

Interest Rates and Price Expectations During the Civil War

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Interest Rates and Price Expectations During the Civil War

Being a brief account of Northern money market conditions from 1861 through 1865, including representative empirical tables, together with yield curves and graphs, and containing speculations on the influence of gold.

WHEN South Carolinean rebels fired on Fort Sumter, April 12, 1861, the United States was on a pure gold standard. All interest and principal repayments on Federal and state government bonds were promised in gold specie. Imports and exports of gold bullion were freely permitted and there were no restrictions on private holding of gold. Many different paper currencies were circulating. Most of these were notes issued by commercial banks under specific authorization from the various state governments and all were convertible into gold specie upon demand.

At the end of 1860, the par value of outstanding Federal debt was about \$65 million. About \$40 million had been borrowed in 1857 and 1858 to offset2 a large decrease in imports and the accompanying decrease in customs duties,3 (which in those days constituted more than 90 percent of Federal government revenue). The balance of Federal debt in 1860 was composed of such miscellaneous obligations as Texas Indemnity Stock* and Navy bonds of 1812.

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was partly supported by The National Science Poundation.

1 U.S. Register's Office, Treasury Department, Statement of Public Debt of the United States, 1789-1885, (1886), p. 75.

2 Rafael A. Bayley, National Loans of the United States (2nd ed.; Washington:

U.S. Government Printing Office, 1882), p. 74.

U.S. Government Printing Office, 1882), p. 74.

3 The decrease in imports was apparently caused by high prices in Europe and Canada. An additional factor in the fall of customs revenues was a lowering of rates early in 1857. Davis Rich Dewey, Financial History of the United States (New York: Longmans-Green and Co., 1934), p. 262. 1858 was the only year between 1851, when the record starts, and 1876 when the U.S. had an international trade surplus. U.S. Bureau of the Census, Historical Statistics of the United States, Colonial Times to 1957 (Washington: U.S. Government Printing Office, 1960), p. 565.

4 Texas Indemnity shares were really 5 percent, 14-year bonds conceded to creditors of the State of Texas in return for Texas relinquishing extensive territorial claims. In debate on the subject, it was said (presumably by a Texan) that "... her southeast corner was at the mouth of the Rio Grande, the region of perpetual flowers, her northwest corner near the South Pass in the Rocky Mountains, a region

flowers, her northwest corner near the South Pass in the Rocky Mountains, a region

By the end of 1865, the Federal debt had risen to \$2,225 million, 3,460 percent in five years.⁵ In addition, \$431 million in legal tender treasury notes (greenbacks) and \$26 million in fractional currency (shinplasters) were outstanding.

The first bill for issuing greenbacks was submitted to the House on January 22, 1862 by Secretary Salmon P. Chase of the Treasury and the ensuing debate, both in and out of Congress, provides some entertaining and instructive reading. For example, the following excerpt from a speech by Senator Fessenden could still be applied to certain topics in the field of finance:

Nobody knows much upon the question of finance, not even those who are most familiar with it; for, sir, I declare today that, in the whole number of learned financial men that I have consulted, I never found any two of them who agree, and therefore, it is hardly worthwhile for us to plead any very remarkable degree of ignorance when nobody is competent to instruct us; and yet such is the fact. I can state to you, Mr. President, that on one day I was advised very strongly by a leading financial man, at all events to oppose this legal-tender clause; he exclaimed against it with all the bitterness in the world. On the very same day I received a note from a friend of his, telling me that we could not get along without it. I showed it to him, and he expressed his utter surprise. He went home, and next day telegraphed to me that he had changed his mind and now thought it was absolutely necessary; and his friend who wrote to me wrote again that he had changed his, and they were two of the most eminent financial men in the country.

of everlasting snows; that she had a front on the Gulf of Mexico of a thousand miles, a front on the Rio Grande of 2000 miles, and undisputed frontier on the Arkansas of 1000 miles, and within these boundaries an area of 350,000 square miles, or more than equal to seven states as large as Pennsylvania," Congressional Globe, 1st session, 31st Congress, p. 165. \$7,750,000 in stock was actually paid to Texas for a few war materials she had had as a republic and for these land claims. Not counting the armaments, this amounts to \$.14 per acre, the current size of Texas being 267,000 square miles.

5 This may be contrasted with the 602 percent rise in federal debt from July.

⁵ This may be contrasted with the 602 percent rise in federal debt from July, 1940 through July, 1945—from \$43 billion to \$259 billion.

⁶ Later, as Chief Justice of the Supreme Court, Salmon P. Chase wrote the majority opinion that ruled greenbacks unconstitutional. He had never liked them and accepted the duty of their issuance in 1862 only under the pressure of First Bull Run and McClellan's pending peninsula campaign. Cf. Bray Hammond, "The North's Empty Purse," American Historical Review, LXVII (Oct. 1961), 2. Also see Irwin Unger, The Greenback Era, A Social and Political History of American Finance, 1865-1879 (Princeton: Princeton University Press, 1964), pp. 174-75. A valuable source about the early Civil War years and the financing arrangements debated in the 37th Congress is Bray Hammond, Sovereignty and an Empty Purse, Banks and Politics in the Civil War (Princeton: Princeton University Press, 1970). Sidney Homer, A History of Interest Rates (New Brunswick: Rutgers University Press, 1963), pp. 306-12 contains a concise history of Federal securities from 1861 to 1865 and draws attention to many unusual bond covenants.

7 Bayley, National Loans . . . , pp. 80-81.

As another example, the most common criticism of Civil War financing by later economists was anticipated by Horace Greeley who thought that " . . . heavy taxing, light stealing, and hard fighting ... " along with " . . . a stirring appeal to the people for a patriotic loan of two or three hundred millions . . . "8 would enable the

government to avoid the issuance of paper currency.

Most of the comments and arguments on the legal-tender bills were more virulent than Greeley's and Fessenden's. The advocates saw greenbacks as the salvation of the Republic while opponents called paper money unconstitutional, unpatriotic, unAmerican, and immoral. Fortunately for this paper, the advocates won and more than \$1 1/2 billion worth of greenbacks were subsequently issued.9 Their existence, the existence of a purely fiduciary currency, simultaneously with a pure metallic money, makes the Civil War years a unique period for studying changes in the value of money and the effect of expectations on interest rates.

In the mid-nineteenth century, United States Government securities possessed much more than the negligible default risk they enjoy today. During the Civil War, every telegraphed account of skirmish or battle set bond prices in motion and major price movements closely followed the fortunes of Union armies. Figure 1 illustrates this with a time series of prices for U.S. 6's of 1861 and 1881, two widely-held 6 percent Federal coupon bonds.10 Among other things, the series suggests that Confederate victories at Second Bull Run and Fredericksburg must have induced as much depression in Boston and New York bondholders as in Generals Pope and Burn-

One may indulge in some intriguing speculations on the ability of bond prices to measure the importance of campaigns and major battles. The capture of New Orleans, for example, was very significant, as it forecast the success of a Union plan to split the South along the Mississippi. The bond price behavior accompanying Farragut's achievement shows that investors agreed on its importance. Accompanying New Orlean's capture was the beginning of McClellan's

9 However, less than \$1½ billion were outstanding at any given time as older

issues were frequently redeemed.

⁸ New York Tribune, January 22, 1862 and February 1, 1862, italics added.

¹⁰ Bond prices from 1862-65 were obtained from various issues of the Commercial Chronicle and Review; 1861 prices came from Banker's Magazine. Since prices were quoted including accrued interest, it was necessary to subtract the accrued amount from each quotation to eliminate discontinuities at coupon payment dates.

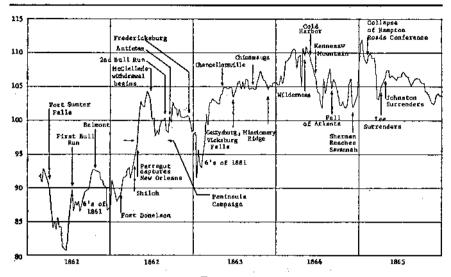


FIGURE 1. FEDERAL BOND PRICES, 1861-1865

peninsula campaign which was expected by everyone but the Rebels to bring a prompt end to the War. But in July, 1862, bond prices plummetted along with hopes for McClellan's success, as the Army of the Potomac reembarked for Washington without having brought Richmond to surrender. Up through the end of 1862, the bond market seemed to reflect Northern optimism. Before each major battle, prices rose, only to fall again after a Confederate success. This pattern was repeated at First Bull Run, Belmont, the Peninsula Campaign, Second Bull Run, Antietam, and Fredericksburg.

After 1863 began, however, bond prices expressed in greenbacks no longer reacted to major events with the same magnitude and intuitive direction. The Northern calamity at Chancellorsville was actually followed by a price increase¹¹ while victories at Gettysburg and Vicksburg caused scarcely a ripple. One may also wonder about the high prices during the 1864 battles between Grant and Lee in the Wilderness and at Spotsylvania, for these were among the War's most costly to Northern soldiery. Nearly 27,000 Union casualties occurred in the week of May 5-12 alone. The bouyant market during this bloody period may have been responding to the arrival at last of a commander-in-chief who was not afraid to commit the North's

¹¹ The Southern victory at Chancellorsville was partly offset by Jackson's wounding and subsequent death.

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vastly superior human resources to battle. ¹² But there were other reasons for high prices then. The premium on gold rose steadily in the first half of 1864, and in the next section, I shall argue that its financial influence was very apparent in the bond market. In fact, bond prices expressed in gold terms *did* react appropriately to major battles such as Gettysburg and Vicksburg; but gold's fluctuating greenback premium masked the movement of bond prices expressed in greenback terms. (The latter data are plotted in Figure 1.)

GOLD, GREENBACKS, AND BONDS

As greenback dollars became widely circulated in 1862, markets arose to facilitate their conversion to gold (specie).¹³ The prevailing conversion price throughout the period reflected their supply and the uncertainty about their future command over commodities; for although they were used in most commodity transactions, their purchasing power fluctuated drastically.

In fact, the gold standard in force throughout the rest of the world along with gold's relatively constant command over domestic commodities must have made it a more reliable standard of value. One of the strongest evidences for gold's stationary purchasing power is the closely parallel movement of the greenback price of gold and the greenback wholesale price index constructed by Mitchell.¹⁴ As Mitchell notes, greenbacks effectively provided flexible exchange that insulated Europe from the financial influences of the Civil War and major European countries enjoyed relatively stable prices during the 1860's.

Mitchell fills many pages testing his hypothesis that immense issues of paper money during the War caused commodity price inflation! No economist today would bother to do more than assert this relation as an obvious fact. But during the Civil War, and for many years thereafter, high prices were blamed by many on wicked gold speculators, who were accused of being agents of Jefferson Davis. Gold trading was actually prohibited by law on June 21, 1864, but the law was repealed on July 1, under pressure from New York businessmen.

¹² Grant became Commander-in-Chief on March 9.

Guaranteed greenback conversion to specie was suspended by the Government from January, 1862 to January, 1879.
 Wesley C. Mitchell, A History of the Greenbacks (Chicago: University of

Wesley C. Mitchell, A History of the Greenbacks (Chicago: University of Chicago Press, 1903) Chart III, p. 277.
 Mitchell, p. 224.

Table 1						
HIGHEST PRICES	OF	GOVERNMENT	BONDS	IN	1864	

Bonda	Approximate Term Until Maturity (Years)	Highest Price (In Greenbacks)
5's, 1865	1	225
6's, 1867	3	147
6's, 1868	4	145
5's, 1871	7	110
6's, 1881	17	118
6's, 1882	18	114
6's, 1884	20	1091/2
Gold Coin	_	285

^a The first number refers to the coupon, payable in specie, and the second number gives the maturity date.

Source: Joseph G. Martin, Martin's Boston Stock Market (Boston: published by the author, 1886), p. 129.

Greenbacks and government bonds shared the common frailty of Federal credit but bonds had the more complex variety of risk. It was deemed unsure whether the principal on many issues would be repaid in gold or in greenbacks! And of course, there was always the possibility that it would not be repaid at all. Just after the fall of Atlanta, on September 3, 1864, U.S. 6's of 1881, which were to mature in only 16 years, traded for 107 in greenbacks. But a gold dollar on the same date sold for 2.54 greenbacks: the bond's gold price was 42.1. One would assume this deep discount of 57.9 percent partly reflected a low probability of the bond being redeemed in specie even though interest had been guaranteed, and continued to be paid, in specie. 16

Throughout the War and afterward, gold prices of bonds followed the gold price of greenbacks.¹⁷ The correlation was not perfect, however. Investors may have been doubtful of government intentions to redeem bonds in gold but they did not regard it as an event of zero probability. To support this assertion, reference is made to the highest prices reached in 1864 by the bonds given in Table 1. The

¹⁶ Charles A. Jellison, Fessenden of Maine, Civil War Senator (Binghamton, New York: Syracuse University Press, 1962), p. 183.

¹⁷ Many authors have lamented the great profit gained by investors, especially English investors, who purchased government bonds when the gold premium was high, Cf. Joseph G. Martin, Martin's Boston Stock Market (Boston: published by the author, 1886), p. 123. When U.S. 6's of 1881 were trading for 42 in gold, their current gold yield was over 14 percent. In hindsight, this looks high because we know the bonds nearly doubled in price within a year. But the investor of 1864 could not have regarded 14 percent as better than a decent return after contemplating the condition of Federal finances.

principal on each was ultimately paid in gold, but in 1864, these issues had between one and 20 years left until maturity.

If investors had not anticipated any chance of gold redemption on the 5's of 1865, the yield-to-maturity would have been less than -50 percent in greenbacks, an impossible number unless greenbacks were expected to appreciate in value more than 100 percent. (And we shall argue next that such anticipations are wholly inconsistent with a reasonably competitive gold market.)

The present greenback price of a bond, p_{B,t}, is related to promised greenback inflows by

$$p_{B,t} = d \left[\frac{p_{g,t+1}^e}{1+R} + \dots + \frac{p_{g,t+n}^e}{(1+R)^n} \right] + \frac{100}{(1+R)^n} [q p_{g,t+n}^e + (1-q)] \quad (1)$$

where symbolic definitions are:

- d, the per-period coupon rate payable in gold;
- pet+1, the expected greenback price of gold in period t+j;
- q, the probability that the principle redemption of \$100 will be paid in gold;
- R, a discount rate appropriate for the risks of these promises;
- n, the bond's term-to-maturity.

One observes that the greenback price must not only reflect the usual factors of coupon and maturity but must also measure the appreciation of gold prices and the forecast actions of government at redemption. We now know that the sequence of gold prices must have had a special structure; for competition would have driven gold's current price toward prices expected in the future until no abnormal gains were available to gold traders. In the most purely competitive market, the gold price would have followed a submartingale sequence 18 such as

¹⁸ This is the standard condition for prices in an "efficient market," Cf. Eugene F. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, XXV (May 1970), 383-88; Richard Roll, The Behavior of Interest Rates (New York: Basic Books, Inc., 1970), ch. ii; and Paul A. Samuelson, "Proof that

$$p_{s,t+n}^e = (1+\alpha)^n p_{s,t}$$

where pg, t is the currently-observed gold price.

What can be said about the value of a, the anticipated rate of change of the greenback price of gold? (It is on the order of an interest rate.) In a world of risk aversion and efficient markets, a must have been negative if gold was less risky than greenbacks! With such opinions about relative risk, investors would have agreed upon an equilibrium gold price which brought a negative expected return in greenback terms. This assertion is proven by first noting that gold coins were as useful in transactions as greenbacks. In fact, they were exacted in the payment of customs duties, a violation of the legal tender status of greenbacks by the very government that required it. For some time, they were also demanded by state tax collectors and were always insisted upon by foreign businessmen. Therefore to an individual, greenbacks could not have been more desirable than gold for transactions purposes. Furthermore, the anticipated command of either greenbacks or gold over future consumption goods must not have exceeded their command over current goods by more than the rate of commodity time preference adjusted for the risk of each asset. Thus, the current prices of both gold and greenbacks expressed in commodity terms should have adjusted until no abnormal gains (in commodity terms) were expected from either one; and if gold was a considerably surer claim on future commodities, the risk premium added to its time preference rate would have been smaller.

Notationally, let $\pi_{g,t}$ express the units of a composite commodity that could have been purchased by a gold dollar in t and let $\pi_{m,t}$ be the units that could have been purchased by a greenback dollar. In an efficient market equilibrium, these prices would have satisfied

$$egin{aligned} \pi_{g,t+n}^e &= (1+lpha_g)^n \pi_{g,t} \ \pi_{m,t+n}^e &= (1+lpha_m)^n \pi_{m,t} \,. \end{aligned}$$

But if greenbacks were more risky in commodity terms than gold, risk averse investors would have accepted a lower return on gold; that is, $\alpha_m > \alpha_g$. Noting that $\pi_{g,t}/\pi_{m,t} = p_{g,t}$ was the greenback price of gold, dividing the two equations shows that $p_{g,t+n}^e = (1+\alpha)^n p_{g,t}$

Properly Anticipated Prices Fluctuate Randomly," Industrial Management Review, VI (Spring 1965), 41-49. The expectation is taken conditional on all current information including the past sequence of prices.

where $\alpha < 0$. (I have neglected the slight complication caused by the ratio of expected prices not having been equal to the expectation of the ratio unless the components were independent.) Note well that this was an ex ante relation. Over the period 1861-1865, there was a net increase in the price of gold expressed in greenbacks. This could have been caused by gold having been *more* risky than greenbacks or simply by erroneous anticipations. The latter are a likely cause of the discrepancy since investors at the time could not have foreseen the very large inflation of greenback prices during 1863 and 1864.

If the analysis above is reasonably accurate, a bond's current price expressed in gold dollars would have satisfied.

$$\begin{split} \frac{p_{B,t}}{p_{g,t}} < d \left[\frac{1}{1+R} + \ldots + \left(\frac{1}{1+R} \right)^n \right] \\ + \frac{100}{(1+R)^n} \qquad \left[q + \frac{1-q}{p_{g,t}} \right] \end{split} \tag{2}$$

and the very substantial price decreases with term-until-maturity displayed by government bonds in 1864 had to be caused by (a) an increase in the discount rate with maturity, (b) a decrease in the probability of gold redemption with maturity, ¹⁹ or (c) a combination of the two. The decrease in anticipated discount rate sufficient to induce the price pattern of Table 1 would have been far greater than what is commonly observed. Thus, we would seem entitled to a belief that investors of 1864 thought gold redemption less likely the further it was scheduled in the future.

The preceding argument's validity depends largely on how close the gold market of the 1860's approached the "efficient" ideal. Evidence on this can be obtained directly from gold prices themselves, for they should have fluctuated unpredictably if strong competition existed among gold traders. This hypothesis has been elaborated in much more detail elsewhere²⁰ but the basic reasoning behind it is as follows: if future gold price changes were predictably related to past price changes, speculators would have been able to use costless trading rules to earn abnormally high profits because serial dependence is easy and cheap to discover. Eager traders would have eliminated

Obviously, bond prices are positively related to the probability of gold redemption as long as there is a premium on gold.
Ose the references in fn.18.

such profits along with the serial dependence that provided them by competing to obtain and act quickly on all information, including such information as past price change patterns. Consequently, a weak measure of competitiveness is a lack of serial correlation in observed gold price changes.²¹

TABLE 2 SERIAL CORRELATION COEFFICIENTS IN COLD PRICE CHANGES, 1862-1865

Differ- encing Interval					Lag (V	Veeks)				
(Weeks)	1	2	3	4	5	6	7	8	9	10
1	.042	050	.061	.181	.035	070	035	.080	.074	11
2	078	.30	19	.16	12	092	.028	.044	.20	14
3	.097	.027	056	14	.083	.13	036	0098	.047	026
4	.26	032	0089	.23	047	029	.042	094	14	23
5	.36	12	044	025	.065	.15	13	38	23	27

Note: Sample Sizes are: Largest Integer Less Than (203/DIFFERENCING INTER-VAL)—LAG e.g., for a Differencing Interval of 3 weeks and a lag of five weeks. The sample size is

INT (203/3) - 5 = 67-5 = 62

Source: Gold prices were taken from various issues of the Commercial Chronicle and Review, except for a few missing observations that were found in Wesley C. Mitchell, A History of the Greenbacks, Appendix A, Table 2.

Table 2 gives serial correlation coefficients observed in gold price changes²² over several differencing intervals and lags during the period 1862-1865. Of the fifty coefficients in the Table, 28 are negative and 22 are positive. Most are close to zero and all five with absolute magnitudes in excess of 0.20 are in the last two rows where sample sizes are relatively small. This negligible degree of serial dependence suggests that competition in the 1860's gold market was keen enough to have precluded any sizable gains from trading on the past price sequence.

DEDUCING ANTICIPATED GOLD PRICE CHANGES FROM BOND PRICES

If two bonds made by the same issuer and promised for the same maturity are being actively traded in a competitive market, their

²¹ For the distinction between strong-form and weak-form tests of competition, see Fama, "Efficient Capital Markets," p. 388.

²² A price change is defined as $p_t \cdot p_{t-1}$, where p_t is the market price at date t and j is the differencing interval.

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prices should be nearly identical. If such bonds differ only by the monetary units in which they are denominated, their current prices expressed in either currency should differ so as to reflect the anticipated ratio of monetary exchange at each future payment date. For example, consider two hypothetical one-year bonds of the Federal Government during the 1860's, one payable in greenbacks and one payable in gold. Let the redemption payment be $100 + d_{\rm J}$ for each and the discount rate in greenback terms be R. The current (in year t) greenback prices of these must be

$$p_{A,t} = (100 + d_A)/(1 + R)$$
 for the greenback bond and

$$p_{B,t} = [(100 + d_B) p_{g,t+1}^e]/(1 + R)$$
 for the gold bond,

where $p_{\mathbf{s},t+\lambda}^{\mathbf{e}}$ is the anticipated greenback price of gold after one

year. As was previously argued, $p_{g,t+1}^e = (1+\alpha_t)p_{g,t}$ in an efficient gold market. Thus, an estimate of α_t , the anticipated price change of gold over the next year, can be obtained from the two bond prices, the gold price and the known redemption payments as follows:

$$\hat{\alpha}_t = \left(\frac{100+d_{\mathtt{A}}}{100+d_{\mathtt{B}}}\right) \cdot \frac{p_{\mathtt{B},t}}{p_{\mathtt{g},t}p_{\mathtt{A},t}} - 1\,.$$

Such an estimate is not functionally related to the past or future gold price sequence and it provides, therefore, an independent measure of anticipations that can be compared with realizations. It differs considerably from commonly used distributed lag measures of inflation and less common, but perhaps more interesting, measures based on survey questionnaires.²³ It is related to survey measures in the sense of being a market-aggregated, resource-weighted, average of bond traders' expectations about gold prices.

Unfortunately, the United States Government was too shortsighted to accommodate our research efforts by issuing bonds identical except for the currency units promised in repayment. It came closest in 1865 when the 5 percent 10-40's began trading. These were the only long-term Federal bonds with both interest and principal definitely payable in gold.²⁴ Outstanding concurrently, and being traded ac-

²³ S. J. Turnovsky, "Empirical Evidence on the Formation of Price Expectations," Journal of the American Statistical Association, LXV (Dec. 1970), 1441-54.
²⁴ Bayley, National Loans . . . , p. 164; Commercial Chronicle and Review, January, 1865, p. 62.

tively, were the "new" 6 percent one-year certificates of indebtedness for which both interest and principal were promised, and paid, in greenbacks. Of course, the problem in using these two bonds to infer an expected gold price change is that one was short-term and one was long-term. Even without differences in currency units promised, we would expect the greenback yields-to-maturity to differ for term structure reasons—for the anticipated short-term interest rate may not have been constant or risk preferences may not have been neutral with respect to maturity. The greenback yield-to-maturity on the long-term bond, R_L , and the short-term bond's yield, R_S , were thus related for any period t by $(1+R_{L,t})=(1+R_{S,t})(1+L_t)$ where L_t was a number depending on aggregated anticipated future yields and risk preferences.

Since the long-term bond paid its bearer coupons and principal in gold, its greenback price in t was related to its greenback yield by

$$\begin{split} p_{L,t} = p_{g,t} \Big\{ d_L \bigg[\frac{1+\alpha_t}{1+R_{L,t}} + \dots \\ & \qquad \qquad \Big(\frac{1+\alpha_t}{1+R_{L,t}} \Big)^n \Big] + 100 \left(\frac{1+\alpha_t}{1+R_{L,t}} \right)^n \Big\} \end{split}$$

 $p_{z,t}$ being the gold price, α_t being the anticipated gold price change, and n being the number of periods until redemption. The solution R. to

$$\frac{p_{L,t}}{p_{g,t}} = d_L \left[\frac{1}{1 + R_t^{\bullet}} + \ldots + \left(\frac{1}{1 + R_t^{\bullet}} \right)^a \right] + 100 \left(\frac{1}{1 + R_t^{\bullet}} \right)^a$$

is the gold yield-to-maturity and can be used to estimate at through

$$1 + \hat{\alpha}_t = \frac{1 + R_{L,t}}{1 + R_{\star}^{\star}}.$$

Unfortunately, the greenback yield R_L on the gold bond is unobservable and we can only obtain a rough estimate of α_t ; (by using the greenback yield on the one-year bond in its place). These yields are presented in Table 3 for weekly observations available during the year 1865 assuming that investors knew the actual redemption date of the 10-40's—the end of 1879. This last assumption is necessitated by another feature of the long-term bonds that has not yet been mentioned: They were callable. The designation "10-40's" refers to

TABLE 3
YIELDS-TO-MATURITY AND ESTIMATED GOLD PRICE CHANGES
FEDERAL OBLIGATIONS, 1865

-	· · · · · ·		Percent Pe	r Annum		
	Estimated Yield on 10-40's One-Year Gold Price		Yield			
Date (1865)	Greenback Yield	Change â	10-Year Maturity	Yield thru 1879	40-Year Maturity	on 6's of 1881
1 7	9.49	-4.57	16.85	14.38	11.72	14.23
1 14	8.13	-5.31	16.15	13.80	11.22	13.48
I 21	8.66	-3.79	14.79	12.69	10.31	12.50
1 28	8.13	-5.30	16.15	13.80	11.21	13.59
2 4	8.27	-4.90	15.80	13.50	10.96	13.38
2 11	7.32	-5.40	15.27	13.07	10.60	12.98
2 18	7,32	-5.40	15.27	13.07	10.59	12.89
2 25	7.72	-4.73	14.89	12.75	10.34	12.60
3 I	7.59	-4.90	14.96	12.80	10.37	12.71
3 8	8.00	-4.42	14.84	12.70	10.28	12.51
3 15	7.59	-3.42	12.85	11.20	9.21	11.15
3 22	9,22	-1.34	12.16	10.63	8.78	10.43
3 29	9.22	-1.01	11.72	10.28	8.52	10,07
4 5	8.13	-1.77	11.37	9.99	8.30	9,68
4 12	7.32	-2.19	10.89	9.61	8.02	9,36 9, 4 6
4 26	6.79	-2.53	10.68	9.44 8.29	7.90 7.10	8.11
5 10 5 17	6,52 7.04	-1.69 -1.19	9.23 9.23	8.29 8.29	7.10	8.04
5 17 5 24	6.92	-1.19 -1.53	9.23 9.52	8.51	7.09 7.24	8.22
5 24 5 31	6.79	-1.53 -2.03	10.03	8.91	7.52	0.22
6 14	6.92	-2.03 -2.15	10.36	9.16	7.69	9,04
6 28	6.13	-2.69	10.07	8.94	7,53	8,82
7 5	6.13	-2.72	10.20	8.97	7.50	8,86
7 12	6.13	-2.86	10.39	9.12	7.59	8,91
7 19	6.13	-3.07	10.69	9.34	7.74	9,00
7 26	8.13	-1.25	10.82	9.45	7.81	9,13
8 5	8.53	-0.89	10.84	9.46	7.81	9,16
8 12	8.39	-0.92	10.72	9.36	7.74	9,11
8 19	7.59	-1.78	10.85	9.45	7.81	9.33
8 26	7.86	-1.53	10.86	9.46	7.81	9.30
9 2	7.86	-2.00	11.19	9.68	7.94	9.29
$\tilde{9}$ $\bar{9}$	7.18	-2.34	11.03	9.64	7.97	9.28
9 16	7.32	-2.15	10.94	9.57	7.92	9.15
9 23	7.32	-2.16	10.97	9.58	7.93	9.20
9 30	7.32	-2.27	11.11	9.69	8.00	9.30
10 7	7.59	-2.22	11.41	9.92	8.16	9.48
10 14	7.86	-1.86	11.27	9.81	80,8	8.36
10 21	8.39	-1.57	11.57	10.04	8,24	9.60
10 28	8.13	-1.85	11.62	10.07	8.26	9.60
11 4	8.39	-1.74	11.81	10.22	8.36	9,75
11 18	8.39	-1.92	12.07	10.41	8.49	9.89
11 25	8.53	-1.88	12.19	10.50	8,55	9.91
12 1	9.08	-1.51	12.41	10.67	8.67	9.96
12 8	8.53	-1.79	12.08	10.41	8.48	9.67
12 15	8.39	-1.78	11.89	10.26	8.37	9.54
12 22	8.53	-1.68	11,93	10.29	8,39	9.68
12 29	7,72	-2.39	11.86	10.23	8,34	9.64

Source: See n. 10 and Table 2. Since bond prices were quoted including accrued interest, the accrued amount was subtracted from each quotation.

their being "redeemable after ten years and payable 40 years from [issue] date [approximately October, 1864]." They were actually redeemed in late 1879 and early 1880.

The reasonability of the above maturity choice is supported, however, by referring to Table 3 and comparing the 10-40 yields to yields on another bond, the 6's of 1881, which were not callable. Also given there are yields-to-maturity for the shortest possible redemption of the 10-40's, ten years, and the longest possible redemption, 40 years. The estimated value of \hat{a}_t , which is the anticipated gold price change, is generally negative irrespective of the maturity date or bond used to calculate the long-term yield. In fact, the estimated gold price change is negative in every period if one uses the 6's of 1881 or either of the two short maturity assumption yields of the 10-40's. Using the yields calculated with an assumption of 10-40 redemption after 40 years, only about 3/4 of the estimated α 's are negative. 28

The 10-40 bond, which was definitely to be redeemed in gold, was not outstanding before the beginning of 1865. However, its very close yield relation to the 6's of 1881 suggests the latter may be useful in estimating anticipated gold price changes for earlier dates. Table 4 presents the gold yield-to-maturity on the 6's of 1881 from May 2, 1863 through December, 1864, assuming that investors believed these bonds would be redeemed in gold (in 1881). It also presents greenback yields on the one-year certificate and estimated gold price changes based on the ratio of the gold yield to the greenback yield. The earliest entry is for May, 1863 because the "old" one-year certificates outstanding before that date were not purely greenback instruments.

Figure 2 plots the data from Tables 3 and 4 along with the level of gold prices. One may observe that both the gold yield (on long-term bonds) and the greenback yield (on one-year certificates) were closely related to the level of gold prices. However, the spread

²⁵ Bayley, National Loans . . . , p. 164.

²⁶ The 6's had a different defect—their principal was not definitely payable in gold.

²⁷ Only a's estimated with the 10-40 yield through 1879 are reported in Table 3.
²⁸ A negative anticipated gold price change persisted long after the end of the Civil War. Gold bonds yielded a higher (gold) return than paper bonds, until resumption of specie payment by the Federal government in 1879. After 1889, gold yields fell considerably below paper yields, an occurrence that Fisher attributes to the Free Silver movement. Irving Fisher, *The Theory of Interest* (New York: The Macmillan Co., 1930), pp. 401-03.

TABLE 4
YIELDS-TO-MATURITY AND ESTIMATED GOLD PRICE CHANGES,
FEDERAL OBLIGATIONS, 1863-1864

	Percent Per Annum		
Date (1863-64)	One Year Greenback Yield	Estimated Gold Price Change	Gold Yield on 6's of 1881
5 2	6.52	-3.01	9,68
5 9	6.79	-2.88	9.81
5 16	6.39	-2.95	9,49
5 23	6.52	-2.80	9.46
5 30	6.52	-2.56	9.21
6 6	8.94	-0.34	9.29
6 13	8.13	-1.22	9.41
6 20	7.45	-1.50	9.02
6 27	9.22	-0.27	9.50
7 11	7.32	-0.95	8.30
7 18	7.04	-0.55	7.62
7 25	8.00	0.30	7.69
8 1	6.92	-1.05	
8 8	6.79	-1.03 -0.91	8.01 7.73
8 15			
8 22	6.66	-1.02	7.72
	6.79	-0.73	7.54
8 29	6.26	-1.2 3	7,54
9 5	6.66	-1.59	8.31
9 12	6.52	-1.37	7.95
9 19	6.66	-1.63	8.36
9 26	6.66	-2.12	8.87
10 3	6.66	-2.32	9.08
10 10	6.55	-2.52	9.29
10 17	6.66	-2.79	9.58
10 24	6.66	-2.29	9.05
11 7	7.18	-2.04	9.31
11 14	7.86	-1.37	9.30
11 21	8.13	-1.38	9.56
11 27	8.13	-0.96	9.14
12 5	8.13	-1.5 5	9.76
12 12	8.13	-1.4 3	9,63
12 19	7.72	-1.87	9.68
12 26	8.13	-1.46	9.66
1 2	8.39	-1. 2 7	9,72
19	8.53	-1.16	9.74
1 16	8.27	-1.61	9,95
1 23	9.22	-0.64	9,89
1 30	8.39	-1.58	10.05
2 6	7.72	-2.33	10.17
2 13	7.45	-2.33	9.90
2 20	6.79	-2.97	9.90
2 27	6.79	-2.94	9.87
3 5	6.79	-3.03	9.97
3 12	6.66	-3,17	9,98
3 19	6.79	-2,98	9,92
3 26	6.79	-2.38 -3.48	10.44
4 2	6.79	-3,57	10,55
4 9	6.92	-3.43	10,53
	0.82	— 3,4 3	10,00

TABLE 4 (Continued)

	Percent Per Annum					
Date (1864)	One Year Greenback Yield	Estimated Gold Price Change	Gold Yield on 6's of 1881			
4 16	7.04	-3.53	10.76			
4 23	9.22	-2.06	11.40			
4 30	8.00	-2.92	11.08			
5 7	8.00	-2.69	10.83			
5 14	8.00	-2.47	10.60			
5 2 1	8.00	-3.22	11.40			
5 2 8	7.86	-3,56	11.63			
6 4	8.13	-3.70	12.06			
6 11	8.27	-4.09	12.61			
6 18	9.22	-3.15	12.56			
6 25	9.76	-3.69	13.70			
7 2	12,57	-2.71	15.49			
7 9	12.57	-4,46	17.41			
7 16	12.28	-3.59	16.17			
7 23	14.31	-2,12	16.61			
7 30	11.15	-4.56	16.07			
8 6	12.28	-4,25	16.90			
8 13	11.43	-4.67	16.49			
8 20	11.57	-4.4 3	16.36			
8 27	11.43	-4.27	16.05			
9 3	13,29	-2.94	16.47			
9 10	13,44	-1.55	15.10			
9 17	13,44	-0.95	14.46			
9 24	12.00	-1.83	13.96			
10 1	11.72	-1.14	12.93			
10 8	12.28	-1.16	13.52			
10 15	11,86	-2.33	14.36			
10 22	11.72	-2.18	14.05			
10 29	11.43	-2.16 -2.96	14.61			
II 5	10,87	-2.50 -4.79	16.04			
1 12	10,73	-4.79 -5.08	16.23			
11 12	9,49	-5.08 -4,35	16.23 14.15			
1 26	9.49					
12 3	8.66	-4.42 -6.41	14.22			
l2 12	8.39	-6.41 -7.07	15.58			
12 12 12 19	8.53	-7.07 0 54	16.02			
12 19 12 26		-6.54	15.57			
LA 40	9.63	-4.47	14.41			

Source: See n. 10 and Table 2. Since bond prices were quoted including accrued interest, the accrued amount was subtracted from each quotation.

between these yields, which is a rough measure of the anticipated gold price decrease, does not seem to be algebraically related to the recent changes in gold prices. In fact, $\hat{\alpha}_t$ is most negative when the gold price rises or falls rapidly.²⁹ In an efficient market context, this

Assuming the absolute value of the gold price change, $\mathbf{Z}_t = \big| p_{g,t} - p_{g,t-1} \big|$

makes sense because these are periods when holding greenbacks is a more risky investment practice than holding gold. Hence, it is precisely these risky periods for greenback holders that should display the greater concession in gold's expected return.

But if the expected gold price change was negative throughout 1863-1865, and even most negative during the greatest periods of inflation, why is the level of yields so strongly related to the level of gold prices? A potential answer may be that commodity prices were changing in gold terms as well as in greenback terms. Figure 3 plots Mitchell's quarterly commodity price index along with quarterly observations of gold prices in greenbacks and calculated gold prices of commodities. Although gold's greenback price rose with the great greenback inflation of January-July, 1864, the fall in gold's price in the last half of 1864 and in 1865 was greater than Mitchell's estimates of the greenback deflation. The result: a 40 percentage point rise in the gold prices of commodities between July, 1864, and October, 1865.

I would be remiss not to emphasize again that other reasonable hypotheses could explain why gold yields were higher than greenback yields even during periods of rapid depreciation of greenback purchasing power. (Note that one would expect gold yields to be higher during periods such as early 1865 when commodity prices were falling in greenback terms but rising in gold terms.) The ratio of short-term greenback yield to long-term gold yield does not identify the anticipated gold price change uniquely. It contains the long-short greenback yield differential and if this fluctuates with the level of greenback yields or with the uncertainty in commodity price inflation, the observed pattern of yields could have resulted.

An additional problem was suggested by Friedman and Schwartz who have argued the gold yield was higher than the paper yield because "...during the war and for some time thereafter, there was

is a measure of uncertainty, a regression between Z and the estimated gold price change, $\hat{\alpha}$, was calculated as follows:

change, $\hat{\mathbf{a}}_t$ was calculated as follows: $\hat{\mathbf{a}}_t = .88 - 1.0 \ Z_{t+1} + 1.3 \ Z_t + .68 \ \hat{\mathbf{a}}_{t-1}$ (.30) (.23) (1.1) (.058) Durbin-Watson d = 1.89, $\mathbf{R}^2 = .77$. Numbers in parentheses are standard errors. The negative, and significant coefficient of \mathbf{Z}_{t+1} , the absolute value of the gold price change observed just after the predicted change $\hat{\mathbf{a}}_t$, supports the contention in the text. The lagged values \mathbf{Z}_t and $\hat{\mathbf{a}}_{t-1}$ were added for econometric reasons—to reduce autocorrelation in the residuals. The coefficient of $\hat{\mathbf{a}}_{t-1}$ is an estimate of the first-order serial correlation coefficient for the residuals from a two-variable regression of $\hat{\mathbf{a}}_t$ on \mathbf{Z}_{t+1} . The coefficient of \mathbf{Z}_t should be approximately equal to minus the product of the coefficients of $\hat{\mathbf{a}}_{t-1}$ and \mathbf{Z}_{t+1} . In fact, it is less than one standard deviation from this number from this number.

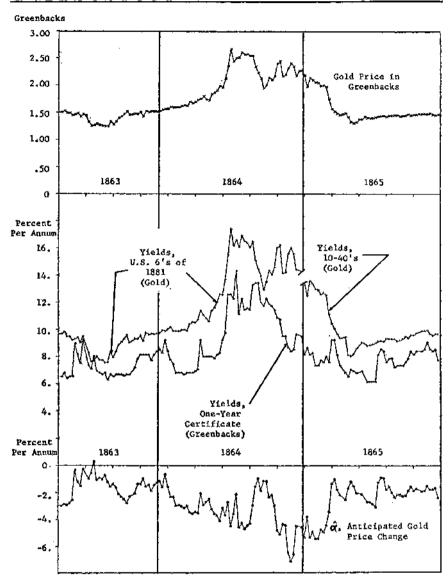


FIGURE 2.
GOLD PRICES, GOLD YIELDS, GREENBACK YIELDS AND ESTIMATED GOLD PRICE CHANGES, FEDERAL OBLIGATIONS, 1863-1865

little confidence that the government would in fact pay principal and interest in gold."80 This did not deter Friedman and Schwartz from believing that speculators of the period anticipated a fall in gold's

³⁰ Milton Friedman and Anna J. Schwartz, A Monetary History of the United States, 1867-1960 (New York: National Bureau of Economic Research, 1963), p. 73.

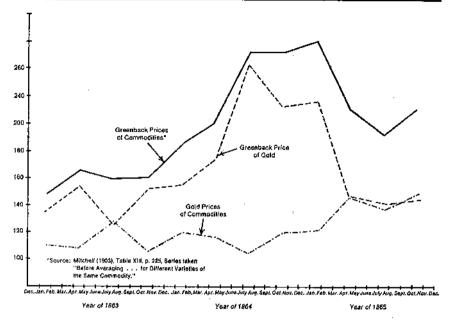


FIGURE 3.
COMMODITY PRICES IN GOLD AND GREENBACK TERMS, 1863-1865

greenback price³¹ or even from suggesting low paper yields on railroad bonds were due to "an inflow (to the U.S.) of speculative capital induced by an expectation of a subsequent fall in the price of gold."³²

³¹ *Ibid.*, pp. 70-73.

³² Ibid., p. 71. In a footnote on page 73, Friedman and Schwartz recognize their position is somewhat inconsistent because an expected fall in the greenback price of gold is sufficient alone to cause a positive spread between gold yields and greenback yields on bonds of similar risk. They remedy the inconsistency by offering the following "... reasons: (1) a fall could occur without taking the [gold] price [in greenbacks] back to prewar par, ..." However, to generate the spread, investors need not have expected a fall all the way back to par. Any anticipated decline would have been sufficient. "... (2) even if it took it back to par, that might occur after repayment of the bond, and even if it occurred before, the coupon payments in the interim might be in greenbacks. ... "Again, any decline in the interim would have brought real gains to holders of greenbacks. "... (3) the purchasers of government securities were a much more mixed and broader group than the speculators in foreign exchange were, so we are dealing with the expectations of two very different groups." However, even if the two groups were very different, their expectations could have been the same. Their incentives were certainly the same and besides, a limited number of speculators active in both markets would have been sufficient to erase the arbitrage opportunities implied by Friedman and Schwartz. The active competition demonstrably existing in both markets implies that a greenback-gold yield spread is not just a fluke of one market's participants paying no attention to the other.

However, most statements of the period refer to doubtful *principal* repayment in gold, leaving the implication that coupons were confidently expected on schedule. This is supported by the patterns of bond prices with respect to term-to-maturity shown in Table I; for if *all* future gold coupons were *equally* as doubtful as gold principal redemption, no such pattern would have emerged.

Nevertheless, it does seem reasonable to hypothesize a lower probability of gold payment for more distant future coupons or principal and, in addition, it seems reasonable to hypothesize an inverse relation between the probability of gold payment for any period and the current greenback price of gold. This can be regarded as a refinement of Friedman and Schwartz's basic hypothesis. The last part was added for the obvious reason of costs: a higher greenback price of gold would have increased the government's expense in maintaining gold coupon payments on its bonds.³³

If this hypothesis is valid, estimated anticipated gold price changes reported in Tables 3 and 4 will be biased downward from their true levels. Unfortunately, the extent of bias is difficult to determine since the actual subjective probability of investors concerning the likelihood of gold payments is unobservable. The only alternative is to construct a model of the hypothesis and to fit its parameters by available data. I have chosen to specify the probability, q, of gold payment on the next scheduled bond coupon by

$$q_t(\beta) = 2 - 2/\{1 + \exp[-\beta(p_{g,t} - 1.)]\}$$

where $p_{g,t}$ is the greenback price of gold at time t. This is a modified logistics curve that has the following desirable properties: (a) as the greenback price of gold approaches unity (greenback = gold), the probability of gold payment of the next coupon approaches one; (b) as the greenback price of gold grows indefinitely larger, that probability approaches zero. The coefficient β determines exactly how the probability changes with the level of gold prices.

For want of a better method, the probability of gold payment was

³⁸ One might think such a cost was illusory because if the greenback price of gold had increased, the government could have simply printed proportionately more greenbacks. Ultimately, of course, this could have resulted in hyper-inflation. It never did, and the price of gold was a real cost to the government, because Congress so effectively limited the total circulation of greenbacks.

³⁴ The bias will be downward only if greenback yields on the one-year bond are greater than greenback yields-to-maturity on the long-term bonds (assuming no gold payments at all). This relation is generally true for 1863 through 1865.

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assumed to decline geometrically with time until payment. Thus, for the second scheduled coupon, the probability of gold payment was q², and so on for the third and subsequent coupons, while gold principal redemption, scheduled after n semi-annual periods, received a probability of qⁿ. It seems unlikely to me that the true probability of gold payment would have fallen off quite so rapidly but the result is to bias upward subsequent calculations of estimated gold price change anticipations and thus to offset previous downward biased estimates.

Using the 6's of 1881 in all calculations, a search was conducted over the possible range of β (zero to indefinitely large), to find the value which minimized the mean-squared prediction error between \hat{a} and the actual change in gold's greenback price during the subsequent week. The minimum was found for $\beta=.059$. As an example of the meaning of this number, for a gold price of two dollars in greenbacks (as in June, 1864) it would imply a probability of .97

 35 The procedure follows: First, for an assumed value of $\beta,$ yields-to-maturity were calculated according to the discounting formula

$$\begin{split} P_{B,t} &= d \left\{ \frac{p_{g,t} \left(1 + \alpha_t \right) \, q_t + \left(1 - q_t \right)}{1 + R_{L,t}} \right. \\ &+ \ldots + \frac{p_{g,t} (1 + \alpha_t) \, q_t^n + 1 - q_t^n}{\left(1 + R_{L,t} \right)^n} \right\} \\ &+ \left[\frac{p_{g,t} (1 + \alpha_t) \, q_t + 1 - q_t^n}{\left(1 + R_{t,t} \right)^n} \right] 100. \end{split}$$

where, as before, $p_{B,t}$ is the current bond price in greenbacks, d is the semi-annual coupon and q_t^j is the probability of gold payment in period j (a function of β and of the current greenback price of gold, $p_{g,t}$). Defining the discounting formula by the function $f(x) = d\{x + \ldots + x^n\} + 100 \, x^n$ a solution R^* to

$$\begin{split} f\bigg(\frac{1}{1+R^{\bullet}}\bigg) &= \Bigg[p_{B,t} - f\bigg(\frac{1}{1+R_{L,t}}\bigg) + f\bigg(\frac{q}{1+R_{L,t}}\bigg)\Bigg] \Big/ p_{g,t} \\ \text{provides an estimate of } \alpha_t \text{ by} \\ &\frac{1}{1+R^{\bullet}} \equiv q_t\bigg(\frac{1+\hat{\alpha}_t}{1+R_{L,t}}\bigg). \end{split}$$

Of course, $R_{L,\ t}$ was measured as before by the one-year greenback yield. Using the set of $\hat{\alpha}$'s thus estimated, the mean-squared prediction error is calculated as

M.S.E.
$$(\beta) = \sum_{t=1}^{N} [\hat{a}_t - (p_{g,t+1} - p_{g,t})/p_{g,t}]^2/N$$

where N is the sample size (in weeks).

that the next coupon payment would be in gold and a probability of about .38 that redemption (in 1881) would be in gold.

Estimated gold price changes obtained with this method were uniformly larger than those reported in Tables 2 and 3, but they were still predominantly negative. The only periods of significant estimated price rises were July-October, 1864, June-August, 1863 and November, 1863-June, 1864. There were 41 weeks with positive estimated changes out of 140 possible, and one should remember these estimates are probably biased upward. Table 5 gives estimates for a

TABLE 5
ANTICIPATED CHANGES IN THE GREENBACK PRICE OF GOLD BEFORE AND AFTER ACCOUNTING FOR PROBABILITY OF CESSATION OF GOLD PAYMENTS ON FEDERAL GOVERNMENT SECURITIES 1863-1865

	Anticipated Gold Price Change (percent per annum)	
Date	Estimate Neglecting Possibility of Gold Payment Cessation ^a	Estimate with Probability of Cessation that Minimizes Squared Prediction Errors
6 20 '63	-1.50	2,35
9 26 '63	-2.12	-0.786
12 26 '63	-1.46	0.624
3 26 '64	-3.48	-2.06
6 25 '64	-3.69	0.207
9 24 '64	-1.83	2.01
12 26 '64	-4.47	-1.96
3 29 '65	-1.01	0.330
6 28 '65	-2.69	-2.39
9 30 '65	-2.27	-1.53
12 22 '65	-1.68	-0.441

^a From Tables 3 and 4.

Source: See n. 10 and Table 2. Since bond prices were quoted including accrued interest, the accrued amount was subtracted from each quotation.

few dates and shows how they have changed from the previous estimates.

Despite the series of caveats above, which are typical of those invariably attached to empirical interpretation, I believe the evidence strongly suggests gold price decreases were expected on average by investors from 1862 to 1865. Subsequent years did not alter these opinions and as long as gold's purchasing power over commodities was considered more stable and less risky than greenback purchasing

power, the return demanded by holders of gold was less that that demanded by holders of greenbacks.

As measured by the lack of serial dependence in gold price changes and the rapidity with which bond prices reflected new pieces of information such as military events, northern gold and bond markets of the 1860's were quite efficient. This is significant for at least two reasons: First, it suggests to economists that a much smaller capital market in a less technologically developed state than our own can behave as efficiently. Second, it confirms to historians the usefulness of capital asset price data in measuring opinions of individuals who are no longer alive to express them directly. In many cases the only record and in every case the most representative record of such opinions is the trace of price movements caused by their actions in the marketplace.

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