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A REPLY TO MAYERS AND RICE (1979)

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Mayers and Rice do not resolve the basic problem in portfolio performance evaluation with the securities market line, the ambiguity introduced by being obliged to choose a market index. Other performance evaluation techniques exist and possess some superior qualities. The Mayers–Rice discussion of my critique of the capital asset pricing model (CAPM) fails to recognize the CAPM's unusual testing implications and ignores the existence of alternative asset pricing theories. Residual analysis should give approximately correct estimates of the abnormal returns caused by specific events if it is conducted with the market model.

1. The securities market line as a device for measuring portfolio performance

Does Mayers' and Rice's model of the securities market line (SML)[1] render it a useful device, competitive with others, for the practical business of portfolio performance evaluation? They consider an atomistic investor who is 'informed' relative to everyone else and they show that his portfolio will on average plot above a particular SML, the one constructed by 'uninformed' investors, *all of whom agree on the index's identity*. Since the uninformed investors consider themselves in a state of equilibrium with their own assessments, and since, by assumption, uninformed investors construct their SML by using what they believe to be an ex ante mean/variance efficient portfolio, every individual asset and every portfolio will appear (to them) to fall precisely on the line. This implies that the informed investor must see deviations from the line because he holds different assessments of at least some parameters. Since he observes these deviations, he can easily construct a portfolio which lies above this SML by simply taking long positions in the

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[1]Mayers and Rice (1979, p. 3) criticise my work on the securities market line (SML) as follows: 'Roll's conclusions are due to his focusing on a 'truly' ex ante efficient index.' Yet, even a casual reading of my (1978) paper reveals no such focus. To the contrary, the paper is an extended discussion of ex-ante *non*-efficient indices. The consequences of inefficient indices are present on 17 of the paper's 19 pages and the words 'non-efficient', 'inefficient' or 'not efficient' appear explicitly on 8 different pages. Mayers and Rice (p. 7) contend also: 'Roll assumes away the possibility of superior performance.' Superior performance cannot be impossible and was never thus regarded in my analysis, which merely questioned whether it can be *detected* by using the SML. It can be detected in the ex post mean/variance space, for instance, and probably by many other devices.

assets which (he knows) lie above and/or short positions in assets which lie below.

In order to preserve 'rational' expectations by both informed and uninformed investors, Mayers and Rice require the informed to have no effect on prices. Furthermore, though the uninformed observe deviations from the SML for single periods, these deviations must cancel out over repeated intervals. This implies that (a) individual assets will plot on the uninformed SML over a long period, and (b) the informed portfolio must have a changing composition over time. The informed investor has superior advance knowledge about observed returns in each period, but on average he holds the same expectations as everyone else.

Mayers and Rice prove that an informed investor will have higher expected utility than an otherwise identical uninformed investor and that he will plot above the SML. This can be demonstrated more easily in the context of Ross' (1976) arbitrage pricing model with a single generating factor. The uninformed (subscript U) will perceive no arbitrage opportunities, so they will think expected returns conform every period to

$$E_U(R_j)=R_F+\beta_j E(R_m-R_F), \qquad j=1,\ldots,N, \tag{1}$$

which is equivalent to the capital asset pricing model because there is just a single generating factor. The informed investor (subscript I) will perceive something a bit different. For period t, he will expect

$$E_I(R_{jt})=\alpha_{jt}+R_F+\beta_j E(R_m-R_F), \tag{2}$$

where the riskless rate R_F, the expected market index return $E(R_m)$, and the individual asset's beta, β_j, are each assumed the same for informed and uninformed. But if $\alpha_{jt}\neq 0$ for some j, Ross showed that a purely riskless position with zero investment can be constructed so as to produce positive cash flow to the arbitrageur (the informed investor in this case). This position is characterized by positive investment in assets with $\alpha_{jt}>0$ and short positions otherwise. If x_{jt} is his investment proportion in asset j on date t, then the arbitrage portfolio satisfies $\sum_j x_{jt}=0$, $\sum_j x_{jt}\beta_j=0$ and $\sum_j x_{jt}\alpha_{jt}$ $\equiv\bar{\alpha}_t>0$. The informed investor's total portfolio can be decomposed into two parts, the zero-beta arbitrage portion which requires no investment and a positive investment portfolio which has some desired level of systematic risk, say β_p. Clearly,

$$E_I(R_{pt})=E_U(R_{pt})+\bar{\alpha}_t>R_F+\beta_p E(R_m-R_F).$$

Since $\bar{\alpha}_t$ is positive in every period, the informed investor expects to plot above the securities market line during any sample an outside evaluator may happen to observe.

The informed investor will not only plot above the uninformed SML, he will plot outside of the sample mean/variance efficient frontier! A non-constant composition portfolio *can* plot outside the efficient frontier. For example, suppose that actual returns could be obtained in advance each period from a clairvoyant. Then if short-selling is permitted, a portfolio could be formed each period with any desired ex post level of return. Conceivably, the return on such a portfolio could be chosen to be the same in every period, thus having zero variance. Yet it would be much larger than the market's riskless return. In contrast, portfolios with constant composition cannot lie outside the efficient boundary.

Since the Mayers/Rice informed investor plots outside the efficient frontier, he will be conspicuous to any performance evaluator using the mean/variance diagram (the 'reward-to-variability' ratio). Similarly, the Cornell (1979) performance measurement technique will estimate $\bar{\alpha}_t$ directly and thus will designate the informed investor as superior. Thus, even accepting the Mayers/Rice framework, there seems to be no advantage of the SML over these two alternative evaluation tools.

The Mayers/Rice framework does not permit what was once considered to be the principal advantage of the SML – its alleged ability to detect abnormally good or bad *individual* assets. In the Mayers/Rice model, neither the informed nor the uninformed can perceive individual assets as under- or over-priced on average; i.e., $\lim_{t\to\infty}\sum_t \alpha_{jt}=0$ for the informed while $\alpha_{jt}\equiv 0$ for the uninformed. (Recall that these conditions are required in order to have rational expectations on the part of both types of investors.)

Finally, the Mayers/Rice analysis is tangential to the major problem in applying the SML as a practical performance evaluation tool; evaluators do not know the identity of the unique index used by the 'uninformed' to construct a securities market line. Although commonly-used indices of equities may produce similar rankings (of equities), broader indices that include bonds, real estate and other types of assets may produce contradictory rankings. At the present, no one yet seems to have done the empirical work required to determine whether this will occur. We only know that much contradictions are possible with the SML while they are not possible with alternative evaluating devices. Thus, why bother with the SML? Since the mean/variance diagram and the Cornell procedure do not require an index to be chosen, they finesse this potentially critical problem.[2]

2. Tests of the capital asset pricing model (CAPM)

Mayers and Rice consider my (1977) conclusions about asset pricing

[2]Of course, alternative evaluating tools are still subject to econometric difficulties, as is the SML. For a discussion of these problems in connection with the mean/variance diagram, see Roll (1979) and Jobson and Korkie (1979).

theory tests as 'severe' (pp. 4, 21) and 'vastly overstated' (p. 3). In conclusion, they (pp. 22–23):

> . . . disagree with Roll in his almost total condemnation of all empirical studies to date, implying that they provide virtually *no* information at all. We believe there is some information in these tests, even with imperfect proxies testing joint hypotheses. More importantly, this is the *best* information available. It does no good to ignore this information without providing some better information in its place.

They did not disagree with the basic technical aspects of my criticisms of the previous empirical work.[3] In fact, they '... sympathize with all 3 problems [I raised] and believe they are of some importance' (p. 21). Instead, they allege falsely that I condemn 'all empirical studies to date'. They do not disclose exactly what information was provided by these studies but they claim it is the '*best*... available'.

Mayers and Rice come to this point after remarking that all econometric studies, including studies of asset pricing, must use proxies for measurement and must actually test joint hypotheses that the theory is correct *and* that the proxies are appropriate. This point was mentioned in my (1977) paper: 'The Black, Jensen and Scholes paper tested a joint hypothesis: The Sharpe–Lintner theory *and* the hypothesis that the portfolio they used as a 'market' proxy was the true market portfolio' (p. 144). But, I also pointed out that '... with joint hypotheses'... 'one never knows what to conclude' (p. 145). If the data reject the joint hypothesis, should *all* of its components be rejected? If not, which one?

The proxy problem in testing asset pricing theory involves choosing a portfolio (or index) to measure the true value-weighted market portfolio of all assets. What seems (at least to me) to make the problem more troublesome here than for most economic theories is the *unique* prediction of Black's (1972) generalized CAPM: that the true market portfolio is ex ante mean/variance efficient. The theory makes no prediction concerning any other quantity. The mean returns, variances, 'betas' and all other attributes of individual (and therefore measurable) assets are left unspecified. The return/beta linearity relationship, which was once considered a second and testable implication of the theory, follows mathematically from the market portfolio's ex ante efficiency.

We do know the attributes of a good market portfolio proxy for testing the theory – it should be a value-weighted index which includes as many assets as possible. But this was not characteristic of indices used in published tests. Equally-weighted indexes were used in some tests and *no* test used an

[3]The criticized papers were Black, Jensen and Scholes (1972), Blume and Friend (1973), and Fama and MacBeth (1973).

index which contained any bond, real estate, mortgage, or any other non-equity form of capital.[4]

Mayers and Rice argue that the proxy problem is universally present. As an example, they cite a model in which the consumer price index (CPI) plays a role in explaining income (Y) and they contend that measuring the CPI is fraught with a difficulty similar to that in developing a proxy for the market portfolio. Their CPI model, however, does not have the same stringent mathematical structure as the asset pricing theory. Consider the differences: In their model, $Y_t = a + b(\Delta CPI_t) + u_t$, and the investigator's objective would be estimation of coefficients a and b.

In the CAPM,

$$E(R_j) = \gamma_0 + \gamma_1 \beta_{jI}, \tag{3}$$

where $E(R_j)$ is the expected return on *individual* asset j and β_{jI} is its 'risk' coefficient for a given index, I. The only relevant question is whether the CAPM is exactly linear as shown. The coefficients γ_0 and γ_1 are not predicted by the theory except that γ_1 is positive if there is risk aversion and if I is the market portfolio of all assets. The alternative to the theory is *not* some other values of γ_0 and γ_1. It is a model of the form

$$E(R_j) = \gamma_0 + \gamma_1 \beta_{jI} + \alpha_{jI}, \tag{4}$$

where α_{jI} is an unspecified but *non-constant* vector (which is, incidentally, different for different choices of the index I). Furthermore, in *every* sample, there will always exist portfolios which could serve as market index proxies and which guarantee that (3) will be true. *In the same sample*, there will always be proxies which guarantee that (4) will be true. Is this a structure common to many theories in economics?

Contrary to Mayers and Rice's contention, the proxy problem is not the only problem in testing the CAPM. Another problem with the published econometric technique is bias against rejection of the two-factor version of the CAPM even if the proxy is perfect.[5] The reason for this bias is discussed in a widely-circulated working paper that was originally Part II of Roll (1977) [it is now Roll (1979)]: The portfolio grouping procedure used in all the criticized tests inhibits the only possibility of rejecting the two-factor theory. Mayers and Rice mention this result, and indicate their low regard for its importance, in a footnote (p. 22):

[4]Recently, however, there have been extensions to include non-equity assets in the indices and to assess the differences, if any, that arise in test results; see Stambaugh (1979).

[5]The Sharpe–Lintner theory could have been rejected correctly, however, if the proxy index were perfect.

> Roll also discussed an econometric aggregation problem in the portfolio grouping technique, but this is not a proxy problem. The aggregation problem is again a familiar one to econometricians.

But just because it is not a proxy problem does not imply it is not a problem at all. It may be 'familiar to econometricians' but I venture to guess that some scholars in finance did not realize its implications for testing the two-factor asset pricing model. Here is what it implies: pick any index whatever, even a single stock can serve, and follow the grouping procedure. That is, calculate individual asset betas against the index, rank by beta, form portfolios from the ranked assets, and then do a cross-sectional fit of mean portfolio returns on portfolio betas, always using the selected index to calculate betas. If the number of individual assets is large, the test will be biased in favor of supporting the hypothesis that the index is ex ante efficient, even if the index is a single stock! This is because the mean value of the α_{jI} term in (4) is zero and thus the α_{pI} for each portfolio p will also tend to zero, thereby assuring that the cross-sectional fit of portfolio return on portfolio beta looks like (3) rather than (4). This effect will be present with *any* index.

So even if the criticized papers had used a market proxy that everyone deemed reasonable, without further analysis we should not have accepted conclusions in support of the two-factor asset pricing theory. Those conclusions were based on data analysis with the grouping procedure, a faulty method in this particular application because of the theory's structure.

Mayers and Rice seem to have overlooked this theory's unusual implications for testing. They therefore conclude that my critique was much more broadly aimed than I intended it to be (p. 21):

> Proxies must be used constantly to test all types of economic theories. Are we to abandon studies of inflation because the change in the consumer price index (*CPI*) is merely a proxy for the inflation rate? Are we to abandon concentration studies because the 4-firm concentration ratio is merely a proxy for the concentration of an industry? Are we to abandon all empirical studies?

Mayers' and Rice's final allegation is that we should not abandon the two-factor capital asset pricing model because Stigler (1966) states: '. . . . It takes a theory to beat a theory'. They claim that 'even evidence itself is not enough to invalidate the theory', unless it is replaced by an alternative, and 'it is far from clear what the suggested alternative is' (p. 23). A few people such as Hakansson (1971), Jensen (1972b, pp. 385–388), Merton (1973), Ross (1976), and even Mayers (1973) must be mighty surprised to learn about the absence of competing theories. Some tests of these and other alternatives to the CAPM have been published. See, e.g., Jensen (1972b), Kraus and

Litzenberger (1976), Roll (1973), Roll and Ross (1979). Furthermore, evidence that contradicts the CAPM with the standard choice of market index is now available [Ball (1978), Basu (1977), Reinganum (1978)].

3. Residual analysis

Mayers and Rice say (p. 17):

> Roll's conclusion (10) can be easily interpreted as being critical of the empirical methodology known as residual analysis.

But I never mentioned this technique. Section 5 of my (1978) paper, eq. (12), supports residual analysis as approximately valid even if the market index proxy is not ex ante efficient.[6]

Because there may be some confusion concerning the difference between residual analysis and performance evaluation with the SML, the following thoughts are offered:

Using the Fama–MacBeth (1973) notation and assuming a two-factor return generating process (actually a single 'market' factor and a 'zero-beta' factor), individual asset j's return at time t can be expressed as

$$R_{jt}=\gamma_{0jt}+\gamma_{1t}\hat{\beta}_j+\varepsilon_{jt}. \tag{5}$$

The 'news' which residual analysis seeks to measure is contained in the true abnormal return ε_{jt}. Notice that the 'intercept' term γ_{0jt} might depend on the asset j; for if the market index used in estimating $\hat{\beta}_j$ is not ex ante mean/variance efficient, $E(\gamma_{0jt})=\alpha_j\neq E(\gamma_{0kt})=\alpha_k$. Specification (5) clearly involves an identification problem in distinguishing γ_{0jt} from ε_{jt}.

In both residual analysis and performance evaluation γ_{1t} is generally measured by a market index, say $\hat{\gamma}_{1t}$, either selected *a priori* or obtained from the data themselves à la Fama–MacBeth. In many recent applications of residual analysis γ_{0jt} has been measured by another return, ostensibly that of the zero–beta portfolio.[7] This approach employs an estimate of γ_{0jt}, say $\hat{\gamma}_{0t}$, that does *not* depend on asset j. So the residual computed is actually

$$\hat{\varepsilon}_{jt}=\varepsilon_{jt}+\gamma_{0jt}-\hat{\gamma}_{0t}=R_{jt}-\hat{\gamma}_{0t}-\hat{\gamma}_{1t}\hat{\beta}_j. \tag{6}$$

[6]Because the constant term in the market model regression should correct for the α_{jJ} term in (4) if the regression is conducted for a 'normal' sample period.

[7]Brenner (1976) shows, however, that the $\hat{\gamma}_{0t}$ resulting from the Fama–MacBeth procedure is generally not uncorrelated with the market index as it should be. Among other problems, this implies that the estimated residual, $\hat{\varepsilon}_{jt}$ in (6), is correlated in the market index return (because $\hat{\beta}_j$ is the simple, not a multivariate, estimate); I shall ignore this problem here. It is not present in residual analysis done with the market model, cf. below.

For an individual asset, we then obtain

$$E(\hat{\varepsilon}_{jt}) = E(\varepsilon_{jt} + \alpha_{jt}),$$

where $\alpha_{jt} = \gamma_{0jt} - \hat{\gamma}_{0t}$. This α_{jt} would be identical to the one in eq. (2) if there were actually a riskless rate of interest and if there were only a single stochastic generating factor. *For a single security*, residual analysis done this way is indeed identical to performance evaluation with the SML.

The same result would be obtained for any given portfolio. But unlike portfolio performance evaluation, residual analysis is usually conducted for the purpose of examining a specific event that has occurred for many different assets at separated calendar dates. The time subscript in (6) is taken relative to an event date and the 'abnormal performance' is defined as $\bar{\varepsilon}_{pt} \equiv \sum_j \hat{\varepsilon}_{jt}/N$ where N is the number of assets that have experienced the event of interest. Unless there is some relation between the event having occurred and the value of α_{jt}, we should obtain $\sum_j \alpha_{jt} \to 0$. Thus, $\bar{\varepsilon}_{pt}$ should be a consistent estimator of the actual effect of the event under study.

Residual analysis can also be done with the venerable market model. Applied to (5), the market model produces the following estimated residual:

$$\hat{\varepsilon}'_{jt} = \varepsilon_{jt} + \gamma_{0jt} - \bar{\gamma}_{0j} = R_{jt} - \bar{\gamma}_{0j} - \hat{\gamma}_{1t}\hat{\beta}_j, \tag{7}$$

where $\bar{\gamma}_{0j}$ is the ordinary least squares estimator of the intercept term in (5) ignoring intertemporal variation in γ_{0jt}. Note that $\bar{\gamma}_{0j}$ is not time dependent. Since $E(\gamma_{0jt}) = E(\bar{\gamma}_{0j}) = E(\gamma_{0t} + \alpha_j)$, the estimated residual $\hat{\varepsilon}'_{jt}$ in (7) is an unbiased measure of information contained in the true abnormal return (ε_{jt}); but $\hat{\varepsilon}_{jt}$ has a larger variance than ε_{jt}. This implies that a standard t-test applied to $\hat{\varepsilon}'_{jt}$ will understate its true significance. Nevertheless, the pattern of $\hat{\varepsilon}'_{jt}$ over time is an unbiased depiction of the pattern of the true ε_{jt}. Again, taking an average across assets that experience the same event on different calendar dates should tend to eliminate the term $\gamma_{0jt} - \bar{\gamma}_{0j}$ in (7) and produce an even more reliable result.

Done with the market model, residual analysis is not the same as performance evaluation even with a single asset. Performance evaluation to determine whether asset j has done better than k involves comparing $\bar{\gamma}_{0j}$ and $\bar{\gamma}_{0k}$. This cannot be done with the estimated residuals from (7) since $\sum_t (\hat{\varepsilon}'_{kt} - \hat{\varepsilon}'_{jt})/T \to 0$. But it could be done with the residuals from (6) since $\sum_t (\hat{\varepsilon}_{jt} - \hat{\varepsilon}_{kt})/T \to \bar{\gamma}_{0j} - \bar{\gamma}_{0k} \to \alpha_j - \alpha_k$. Residual analysis done with the portfolio matching technique[8] is equivalent to method (6) and thus can produce biased estimates of the abnormal news event.

[8]I.e., setting betas equal on an 'experimental' and a 'control' portfolio and calculating the residual as the difference in the two portfolios' returns.

mmary, whether residual analysis and performance measurement are nt depends on the treatment of the intercept term γ_{0jt} in (5). If the t is estimated from the data instead of specified according to the s predictions, residual analysis should give an unbiased estimate of e of information associated with the event under study.

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