Expectations and the Demand for Bonds: Comment

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John H. Wood concluded his recent article in this *Review* by stating

> . . . the expectations hypothesis [of the term structure of interest rates] is logically invalid . . . the Hicks and Lutz equations rely on sub-optimal decision rules and, consequently, are without behavioral significance. . . . The awkward Hicks and Lutz formulations have hindered inquiries into the effects of uncertainty on the term structure . . . our results have made it possible for the study of the structure of rates under such conditions to proceed on a sound theoretical basis. [pp. 529–30]

The purposes of this comment are to determine the historical accuracy of Wood's interpretation of Irving Fisher (pp. 273–74), J. R. Hicks (pp. 144–47), and F. A. Lutz (pp. 499–529) and to examine the economic validity of Wood's theory.

I. Fisher, Hicks, and Lutz

According to Wood, the traditional expectations hypothesis of the term structure implies a decision rule of the following form: A trader[1] should ". . . prefer n-period bonds, be indifferent between n- and one-period bonds, or prefer one-period bonds, when"

$$(1)\quad (1 + R_n) \gtreqless \cdot[(1 + R_1)(1 + {}_1r_1) \ldots (1 + {}_{n-1}r_1)]^{1/n} \quad \text{(p. 524.)}$$

where R_k is the current observed market yield on k-period bonds and ${}_jr_1$ is the yield currently expected by the trader to prevail on one-period bonds j periods hence.

At first glance, expression (1) might seem an accurate algebraic representation of the expectations hypothesis. A one-period forward rate applicable j periods hence is, by definition,

$$(1 + {}_{j-1}\rho_1) \equiv \frac{(1 + R_j)^j}{(1 + R_{j-1})^{j-1}}$$

so that a current market yield in terms of forward rates is

$$(1+R_n) \equiv [(1+R_1)(1+{}_1\rho_1) \ldots (1+{}_{n-1}\rho_1)]^{1/n}$$

The "pure" expectations hypothesis states that each observed market forward rate is equal to its corresponding expected future spot rate, (see David Meiselman, p. 10) ${}_j\rho_k = {}_jr_k$, so that the equality form of (1) is the market equilibrium equation.[2] Indeed, Wood indicates this fact and then destroys our belief in the logic of the expectations hypothesis by presenting an alternative decision rule which earns a higher expected return.

The trader using Wood's decision rule will ". . . prefer n-period bonds, be indifferent between n- and m-period bonds, or prefer m-period bonds, when"

$$(2)\quad (1+R_n) \gtreqless (1+R_m)^{m/n} = \frac{(1+{}_1r_{n-1})^{(n-1)/n}}{(1+{}_1r_{m-1})^{(m-1)/m}} \quad \text{(p. 525.)}$$

Decision rule (2) is based on a strategy of maximizing return one period at a time. In the first period, period 0, the investor should make pairwise comparisons of all bonds and select the bond whose expected holding period yield during the first period is highest. Wood proves that decision rule (2) is superior to (1). Rule (1) is sub-optimal because it prompts the trader, who is indifferent to risk, into basing current decisions partly on an-

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[1] We shall assume (with Wood) that this trader operates in a bond market free of transaction costs, maximizes return over a horizon as long or longer than the longest-term bond outstanding, and is indifferent to risk.

[2] This interpretation of (1) requires the r's to be rates expected by some hypothetical composite market trader.

ticipated future decisions. There is no doubt that (2) is a rule superior to (1). The question, however, is whether decision rule (1) can be ascribed to the expectations hypothesis.

An examination of Fisher, Hicks, and Lutz will show that no such decision rule was ever recommended by them. In fact, Fisher and Lutz presented the same example that Wood used to show the inferiority of rule (1). This example, to be discussed in detail in the next section, involves a trader who expects a rise in the rate on long-term bonds. As Wood shows, such a trader will prefer short-term bonds now (p. 527) and will wait until rates have risen to buy long-term bonds. Compare Fisher's analysis:

> Those who expect the [long-term] rate of interest to fall will prefer to invest in long-time securities at the present market rates, even when those rates are less than on securities of shorter time, *while those who expect the [long-term] rate of interest to rise will prefer short-time securities.* (italics added) [p. 274]

and Lutz':

> The second possibility is that the investor may expect the yield on the bond at some intermediate date to exceed the average of the short rates from that date onwards, i.e., he expects the market price of the bond to be relatively *low* at that date. He will then contemplate going into the short market now and into the long market later. [p. 514–15]

Fisher and Lutz were clearly aware that decision rule (1) leads to incorrect actions.

Hicks, the third author Wood associates with rule (1), should probably be left out of the discussion entirely. His theory of the term structure is intricately connected to risk which we have assumed away here.[3] Let it suffice to note that decision rule (1) cannot be found in his book.

Finally, we must mention that all three writers, Fisher, Hicks, and Lutz, were discussing the term structure as a market equilibrium phenomenon and not as a normative theory for the guidance of investors. It is quite easy to be misled by the equality form of (1), which is the market equilibrium condition, into accepting the inequalities, which have nothing whatever to do with the theory. A thorough reading of these early articles will demonstrate the unfairness of Wood's criticism, "Discussions of the expectations theory of the term structure of interest rates have tended to be rather mechanical, ignoring the microeconomic foundations of market equilibrium solutions" (p. 522). Nothing could be less true. All three authors owe the frequent references to their work to a *concentration* on the foundations of market equilibrium and a style of expression that is lucid and *non*-mechanical.

II. A Revised Bond Investor's Decision Rule

We now turn from Wood's interpretation of history to a discussion of his decision rule. This section intends to prove that neither Rule (1) *nor* Rule (2) is optimal.

The correct decision rule can be demonstrated with Wood's three-period example. He assumed that two risk-indifferent traders, G and H, held expectations depicted by[4]

$$\text{(H.1)} \quad (1+R_2)^2=(1+R_1)(1+{}_1r_1)$$

$$\text{(H.4)} \quad (1+R_3)^3>(1+R_1)(1+{}_1r_1)(1+{}_2r_1)$$

$$\text{(G.4)} \quad (1+R_3)^3<(1+R_1)(1+{}_1r_2)^2$$

The lower case r's denote the trader's expected future spot rates and upper case R's denote market spot rates at period 0. Trader G's inequality (G.4) was Wood's decision rule (2) and Trader H's inequality (H.4) was intended to be the decision rule (1) implied by the expectations hypothesis. Wood assumed, via (H.1), that both G and H were indifferent between one- and two-period bonds.[5] According to these decision rules, H prefers three-period over one-period bonds and G prefers the opposite. Wood showed

[3] Lutz was also concerned with risk and devoted much of his article to its discussion. Hicks, however, discussed practically nothing else.

[4] The H and G notations are Wood's.

[5] Both H and G were assumed to have made a decision to commit their funds for at least three periods.

that G's one-period gain would indeed be greater than H's. H is led into error because he "... expects at time 0 to prefer two-period over one-period securities at time 1. Because of his use of decision rule (2), he is influenced by this expected future preference in his portfolio decision at time 0" (p. 527).

Neither the rule Wood recommended (2) nor the rule he attributed to the expectations hypothesis (1) will lead to optimal actions by a risk-indifferent trader. The correct rule is (a) *make one-period spot loans now with available resources and* (b) *make forward loans now if the forward rate is greater than the corresponding expected future spot rate.* In the present example, the forward rates are

$$(3) \qquad (1 + {}_1\rho_1) = (1 + R_2)^2/(1 + R_1)$$

$$(4) \qquad (1 + {}_2\rho_1) = (1 + R_3)^3/(1 + R_2)^2$$

$$(5) \qquad (1 + {}_1\rho_2)^2 = (1 + R_3)^3/(1 + R_1)$$

where ${}_j\rho_k$ is the k-period forward rate to begin j periods hence.

To determine the optimum investment at time zero, the trader must compare forward rates to expected spot rates as follows: ${}_1\rho_1$ to ${}_1r_1$, ${}_1\rho_2$ to ${}_1r_2$, and ${}_2\rho_1$ to ${}_2r_1$.

Using equations (H.1) and (3), we obtain for the first comparison,

$$(6) \qquad 1 + {}_1r_1 = 1 + {}_1\rho_1$$

which indicates that now, in period 0, the trader is indifferent between making one-period forward loans to begin one period hence and waiting until period 1 to make one-period spot loans.

Using the inequality (G.4) and equation (5), we have

$$(1 + {}_1\rho_2)^2 = \frac{(1 + R_3)^3}{(1 + R_1)} < (1 + {}_1r_2)^2$$

or

$$(7) \qquad {}_1\rho_2 < {}_1r_2$$

Since the two-period forward rate to begin one period hence is less than the two-period expected spot rate, the trader should issue forward loans now. He should borrow forward. In a world of perfect capital markets and zero transaction costs, he can do this by selling three-period bonds short and buying one-period bonds with the proceeds. At the beginning of period 1, he will receive an expected capital gain of

$$(8) \qquad d_1\left[(1 + R_1) - \frac{(1 + R_3)^3}{(1 + {}_1r_2)^2}\right]$$

where d_1 is the dollar amount of three-period bonds sold short and one-period bonds purchased. By referring to inequality (G.4), one can verify that the expected gain, represented by (8), is indeed a positive quantity.

Using equations (H.1) and (4) and inequality (H.4), we obtain the third comparison,

$$(1 + {}_2\rho_1) = \frac{(1 + R_3)^3}{(1 + R_2)^2} > 1 + {}_2r_1$$

or

$$(9) \qquad {}_2\rho_1 > {}_2r_1$$

Inequality (9) implies that the trader should now make one-period forward loans for two periods hence. Again, in the perfect world of this example, the transaction can be accomplished by selling short two-period bonds and using the proceeds to buy three-period bonds. *After two periods*, this will bring an expected capital gain of

$$(10) \qquad d_2\left[\frac{(1 + R_3)^3}{(1 + {}_2r_1)} - (1 + R_2)^2\right]$$

where d_2 is the dollar amount of both the long and short transaction. By substituting for $1+R_2$ from (H.1) and using inequality (H.4), one can verify that (10) is also positive.

In summary, the present decision rule instructs a trader to make the following transactions at period zero:

(a) Buy one-period bonds with available resources.
(b) Sell short three-period bonds and use the proceeds to buy one-period bonds.
(c) Sell short two-period bonds and use the proceeds to buy three-period bonds.

Transaction (b) is kept open for one period, period 0 to 1, and transaction (c) is kept open for two periods. Only transaction (a) was recommended by Wood's decision rule (2) and none of the three transactions were recommended by (1). The positive expected capital gains of (8) and (10) that accrue to a trader using the revised decision rule prove that rules (1) and (2) are sub-optimal.

The quantities of bonds bought and sold in transactions (b) and (c) are unspecified. This is a very important fact that requires elaboration. If the trader is truly risk indifferent, and really wants to maximize *expected* return, he would attempt to make d_1, in expression (8) and d_2 in (10) as large as possible. Only by transacting an infinite quantity of bonds would he maximize expected return. The fact that we rarely observe investors attempting to trade infinite amounts brings out the unrealism of the preceding example. Markets are not perfect in the special sense used there and traders can neither grant nor issue unlimited quantities of loans. Even if they could, it is likely that none *would* because no trader operating with unlimited liability is completely indifferent to the risk of total ruin.

The example is important, however, in clarifying the central point of this comment: Expected future spot rates do have behavioral significance. By comparing them to current forward rates, the bond trader chooses an optimal investment strategy. With the introduction of uncertainty, their comparison to forward rates acquires an even more crucial role. Assuming no risk of default, the forward rate is perfectly certain whereas the corresponding future spot rate is a random variable.[6]

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[6] The implications of this environment have been worked out in my doctoral thesis.