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The Fiscal and Monetary Linkage between Stock Returns and Inflation

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ABSTRACT

Contrary to economic theory and common sense, stock returns are negatively related to both expected and unexpected inflation. We argue that this puzzling empirical phenomenon does not indicate causality.

Instead, stock returns are negatively related to contemporaneous changes in expected inflation because they signal a chain of events which results in a higher rate of monetary expansion. Exogenous shocks in real output, signalled by the stock market, induce changes in tax revenue, in the deficit, in Treasury borrowing and in Federal Reserve "monetization" of the increased debt. Rational bond and stock market investors realize this will happen. They adjust prices (and interest rates) accordingly and without delay.

Although expected inflation seems to have a negative effect on subsequent stock returns, this could be an empirical illusion, since a spurious causality is induced by a combination of: (a) a reversed adaptive inflation expectations model and (b) a reversed money growth/stock returns model.

If the real interest rate is not a constant, using nominal interest proxies for expected inflation is dangerous, since small changes in real rates can cause large and opposite percentage changes in stock prices.

THERE IS A WELL-documented but puzzling empirical relation between stock returns and inflation. Expected inflation, unexpected inflation, and changes in expected inflation are all negatively related to stock returns. See Fama and Schwert [12] and the less comprehensive but consistent work by Lintner [26], Jaffe and Mandelker [23], and Nelson [32]. We offer here an explanation for this phenomenon and present evidence supporting the explanation.

The empirical results merit attention because they appear to be in conflict with both economic theory and common sense, according to which stock returns should be *positively* related to both expected and unexpected inflation. A positive

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stock price reaction to unexpected inflation is suggested by the traditional idea that equities are "hedges" against (unanticipated) inflation because they represent claims to real assets. Stock returns should be positively related to expected inflation according to the Fisherian theory of interest; the nominal expected return on any asset equals real interest and a real risk premium (if appropriate), plus expected inflation; yet, the empirical results imply that neither of these arguments is valid.

In the Fama/Schwert paper, expected inflation was measured by the Treasury bill rate at the beginning of the period. The change in expected inflation was simply the change in the T-bill rate, and unanticipated inflation was the ex post difference between the actual inflation rate and the beginning-of-period T-bill rate. Empirical support for the validity of these measures is present in the early work of Fama [10] and the basic support continues to be strong after a series of tests (cf., Hess and Bicksler [22], Joines [24], Nelson and Schwert [34], Fama [8], and Fama and Gibbons [11]).

It seems natural to consider first whether the Fama/Schwert results indicate a causative influence of inflation on stock returns. The authors used inflation as the "explanatory" regression variable; but there are several reasons to suspect either a reversed causality or no causality at all. The estimated effect of inflation on stock returns was far too large to be plausible (an increase in expected inflation of ten percent would cause a decline in expected stock returns of fifty percent). There is one good theory based on the demand for money, (Nelson [33] and Fama [9]) that the relation is partly spurious. Both the possibility of reversed causality and the possibility of spurious correlation will be discussed in detail below.

There is also a puzzle in that two separate phenomena seem to be at work in these results: the negative relationship between stock returns and contemporaneous *changes* in expected inflation seems inconsistent with the negative correlation between stock returns and the *level* of expected inflation at the beginning of the period. To see the inconsistency, imagine that a reasonable theory were developed for a negative relationship between expected inflation and the real risk premium on stocks. Then, if expected inflation increased, ceteris paribus, the real risk premium should decrease, thereby causing an increase in stock prices. But the empirical evidence indicates that when expected inflation increases, stock prices actually fall. Perhaps anticipated future real cash flows fall by an amount sufficient to more than offset the fall in the real risk premium; but, of course, a second theory would be required to explain this fall in expected cash flows.

On the other hand, the negative relation between stock returns and *changes* in expected inflation can be easily reconciled with the observed negative relation between stock returns and *unexpected* inflation; changes in expected inflation are likely to be positively correlated with (and caused by) unexpected inflation. The positive co-movement could be modeled, for instance, by an adaptive expectations model,

$$\bar{I}_{t+1} = \bar{I}_t + \gamma(I_t - \bar{I}_t) + \epsilon_t \tag{1}$$

where I_t is the actual inflation rate, γ is the speed of adjustment parameter for expected inflation, $(\bar{I})$, and ϵ is a random disturbance.

As the following timing schematic shows, expectations subscripted "t" which apply to a period from $t - 1$ to t, are actually held at "$t - 1$," while realizations subscripted "t" are not finally observed until instant t.

Observational Timing

Observed at $t - 1$:		Observed at t:
RF_{t-1}		RF_t,
and Expectations		Realizations of
$(\overline{RS}_t, \bar{I}_t)$		(RS_t, I_t)
	$t - 1$ ———————— t	and Expectations
		$(\bar{I}_{t+1}, \overline{RS}_{t+1})$

Here RF is the Treasury bill rate, RS is the nominal stock return, and I is the inflation rate. Superior bars, —, denote expectations. If the true negative influence on stock returns is the change in expected inflation, $\bar{I}_{t+1} - \bar{I}_t$, the "unexpected inflation" $I_t - \bar{I}_t$ could serve as a proxy; albeit with error. In fact, Fama/Schwert find that when both changes in expected inflation $\bar{I}_{t+1} - \bar{I}_t$ and unexpected inflation $I_t - \bar{I}_t$ are introduced into the same multiple regression to explain stock returns, unexpected inflation is not usually statistically significant (with the single exception of quarterly data and equally-weighted portfolio returns). Using monthly value-weighted portfolio returns, the estimated marginal impact of $I_t - \bar{I}_t$ on stocks is $-.91$, while the marginal impact of $\bar{I}_{t+1} - \bar{I}_t$ is -17.7. Ignoring the error term in Equation (1), this implies a speed of adjustment coefficient γ of about .05. Apparently, $I_t - \bar{I}_t$ and $\bar{I}_{t+1} - \bar{I}_t$ are simply two empirical measures of the same basic underlying influence and $\bar{I}_{t+1} - \bar{I}_t$ is probably the better one of the two.

Several explanations have been offered in past work for a negative inflation/stock return relationship. Kessel [25] pointed out that unanticipated inflation benefits net debtors at the expense of net creditors. This implies that equity returns of only those firms which are net creditors would be negatively related to unexpected inflation so that an aggregate negative relation for all stocks would require equity holders to be net creditors on average. Since most nonfinancial corporations appear to have more fixed nominal liability commitments than fixed nominal assets, they are net debtors and Kessel's argument is not empirically compelling.

Lintner [26] has argued that inflation, whether anticipated, or unanticipated, increases the external financing required by corporations; this is purported to dilute the returns to old equity shares. Lintner argues that firms with fixed gross profit margins and fixed dividend payout ratios require a higher fraction of noninternally-generated funds during periods of inflation in order to sustain working capital in a fixed proportion to sales. He assumes implicitly that the augmented working capital resources do not earn the cost of capital (which is why they "dilute" returns). Cash balances, for instance, receive zero interest and accounts receivable apparently do not influence sales revenues.

It seems rather implausible (to us) that managers are so obstinate or inflexible that they obtain external funds and invest them in subpar assets. To the contrary, corporate treasurers respond rather aggressively to increased inflation by cutting cash balances, tightening the terms of trade credit, delaying payments, and by numerous other devices detailed in working capital management textbooks and corporate handbooks. Thus, Lintner's theory also seems unlikely to explain the phenomena under study.

Modigliani and Cohn [31] believe that investors "are unable to free themselves from 'money illusion' and that as a result, [they] price equities in a way that fails to reflect their true economic value" (p. 4). Of course this conflicts directly with rational expectations and market efficiency. It suffers the typical defect of a theory based on irrationality and concocted after the data are observed. Some such theory is always available for any possible set of observations.

Summers [40] contrasts two opposing hypotheses about the impact of inflation on market valuation. The "inflation illusion" hypothesis that investors are not able to see through the nominal accounting statements and respond to reported rather than real profits is compared to the "tax effects" hypothesis: firms which report high profits due to inflation are penalized by an extra tax burden. Summers's results support the tax effects hypothesis. This conflicts, however, with the results of French, Ruback, and Schwert [13], who find no significant effect of nominal contracting on stock returns.

Nelson [33] and Fama [9] both argued that the money demand theory implies a negative relation between the actual inflation rate and the growth rate of real activity. Since stock returns predict real activity, a negative but spurious correlation is induced between stock returns and inflation. This argument is certainly plausible and is supported by compelling empirical evidence. However, something of a puzzle still remains in Fama's [9] empirical results. Various measures of real activity did not, by themselves, entirely eliminate the negative inflation/stock returns relation. With monthly data, the effect of unexpected inflation (which, as we argued above, is probably the same effect as that of changes in expected inflation; a variable not included in Fama [9]) is never eliminated. Both expected and unexpected inflation are eliminated in regressions with annual data but only when the growth rate of the monetary base is included as another explanatory variable.

Neither Nelson nor Fama offers a reason for including the monetary base in the regression and no reason is provided by money demand theory. As Fama points out (p. 562), the money base growth rate and his proxy for expected inflation (a smoothed Treasury bill rate), are strongly related; so there remains a suspicion that one proxy for expected inflation has simply replaced another and that the underlying relationship remains. This possibility is troubling enough for Fama to offer another "less profound explanation, in particular that the [negative stock returns/money base growth] relations are spurious ... Inclusion of the slowly wandering base growth rate ... has the effect of 'twisting' the regression residuals toward lower values in the early years and higher values in the later years" (p. 562).

But whether the residuals are spuriously "twisted" or not, Fama notes, "even a full guarantee that this viewpoint is correct would leave us with the uncomfortable fact that there is a downward drift in expected real stock returns during the post-1953 period which is not fully explained by our story about real activity" (p. 562).

Our paper supplements and extends Nelson's and Fama's theoretical, and Fama's empirical arguments. First, we offer a theory in which uncertain stock returns signal changes in expected inflation; where "signal" is taken in the econometric "causality" sense of Granger [19] and Sims [39]. There is an economic reason for the negative connection between realized stock prices and the growth rate of the money base. We argue that this money-supply connection aids and abets the Nelson and Fama explanation that works through money demand.

The money supply possibility is strongly suggested by Fama's own empirical results—for if the money base growth rate is significant for anything *other* than a spurious reason, the money supply process is implicated. Furthermore, the sheer size of the marginal effect on stock returns of the money base growth rate suggests that the causative arrow may be pointing in the wrong direction in Fama's regressions. (N.B., he did not argue that there *is* a causative relationship). In his annual results, a one percent increase in the base growth rate is associated with nearly a two percent decline in real stock returns, after taking into account the future level of real activity. Surely there is some value in searching for reasons to support an opposite causality. We discovered that they are not difficult to uncover.

Second, we suggest that stock returns may be negatively correlated with changes in the Treasury bill rate, the *proxy* for expected inflation, even when there is no connection between stock returns and inflation. This can happen if positive (negative) stock returns are associated with negative (positive) changes in the real interest rate.

A change in the real rate of interest should be a true cause of ex post stock returns, because an increase (decrease) in the real interest rate induces a reduction (increase) in all asset values. Thus, to the extent that changes in the Treasury bill rate are due to changes in its real interest component and not to changes in expected inflation, we would anticipate a contemporaneous stock return of the opposite sign. However, extending this line of argument leads to the conclusion that there should be a *positive* relation between the beginning-of-period T-bill rate's real interest component and the subsequent (expected) stock return. Thus, real interest variability cannot explain the negative relation found between ex ante T-bill rates and subsequent equity returns.

Furthermore, during most of the postwar U.S. history, there is evidence of only a small variability in real interest, (cf. Nelson and Schwert [34], Fama and Gibbons [11]). Until the latter part of 1980, Fama's [10] simple inflation prediction model, that assumed that the real rate of interest was constant and that variability in the T-bill rate was caused entirely by revisions in expected inflation, performed very well indeed. Since December 1980, this simple model has consistently overpredicted the observed rate of inflation, implying that the real interest rate has increased. In this recent period, therefore, much of a negative stock return/Treasury-bill change correlation might be due to the impact of real interest rate changes on equity values. Even in the earlier period, real interest changes, although very minute, could have had an important and measurable influence. We discuss the reason in detail below.

Finally, to be complete, we must mention the possibility that stock returns signal, but are not caused by, changes in the real rate of interest. There are at least two theoretical arguments which imply a negative signalling relation (and we shall discuss them in the next section). Of course, if stock returns signal changes in real interest, there should also be a causative feedback effect. Again, the low variability of real interest rates over most of the available data period might seem to render this possibility unpromising. Nevertheless, we shall test for real interest rate effects in the empirical section.

I. A Theory of Stock Returns, Money Supply, and Interest Rates

A. Stock Returns, the Money Supply Process, and the Inflation Component of Nominal Interest Rates

We argue that stock market returns signal changes in the inflationary process because of the following chain of macroeconomic events.

First, the government's principal revenues are personal and corporate taxes. When stock prices increase or decrease in response to anticipated changes in economic conditions, personal and corporate incomes move in the same direction, inducing a similar change in government revenue. Thus, fluctuations in government revenue are closely related to stock market movements.

Second, if government expenditures do not accommodate themselves to changes in revenue, fluctuations in revenue will be reflected in deficits. Recent government deficits have paralleled a rapid rise in the fixed portion of government expenditures, now called "entitlements." In fact, these entitlements (so-called uncontrollable expenses), have grown to be about 80 percent of the Federal government's budget.[1] To the extent that such expenditures really are fixed, changes in economic conditions should be followed by opposite changes in the deficit.

Third, when a deficit occurs, the Treasury is obliged to borrow. It could repay the debt during later surplus periods provided that direct tax revenues increased or expenditures decreased enough to generate such a surplus. Instead, the typical *modus operandi* in recent years has been to have the Federal Reserve System "monetize" the debt by printing currency or expanding bank reserves. This effectively generates the required surplus by *indirect* taxation through the inflation caused by an increased rate of monetary growth.

To recapitulate, a change in stock returns predicts a change in government revenues. Given largely fixed government expenditures, fluctuating revenues lead to periodic government deficits and concomitant increases in government debt. The larger debt causes an increase in expected future indirect tax liabilities, both personal and corporate, because of debt monetization and its consequence, inflation. With this linkage in mind, reconsider the relations mentioned at the beginning of this paper.

When stock prices decline, the government will tend to run a deficit; then, given the practice of monetization (which will be anticipated by rational citizens), expected inflation will rise. Thus, stock market price changes, which are caused by changes in anticipated economic conditions, will be negatively correlated with changes in expected inflation. It is well accepted, moreover, that changes in the

[1] See *Fortune*, 22 September 1980, p. 83.

expected inflation rate cause a more than proportional change in the immediate actual inflation rate. The "Friedman surge" [16] in actual inflation follows as citizens alter real money balances in response to altered inflationary expectations. This implies that stock market price changes will also be correlated negatively and contemporaneously with unanticipated actual inflation.

In order to incorporate the money demand explanation into our perspective, consider an economy with a perpetually balanced budget. In such an economy, some inflation would occur if stock prices fell even though no deficit were possible. As Nelson [33] and Fama [9] have shown, a decline in real activity will reduce the demand for money, and if the supply of money remains unchanged, prices must rise.

While the logic of this money demand approach is correct, the magnitude of the effect it predicts may not be sufficient to explain the observed negative relation between real activity and inflation. One piece of evidence against a pure money demand explanation is that in predeficit days, real activity and inflation were either unrelated or at times positively related.[2] Since money demand theory applied in those years also, the absence of a consistent negative relation suggests another force at work. A second problem with the money demand explanation is that, (by omission), it ascribes a purely passive role to the government. Yet in periods of prolonged deficits, during which the negative relation has been most noticeable, the money supply has not been constant, but instead has significantly increased.

We are by no means denying that the demand for money can create a negative relation between stock returns and inflation. We hope merely to show that the money supply process can produce a similar effect. It seems highly improbable that anyone will be able to identify precisely the relative importance of demand and supply. Later in the paper, however, we document the empirical plausibility of the supply explanation.

So far, we have said nothing about what Fama calls "the most anomalous of the negative stock return-inflation relations, that between ex post real stock returns and ex ante expected inflation rates" [9, p. 560].[3] We believe, however, that the money supply response to stock returns can help explain this anomaly as well.

To understand how this can be, recall first that the proxy for expected inflation used by most recent investigators is the beginning-of-period Treasury Bill rate. Its validity as such a proxy finds ample theoretical and empirical justification in Fama [10] and in Fama and Schwert [12]. There is also good evidence that the inflation rate and the monetary base growth rate are closely associated; see Fama [7]. In fact, a simple and plausible adaptive expectations model for expected inflation, very similar to the one mentioned previously,[4] might very well use the newly-formulated expected money base growth rate as follows:

[2] See Friedman and Schwartz [17, p. 678, Chart 62, and p. 689].

[3] In a recent paper, Schwert [38] noted again the widespread bewilderment over this phenomenon: "the most puzzling result of all is still unexplained: why are aggregate stock returns negatively related to the level of expected inflation" (p. 28).

[4] Equation (1) above.

$$\bar{I}_{t+1} = \bar{I}_t + \gamma(\bar{\tilde{M}}_{t+1} - \bar{I}_t) + \epsilon_t \tag{2}$$

where $\bar{I}_{t+1}$ indicates expected inflation at the end of period t and $\bar{\tilde{M}}_{t+1}$ is the market's expected money base growth rate as held at instant $t + 1$. (See the timing schematic on p. 3); γ is a speed of adjustment coefficient for expected inflation, and ϵ_t is either a random disturbance or the amalgamated effect of all other variables (such as real variables working through money demand).[5]

We assume that the reverse causality notion of unexpected stock returns signalling the expected money base growth rate can be written as a simple linear model

$$\bar{\tilde{M}}_{t+1} = a + b(RS_t - \overline{RS}_t) + \xi_t \tag{3}$$

where the response coefficient b is negative and is probably quite small. The Fisher equation for risky assets in terms of expectations can be written as

$$\begin{aligned} \overline{RS}_t &= r_t + \bar{\rho}_t + \bar{I}_t \\ &= \bar{\rho}_t + RF_{t-1} \end{aligned} \tag{4a}$$

where r_t is the real riskless rate and $\bar{\rho}_t$ is the expected risk premium on stocks. The Fisher equation for the Treasury bill rate has the same form but a zero risk premium

$$RF_{t-1} = r_t + \bar{I}_t \tag{4b}$$

Substituting from Equations (4a) and (4b) into (3) and (2),

$$RF_t - RF_{t-1} = \alpha_t - b\gamma\bar{\rho}_t + \gamma[bRS_t - (1 + b)RF_{t-1}] + \gamma\xi_t + \epsilon_t \tag{5}$$

where $\alpha_t = a\gamma + r_{t+1} - (1 - \gamma)r_t$.

Tests of this reverse causality model for expected inflation as a function of stock returns are presented in the next section.

If the reverse causality argument is correct and uncertain stock returns do signal changes in the expected money base growth rate, and if the simple adaptive expectation model for expected inflation is plausible, then rearranging Equation (5) so that stock returns are the dependent variable yields

$$RS_t = \frac{-\alpha_t}{b\gamma} + \bar{\rho}_t + \left(1 + \frac{1}{b}\right)RF_{t-1} + \frac{1}{b\gamma}[RF_t - RF_{t-1}] - \frac{\gamma\xi_t + \epsilon_t}{b\gamma} \tag{6a}$$

$$= \beta_{ot} + \beta_1 RF_{t-1} + \beta_2[RF_t - RF_{t-1}] + u_t \tag{6b}$$

where $u_t = -(\gamma\xi_t + \epsilon_t)/b\gamma$

If it had been written down directly, Equation (6b) would have seemed a logical vehicle for testing whether stock returns were a "hedge" against both expected inflation and changes in expected inflation. A third term, $I_t - \bar{I}_t \equiv I_t - RF_{t-1}$, could have been added to the right side of Equation (6b) to test whether stock

[5] Model (2) could actually be a better adaptive expectations model than (1), in which observed inflation is used as the driving variable, because the latter contains a greater number of current and past random shocks, whereas the market's assessment of the money base growth rate may be more directly relevant for future inflation.

returns also hedged against *unexpected* inflation. In fact, this was exactly the Fama/Schwert [12] equation; and they found that the unexpected inflation term, which is missing in (6b), was empirically insignificant.

Note that both the level and change effects of the Fama/Schwert Equation (6) are direct algebraic results of reversing the "causality" between stock returns and inflation. When the simple adaptive expectations model for expected inflation [Equation (5)] is reversed, the Fama/Schwert result is induced. The reverse causality explanation implies nothing about the stock market's risk premium, $\tilde{\rho}$. Both the level and the change coefficients, β_1 and β_2, do not apply to the risk premium. Thus, although an asset pricing model might suggest that the level effect can only be induced by a changing risk premium [see Equation (4a)], the reverse causality explanation actually is independent of this implication. Furthermore, if there is concern over the independence of the error terms, note that u_t is not related to RF_{t-1} and its relation to $(RF_t - RF_{t-1})$ should be small, since (2) probably has a very high R^2 (it contains only expectations). Finally, nonstationarity in the intercept term β_{ot} is probably inconsequential because it depends only upon intertemporal shifts in the expected real riskless rate and the expected risk premium on stocks. Even if their variations were large, there is no reason to anticipate that the estimated coefficients of the level, β_1 and of the change β_2, would be biased from their true values of $(1 + 1/b)$ and $1/b\gamma$, respectively.

The coefficient for the "level effect," β_1, is greater than $1/b$ by the Fisher coefficient, +1. Since stock returns signal a significant but small negative change in $\bar{M}$, the coefficient in (6) of ex ante expected inflation, $(1 + 1/b)$, is a *large* negative number. This coefficient was estimated to be about −5 in the Fama/Schwert [12] or Fama [9] monthly results which implies that b is about −.17, not an implausible coefficient in the growth response relation (3). Clearly, $\beta_1 = -5$ cannot be taken seriously as a causative value, since a rise in the Treasury-bill rate of only five percent above its historical level would have plunged expected returns on stocks by 25 percent. Recent T-bill rates have been above fifteen percent, thus implying negative expected stock returns.

The reverse causality argument, to the contrary, is much more plausible. A Fama/Schwert regression such as (6) may not indicate causality, but it still can display a very large coefficient on $RF_t - RF_{t-1}$. If b were approximately −.17 and the speed of adjustment coefficient γ in (2) were at some reasonable level such as .30, the Fama/Schwert coefficient on $RF_t - RF_{t-1}$ would be −20.

We are aware that this story is difficult to accept because (6b) seems to imply that RF_{t-1}, the Treasury-bill rate observed at the beginning of the period accomplishes the impossible feat of predicting the "unexpected" component of the subsequent stock return. An illusion such as this *can* arise in a multiple regression. Consider a two-equation system

$$y = \gamma(x - z) + \epsilon$$

$$x = bw + \xi$$

where the true causative influence runs from $(x - z)$ to y and from w to x; (ϵ and ξ are random disturbances). If, being unaware of the state of nature, we regressed

w on y and z, we would find an *apparent* influence of both latter variables on w. In the regression model

$$w = z/b + y/b\gamma - [(\epsilon + \gamma\xi)/b\gamma]$$

both z and y would be "significant." Note in particular that z would have an apparent influence on w even though there is no relation whatever between them.

In the case of the Fama/Schwert equation, RF_{t-1} could derive some of its observed strength from such a source.[6] We must admit, however, that this explanation based on a spuriously-induced correlation may not be the whole story. The reason is simple: if RF_{t-1}'s negative influence on RS_t were due entirely to the explanation above, a simple regression of RS_t on RF_{t-1} *alone* should find only a weak positive effect (because of the influence RF_{t-1} has on the expected stock return, $\overline{RS}_t$, via (4a)). However, Fama/Schwert [12, p. 135] report a negative simple correlation between RF_{t-1} and RS_t. The correlation is weak, (their trading rule based on RF_{t-1} as a predictor did not earn abnormal profits), but it *is* negative and it would be positive if our spuriously-induced effect were the whole explanation.

B. Stock Returns and Real Interest Rates

Since the empirical proxy for expected inflation is the Treasury-bill rate (RF) at the beginning of the period, changes in stock prices could also be associated with opposite changes in the proxy if *real* interest rate changes are negatively correlated with stock returns. Theoretically, there is a direct causative effect: an increase in the real rate of interest should cause a decline in *all* asset values. In the case of equities, which are long-term assets, we can model the real interest effect without loss of generality by a simple perpetuity formula

$$p_t = \bar{c}/r_t$$

where $\bar{c}$ is the certainty-equivalent real cash flow perpetuity and r_t is the real interest rate. Assuming no change in the certainty-equivalent real cash flow, a given percentage change in the real interest rate is matched by an equal but opposite contemporaneous asset return since

$$dp/p = -dr/r \tag{7}$$

The Fama/Schwert regressions employed the algebraic change in Treasury-bill rates over a month, not the percentage change in the real rate, as one of the explanatory variables. Used in place of the real rate of interest, the T-bill rate could be thought of as containing a large measurement error (equal to the expected rate of inflation). Thus, in terms of a real rate interpretation, the regression actually run was something like

$$dp/p = \beta_0 + \beta_1\,(r + \tilde{\epsilon}) + \beta_2\,(dr + d\tilde{\epsilon}) \tag{8}$$

where $\tilde{\epsilon}$ is the "measurement error."

[6] The appendix contains a further econometric analysis of this issue and includes an examination of the biases induced by statistical dependence between the disturbances and regressors.

If (7) is the correct causative model, regression (8) could result in a large negative value for β_2. For example, imagine that the error ($\tilde{\epsilon}$) were negligible and that the presence of r in (8) causes no econometric problem; then, β_2 should approximate $-1/r$. If r is a very small number, say much less than .1 percent per month, β_2 should be large and negative, say -1000 or larger. The presence of the measurement error in (8) probably biases the estimate $\hat{\beta}_2$ towards zero, which could explain why the observed $\hat{\beta}_2$ was only -18 rather than, say, -1000.

This real rate-based argument, however, cannot explain the negative interest rate *level* coefficient, i.e., the observed negative β_1 in (8). To the contrary, if changes in the real rate move stock prices in the opposite direction, the ex ante real rate should be positively related to the expected stock return and thus also be positively related to the observed stock return.

In addition, there are empirical grounds for concluding that real rate movements may not be the explanation of the observed negative coefficient, $\hat{\beta}_2$ of the change in interest rates. Although Fama/Schwert used the nominal interest rate as a proxy for expected inflation and thus open their findings to a real rate explanation, other authors used actual or forecasted inflation directly and found a similar negative relation. Lintner [26], for example, reported a strong negative correlation between actual stock returns and actual inflation. Jaffe and Mandelker [23] report negative coefficients between actual stock returns and actual inflation; both contemporaneous and leading, (cf. their Table 3, p. 453). They do, however, find a stronger correlation between stock returns and the Treasury bill rate than between stock returns and inflation. This suggests that the real rate of interest could be partly responsible. On the other hand, if the T-bill rate is a better measure of *expected* inflation than is the actual inflation rate, and if it is really expected inflation which is negatively related to stock returns, the real rate might not be playing a role after all.

Nelson [32] constructed a time series model of actual inflation and related the "innovation" in the series to stock returns, the idea being to use unanticipated inflation as an explanatory variable. Again, he found an impressive negative relation.

We draw the conclusion from these other studies using inflation, that changes in the real rate cannot be the entire explanation of the negative contemporaneous relation found by Fama/Schwert between stock returns and changes in the nominal rate of interest.

There is also a real rate/stock return interaction which is not causative. If stock returns are negative, the government's deficit will tend to increase because expenditures do not fall dollar for dollar with tax revenues and the Treasury must borrow to finance the deficit. If the Federal Reserve System does not completely monetize this increased public debt, real interest rates might increase. Perhaps the simplest way to understand this effect is to consider the Treasury as just another (very large) consumer that refuses to alter its intertemporal consumption pattern in response to changes in transitory income. Even though its income falls, the Treasury keeps right on consuming; to finance that consumption, however, it is obliged to offer higher (real) interest rates in order to induce other consumers to save (lend). The *fraction* of aggregate output consumed by the Treasury increases.

There is a counterargument by Barro [4] to this Keynesian theory: if nongovernment consumers are rational, they will realize that future tax liabilities must increase to pay off the future value of the current deficit. Thus, the purchaser of a Treasury security may anticipate delivering it next period to satisfy his tax liability; indeed, he might consider the tax as effectively being paid currently in the form of a bond purchase. After all, the purchase is a net cash outflow and it occasions a reduction in his current consumption.

Whether rationally-anticipated taxes fully offset the direct effect on real interest rates is an empirical question; but there may be reasons to doubt a full offset. One obvious reason is that future taxes need not be payable by the same individual current savers and Treasury bond purchasers. Future taxes could be paid by later monetization or by direct taxes on future generations. The recent experience from a period (1981–82) characterized by large deficits and a low rate of Federal Reserve monetization seems to be that real interest rates have increased dramatically. At least, journalists, investment bankers, and foreign government leaders seem to think so.

In periods of partial debt monetization, a decrease in stock prices could be associated with an increase in Treasury-bill interest rates because both the real interest rate and the anticipated inflation rate increase, the latter because of the anticipated increase in the growth rate of money and the former because of the anticipated effect of increased Treasury borrowing.

II. Empirical Evidence

In the last section, we traced a possible causative chain that explained why stock market returns are negatively associated with nominal interest rates. If investors can forecast changes in future real activity, stock market movements should foretell such changes. Given corporate and personal income tax statutes, changes in real activity will then bring about increases or decreases in Federal Government tax receipts. But government expenditures are not very responsive to receipts, so the Federal Treasury must initiate a change in borrowing. Federal Reserve System behavior leads to a partial "monetization" of the change in debt shortly thereafter. The concomitant change in base money brings about an eventual change in the price level. Rational investors predict higher inflation and impound their predictions into nominal interest rates. To the extent that monetization is not complete, the real interest component of nominal interest rates might also increase.

Our purpose in this section is to provide empirical evidence that each element in this chain of events actually occurs. Most of our results are present in various empirical papers scattered in the economics, finance, and accounting literature. The authors of these papers were occupied with a detailed examination of a particular link and apparently did not intend to examine the larger chain under investigation here. We have found no mention in any of these papers of the overall route via the government budget from stock market returns *to* inflation. Nevertheless, the previous work affords us an expositional luxury, since we can present some relatively simple and unsophisticated empirical results for each link and rely on the more detailed previous publications for rigorous support.

A. *The Stock Market Forecasts Changes in Economic Activity*

Table I shows that lagged stock market returns are statistically significant predictors of changes in two indices of economic activity, corporate earnings, and employment rates.[7]

Using a simple linear regression model (reported in the first part of Table I), the stock market's return is a statistically significant predictor of next quarter's change in the unemployment rate. The relationship is negative: a high stock market return foretells a reduction in the rate of unemployment during the subsequent quarter. Most of the predictive content of unemployment by stock market returns is in the first lagged quarter but the second lag is marginally significant.

In the case of corporate earnings, the first quarter's lag contains most of the predictive content. A regression of the growth rate in earnings on the first quarter-lagged stock market return alone (see the last column) actually has a higher adjusted R^2 than the regression which includes contemporaneous and four lagged terms.

Although the stock market predicts corporate earnings with statistical significance, the level of accuracy is quite low; the adjusted R^2 is only about 10 percent. Perhaps this is not too surprising, however, in view of the well-documented high volatility and randomness in corporate earnings.[8]

Since we have no theory of the intertemporal relation between earnings and stock returns nor between unemployment and stock returns (other than that stock returns should lead earnings and unemployment), an autoregressive integrated moving average (ARIMA) transfer function model[9] was also fit to these data. The results are reported in the second part of Table I.

The ARIMA fit of unemployment disclosed an extremely strong seasonal, a positive correlation at the fourth-quarter lag, and negative correlations at intervening lags. This was missed completely by the regression and the standard Durbin-Watson statistic, which indicated only a marginally significant negative first-quarter lag correlation (remember that these are already growth rates; first differences of logs).

The ARIMA results indicate strong first and second quarter lag effects of stock returns on the unemployment growth rate. The ARIMA residuals were examined for further dependencies and none were found. However, the residuals did indicate a modest right skewness.

For corporate earnings, much less intertemporal dependence was detected in the series. No moving average component was required and the autoregressive component was strongly present with only a two-quarter lag. The four-quarter (annual) seasonal was marginally significant. As in the regression results, stock prices have a strong effect on the next quarter's earnings but no other lags are

[7] Fama [9] documented the strong correlation between current stock market returns and future growth rates of real GNP and industrial production. We add earnings and unemployment merely because they are more obviously related to taxes, which we investigate next.

[8] The first paper demonstrating such randomness in British earnings was by Little [28]. American data with similar characteristics were analyzed by Lintner and Glauber [27], by Ball and Watts [3]. Cf. also Foster [14, ch. 4].

[9] See Box and Jenkins [6].

Table I

Stock Market Predictions of Corporate Earnings and Unemployment
Quarterly Data, 1947:1–1980:1

LINEAR REGRESSION

Lag (Quarters)	Change in Unemployment		Change in Corporate Earnings	
0	.267 (.978)	—	.0463 (0.395)	—
1	−.692 (−2.31)	−.771 (−3.09)	.326 (2.54)	.404 (3.82)
2	−.508 (−1.67)	—	.161 (1.24)	—
3	−.122 (−.408)	—	−.0526 (−.412)	—
4	−.110 (−.407)	—	−.491 (−.425)	—
t-statistics are in parentheses				
Durbin-Watson	2.43	2.36	1.91	1.84
F-Statistic (Probability Level)	3.30 (.992)	9.57 (.999)	3.31 (.993)	14.6 (.999)
Adjusted R^2	.0832	.0627	.0834	.0949

The Regression equation was

$$y_t = a_0 + b_1 R_t + b_1 R_{t-1} + \cdots + b_4 R_{t-4}$$

where

R_t = Stock Market Return $= \log_e(\text{S\&P}_t/\text{S\&P}_{t-1})$

y_t = growth rate in unemployment $= \log_e(U_t/U_{t-1})$

or

growth rate in earnings $= \log_e(\text{EPS}_t/\text{EPS}_{t-1})$

$\text{S\&P}_t$: Standard and Poor's 500 Composite average at the end of quarter t.
U_t: Unemployment Rate—Civilian Workers, Percent, Seasonally Adjusted from U.S. Department of Labor, Bureau of Labor Statistics, during Quarter t.
EPS_t: Average value of earnings per share, Standard and Poor's 500 Composite Stocks

ARIMA

Lag	MA	AR	TR	MA	AR	TR
(Quarters)	Change in Unemployment			Change in Corporate Earnings		
1	−.537 (−3.78)	−.337 (−2.59)	−.448 (−3.10)	—	.0481 (.54)	.291 (2.50)
2	−.220 (−1.72)	−.253 (−2.24)	−.631 (−4.38)	—	−.212 (−2.34)	.129 (1.12)
3	—	−.244 (−2.77)	−.143 (−1.03)	—	—	—
4	—	.529 (5.40)	—	—	−.159 (−1.75)	—

Table I—*continued*

t-statistics are in parentheses

F-Statistic (Probability Level)	33.2 (>.999)	5.63 (>.999)
Adjusted R^2	.682	.133

MA: moving average component

AR: Autoregressive component

TR: Transfer Function Component, the independent variable was R_t (see above).

The ARIMA estimation was by the nonlinear unconditional least squares method (See Box and Jenkins [6, Ch. 7]), using the BMDQ2T software package developed by Liu [29]. The fitted model was of the form

$$(1 - \hat{\Psi}_1 B - \hat{\psi}_2 B^2 - \cdots)y_t = (\Omega_0 + \Omega_1 B + \Omega_2 B^2 + \cdots)R_t + (1 - \hat{\theta}_1 B - \hat{\theta}_2 B^2 \cdots)\epsilon_t$$

where B is the backshift operator and ψ, Ω, θ are parameters. This is a transfer function model without feedback.

significant. The ARIMA residuals are serially uncorrelated and appear to be normally distributed.

A more detailed corroborative study of the predictive ability of stock market returns for corporate earnings was conducted by Ball and Brown [2]. They showed that unanticipated earnings changes had a strong influence on stock prices and that such unanticipated changes were preceded by stock price movements in the same direction; i.e., that stock price movements forecast future changes in earnings.

An interesting implication of the Ball and Brown study is that an overall negative linkage between stock prices and inflation must be due solely to random shocks in real activity and not to forecastable changes. Any change in real activity reflected by aggregate corporate earnings which could have been anticipated long in advance would not be associated with a contemporaneous or immediately-preceding stock market movement. As Ball and Brown demonstrated, the market reacts only to unexpected results. Thus, whatever the source of the shock in real activity that ultimately brings a change in inflation, if it is to be associated with stock market movements, it must be unexpected.

B. *Changes in Unemployment and in Corporate Earnings Strongly Influence Changes in Federal Tax Collections*

The second link relating stock price movements to inflation is the impact on Federal Government tax collections of changes in real activity. In Table II, we report the contemporaneous and lagged influence of the growth rates in unemployment and in corporate earnings on three different tax measures.

The unemployment rate, used as a proxy for personal income, has a strong effect on personal taxes with the strongest lag being one quarter. There are also significant contemporaneous and two-quarter-lag effects. Corporate earnings have a very strong contemporaneous effect on corporate tax collections and seem to have little lagged effect.

The regressions (II.1) and (II.3) probably reflect the typical lag structure relating taxes to incomes, both personal and corporate. It is tempting to conclude that corporations pay their tax bills more promptly than individuals; but such

Table II

Tax Collections and Economic Activity. Growth Rate in Taxes on Growth Rates in Unemployment and Corporate Earnings, Quarterly Data, 1947:1–1980:1

LINEAR REGRESSION

	Unemployment		Corporate Earnings	
Lag (Quarters)	Personal Taxes (1)	Total Taxes (2)		Corporate Taxes (3)
0	−.0996	−.149	.165	.755
	(−2.81)	(−3.61)	(2.41)	(9.06)
1	−.127	−.0792	.0291	.0558
	(−4.48)	(−2.93)	(.461)	(.701)
2	−.0595	−.0279	.0292	.0774
	(−2.25)	(−1.07)	(.467)	(.938)
3	−.0417	−.0248	−.0779	−.0995
	(−1.52)	(−.918)	(−1.27)	(−1.23)
4	.0180	.111	.0379	.0411
	(.525)	(2.79)	(.612)	(−.501)
	t-statistics are in parentheses			
Durbin-Watson	2.45	2.18		1.56
F-Statistic (Probability Level)	6.59 (>.999)	6.25 (>.999)		19.2 (>.999)
Adjusted R^2	.185	.299		.417

All variables were in rates of growth form, i.e., $\log_e(x_t/x_{t-1})$. Regressions were:

$$T_{P_t} = a + b_0 U_t + \cdots + b_4 U_{t-4} \tag{II.1}$$

$$T_{T_t} = a + b_0 U_t + \cdots + b_4 U_{t-4} + c_0 EPS_t + \cdots + c_4 EPS_{t-4} \tag{II.2}$$

$$T_{C_t} = A + c_0 EPS_t + \cdots + c_4 EPS_{t-4}. \tag{II.3}$$

T_{j_t}: Growth Rate of Tax Collections of type j for quarter t, j = Personal, Total, Corporate, Federal Government Tax Accruals, seasonally adjusted, U. S. Department of Commerce, Bureau of Economic Analysis.

U_t: Growth Rate of Unemployment Rate for quarter t-Civilian Workers, Percent, Seasonally Adjusted, U.S. Department of Labor, Bureau of Labor Statistics.

EPS_t: Growth rate of earnings per share, Standard & Poor's 500 Composite Stocks.

ARIMA

	MA	AR	TR	MA	AR	TR
Lag (quarters)	Personal Taxes			Corporate Taxes		
0	—	—	−.109	—	—	.751
			(−4.09)			(9.96)
1	−.615	−.839	−.132	.939	1.088	.0779
	(−20.0)	(−8.92)	(−5.09)	(11.9)	(9.32)	(1.03)
2	−.965	−1.10	−.0568	.0377	−.157	—
	(−58.5)	(−9.00)	(−2.20)	(.37)	(−1.07)	
3	—	−.274	−.0421	—	—	—
		(−2.24)	(−1.61)			
4	—	−.130			−.0952	
		(−1.34)	—	—	(−1.58)	—

Table II—*continued*

t-statistics are in parentheses		
F-Statistic (Probability Level)	5.55 (>.999)	20.4 (>.999)
Adjusted R^2	.254	.488

MA: Moving Average Component
AR: Autoregressive Component
TR: Transfer Function Component; for personal taxes, the independent variable was U_t; for corporate taxes, it was EPS_t.
See Table I notes for estimation method.

speculations are neither necessary nor germane to our subject here. Suffice it to note that taxes respond strongly to these indicia of economic activity.

In the middle panel of Table II, we report the effect on total Federal tax collections of both unemployment and corporate earnings. The influence of both variables on total taxes is weaker, of course, than the separate influences on their more associated subcategory of tax. Nevertheless, the generally significant pattern remains with the proper signs for both variables.[10]

Again, due to a lack of any guiding theory, we estimated an ARIMA transfer function model for these data. The results show strong intertemporal dependencies in both personal and corporate tax growth rates. After the moving average and autoregressive components in the tax series are accounted for, the effects of unemployment and of earnings are quite similar to those obtained by ordinary regression. The growth rate of unemployment has a strong contemporaneous, first-lagged, and second-lagged effect on the growth rate of personal taxes. The growth rate of earnings has only a contemporaneous effect on the growth rate of corporate taxes but it is very strong.

C. *Increases in the Deficit Elicit Increases in Federal Government Debt*

The link between a Federal deficit and an increase in the level of total Federal debt needs no statistical investigation. It is an accounting identity. The identity can be verified by examining the appropriate item on the Treasury's balance sheet and income statement which are published in the first table of each month's *Treasury Bulletin.* Table FFO-1, "Summary of Fiscal Operations," presents the deficit for recent months and recent years and also shows exactly how it was financed.

The biggest single financing item is "Borrowing from the public," but there are six other items such as changes in Treasury cash balances, special drawing rights, etc. The only one of any significance is "Transactions not applied to year's surplus or deficit." This can be quite large. For instance, during fiscal 1980, the stated deficit was \$58.96 billion. Borrowing from the public was \$70.52 billion and the "Transactions not applied" entry was \$−12.6 billion. Only 58.96 + 12.63 − 70.52 = \$1.07 billion was financed by other items. The "Transactions not applied to this year's surplus or deficit" appears to be simply an accounting ruse. Some of the items included in this category are "net outlays of off-budget federal agencies," "housing for the elderly and handicapped fund," and "Federal financing bank,"

[10] With the anomalous exception of the fourth-quarter lag for unemployment.

Table III

"True" Deficits and Borrowing by the Treasury from the Public, 1972–1980.

Year	Current year's deficit plus transactions not applied to this year.	Public Borrowing
	($ Billions)	($ Billions)
1972	21.93	19.44
1973	14.51	19.28
1974	4.24	3.01
1975	52.50	50.85
1976	73.16	82.91
1977	53.28	53.52
1978	58.33	59.11
1979	36.55	33.64
1980	71.59	70.52

Source: *The Treasury Bulletin*

(footnote to Table FFO-1). These should be counted as part of the true deficit even though they may consist of payments which were contracted in earlier years, or, for some other reason, do not appear in the current year's budget.

The actual deficit plus the "transactions not applied" and public borrowings are shown in Table III for 1972–1980.

Clearly, there is a very close connection between borrowing and the true deficit. The only year with a substantial difference, 1976, is explained by almost a doubling in the Treasury's operating cash, from $7.59 billion in 1975 to $14.84 billion in 1976.

At this point, the causative linkage required to explain the impact of stock returns on *real* interest rates has been successfully traced. Even if there were no monetization of Treasury debt and consequently no impact of stock returns on money growth and expected inflation, we might still find that stock returns were negatively related to changes in nominal interest rates, if there is a real effect of Treasury borrowing.

D. *The Federal Reserve System Buys Part of the Increases in Treasury Debt*

The Federal Reserve System "monetizes" the national debt by purchasing new Treasury issues with newly-created base money. This is a foregone conclusion to many economists; e.g., Hein [21], who points out that the fraction of public debt held by the Federal Reserve, although declining slowly since 1974, is maintained, nevertheless, at a very significant level.[11]

Past attempts to demonstrate the process of monetization over short intervals, however, have been rather strained. For example, Barro [5] needed to fit a complex polynomial in order to find a significant connection between the money supply growth rate and the difference between actual and what he called the "normal" level of Federal expenditures.

[11] It rose from around 12% in 1961 to over 23% in 1974. It had fallen to 16.5% by the end of 1980. These movements were quite smooth over time.

We, too, found an apparently very weak relationship between the monthly growth rates in Treasury debt and the growth rates in Federal Reserve holdings of Treasury debt. A simple regression with up to 18 monthly lags is reported in Table IV. A first look at these results suggests little, if any, Federal Reserve monetization, month-by-month. The only really significant coefficient is for a lag of two months and it is negative!

The sum of the first three lagged coefficients is approximately zero, a marginally significant contemporaneous and first lag being offset by a second lag. Then, the third lag is again marginally significant, only to be offset by the fourth. The sum of all coefficients is only +.199, hardly evidence of material monetization.

However, a closer look at the results suggests some serious econometric problems. The Durbin-Watson statistic indicates negatively autocorrelated residuals, but a more ominous problem for simple regression is the pattern of current and lagged coefficients between the growth rates of Treasury debt and of Federal Reserve holdings of this debt (See Table IV, Columns 4 and 8). The signs are ++−, ++−, ++−, ++−, ++−, (−+−), +. The only deviation from the repeating triplet pattern, ++−, is in the fifteenth lag, and its coefficient is very small. The pattern suggests that one or both of the variables are strongly seasonal with quarterly frequencies. This was verified by the partial autocorrelation functions of each variable, shown in Figure 1, which indicate that the growth rate of Treasury debt is, indeed, strongly seasonal.

The growth rate of Federal Reserve holdings of Treasury debt also displays strong serial dependence but the lags are not the same and the signs of the partial autocorrelation coefficients are negative. Given these patterns, it is very hard to ascertain without more powerful tools whether there is any causative relationship between Treasury borrowings and Federal Reserve issues of base money.

The ARIMA technique is perfectly suited to sort this out. We estimated an ARIMA model for Treasury Debt growth rates, filtered Fed holdings growth rates with the same model, and examined the cross-correlation function of the residuals. *The only significant coefficient was contemporaneous.*

The parsimonious transfer function model reported in the second panel of Table IV was the result: the partial autocorrelation function of the residuals from this model is shown in the bottom panel of Figure 1 and it indicates no significant coefficients out to 18 lagged months.[12]

The results show that there is definitely a strong *and contemporaneous* connection between Treasury borrowing and Federal Reserve issuance of base money. After the seasonals were removed, the contemporaneous coefficient is almost .8 with a t-statistic of 5.4.

E. *Federal Reserve Purchases of Treasury Debt Increase the Monetary Base*

The long-run connection between Federal Reserve holdings of Treasury debt and the level of base money, (currency plus reserves) is very close. Most of the

[12] The moving-average coefficient at lag 7 is only marginally significant. If it is deleted, the partial autocorrelation function of the residuals displays a significant coefficient at lag 7. The other coefficients, however, are almost unchanged—the debt transfer function contemporaneous coefficient becoming .794 with a t-statistic of 4.41. The MA(1) and AR(12) are changed only slightly. Their t-statistics being 5.44 and 2.24 respectively.

Table IV

Growth Rate in Federal Reserve Holdings of U.S. Treasury Debt on Growth Rates of U.S. Treasury Debt
Monthly Data: August, 1968–December, 1980[a]

Lag (Months)	Regression Coefficient	t Statistic	Correlation Coefficient	Lag (Months)	Regression Coefficient	t Statistic	Correlation Coefficient
LINEAR-REGRESSION[b]							
0	.637	1.41	.188	10	.0691	.176	.0421
1	.387	.856	.0925	11	−.713	−1.83	−.235
2	−1.02	−2.27	−.245	12	−.143	−.328	.0692
3	.776	1.72	.123	13	−.179	−.408	.0489
4	−.426	−.937	.0570	14	.138	.317	−.152
5	.200	.436	−.0516	15	−.841	−1.98	−.0626
6	−.243	−.540	.00552	16	.684	1.63	.106
7	.246	.597	.0443	17	−.372	−.888	−.112
8	.262	.644	−.0259	18	.433	1.05	.0439
9	.0691	.176	.0895				
F-Statistic		= 1.45		Adjusted R^2		= .0617	
Probability Level		< .90		Durbin-Watson		= 2.76	

[a] This was the period of data availability on our source, DRI.
[b] The regression model was

$$g_{F,t} = a + b_0 g_{T,t} + b_1 g_{T,t-1} + \cdots b_{18} g_{T,t-18}$$

where g_{Ft} is the continuously compounded growth rate ($\log_e$ first difference) of Federal Reserve holdings of Treasury debt during month t and g_T is the growth rate of total Treasury debt for month t. Both are not seasonally adjusted.

Source: DRI data base derived from the *Federal Reserve Bulletin* and the *Treasury Bulletin*, respectively.

Lag (Months)	MA	AR	TR
ARIMA			
0	—	—	.797 (5.43)
1	.426 (5.44)	—	—
7	.136 (1.72)	—	—
12	—	.186 (2.13)	—
t-statistics are in parentheses			
F-statistic (Probability Level)		9.83 (>.999)	
Adjusted R^2		.164	

MA: Moving Average Component
AR: Autoregressive Component
TR: Transfer Function Component
(The dependent variable was g_F.
The independent variable was g_T.)
See Table I notes for estimation method.

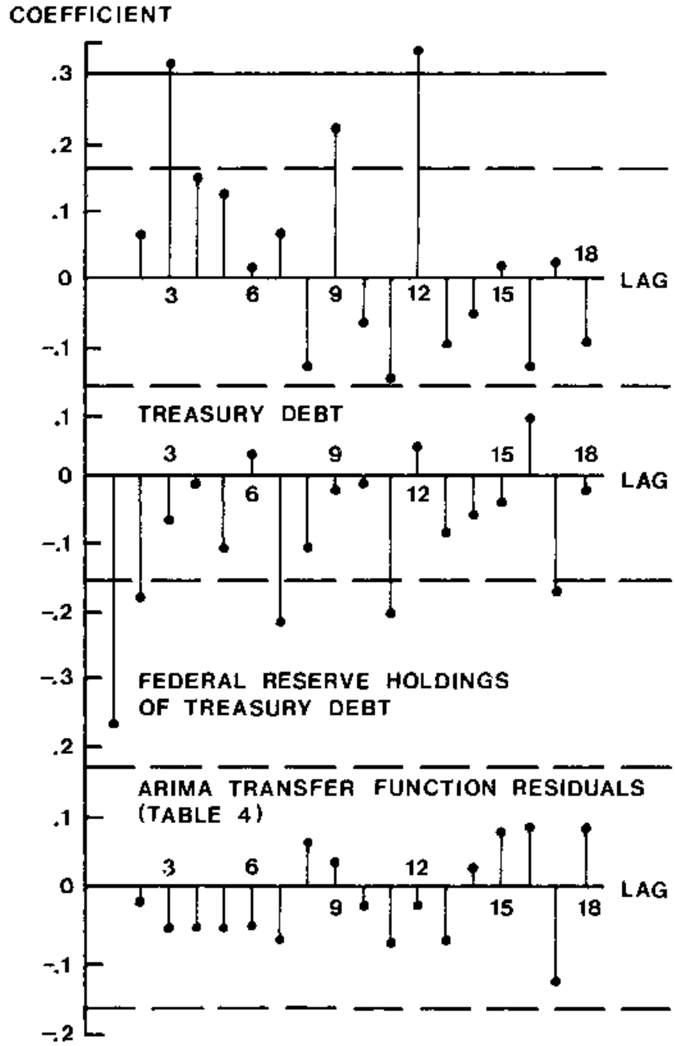

Figure 1. Partial Auto-Correlation functions of growth rates in treasury debt, federal reserve holdings of treasury debt, and residuals from transfer function model of treasury debt on federal reserve holdings[a/b]

[a] For variable definitions, see Table IV.

[b] Dashed lines indicate 95% confidence region for no partial autocorrelation

Federal Reserve's transactions with outsiders involve straight exchanges of base money for Treasury securities.

The Federal Reserve's balance sheet is reported by week in Table A 11 of the *Federal Reserve Bulletin*. The balance sheet for the end of June 1981 was as follows:

Percent of Total Assets

Assets		Liabilities	
Gold Certificates	6.75	Currency	75.57
U.S. Treasury Debt	74.61	Reserves	15.76
Federal Agency Debt	5.48	Other	
Loans to Banks	0.62	Liabilities	6.97
Float	4.29	Equity	1.70
Other	8.25		
Total	100.00		100.00

The "other" asset account consists of SDRs, coins, Banker's Acceptances, bank promises, and foreign-denominated assets. The "other" liability account consists of U.S. Treasury deposits, foreign-owned deposits, deferred availability cash items ("float"), and accrued dividends.

The "other" items are small fractions of the total and do not change much. The gold certificate account does not change at all. Thus, the only outlet for increases in base money other than purchases of Treasury debt are purchases of Agency debt, loans to depository institutions at the discount window, float, and Treasury balances.

Although it seems obvious that long-term increases in the Fed's holding of Treasury securities are associated with increases in base money, the short-term relationship is hidden in a fog of transactions (cf. Friedman) [15]. During 1980, Federal Reserve holdings of Treasury securities increased by $3.87 billion; but this was accomplished only after total transactions, purchases and sales, of $1,596 billion, over 400 times the net increase! The vast bulk of these purchases and sales were "matched transactions" ($1,349 billion) and repurchase agreements ($227 billion). Although these types of transactions may not lead to long-run changes in the money base growth rate, they inject a substantial amount of noise into short-term changes. Imposed onto this noisiness is the seasonality of Fed holdings (see Section D).

Because of the large noise component, the detectable effect of Federal Reserve System Treasury debt holdings on the Fed's issuance of base money is very small in estimated magnitude; however, it *is* significant. The data were analyzed in three ways and the results for each method are reported in Table V. First, simple lagged regressions were computed for the effect of Federal Reserve holdings on the "source" money base, the currency component of the base, and the "adjusted" base.[13]

The currency component and the adjusted base display significant contemporaneous coefficients but the source base displays coefficients which are significant only at lags of three, five, and six months. We do not have an explanation for this pattern.

[13] The reserve adjustment magnitude (RAM) is computed by the St. Louis Federal Reserve Bank to correct the money base for changes in reserve requirements. The RAM factor is applied to the actual base to derive the "adjusted" base.

Using the source base, the least successful variable in the simple regressions, an ARIMA transfer function model was estimated and the results are given in the second panel of Table V. Again, the effect of Fed holdings shows up only with a lag, but now, after the ARIMA expurgation of seasonals, lags of 2, 3, 5, and 6 months are all highly significant. Apparently, Federal Reserve behavior is to string out the permanent increase in the money base after a permanent increase in holdings of Treasury debt.

Finally, in an attempt to overcome the excessive noise induced by the Federal Reserve's enormous volume of open-market transactions, a signal extraction technique suggested by Lucas [30] was implemented. With this technique, the growth rate of the money base and of Fed holdings of Treasury debt are "smoothed" by a two-sided exponentially weighted moving average. A simple regression was then fitted to the contemporaneous smoothed values. The results are reported in the third panel of Table V.

As we anticipated, this method greatly increases the significance level because it amplifies the underlying "signal," the actual long-run impact of Fed holdings on the money base, relative to the "noise," the month-to-month transitory fluctuations in Fed holdings occasioned by open market operations. However, this technique is rather new and we are not certain that the t-statistics can be interpreted in the usual way.

We anticipated that this last link in the chain would be among the easiest to document. A priori, we expected a strong and direct connection between the principal Federal Reserve liability (currency and reserves) and the principal asset (Treasury securities). But this a priori opinion failed to reckon with the incredible short-term churning of the Fed's asset portfolio. Nevertheless, detectable in the noise is the expected pattern, the pattern that must be expected also by stock and bond market participants who cause changes in short-term interest rates to be contemporaneously and negatively related to stock market returns.

F. *The Bottom Line*

Interest rates are determined by market participants who realize that stock returns predict changes in Treasury borrowing and a possible change in base money. Although these latter effects may evolve slowly, they will be anticipated and impounded into current market rates. Even though stock returns signal interest rate changes because other macroeconomic variables react with a lag, stock returns and interest rate changes should be contemporaneously correlated. The true signalling link is: stock returns forecast real activity and *anticipations* of macroeconomic changes which cause interest rate changes. It still remains to demonstrate that this last relationship, working through anticipations of macroeconomic factors rather than their actual values, is empirically supported.

This requires estimation of our reversed causality model (5) of stock returns on changes in Treasury bill rates, which we rewrite, reparameterized, as

$$RF_t - RF_{t-1} = \Gamma_{ot} + \Gamma_1 [\beta RS_t - RF_t] + \Delta_t \tag{5'}$$

where Γ_1 and β are parameters which should be small and of opposite signs (β

Table V

Growth Rates of the Money Base on Growth Rates of Federal Reserve Holdings of U.S. Treasury Debt. Monthly Data: August, 1964–December, 1980[a]

	Money Base Measure		
Lag (months)	St. Louis Fed Source Base	St. Louis Fed Adjusted Base	Currency
	Linear-Regression[b]		
0	.00694	.0527	.0708
	(.475)	(2.01)	(2.41)
1	.0181	.0773	.0252
	(1.14)	(2.71)	(.791)
2	.0185	.0103	.0290
	(1.15)	(.354)	(.897)
3	.0311	.0163	.0173
	(1.93)	(.562)	(.535)
4	−.04631	.00107	−.0299
	(−.00392)	(.0369)	(−.925)
5	.0324	−.0471	−.0488
	(2.04)	(−1.65)	(−1.53)
6	.0326	−.0139	.00947
	(2.21)	(−.526)	(.320)
	Regression Fit		
F-Statistic	1.76	1.96	1.66
Probability Level	<.95	<.95	<.95
Adjusted R^2	.0263	.0332	.0230
Durbin-Watson	2.25	1.60	1.89

[a] This was the period of data availability on our source, DRI.

[b] Coefficients with t-statistics in parentheses.

[b] The regression model was

$$g_{m,t} = a + b_0 g_{F,t} + b_1 g_{F,t-1} + \cdots + b_6 g_{F,t-6}$$

where $g_{m,t}$ is the continuously-compounded growth rate (log first difference) of the money measure and $g_{F,t}$ is the growth rate of Federal Reserve holdings of Treasury debt for month t.

Source of Data: DRI, Derived from *Federal Reserve Bulletin* and *Federal Reserve Bank of St. Louis Review*.

ARIMA (St. Louis Fed Source Base)			
Lag (Months)	MA	AR	TR
1	.115	—	.0140
	(1.51)		(1.11)
2	—	—	.0280
			(2.16)
3	—	.307	.0425
		(4.63)	(3.28)
4	—	—	−.0008
			(−.06)

Table V—*continued*

ARIMA (St. Louis Fed Source Base)

Lag (Months)	MA	AR	TR
5	—	—	.0530
			(4.10)
6	—	.524	.0416
		(7.65)	(3.25)

t-statistics are in parentheses

F-Statistic (Probability Level)	41.4 >.999
Adjusted R^2	.630

MA: Moving Average Component
AR: Autoregressive Component
TR: Transfer Function Component (The dependent variable was g_m. The independent variable was g_F).
See Table I notes for estimation method.

Signal Extraction Smoother

Smoothing Parameter	Contemporaneous Correlation Smoothed Values	Regression Coefficient (t-statistic) Smoothed Values
.9	.428	.0666
		(6.01)
.8	.425	.0963
		(5.96)
.7	.356	.0978
		(4.84)

Both growth rates g_m(using the source base measure), and g_F were smoothed with the same constant. The first and last twenty values of the smoothed series were discarded to avoid a problem with starting and ending the smoother. See Lucas [30] for details of the method.

< 0). Nonlinear methods are necessary to obtain separate estimates of β and Γ_1. We shall also assume that intertemporal variation in Γ_{ot} can be ignored.

Finally, we shall attempt to provide some evidence on the relative importance of the real interest rate component versus the anticipated inflation component in nominal interest rate changes signalled by stock returns. It is quite probably impossible to separate expected real rates and expected inflation rates with any great degree of confidence, but consider the following line of reasoning: if interest rate markets are efficient, the nominal rate RF_{t-1} on date t is decomposable into a real component, r_t, and an expected inflation component, $\bar{I}_t$.

The change in riskless real rates is

$$r_{t+1} - r_t = RF_t - RF_{t-1} - (I_{t+1} - I_t) + v_t$$

where $v_t \equiv (I_{t+1} - \bar{I}_{t+1}) - (I_t - \bar{I}_t)$ is the difference between two successive inflation forecasting errors.[14] The influence of stock returns on real rate changes can, therefore, be estimated from a model such as

$$r_{t+1} - r_t = cRS_t + \epsilon_t'$$

or, in terms of observable variables.

$$(RF_t - I_{t+1}) - (RF_{t-1} - I_t) = cRS_t + \epsilon_t' - v_t \tag{9}$$

The difference in β from (5) and c from (9) should measure the marginal influence of stock returns on the anticipated inflation component of nominal interest rates because of the effect of stock returns on the money base growth rate.

An econometric problem can be anticipated in fitting (9). Although the compound forecast error, v_t, should have mean zero, it will be negatively autocorrelated because it is composed of successive first differences. This can be combatted with a simple transfer function model.

The empirical results are reported in Table VI. The original Fama/Schwert model is shown in the first column. Their estimation period is given in the second panel. We were not able to replicate their results exactly but we did come close.[15] The discrepancy is due to different data sources for Treasury bill rates (the stock returns and inflation rates were identical). We collected end-of-month T-bill prices from the *Bank and Quotation Record* and used the average of the bid-asked quotes. The small difference in results should not be materially relevant to the interpretations below.

The second column of Table VI presents our "bottom line" model, i.e., Fama/Schwert with reversed causality. Estimation was by the method of maximum likelihood. The speed of adjustment coefficient, Γ_1, is very small during the 1953–71 period and during the overall period (1953–80). Nevertheless, it *is* very significant. The coefficeint β_1, representing the marginal impact of stock returns on nominal rate changes via the macroeconomic linkages described earlier, has the anticipated sign and is in a reasonable range. However, our estimation method does not produce a t-statistic for this variable.

The last panel of Table VI reports the reversed causality model estimated by ordinary least squares using stock returns and beginning-of-period bill rates as separate regressors. Apparently, the estimation method makes little difference because OLS estimates of Γ_1 are virtually unchanged and OLS estimates of $\Gamma_1\beta$ are very close to the product of the nonlinear estimates of Γ_1 and β. The OLS

[14] $\bar{I}_{t+1}$, like RF_t, is observed at period t. It forecasts the actual inflation from t to $t + 1$, which is denoted I_{t+1}.

[15] They did not report a regression that excluded the unanticipated inflation variable, $I_t - RF_{t-1}$. Our replication regression, including this variable, resulted in the following comparison:

Coefficient for	Fama/Schwert Original	Fama/Schwert with our T-bill Measure
RF_{t-1}	−6.03	−5.60
	(−1.84)	(−3.09)
$RF_t - RF_{t-1}$	−17.7	−17.6
	(−7.43)	(−2.27)
$I_t - RF_{t-1}$	−.91	−.89
	(1.21)	(−.734)

t-values are in parentheses.

Table VI

Stock Returns and Interest Rates: Fama/Schwert Models, Reversed Causality Models, and Real Rate Models*

				b_1		
$\hat{b}_1$	$\hat{b}_2$	$\hat{\beta}_1$	$\hat{\Gamma}_1$	OLS	ARIMA	$\hat{\theta}^a$
Fama/Schwert $RS_t = b_0 + b_1 RF_{t-1} + b_2(RF_t - RF_{t-1})$		Reversed Causality $RF_t - RF_{t-1} = \Gamma_0 + \Gamma_1(\beta_1 RS_t - RF_{t-1})^b$		Real Interest $RF_t - RF_{t-1} - (I_{t+1} - I_t) = b_0 + b_1 RS_t + (1 - \theta B)\epsilon_t$		
January 1953–December 1980						
−2.96 (−3.90)	−9.37 (−2.49)	−.0836	.0233 (2.77)	−.00233 (−.584)	−.0004 (−.26)	.832 (24.7)
R^2	.0338	.0196		−.00197	.353	
F-statistic (Probability Level)	6.86 (>.99)	7.70 (>.99)		.342 (<.9)	183.9 (>.999)	
January 1953–July 1971 (Fama/Schwert Period)						
−5.56 (−3.08)	−17.3 (−2.24)	−.0961	.0134 (2.28)	.00792 (1.65)	.0002 (.14)	.874 (27.5)
R^2	.0518	.0186		.00777	.390	
F-statistic (Probability Level)	7.07 (>.99)	5.20 (>.99)		2.74 (<.95)	143.0 (>.999)	
August 1971–December 1980						
−2.81 (−1.33)	−7.56 (−1.55)	−.0369	.0773 (2.36)	−.0160 (−2.30)	−.0031 (−.77)	.749 9.90)
R^2	.0141	.0393		.0367	.320	
F-statistic (Probability Level)	1.80 (<.95)	5.58 (>.99)		5.27 (>.99)	53.8 (>.999)	

Reversed Causality Estimate by Ordinary Least Squares
$RF_t - RF_{t-1} = \Gamma_0 + \hat{\Gamma}_1\beta RS_t - \hat{\Gamma}_1 RF_{t-1}$

Period	$\hat{\Gamma}_1\beta$	$-\hat{\Gamma}_1$	R^2	F-statistic (Probability Level)
1/53–12/80	−.00194 (−2.49)	−.0234 (−1.58)	.0166	3.84 (>.95)
1/53–7/71	−.00129 (−2.24)	−.0134 (−.844)	.0141	2.59 (<.95)
8/71–12/80	−.00284 (−1.55)	−.0775 (−1.91)	.0305	2.76 (<.95)

[a] Moving-Average Component. Symbol "B" is the backshift operator. The OLS model was fitted with the constraint, $\theta = 0$.

[b] The non-linear model was estimated by the method of maximum likelihood, i.e., $\hat{\beta}_1$, was the estimate which minimized residual variance.

* t-statistics are in parentheses below regression coefficients.

results suggest that the speed of adjustment coefficient is significantly only in the last period (1971–80). However, since the correct model is nonlinear, we should not rely on the significance levels from the OLS fit.

Neither the Fama/Schwert models nor the Reversed Causality models displayed evidence of autocorrelated residuals for any lag.

The last column of Table VI presents evidence about whether stock returns signal changes in *real* interest rates. The dependent variable is the change in ex post real rates which is very noisy since it contains two successive inflation forecasting errors. Looking first at the OLS calculations, we do find a significant negative signal of stock returns for real rates during the 1971–80 period. The overall period has a negative coefficient but is not significant, and the 1953–71 period actually has a positive and marginally significant coefficient.

In each of these OLS models, the Durbin-Watson statistic was around 2.9. As we anticipated, the dependent variable was constructed in such a way that it behaves as a first-order moving average process. In an attempt to overcome this problem, an ARIMA transfer function model was estimated with a single moving average component and a contemporaneous transfer function component for *RS*, the stock returns variable; this is analagous to the standard (Koyck) transformation to account for first-order serial dependence in the disturbances.[16]

As the results show, the moving average component is highly significant but the estimated effect of *RS* on real rate changes, although it retains the same sign, is not significant. If there is a real rate effect, it seems to be quite small over these long periods. Of course, there could still be a period such as 1980–81 when the real rate effect was larger due to a very low degree of debt monetization by the Federal Reserve.

III. Summary and Conclusions

The negative relationship between stock returns and (a) beginning-of-period short-term interest rates (b) contemporaneous changes in short-term interest rates, and (c) unanticipated inflation, were the subjects of this paper. We argue that relations (b) and (c) are really two ways to measure the same thing. Numerous explanations for the remaining relations have been offered by other authors, but only Nelson's [33] and Fama's [9] money demand explanation is logically consistent and it seems unable to fully explain all of the empirical phenomena.

We offer a supplemental explanation consisting of the following argument: a random negative (positive) real shock affects stock returns which, in turn, signal higher (lower) unemployment and lower (higher) corporate earnings. This leads to lower (higher) personal and corporate tax revenues. Government expenditures do not change to accommodate the change in revenues so the Treasury's deficit increases (decreases). The Treasury responds by increasing (decreasing) borrowing from the public. The Federal Reserve System purchases some of the change

[16] Fama and Gibbons [11] use a very similar model for the change in the real interest rate. Their dependent variable and moving average component are identical but they omit the transfer function part involving the contemporaneous stock return.

in Treasury debt and eventually pays for it by expanding (contracting) the growth rate of base money. Higher (lower) inflation is induced by the altered money base growth rate. Rational investors realize that a random real shock signalled by the stock market will trigger this chain of fiscal and monetary responses. Thus, they alter the prices of short-term securities contemporaneously with the stock return signal. To the extent that an increased (decreased) deficit, triggered by a real shock, is not expected to be "monetized" by the Federal Reserve, the Treasury's increased (decreased) supply of debt securities can also cause an increase (decrease) in real interest rates. Investors decide collectively on whether a particular stock return signifies a change in real rates, in expected inflation rates, or in both. Regardless of the mix between real rate and expected inflation, nominal interest rates must change.

Other papers have established that the beginning-of-period short-term interest rate is negatively related to stock returns. We argue that this finding is at least partly due to a reversal of an adaptive expectations model; the difference between a stock return and the beginning-of-period short-term interest rates causes a negative contemporaneous change in T-bill rates; but if the equation is reversed, it appears that the beginning interest rate level as well as the change in rates cause changes in stock prices.

With data from the past three decades, we have examined every link in the causative chain described above and have found supporting evidence in each case. The fiscal and monetary linkage from stock returns to money base growth is firmly in place. Thus, stock returns signal change in nominal interest rates and changes in expected inflation. There is little evidence for a real rate effect but the data do suggest that such an effect is more likely to have been present in recent periods.

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APPENDIX

Some Econometrics of Reversed Causality

In this appendix, we demonstrate how a regression model containing a reversed causality can produce strange results. We also derive the large sample coefficient biases for the Fama/Schwert reverse causality model, Equation (5) of the text.

An Illustration of Reversed Causality

A simple setup convenient to illustrate the problem is the following "true" time-series model

$$Y_t = X_t - Z_{t-1} + \epsilon_t \tag{A.1}$$

where Y and X are observed at the end of a period and Z_{t-1} is observed at the beginning. The disturbance term ϵ is assumed to be independent of X_t and Z_{t-1}. This could be a "rational expectations" model, Z_{t-1} being an expectation and X_t being an "unexpected" outcome. Y_t would then be a reaction of some third variable to the difference between the outcome and the expectation. Rational expectations would imply, and we assume for illustration, that X_t and Z_{t-1} are independent and $\mathrm{Cov}(X_t, Z_{t-1}) = 0$. We assume also for convenience that X_t and Z_{t-1} are intertemporally uncorrelated and have zero means.

Not knowing the truth of (A.1), an experimenter decides to take X_t as the "dependent" variable and fits the regression model

$$X_t = \hat{\beta}_0 + \hat{\beta}_1 Y_t + \hat{\beta}_2 Z_{t-1} + \hat{\epsilon}_t \tag{A.2}$$

What will be the asymptotic values of the fitted coefficients?

One might think that $\hat{\beta}_2$ would be zero because there is no connection between X_t and Z_{t-1}. This is not true, however, as we now demonstrate. There are two reasons why it fails: first, the regression (A.2) is a multiple regression and its explanatory variables are correlated and, second, the true disturbance term is not distributed independently of the explanatory variables in (A.2).

In terms of the parameters of the true causative model, the reversed model is

$$X_t = Y_t + Z_{t-1} - \epsilon_t \tag{A.3}$$

So the population values of the coefficients in (A.2) are $\beta_o = 0$, $\beta_1 = 1$, $\beta_2 = 1$. The intuition that Z_{t-1} does not have a measured influence on X_t requires that the true (but noncausative) coefficient $\beta_2 = 1$ be completely offset by the econometric bias introduced through the dependence of Y_t on ϵ_t.

In a general model, $y = X\beta + \epsilon$, where the disturbance is related to the independent variables, the large sample coefficient estimates satisfy

$$\operatorname{plim} \hat{\beta} = \beta + \operatorname{plim}\,[(X'X)^{-1}X'\epsilon] \tag{A.4}$$

See Goldberger, [18, p. 270].

In our model (A.3),

$$(X'X)^{-1}X'\epsilon = \begin{bmatrix} \sigma_z^2 & -\sigma_{yz} \\ -\sigma_{yz} & \sigma_y^2 \end{bmatrix} \begin{bmatrix} \sigma_{y\epsilon} \\ 0 \end{bmatrix} \frac{1}{\sigma_y^2\sigma_z^2 - \sigma_{yz}^2}$$

where, for short, $\sigma_{ab} = \mathrm{Cov}(a, b)$.

From (A.1), we note that $\sigma_{yz} = \mathrm{Cov}[(X - Z) + \epsilon, Z] = -\sigma_Z^2$ since $\sigma_{xz} = 0$ by assumption. This implies that the determinant of the matrix is $\sigma_z^2(\sigma_z^2 - \sigma_z^2) = \sigma_z^2(\sigma_x^2 + \sigma_\epsilon^2)$. Also, note from A.1 that $\sigma_{y\epsilon} = \sigma_\epsilon^2$. Thus,

$$\text{plim}\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{bmatrix} \sigma_z^2 & \sigma_z^2 \\ \sigma_z^2 & \sigma_y^2 \end{bmatrix} \begin{bmatrix} -\sigma_\epsilon^2 \\ o \end{bmatrix} \frac{1}{\sigma_z^2(\sigma_x^2 + \sigma_\epsilon^2)}$$
$$= \begin{pmatrix} 1 - k \\ 1 - k \end{pmatrix}$$

where $k = \sigma_\epsilon^2/(\sigma_x^2 + \sigma_\epsilon^2) < 1$.

In particular, the asymptotic value of $\hat{\beta}_2$ is nonzero, which, if taken literally, would imply that Z_{t-1}, a variable observed at the beginning of the period, has an impact on an unanticipated variable (X_t) observed at the end! In reality, Z_{t-1} contains no information about the unanticipated outcome, but the reversed regression model makes it seem to contain such information because the "independent" variables are incorrectly interpreted as causative.

Coefficient Bias in the Fama/Schwert Regression

If the Fama/Schwert regression actually results from a reversal of the adaptive expectations model [Equation (2) in the text] and if the expected money growth reacts negatively to stock returns [Equation (3)], the fitted model is parameterized as

$$RS_t = \beta_{ot} + \beta_1 RF_{t-1} + \beta_2[RF_t - RF_{t-1}] + \mu_t \tag{A.5}$$

where the true coefficients are

$$\beta_1 = 1 + 1/b, \beta_2 = 1/b\gamma$$

and the disturbance is $\mu_t = -(\xi_t/b + \epsilon_t/b\gamma)$. The equation is the same as (6b) of the text.

We assume now that β_{ot} is, in fact, a constant over time. This implies that both the real interest rate and the risk premium on equities are intertemporal constants. Even if they vary, the argument below is unaffected provided that their variation is independent of RF_{t-1} and $RF_t - RF_{t-1}$.

The asymptotic values of the fitted coefficients in (A.5) are

$$\text{plim}\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{bmatrix} 1 + 1/b \\ 1/b\gamma \end{bmatrix} + \begin{bmatrix} \sigma_{RF}^2 & \sigma_{RF,\Delta RF} \\ \sigma_{RF,\Delta RF} & \sigma_{\Delta RF}^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -\sigma_\epsilon^2/b\gamma \end{bmatrix}$$

where shorthand notation is,

$$\Delta RF \equiv RF_t - RF_{t-1}$$

There is a good reason to argue that the off-diagonal elements, $\sigma_{RF,\Delta RF}$ in the moment matrix, are equal to zero. With a constant riskless real rate, the adaptive expectations model becomes

$$RF_t - RF_{t-1} = a + \gamma(\bar{\dot{M}}_{t-1} - RF_{t-1}) + \epsilon_t$$

A nonzero value for $\text{Cov}(RF_t - RF_{t-1}, RF_{t-1})$ would imply that $\text{Cov}(\dot{M}_{t+1} - RF_{t-1}, RF_{t-1}) \neq 0$ or that $\text{Cov}(\bar{\dot{M}}_{t+1}, RF_{t-1})/\text{Var}(RF_{t-1}) \neq 1$. But the expected money growth rate $\bar{\dot{M}}_{t+1}$ should react one-for-one to RF_{t-1}, i.e., the OLS slope coefficient of $\bar{\dot{M}}_{t+1}$ on RF_{t-1} should equal unity. If this be true, then

$$\text{plim}\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = \begin{pmatrix}1 + 1/b\\ R^2/b\gamma\end{pmatrix}$$

where R^2 is the simple coefficient of determination in the adaptive expectations model.[17] The coefficient $\hat{\beta}_1$ of RF_{t-1} in the Fama/Schwert model (A.5) is actually unbiased with respect to its reversed causality value $1 + 1/b$. It will indicate a strong negative influence of RF_{t-1} on RS_t because b is negative and very small. Even so, there is actually no causative relation between RF_{t-1} and the unexpected part of the observed stock return.

The coefficient $\hat{\beta}_2$ of the change in interest rates is biased but the bias is probably quite small because the R^2 should be close to unity in the basic adaptive expectations model, (2).

[17] *Proof:*

$$(x'x)^{-1}x'\epsilon = \begin{bmatrix}\sigma^2_{\Delta RF} & 0\\ 0 & \sigma^2_{RF}\end{bmatrix}\begin{bmatrix}0\\ -\sigma^2_\epsilon/b\gamma\end{bmatrix}\frac{1}{\sigma^2_{RF}\sigma^2_{\Delta RF}}$$

$$= \begin{pmatrix}0\\ -\frac{1}{b\gamma}\cdot\frac{\sigma^2_\epsilon}{\sigma^2_{\Delta RF}}\end{pmatrix} = \begin{bmatrix}0\\ -\frac{(1-R^2)}{b\gamma}\end{bmatrix}$$