# A PURE FOREIGN EXCHANGE ASSET PRICING MODEL

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If consumption tastes differ among countries, a position in foreign-denominated nominally riskless bonds is risky in real terms. Risk averse and rational consumer-investors facing such a situation would generally seek a diversified portfolio of foreign bonds. They would demand risk premia in accordance with portfolio (covariance) risk. A model is specified to portray this behavior and it is tested with data from eight countries. The results indicate that the actual premia earned in foreign risky positions are positively related on average to portfolio risk measures; but the premia deviate significantly from those predicted by the model.

## 1. Introduction

The traditional theory of flexible exchange rates is based on trade and balance of payments adjustments, exchange rates being the prices that equilibrate current accounts. Recently, some monetary economists have emphasized that exchange rates are determined in asset markets [Doinbusch (1975), Frenkel and Rodriguez (1975)]. However, their portfolio models assume that the world consists of only two countries and, more importantly, that everything is known with certainty. A realistic theory of foreign exchange equilibrium should explicitly consider uncertainty and exchange risk within a multicountry framework. The purpose of this paper is to contribute to the growing literature by presenting an empirical examination of foreign exchange pricing.

The standard theory of asset pricing has recently been internationalized [see, e.g., Solnik (1973), Adler and Dumas (1975) and Grauer, Litzenberger and Stehle (1976)]. This approach implies an equilibrium relationship among interest rates, exchange rates and inflation assuming an efficient international capital market. Traders and investors are presumed to be rational, risk-averse and price takers. When investors take positions in currencies, their expected return is the interest rate for the currency and period considered plus the expected exchange rate variation. The risk borne is exchange risk. The theoretical

existence of exchange risk and its influence on equilibrium pricing has often been questioned (e.g. Grauer, Litzenberger and Stehle). Since this article will investigate the existence and importance of exchange risk, it might be useful to first stress why exchange risk is relevant.

If markets were perfect, or nearly so, and if the same consumption of goods were produced and consumed in the same proportions in all countries of the world; if anticipation were homogeneous and if transportation were costless and instantaneous; then the international asset pricing theory would be indeed a trivial extension of the standard domestic model. Under these circumstances, the fact that francs were used in one location and pounds, yen, or cruzeiros used in others would only constitute a multinational version of the 'veil of money.' Real interest rates would be equal everywhere as would the real price of risk, and capital asset pricing relations (written in commodity rates of return) would be identical for the residents of all countries. In such idealized circumstances, real exchange risk would be absent because exchange rates would equal the ratios of commodity prices at every instant.

In the face of this ideal world, we have the observed reality: contracts are written in nominal terms almost everywhere. Forward exchange markets exist to provide insurance against the real effects of exchange rate changes. Tastes and consumption patterns (of multiple commodities) differ by country and anticipations probably do too. Production is stochastic, resulting in different levels of output from the same levels of factor inputs. International shipments use real resources. Taxes are differential. These would seem to constitute sufficient reasons to study the international problem in the expectation of finding results which differ from the standard problem.

In a previous paper, Solnik (1973) developed a model of equilibrium asset returns under the assumption that each country consumes a different good. In this model, production can take place for foreign consumption and thus a form of international trade can occur; but cross-shipment of the same good is excluded and the final destination of each good is fixed. This is a severe assumption and it lacks realism in the sense that many goods are, in fact, imported and exported multilaterally.<sup>1</sup>

More recently, multiconsumption models have been developed where countries are defined as sets of individuals having similar tastes [e.g. Grauer, Litzenberger and Stehle (1976), Solnik (1976)]; and the derived pricing relations are of the same form as in the simpler model mentioned above. Even with

<sup>&</sup>lt;sup>1</sup>It might at first seem that a model of restricted international trade would have little need of international financial transactions. This is false for two reasons: first, even if the aggregate real production of a country were unaffected by the existence of foreign financial assets, because only locally-produced commodities were consumed, individual claims to those commodities might be greatly altered by the opportunity to borrow and lend abroad. Secondly, only by chance would aggregate production decisions actually be unaffected by foreign loan opportunities.

instantaneous and costless international trade, real exchange risk will be present as long as citizens of various nations<sup>2</sup> have different consumption preferences and *relative* commodity prices fluctuate.

We believe that these arguments imply the worthiness of empirical work based on the multinational asset pricing theory (and on the existence of exchange risk). So this paper reports on a particular empirical model derived from the theory.

## 2. A capital asset pricing model for foreign exchange

To develop an asset pricing relation for rates of exchange, we begin with one of the equilibrium conditions derived in Solnik (1973) as part of a broader theory of international asset pricing. This equation explains the difference between riskless nominal interest rates in two countries by '... the expected change of parities between these two countries plus a term depending on exchange risk covariances' (p. 37).

This means that, in general, the interest rate differential (or forward exchange discount or premium) will be a biased predictor of the subsequent change in the spot exchange rate. The bias between the expected exchange rate and the forward rate is caused by exchange risk and depends on covariances between the spot rate in question and the spot rates of all other countries.

Formally, the interest rate differential takes the form

$$r_n + r_i = \mu_{ni} + b_i \sum_{j=1}^{N} w_j (r_n - r_j - \mu_{nj}), \text{ for } i = 1, ..., N,$$
 (1)

where  $r_j$  is the riskless interest rate for country j, (expressed in units of currency of country j),  $b_i$  is a constant,  $w_j$  is a weight applicable to country j, and

$$\mu_{nj} = E(\widetilde{S}_{j,t+1} - S_{j,t})/S_{j,t}$$

<sup>2</sup>We realize, of course, that the use of geographic (and currency) regions to delineate differences in tastes is as much a convenience as a depiction of reality. Ideally, the world should be partitioned into regions of identical ordinal preferences for commodities, regardless of the residential site or currency employed by the individuals in each partition. However, since there are some minor measurement difficulties in achieving this ideal, we have used easily recognisable boundaries, despite their a priori imprecision vis-à-vis the best theoretical partition.

<sup>3</sup>In common with most asset pricing models,  $b_i$  should depend, to a first-order approximation, on the covariances of the exchange rate change for country i with the index of changes whose expectation is

$$\sum_{j=1}^{N} w_j \mu_{nj}.$$

\*Theoretically,  $w_j$  should be the net value of capital of country j which is foreign-owned as a proportion of the total value of world capital (expressed in a common currency, of course). See Solnik (1973, pp. 34-35).

is the expected rate of change in the exchange rate – currency units of the reference country (n) per unit of country j's currency – from the current period t until the next period t+1. N is the total number of distinct currencies and countries. Since the full derivation of (1) is published and available, there is no need for algebraic reproduction here. It should suffice to mention the required assumptions. In the last section, we mentioned the critical assumption: different consumption preferences across countries. The other assumptions are standard in the asset pricing literature: perfect capital markets and capital mobility, no differential taxes or transactions costs, continuous trading, homogeneous expectations, constant equilibrium. A fuller discussion of these requirements can be found in Solnik (1973, p. 16).

Interest rates in (1) can be replaced by spot and forward exchange rates using the interest rate parity 'theorem:'

$$r_j - r_n = (S_{j,t} - F_{j,t})/S_{j,t},$$

where  $F_{j,t}$  is the forward rate of exchange (units of country n per j unit) for one period hence as of period t.<sup>5</sup>

There has been some controversy over the validity of interest rate parity, which in principle is a pure arbitrage condition and thus should hold all the time. Aliber (1973) and Frenkel and Levich (1975) emphasize that much of the past supposed empirical failure of interest rate parity was due to noncomparability of the four variables under measurement. The interest rates must be for the same term as the forward exchange rate and they must be completely free of default risk in their own currency. We also have made a modest empirical study using Eurocurrency rates and have found that the relation is invariably within the bounds where trading costs preclude a profit opportunity.

Given the seeming validity of interest rate parity when its components are properly measured, the combination of (2) and (1) gives an asset pricing model expressed in spot and forward exchange rates alone:

$$E(\tilde{S}_{i,t+1} - F_{i,t})/S_{i,t} = b_i \sum_{j=1}^{N} w_j E(\tilde{S}_{j,t+1} - F_{j,t})/S_{j,t},$$
for  $i = 1, ..., N$ . (3)

<sup>5</sup>Strictly speaking, the interest rate parity expression should be written

$$(1+R_i)/(1+R_n) = F_{ji}/S_{ji}$$

or

$$(R_j - R_n)/(1 + R_n) = (F_{jt} - S_{jt})/S_{jt}.$$

However, since the basic model (1) is based on a continuous time assumption and since our data are for very short periods, we simply ignore the denominator on the left side of the exact expression for interest rate parity and use the approximate version, (2).

The notation of (3) can be simplified by defining the variable,

$$\tilde{R}_{j,t+1} = (\tilde{S}_{j,t+1} - F_{j,t}) / S_{j,t},$$
(4)

which is the 'extraordinary exchange return' for country j gained by a citizen of the reference country n. This return is dubbed 'extraordinary' because it is a rate earned above and beyond the hedged return knowable in advance from the forward exchange rate. Notice that the uncovered return from a purchase of j's currency by a resident of n is  $(\tilde{S}_{j,t+1} - S_{j,t})/S_{j,t}$ . The hedged return is  $(F_{j,t} - S_{j,t})/S_{j,t}$ . The difference is the risky nominal return from a currency position in j. Substituting the definition (4) into (3) and assuming that expectations can be replaced by observations plus an additive error (denoted  $\varepsilon$ ), we obtain the observable linear index model:

$$R_{i,t} = a_i + b_i \sum_{j=1}^{N} w_j R_{j,t} + \varepsilon_{i,t},$$
 (5)

with  $a_i = 0$ , i = 1, ..., N. This model is subjected to empirical scrutiny in the next section (through time-series and cross-sectional investigation). The economic theory developed above implies that  $a_i$  is equal to zero for all currencies. In other words, forward exchange rates should not deviate from expectations about future spot rates by a simple constant term. Any systematic bias should be explained by the portfolio risk of the foreign currency (relative to the reference currency) as measured by  $b_i$ . This exchange risk factor  $b_i$  will be significantly different from zero and vary among currencies to the extent that exchange rate risks vary. (For example, we might expect the risk measure to be relatively low for the Canadian dollar and high for the Deutsche mark for a U.S. investor.) The empirical observation of non-zero risk coefficients (b's) could be interpreted as implying the existence of exchange risk.

# 3. Empirical tests with eight countries

## 3.1. The data

The data source is *International Financial Statistics* for various years. A magnetic tape containing the data was available for monthly observation up through May 1973. Series were extended through January 1975 by adding observations published in later volumes. A filter program was used to detect errors and suspect observations were checked and corrected.

Spot exchange rates were available monthly from January 1956 and threemonth forward exchange rates were available monthly after January 1961. However, we do not report the results here for the entire period because a system of fixed parities was in effect until the first part of 1971. Because of this, we decided to begin the sample with July 1971. The maximum sample was thereby limited to 43 months (July 1971–January 1975) and the actual sample size was slightly smaller for several countries because of missing observations.

The eight countries for which both forward and spot exchange were available are listed hereafter with their international license plate designators that we use as an identifying symbol in the tables and figure.

	Country	Designator
.)	Great Britain	UK
)	Belgium	В
)	France	F
)	Germany	D
	The Netherlands	NL
•	Switzerland	CH
)	Canada	CDN
	United States	USA

The basic model contains two variables that cannot be directly observed:  $F_{t,j}$ , the 1-month forward exchange rate, and  $w_j$ , the weights. Theoretically, the weights should be proportional to the net value of foreign investment for country j expressed in some common currency. We tried several proxies for these weights (real GNP, national income, etc.) and we also tried an unweighted average ( $w_j = 1/7$  for all j). The results were virtually identical for all weighting schemes so we just report the unweighted average index here. The estimation of  $F_{t,j}$  is explained in the note.

Before looking at the results, one final clarifying word about the model is in order. Remember that the S's and F's in (3) are in units of U.S. dollars per

<sup>6</sup>Of course, even after many central banks had announced a policy of floating rates, they continued to 'stabilize' the market. We have no information on their actual transactions, however, which undoubtedly differed across countries. So, we simply assume that the period after July 1971 was characterized by relatively free exchange markets. We have made calculations concerning the entire period of available data and these were available to interested readers.

<sup>7</sup>The source document provides monthly observations but the forward exchange rate given is for a three month duration (denoted  $F_i^{(3)}$ ). In an attempt to avoid an induced error, we used the following procedure: the extraordinary exchange return in (5) can be written as

$$R_{i,t} = [S_{i,t} - S_{i,t-1} - (F_{i,t-1} - S_{i,t-1})]/S_{i,t-1}.$$

The second part of this term,  $(F_{t,t-1} - S_{t,t-1})/S_{t,t-1}$ , is the *one* month forward 'agio' at period t-1. The three-month forward agio is  $(F^{(3)}_{t,t-1} - S_{t,t-1})/S_{t,t-1}$ . It should be equal to three times the one-month agio, naively extrapolating. Therefore, we estimated the one-month agio by one-third the observed three-month agio, and actually measured the return in (4) by

$$R_{t,t} = [S_{t,t} - S_{t,t-1} - \frac{1}{3} (F^{(3)}_{t,t-1} - S_{t,t-1})] / S_{t,t-1}.$$

currency of another country. Thus, if Germany is the base country, and France is the country being considered in a particular regression, i = F, then the units of the ratio  $S_F$  and  $F_F$ , are Deutsche marks per franc Français. The units of  $S_j$ in (3) (j = UK, B, etc.) are Deutsche marks per English pound, per Belgian franc, etc. The 'return'  $R_{\rm F,r}$  in (5) is the 'extraordinary' percentage return that would have been obtained by a German investor between periods t-1 and tfrom purchase of French francs in t-1 and a reconversion to marks in t. We use the term 'extraordinary' to indicate that this investment could have been decomposed into two parts. The total uncovered return would have been  $r_{\rm u} = (S_{F,t} | S_{F,t-1}) - 1$  and the covered or hedged position would have returned  $r_c = (F_{F,t-1}/S_{F,t-1}) - 1$ . Thus, part of the total return to an uncovered investment in francs would have been certain since it could have been calculated from the forward market rates in advance. This decomposition is necessary since exchange risk should explain only the return above and beyond the risk free (or covered) return (i.e.  $r_u - r_c$ ). Again it should be stressed that the risk free exchange rate for the end of period t is the forward rate  $F_{F,t-1}$  and not the current spot rate  $S_{F,r-1}$ .

To clarify this further, suppose that this German 'investor' was an exporter who had sold goods in France at a franc-denominated price; and suppose his receipt of francs was delayed for one month. If he had 'hedged' the account receivable by a forward exchange transaction, he would have received the covered return  $(r_c)$  just mentioned (which could have been negative, of course). If he had done no trading in the exchange markets and thus had not hedged his account receivable, his total mark-denominated return would have been the uncovered return,  $r_u$ , above. The difference between these two,  $r_u-r_c$ , is the expost reward for having taken a risky (to the German) position in French currency.

The model was estimated with each of the eight available countries as the base or home country. For each one, seven 'foreign' countries were used in the index and seven separate regressions were fitted with the extraordinary exchange return for an individual foreign country as the dependent variable in each one. Thus, a total of 56 separate regressions were computed.

# 3.2. Time series estimates

Before presenting the regression results, let's first look at how the various currencies fared during the period considered. Table 1 presents the mean monthly extraordinary exchange returns  $(R_j)$  for the period July 1971 through January 1975 expressed in percent per annum. These are means of the regressions' dependent variables. A positive mean return implies that the particular base

<sup>8</sup>The number of observations is 43 for country combinations of UK, B. D, USA. Due to missing data, the number of observations is reduced by one for CH, two for NL and 7 for CDN. The missing observations are additive. Thus the combination CDN-NL has only 34.

Mean monthly 'extraordinary' returns on positions in foreign currency, July 1971-January 1975 (percent per annum).

				Fo	oreign country			
Home	UK	В	Ш	D	N.	CH	CDN	USA
UK	0.0	6.78	4.84	7.02	6.45	8.69	-5.39	-2.33
æ	-6.17	0.0	- 1.88	0.151	-0.733E-01	2.10	-12.7	-8.47
Į.	-4.02	2.21	0.0	2.38	9260	4.24	-11.2	-6.37
Ω	-6.10	-0.349E-02	-1.87	0.0	-0.534	1.92	9.82	8.41
z	-5.63	0.155	0.522	0.731	0.0	3.43	-10.1	-7.89
CH	-7.69	-1.76	-3.60	-1.50	-3.01	0.0	-12.5	-9.55
CDN	5.75	13.4	12.0	10.7	10.9	13.9	0.0	0.101
USA	2.76	9.58	7.59	9.81	9.17	11.0	-0.531E-01	0.0

currency (given in the left column) was weak relative to the other currency (given across the top row), where 'weak' means a decline in relative purchasing power on average over the sample period. For example, the French franc declined on average 2.38 percent per annum relative to the West German mark from July 1971 through January 1975, in addition to its hedged position decline. Similarly, negative numbers mean a relatively strong currency, (e.g. the Swiss franc had an 'extraordinary' gain of 1.50 percent per annum against the mark over this same period). With hindsight, German exporters should have hedged their accounts receivable in France but should have taken speculative positions in Switzerland, ceteris paribus, A glance at this table shows that the order of currencies from strongest to weakest during this period was: Switzerland, Germany, Belgium, Netherlands, France, Great Britain, United States, Canada. Note that the table is not symmetric only because the mean reciprocal of a random variable is not usually the reciprocal of the mean.

Table 2 gives the estimated slope coefficients and t-ratios for model (5). In most cases, the estimated slope seems to make intuitive sense. For example, it is well known that the Belgian central bank intervenes in the exchange market to keep the Belgian franc aligned with the mark and the guilder. Thus, a Belgian resident would not regard the mark or guilder as very risky insofar as their commands over goods within Belgium were concerned. These currencies should have had and do have low estimated slope coefficients in model (5) with Belgium as the home country. They are not as risky in Belgian franc terms as say the Canadian and U.S. dollars.

Looking at some other cases, the Canadian dollar and U.S. dollar are so closely related that a resident of either country would regard the other currency having virtually no purchasing power risk. British citizens would regard the two dollars as less risky than continental currencies and the North Americans would reciprocate with pounds. As for France, Germany, the Netherlands and Switzerland, they would consider each other and Belgium as relatively low risk compared to Britain and the North American countries and they would regard Britian as slightly less risky than the U.S. and Canada. Another coherent pair is Germany and the Netherlands which have low reciprocal risk.

## 3.3. Cross-sectional examination

A standard test of capital asset pricing type models is conducted by comparing the ex post observed return against the estimated slope coefficient with the benchmark index. This would mean running (for each base country) a cross

<sup>9</sup>The only surprise here is the position of Great Britain with a stronger currency than those of the U.S. and Canada. This is no doubt attributable to using arithmetic rather than geometric means in table 1. The recent extremely poor performance of the pound is thus understated as is its total decline over the sample period.

Estimated slope coefficients and t-ratios for linear index model of foreign exchange, July 1971-January 1975 (monthly observations).\*

Home country Slope estimates B F F	UK				roicigii counu y			
Slope estima UK B F		æ	<u>[</u>	D	NL	CII	CDN	USA
M m m c	les							
ಹಗಳ	0.0	1.06	1.11	1.30	1.21	1.18	609.0	0.358
. II. C	1.39	0.0	0.618	0.237	0.213	0.480	1.94	2.31
	1.13	0.778	0.0	0.865	0.858	0.849	1.22	1.34
1	1.42	0.531	0.867	0.0	0.449	0.595	1.74	1.79
z	1.47	0.424	0.839	0.319	0.0	0.643	16.1	1.88
CH	1.14	0.740	098.0	0.706	0.766	0.0	1.53	1.33
CDN	0.679	1.19	1.13	1.22	1.20	1.63	0.0	-0.153E-01
USA	0.551	1.20	1.14	1.28	1.20	1.21	0.121	0.0
t-ratios for estimates ab	estimates above	•						
1JK	0.0	14.0	9.17	13.8	14.3	8.72	3.55	2.34
: =	68.9	0.0	2.98	1.47	1.58	2.12	9.47	13.6
ı	8.65	9.12	0.0	7.25	8.02	5.79	7.29	8.24
Ω	10.9	6.28	5.58	0.0	4.35	3.62	9.35	12.1
z	9.35	4.78	4.69	2.38	0.0	3.33	9.17	10.5
CII	80'6	9.48	6.71	6.27	7.58	0.0	17.4	8.57
CDN	5.86	20.3	9.92	17.2	19,5	17.5	0.0	-0.270
VSII	604	31.1	12.7	20.3	20.9	12.0	2.51	0.0

\*Adjusted R2 for these regressions ranged from --0.027 (for CDN-USA) to 0.958 (USA-B); 42 of 56 were above 0.40, 32 above 0.60.

sectional regression between the mean observed return on each currency and its risk measure  $(b_i)$  estimated previously. The theory outlined in section 2 predicts that mean 'extraordinary' returns are proportional to the currency risk measures.

That is, in the cross-sectional relation,

$$R_i = a_0 + a_i b_i + \varepsilon_i, \tag{6}$$

between the mean returns (R) and the estimated risk coefficients (b) the intercept  $(a_0)$  should be insignificantly different from zero. We are well aware of the many econometric difficulties encountered in such tests<sup>10</sup> and we intend to do no more here than provide a plot of the relevant data and discuss them briefly without laying claim to any formal statistical test results.

For each base country, an 'exchange capital market line' is plotted in fig. 1. The data points are simply the ex post returns and estimated slopes from tables 1 and 2 and the plotted lines are fitted cross-sectional linear regressions

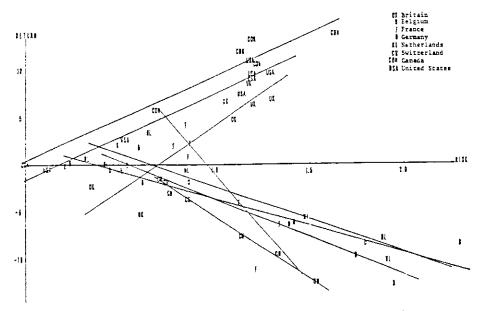


Fig. 1. Exchange capital market lines for eight countries (1971-75).

<sup>10</sup>Two well-known empirical papers that used data for equity assets are Black, Jensen and Scholes (1973) and Fama and MacBeth (1973). For reasons that are too lengthy to discuss here, we are skeptical that these papers actually contain valid tests of any scientific theory. Nevertheless, we have reported similar calculations for exchange rates on the grounds that these familiar calculations provide some descriptive information even if they do not constitute unambiguous empirical tests. A complete discussion of the issues involved in testing *any* asset pricing model can be found in Roll (1977).

to these points. The lines plotted correspond to the 'uncorrected' results presented in table 3.

The theoretically-anticipated slope of the cross-sectional regression of figure 1 and table 3 is the anticipated index return for the base country in question. An estimate of this is the ex post mean return of the index. Since the index units are in quantity of base country currency per 'foreign' country averaged over the foreign countries in the sample, the slope of the exchange capital market line should be inversely related to the strength of the home currency. Thus, we should observe the same ranking in the cross-sectional slopes as we had observed in the matrix of mean returns themselves.

Certain discrepancies from this prediction are evident in the results. France has too negative an estimated cross-sectional slope, for example. It changes sign; its t-ratio falls precipitously after a bias correction (explained below and reported as the 'corrected' results in table 3). All but one of the uncorrected slope estimates are further from zero than their predicted values (the average index). Five of eight are more than two standard errors away and this could imply a false model. But since we are mainly interested in presenting a description of the evidence without formal hypothesis testing, we must say that the model's predictions are at least qualitatively in the right direction – that is, the relation between ex post return and estimated time series slope is highly correlated and the relation seems to be linear. (Further evidence on linearity is given in the appendix.)

We have tried to correct for errors-in-variables bias in the cross-sectional regressions. This bias is the familiar result of the explanatory variable b being itself an estimate. For example, the t-ratios of table 2 for France as home country show that the standard error of b is on the order of 1/8th the size of the coefficient (which is on average equal to 1.0). The cross-sectional variance of b for France is 0.0485. So, if the estimation error of the time series slope (b) is independent cross-sectionally of the true b, the attenuation bias in the cross-section slope of fig. 1 for France is about 30 percent. This is certainly substantial enough to warrant a correction effort.

A simple correction method could be based on an assumption of independence between the true b and its estimation error, as above. Then, the computed cross-sectional variance of b would be composed of two parts such that

$$\operatorname{Var}(b) = \operatorname{Var}(\hat{b}) - \operatorname{Var}(\xi_b),$$

where  $\xi_b$  is the time-series estimation error in  $\hat{b}$ . Its variance can be measured for each individual coefficient by the computed standard error (squared). A bias-corrected estimate of the individual slope coefficient for country j would then be given by  $b^*$  in

$$\hat{b}^* = \hat{b}_{ji} \{ 1.0 + \widehat{\mathrm{Var}} (\xi_{\hat{b}j}) / [\widehat{\mathrm{Var}} (\hat{b}) - \widehat{\mathrm{Var}} (\xi_{\hat{b}j})] \},$$

Cross-sectional exchange capital market lines for eight countries, July 1971-January 1975 (monthly observations).

		Slope e	Slope estimates	Interce	Intercept estimates	γ	Adjusted R <sup>2</sup>
Country	Mean index return" (% annum)	Ordinary least squares (b)	Corrected (b)	Ordinary least squares	Corrected	Ordinary least squares	Corrected
CH	-5.53	-13.1	-14.8	7.58	7.04	0.993	0.905
D	-3.41	(29.3) - 7.53	(-1.62) -8.38)	(16.1) 4.42	(4.64) 4.61	0.919	0.910
æ	-3.73	(-8.30) -5.75	(7.85) 6.06	(4.10) 2.04	(3.98) 1.99	0.796	0.787
Z	-2.66	(4.94) 6.98	(4.81) $-7.77$	(1.35) 4.64	(1.29) 4.80	0.852	0.852
<u> </u>	-1 54	(-5.97) -22.9	(-5.95) 16.9	(3.19) 21.3	(3.25) 10.9	0.761	-0.0685
. =	3.89	(-4.48)	(0.748)	(4.07)	(-0.911) $-8.16$	0.771	0.792
VSII	7.31	(4.60) 9.10	(4.88) 9.33	(-3.15) $-1.60$	(-3.13) -1.52	0.946	0.938
CDN	9.52	(10.3)	(9.60) 9.33	(-1.73) 0.426	(1.55) 0.468	0.921	0.922
	!	(8.43)	(8.51)	(0.354)	(0.394)		

"Across seven countries and over time. For each home country, the index differs slightly for each foreign country because of small differences in sample size. See note 8.

\*restatisties for the coefficients are in parentheses.

where  $\widehat{\text{Var}}(b)$  indicates (the observed) cross-sectional variance and  $\widehat{\text{Var}}(\xi_{bj})$  is the computed slope's squared standard error for country j in the original time-series model [see Johnston (1963, pp. 148–162)].

These corrected estimates  $(b^*)$  of the time-series slopes were used in a second cross-sectional regression and the results are given in table 3. The results show no major changes except for France whose corrected capital market line (6) is virtually without statistical significance.

A second standard asset pricing model tests involves measurement of the intercept in the cross-sectional model, in order to ascertain whether it deviates from zero, the theoretical prediction. This prediction is not supported empirically by the cross-sectional evidence because most of the intercepts have rather large absolute *t*-ratios (see table 3). The bias correction seems to change the *t*-ratios only slightly, except for France, but the French results are too bizarre to constitute a reliable test.

An alternative test of the intercept and its deviation from zero can be obtained directly from the time-series estimates. The *t*-ratios for these estimated intercepts are given in table 4. Few of them are large in absolute value. The only systematic exception is Canada, which, when a dependent variable, displays rather large negative intercepts for four of seven base countries. These estimates may very well be statistically dependent across base countries so no reliance can be placed on a statistic calculated from the number of a given sign. Nevertheless, the general impression and best judgement would be an intercept for model (5) equal to zero with Canada marked as possible exceptions.

We conclude that the descriptive empirical evidence conforms to the qualitative implications of the basic model and that the relations among individual slope coefficients conform to a priori ideas about the connections among the several currencies. To obtain a reasonable level of confidence with these conclusions, further checks on the model specifications have been performed. The results are reported in the appendix and they are broadly in agreement with the above conclusions.

# 4. Summary and conclusions

An asset pricing model of foreign exchange has been presented and fitted to data. The model derives from Solnik's (1973) international asset pricing theory for riskless interest rates and from the interest rate parity arbitrage condition. The resulting equation relates the 'extraordinary exchange' return to an index of such returns. These 'returns' are the unhedged parts of the relative rates of change in exchange rates between pairs of countries.

<sup>&</sup>lt;sup>11</sup>Canada had about seven fewer observations than the other countries. Five of these were caused by missing forward exchange rates from August-December 1973. Thus, the atypical Canadian results are possibly attributable to this period being different.

Fratios of intercept estimates for linear index model of foreign exchange, July 1971. January 1975 (monthly observations).

Home	UK	=	<u>:</u>	a	ž	E	CDN	USA
C.F.	0.0	1.42	0.150	0.832	0,634	1.19	-2.10	-1.02
	-0.409	0.0	0.113	0.429	0.287	1.12	-0.647	0.721E -0
	0.983	1.61	0.0	1.25	1.05	1.49	-1.97	- 1.35
	-0.385	1.17	0.427	0.0	0.512	1.23	2.11	0.661
	-0.543	0.875	0.592	00.700	0.0	1.50	-2.33	- 0.829
=	-0.557	1.14	0.301	0.822	0.676	0.0	-1.33	0.691
Z	-0.238	1.38	0.412	0.513	-0.840	-0.137	0.0	0.168
VS	- 0.384	1.02	-0.134	0.423	0.285	0.976	0.894	0.0

The data were monthly spot and forward exchange rates for 6 European and two North American countries. An exchange capital market line was fitted for an investor from each country regarding the other seven countries as foreign and their currencies as risky relative to his own. The results confirmed the qualitative implications of the theory – the weaker a currency, the higher the slope of its exchange capital market line. However, if the observed mean return can be used as a proxy for the anticipated return, the observed slopes of capital market lines are larger in absolute value than the theory predicts.

These empirical results suggest that uncertainty and risk aversion can and should be taken into account in any realistic theory of international trade. Although the classic theories of deterministic equilibrium may provide accurate insights into the long-term behavior of international capital markets, short- and medium-term fluctuations in relative prices should cause risk to be a relevant factor in exchange rate equilibrium.

## Appendix: Further checks on model specification

Even though the model provides a reasonable description of the underlying relations in the data, and even though it is always tempting to terminate empirical work upon achieving a reasonable set of calculations, there are several econometric and conceptual difficulties that should be investigated to insure that no significant relationships have gone undiscovered and that no measured relationships were obtained spuriously.

The model's linearity can be checked by the Fama-MacBeth (1973) procedure of introducing nonlinear functions of  $\hat{b}$  in the cross-sectional regressions. Of course, there are very few remaining degrees of freedom when extra regressors are added (with only seven cross-sectional observations for each home country) and we do not want to be recorded as having made the assertion that nonlinearity is totally absent. Nevertheless, there is little evidence of its presence. The coefficients of  $\hat{b}^2$  terms for the eight home countries had the following t-ratios: UK = 0.658, B = 0.547, F = 0.570, D = 0.779, NL = -1.49, CH = 0.425, CDN = -0.756, USA = 0.939. Using the bias-corrected estimates, (squared  $\hat{b}^{*2}$ ), gave essentially the same results. Only for the Netherlands is there anything approaching an indication of nonlinearity, and even in this case the adjusted  $R^2$  rises only slightly (from 0.852 without the  $\hat{b}^{*2}$  term to 0.881 with it).

A second standard check of the model's specification involves possible serial dependence in the disturbances. For reasons of space, we will not present the full matrix of 56 Durbin-Watson statistics. The minimum observed Durbin-Watson was 1.23 and the maximum was 2.07. Seven of 56 were below 1.4 and only 12 were above 1.9. This may indicate a small degree of positive first-order serial dependence in the residuals, a rather common finding with speculative return index models. We should remember again, however, that no presumption

can be made about mutual independence across the 56 regressions and thus no inference can be drawn from these category counts.

A third check, which is important if the *t*-ratios are to be used for hypothesis testing, is the normality of residuals. Studentized ranges were computed for the residuals of all 56 regressions. For these sample sizes, the 95th percentile of the studentized range under a Gaussian null hypothesis is about 5.40. Seventeen of the 56 observed values were larger than 5.40; which indicates a departure from normality. Normal probability plots confirmed that this was due to thicktails in the distribution, another common result in data analysis of speculative prices. The 99th percentile of the null distribution for the studentized range is about 6.3 and only three computed values exceeded that level. This implies that the residuals are not very non-normal, roughly speaking, and that *t*-values of the size observed are unquestionable indications of significance.

Fourth, we were somewhat bothered by the incorporation of the dependent variable as one-seventh of the explanatory variable in each time series regression. Recall that the index is an equally-weighted average of all foreign currency extraordinary returns including the particular foreign currency which is the dependent variable. To the extent that measurement errors exist in the data, the time-series slope coefficients would be biased positively and the explanatory power of the regressions would be overstated. So we recomputed everything while constructing a separate index in each regression, an equally-weighted index of the six returns that were not dependent variables. Not surprisingly, the results were a reduction in the absolute size of slope coefficients and a slight reduction in adjusted  $R^2$ . But 30 of 56 adjusted  $R^2$ 's were still above 0.40 (as opposed to 42 of 56 with the seven-element index), and the average reduction in  $\hat{b}$  was only about 0.10. The t-ratios for  $\hat{b}$  also declined, of course, but 45 of 56 remained larger than 3.0 and most were very large. 12 We infer that some bias was introduced by measurement error but that all qualitative conclusions ought to remain unchanged. The total effect on the cross-sectional capital market lines is summarized in table A.1, which presents results corresponding to table 3 but with the dependent variable omitted from the index in computing the time series estimates. For space saving, only the corrected estimates are given. Uncorrected estimates are quite close to these, except for France, which follows the pattern of table 3. France represents the only significant difference caused by constructing an index without the dependent variable. The cross-section slope for France drops from 21.3 to 14.2 and the  $R^2$  increases from virtually zero to 0.537. Nevertheless, France's capital market line seems to be still quite far from its theoretically predicted position.

Fifth and finally, we reiterate our skepticism in interpreting the results found here as formal tests of hypotheses. We know that there are internal constraints

<sup>12</sup> The full table of results for the 6-element index is available on request.

Table A.1

Cross-sectional exchange capital market line slopes for eight countries, July 1971-January 1975 (with dependent variable deleted from index).

	Adjusted R <sup>2</sup>
-12.4	0.751
<del>-</del> 8.39	6.839
-5.63	0.766
-8.44	0.877
14.2	0.537
12.5	0.757
3.87	0.918
8.77 (7.34)	0.898
	(-4.37) -8.39 (-7.15) -5.63 (4.54) -8.44 (-6.62) 14.2 (2.82) 12.5 (4.44) 8.87 (8.27) 8.77

<sup>\*</sup>t-ratios are in parentheses.

on the calculations that make the sampling variations difficult to interpret. For example, it has recently been emphasized by Ross (1976) and Roll (1977) that the cross-sectional return risk relations would be exact, by construction, if the index were a mean/variance efficient portfolio. The fact that the data points in fig. I do not fall exactly along a line can be interpreted for each home country as implying that the index was not exactly ex post mean/variance efficient. Nevertheless, even if the index is not exactly efficient, it still can serve as a benchmark against which the risks of each country's currency can be compared. But some other benchmark, it must be admitted, could have yielded completely different results: and there would have been no way to tell which benchmark, and which set of descriptive information, gave results more representative of the riskiness perceived by investors.

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<sup>\*</sup>Using corrected b\*.

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