that if $\alpha > 0$ in equilibrium (so that $E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$ is equal to $E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$), then the weak analyst strictly prefers to report f_l when observing y_l. From relations (c) and (d) it follows that if $\alpha = 0$ in equilibrium, then the weak analyst also prefers to report f_l with probability 1 when observing y_l. This verifies that the clients' conjecture for the weak analyst's behavior upon observing y_l is fulfilled by his action. Finally, it is straightforward to show that, just as when y_b is observed, the strong analyst has a greater gain from reporting a forecast consistent with his information than does the weak analyst, conditional on observing y_l. Therefore, he also releases f_l with probability 1 if he observes y_l. This verifies that the clients' conjecture for the strong analyst's behavior upon observing y_l is fulfilled by the analyst's action in equilibrium.

Finally, the off-the-equilibrium-path beliefs of the clients, that a forecast other than f_l or f_b results from a weak analyst, is consistent with observed actions. It is also a reasonable belief given that the strong analyst receives a greater benefit by reporting f_l or f_b, depending on the signal he has observed, than does the weak analyst. This verifies that the remaining client conjecture is fulfilled by the analyst's action in equilibrium. Q.E.D.

Proof of Proposition 3

Part a. Differentiating $E[p(s \mid f_l, \tilde{e}) \mid y_b, w]$ and $E[p(s \mid f_b, \tilde{e}) \mid y_b, w]$ with respect to α and t reveals that

1. $\partial E[p(s \mid f_l, \tilde{e}) \mid y_b, w]/\partial\alpha < 0$.
2. $\partial E[p(s \mid f_l, \tilde{e}) \mid y_b, w]/\partial t > 0$.
3. $\partial E[p(s \mid f_b, \tilde{e}) \mid y_b, w]/\partial\alpha > 0$.
4. $\partial E[p(s \mid f_b, \tilde{e}) \mid y_b, w]/\partial t < 0$.

Given these relations and the fact that the equilibrium α is the value that equates $E[p(s \mid f_l, \tilde{e}) \mid y_h, w]$ and $E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$ (unless $E[p(s \mid f_l, \tilde{e}) \mid y_h, w]$ is less than $E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$ at $\alpha = 0$, in which case the equilibrium α is equal to zero), it follows that $\partial\alpha/\partial t \geq 0$.

Part b. Differentiating $E[p(s \mid f_l, \tilde{e}) \mid y_h, w]$ and $E[p(s \mid f_h, \tilde{e}) \mid y_h, w]$ with respect to α and p reveals that

1. $\partial E[p(s \mid f_l, \tilde{e}) \mid y_h, w]/\partial\alpha < 0$.
2. $\partial E[p(s \mid f_l, \tilde{e}) \mid y_h, w]/\partial p < 0$.
3. $\partial E[p(s \mid f_h, \tilde{e}) \mid y_h, w]/\partial\alpha > 0$.
4. $\partial E[p(s \mid f_h, \tilde{e}) \mid y_h, w]/\partial p > 0$.

Analogous to the reasoning in the proof to part (a), the signs of these derivatives imply that $\partial\alpha/\partial p \leq 0$. Q.E.D.

Proof of Observation 4. The proof is given for the case of a forecast release of either f_l^+ or f_l^-. The proof for the case of a release of either f_h^+ or f_h^- follows along similar lines. The covariance between the forecast released and the subsequent forecast error is given by

$$\begin{aligned}&\text{cov(forecast, error)}\\&\quad = E(\text{forecast}\cdot\text{error}) - E(\text{forecast})\cdot E(\text{error}).\end{aligned} \tag{B3}$$

Since $f_l^+ = -f_l^-$, E(forecast) is equal to zero. Therefore, the sign of the covariance in (B3) is the same as the sign of E(forecast·error). This expectation is equal to

$$f_l^+[E(\tilde{e} \mid f_l^+) - f_l^+]\text{prob}(f_l^+) + f_l^-[E(\tilde{e} \mid f_l^-) - f_l^-]\text{prob}(f_l^-). \tag{B4}$$

Using Observation 3, it follows immediately that (B4) is positive. Noting that the price change at the time of the forecast release is of the same sign as the forecast (positive when the forecast is greater than the prior earnings expectation of zero and negative, otherwise), it also follows that the covariance between the price change and the subsequent forecast error is also positive. Q.E.D.

Proof of Observation 5. Noting that the strong analyst reports truthfully, the bias must come from the weak analyst. Assume for a moment that, conditional on observing a signal of y_h^z, $z \in \{+, -\}$, the weak analyst never announces a forecast of f_l^z. In such a case the expected bias conditional on observing signal y_i^z, $i \in \{h, l\}$, is given by the difference between the analyst's earnings expectation given observation of y_i^z and his reported forecast of f_i^z, $E(\tilde{e} \mid y_i^z, w) - f_i^z$. By (4) and the definition of f_i^z, the expected bias (where the expectation is taken over all i) is given by

$$(e_h^z - e_l^z)\{[p(e_h^z \mid y_l^z, w) - p(e_h^z \mid y_l^z, s)](2t) + [p(e_h^z \mid y_h^z, w) - p(e_h^z \mid y_h^z, s)](1 - 2t)\}, \quad \text{(B5)}$$

which is equal to zero for $z \in \{+, -\}$.

Taking into account, now, that with positive probability the weak analyst issues the forecast f_l^+ (f_l^-) after observing signal y_h^+ (y_h^-), there is expected to be a downward (upward) bias for a forecast that is greater (less) then the prior earnings expectation. Q.E.D.

Appendix C

I demonstrate here that allowing for a positive probability that the analyst is incorrect in his prediction of the sign of the period's earnings does not affect the nature of the equilibrium analyzed in the text. The notation corresponds to that used there.

Let the relation between the analyst's private information and the period's earnings be as follows:

$$\text{prob}(y_i^z \mid e_i^z) = k_1, \qquad i \in \{h, l\}; \qquad z \in \{+, -\}, \quad \text{(C1)}$$

$$\text{prob}(y_j^z \mid e_i^z) = k_2, \qquad i, j \in \{h, l\}; \quad i \neq j; \ z \in \{+, -\}, \quad \text{(C2)}$$

$$\text{prob}(y_i^{z'} \mid e_i^z) = k_3/2, \qquad i \in \{h, l\}; \quad z,z' \in \{+,-\}; \quad z \neq z' \quad \text{(C3)}$$

$$\text{prob}(y_j^{z'} \mid e_i^z) = k_3/2, \qquad i, j \in \{h, l\}; \quad z,z' \in \{+, -\}; \quad i \neq j; \ z \neq z', \quad \text{(C4)}$$

where $k_i = g_i$ (b_i) for a strong (weak) analyst. Then k_3 is the probability that the analyst's private information is different in sign from the period's earnings. The following sets of assumptions are made with respect to these probabilities:

(i) $k_1 > k_2 > k_3/2$;
(ii) $g_1 > b_1$, $g_3 < b_3$;
(iii) $g_1/b_1 > g_2/b_2 > g_3/b_3$.

Assumption (i) implies that the analyst's private information is most likely to equal the realized earnings and least likely to be of opposite sign to those earnings. Assumption (ii) reflects the notion that a stronger analyst is more likely to receive private information consistent with the earnings to be realized and is less likely to be mistaken about the sign of those earnings. Assumption (iii) can be shown to be equivalent to the conditions

$$\text{prob}(s \mid y_i^z, e_i^z) > \text{prob}(s \mid y_j^z, e_i^z)$$
$$> \text{prob}(s \mid y_i^{z'}, e_i^z) = \text{prob}(s \mid y_j^{z'}, e_i^z),$$

where $i \neq j$ and $z \neq z'$. These conditions imply that if clients could observe the analyst's private signal, they would have the most favorable assessment of the probability that he is strong if the signal and the realized earnings are of the same sign and are either both high or both low and the least favorable assessment if the signal is of opposite sign to the realized earnings.

From (C1)–(C4), an analyst of type m, $m \in \{s, w\}$, has a posterior probability for each of the four earnings levels, conditional on his private information, that is given by

$$p(e_l^z \mid y_l^z, m) = \frac{k_1 t}{k_1 t + k_2(.5 - t) + k_3/4}, \tag{C5}$$

$$p(e_h^z \mid y_l^z, m) = \frac{k_2(.5 - t)}{k_1 t + k_2(.5 - t) + k_3/4}, \tag{C6}$$

$$p(e_l^{z'} \mid y_l^z, m) = \frac{.5k_3(.5 - t)}{k_1 t + k_2(.5 - t) + k_3/4}, \tag{C7}$$

$$p(e_h^{z'} \mid y_l^z, m) = \frac{.5k_3 t}{k_1 t + k_2(.5 - t) + k_3/4}, \tag{C8}$$

$$p(e_l^z \mid y_h^z, m) = \frac{k_2 t}{k_1(.5 - t) + k_2 t + k_3/4}, \tag{C9}$$

$$p(e_h^z \mid y_h^z, m) = \frac{k_1(.5 - t)}{k_1(.5 - t) + k_2 t + k_3/4}, \tag{C10}$$

$$p(e_l^{z'} \mid y_h^z, m) = \frac{.5k_3 t}{k_1(.5 - t) + k_2 t + k_3/4}, \tag{C11}$$

$$p(e_h^{z'} \mid y_h^z, m) = \frac{.5k_3(.5 - t)}{k_1(.5 - t) + k_2 t + k_3/4}, \tag{C12}$$

where $z \neq z'$.

Given the clients' conjectures of the analyst's reporting strategy (from Section 2), their posterior assessment of the probability that he is strong, as a function of the announced forecast and realized earnings, is as follows:

$$p(s \mid f_l^z, e_l^z) = \frac{g_1}{g_1 + (\alpha b_2 + b_1)p}, \tag{C13}$$

$$p(s \mid f_l^z, e_h^z) = \frac{g_2}{g_2 + (\alpha b_1 + b_2)p}, \qquad \text{(C14)}$$

$$p(s \mid f_l^z, e_l^{z'}) = p(s \mid f_l^z, e_h^{z'}) = \frac{g_3}{g_3 + (1 + \alpha) b_3 p}, \qquad \text{(C15)}$$

$$p(s \mid f_h^z, e_l^z) = \frac{g_2}{g_2 + (1 - \alpha) b_2 p}, \qquad \text{(C16)}$$

$$p(s \mid f_h^z, e_h^z) = \frac{g_1}{g_1 + (1 - \alpha) b_1 p}, \qquad \text{(C17)}$$

$$p(s \mid f_h^z, e_l^{z'}) = p(s \mid f_h^z, e_h^{z'}) = \frac{g_3}{g_3 + (1 - \alpha) b_3 p}, \qquad \text{(C18)}$$

where $z \neq z'$.

From the preceding expressions, it is straightforward to show the following:

(i) $E[p(s \mid f_i^z, \tilde{e}) \mid y_i^z, m] > E[p(s \mid f_i^{z'}, \tilde{e}) \mid y_i^z, m]$ for $z \neq z'$,

(ii) $E[p(s \mid f_i^z, \tilde{e}) \mid y_i^z, m] > E[p(s \mid f_j^{z'}, \tilde{e}) \mid y_i^z, m]$ for $i \neq j$, $z \neq z'$,

(iii) $E[p(s \mid f_l^z, \tilde{e}) \mid y_h^z, w] > E[p(s \mid f_h^z, \tilde{e}) \mid y_h^z, w]$ at $\alpha = 0$ if and only if $p(e_l^z \mid y_h^z, w) > p(e_h^z \mid y_h^z, w)$,

(iv) $E[p(s \mid f_l^z, \tilde{e}) \mid y_l^z, m] > E[p(s \mid f_h^z, \tilde{e}) \mid y_l^z, m]$,

(v) $E[p(s \mid f_h^z, \tilde{e}) \mid y_h^z, s] > E[p(s \mid f_l^z, \tilde{e}) \mid y_h^z, s]$,

where $m \in \{s, w\}$.

Conditions (i) and (ii) imply that an analyst observing y_i^z does not report either $f_i^{z'}$ or $f_j^{z'}$, where $i \neq j$ and $z \neq z'$. Condition (iii) is similar to Proposition 2 in the text in that a weak analyst observing y_h^z reports f_l^z with positive probability if and only if it remains more likely that earnings of e_l^z rather than e_h^z will occur. From condition (iv), an analyst observing y_l^z reports forecast f_l^z. Finally, condition (v) ensures that a strong analyst always truthfully reports his private information. These conditions directly imply that the clients' conjectures are fulfilled by the analyst's actions. Introducing a positive probability that the analyst is incorrect in his prediction of the sign of the period's earnings does not affect the nature of the equilibrium in this economy.

Without giving a formal proof, it is straightforward to show that the empirical implications reflected in Observations 4 and 5 continue to hold in this setting as long as the probability that the analyst's private information is of opposite sign to realized earnings, k_3, is not too large. That this must be true follows directly given that (a) these results hold in the extreme case where k_3 is zero (this is the case analyzed in the text) and (b) both the covariance between the ex post forecast error and the price change at the time of forecast release and the expected bias in the analyst's forecast are continuous functions of k_3. If k_3 is large enough, however, it is possible to find some sets of parameter values for which these empirical implications continue to hold and others for which they do not.

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