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Over-the-Counter Option Market Dividend Protection and "Biases" in the Black-Scholes Model: A Note

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ABSTRACT

Most options are traded over-the-counter (OTC) and are dividend "protected;" the exercise price decreases on the ex date by an amount equal to the dividend. This protection completely inhibits the early exercise of American call options. Nevertheless, OTC-protected options have market values which differ systematically from Black-Scholes values for European options on non-dividend paying stocks. The pricing difference is related to both the variance of the underlying stock return and to time until expiration of the option, but it is quite small in dollar amount.

PRIOR TO THE OPENING of the Chicago Board Options Exchange (CBOE) in April 1973, put and call options on U. S. common stocks were traded in an over-the-counter (OTC) market. Today, options are listed on several different exchanges. However, the Securities and Exchange Commission (SEC) has allowed the listing of options for only about 200 stocks, a small percentage of those traded on the New York and American Exchanges. No stock trading exclusively over-the-counter or on smaller exchanges has listed options. Virtually all of these stocks can, however, still be optioned in the over-the-counter market. Thus, although the dollar volume of options trading over-the-counter may be small relative to listed option trading, there are substantial off-exchange trading possibilities.

The exchange-listed options offer no protection against either stock or cash dividends but, traditionally, options traded in the over-the-counter market have offered cash dividend protection clauses. The typical adjustment, which we term "OTC protection," is to reduce the option's exercise price by the amount of the dividend.

Merton [5] has shown that this OTC adjustment is incorrect and has derived exact protection. An option which has the OTC protection against cash dividends will not have a value equal to an otherwise equivalent option (European or American) on a stock paying no dividends. Assuming that the stock price drops by the amount of the dividend on the ex-dividend date, both put and call options with OTC protection will have lower market values than otherwise equivalent

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options on stocks without dividend payments, at least to the first-order, Taylor series approximation used by Merton. Although this approximation to the loss is useful, it is not accurate when more than one dividend remains to be paid. Thus, practitioners and academics would be interested in the exact amount of the loss.

In Section I, we report on the exact magnitude of loss in option value attributable to OTC protection. We also demonstrate that, while the OTC protection decreases the value of a call option, it does provide the benefit of inhibiting early exercise. It never pays to exercise an OTC-protected American call option prematurely. For put options, to the contrary, the OTC protection increases the probability of early exercise.

There is an important associated empirical issue regarding a variance bias detected by Black and Scholes [2]. In Section II, we analyze the consequences of the fact that the stock price actually drops by less than the amount of the dividend on the ex-dividend date. We find in this case that the Black-Scholes European option pricing model [1] has systematic biases in valuing OTC-protected American call options. The Black-Scholes model overvalues call options on relatively high-variance stocks while undervaluing options on low-variance stocks. These biases were found empirically by Black and Scholes [2] in their test using OTC data.

I. OTC Payout Protection

Because of the stock price decline on the ex-dividend date, cash dividend payments reduce the value of call options and raise put option values, regardless of whether the option is European or American. In frictionless markets, (i.e., without taxes or transaction costs), the stock price should decline by exactly the amount of the dividend on the ex date. In recognition of this possibility, the over-the-counter market attempts to protect option traders by reducing the option's exercise price by exactly the amount of the dividend on the same date.

Since the OTC adjustment attempts to protect against dividend payments, the natural standard for comparison is an otherwise equivalent option on a stock with no dividend payouts. Merton [5] has demonstrated that the OTC adjustment offers partial but not perfect protection. The difference in values can be approximated with a Taylor series expansion. If $C(S, X, \tau)$ is the price of an American call option on a stock whose price is S with time τ until expiration and with exercise price X, then at the ex-dividend date, τ_D, the OTC adjustment can be characterized as $C(S - D, X - D, \tau_D)$, where D is the amount paid out. The difference between the OTC-adjusted call and an American call on a stock with no dividend payouts (i.e., a European call) is

$$L \equiv C(S - D, X - D, \tau_D) - C(S, X, \tau_D) \tag{1}$$

Merton showed that $L \cong -(D/S)C(S, X, \tau_D)$, to a first-order approximation when the option is at-the-money. Merton's analysis holds also for OTC put protection and it results in the same approximate difference values. This approximation to the OTC loss either over- or underestimates its exact magnitude given in Tables I and II (cf. Footnote 2).

The only difference between an American and European option is the privilege of exercising the American option early. Since the OTC-protected American call

Table I

OTC-protected American Call Option Values

$S = \$40.00$, $\sigma = 0.3$

			Expiration (Months)					
			No Dividend (Black-Scholes)			OTC-protected		
r	X	D^*	3	4	6	3	4	6
0.05	40	0.50	2.63	3.07	3.84	2.60	3.03	3.78
	40	1.00	2.63	3.07	3.84	2.57	3.00	3.71
	40	2.00	2.63	3.07	3.84	2.52	2.92	3.59
0.20	40	0.50	3.36	4.02	5.27	3.30	3.97	5.17
	40	1.00	3.36	4.02	5.27	3.27	3.92	5.09
	40	2.00	3.36	4.02	5.27	3.20	3.82	4.92

* The ex-dividend dates are 0.5 and 3.5 months.

Table II

OTC-protected American Put Option Values

$S = \$40.00$, $\sigma = 0.3$

			Expiration (Months)					
			No Dividend (American)			OTC-protected		
r	X	D^*	3	4	6	3	4	6
0.05	40	0.50	2.17	2.474	2.955	2.17	2.446	2.934
	40	1.00	2.17	2.47	2.96	2.17	2.41	2.91
	40	2.00	2.17	2.47	2.96	2.16	2.36	2.87
0.20	40	0.50	1.69	1.86	2.09	1.69	1.84	2.08
	40	1.00	1.69	1.86	2.09	1.69	1.82	2.07
	40	2.00	1.69	1.86	2.09	1.68	1.78	2.04

* The ex-dividend dates are 0.5 and 3.5 months.

does not have a value equal to an otherwise equivalent European call on non-dividend-paying stock, one might wonder whether the OTC adjustment fully protects the call writer against early exercise. The following theorem and proof show that it does.

1A. Theorem

If the dividend is less than the exercise price, and if the stock price falls by the dividend on the ex-dividend date, an OTC-protected American call option will not be exercised prematurely.

1B. Proof

Consider an American call on a stock with only one scheduled dividend payment before the option expires. Denote $-$ as the instant after the ex-dividend date and + as the instant before. Then, after the ex-dividend date, the option has the same value as a European option, and Merton [5] has shown by arbitrage that the European call satisfies $C(S^-, X^-, \tau^-) \geq S^- - X^- e^{-r\tau^-}$. Since $S^- - X^- e^{-r\tau^-} \geq S^- - X^- = (S^+ - D) - (X^+ - D) = S^+ - X^+$, the European call value after the ex-dividend date must also exceed $S^+ - X^+$, the amount received by exercising the American call the instant before the ex-dividend date. Thus, an OTC-protected American call will not be exercised early.[1]

It is intuitive that OTC protection fully inhibits early exercise. Since the call holder can receive the same amount (i.e., $S^+ - X^+ = S^- - X^-$) the instant before or after the ex-dividend date, there is no incentive to exercise in order to "get the dividend." Merton's demonstration that OTC protection reduces value may therefore seem paradoxical since the possibility of early exercise is the only difference between an American and European option. The loss in value seems to imply that the OTC adjustment "overprotects" against dividend payments. In fact, however, the European call is simply a different security than an OTC-protected call, since the latter includes the added feature of a discrete "jump" in both the stock price and the exercise price on a specific date.

The following tables document the exact magnitude of the loss attributable to OTC protection. Both the absolute and relative stock price declines depend upon whether the option is in- or out-of-the-money. The percentage of loss is greatest for out-of-the-money options because the option value is small. However, the absolute size of the loss is rarely greater than one cent except for near-the-money options. In fact, for in- and out-of-the-money options and for a variety of dividend schedules and sizes, interest rates, and expiration dates, the OTC adjustment is an excellent approximation to correct protection. Thus, the tables only present at-the-money values where the absolute size of the difference is large. The OTC-protected option values were computed for puts using an explicit finite difference approximation and for calls using the binomial method (see Geske and Shastri [4]).

Table I illustrates OTC-protected American call option values. The absolute size of the OTC loss is an increasing function of the size of the dividend, the interest rate, and the expiration date. For example, for a \$0.50 quarterly dividend (which is a 5% annual yield when S = \$40) the maximum error is 10 cents (i.e., \$5.27 − \$5.17) or 2% of the true OTC price. The largest tabulated loss is 35 cents (i.e., \$5.27 − \$4.92), or 7% of the OTC price, when the quarterly dividend is \$2.00.

The values of OTC-protected American put options are presented in Table II. Here the absolute size of the OTC loss is an increasing function of the size and number of dividends and a decreasing function of the interest rate and expiration date. For example, the maximum tabulated loss for puts is 11 cents for D = \$2.00, $r = 0.05$, $\tau = 4$ months (i.e., \$2.47 − \$2.36), or 5% of the true OTC price.[2]

[1] Typically the dividend is for less than the exercise price. Sources trading OTC options state that the converse has never occurred. However, if the dividend were ever to exceed the exercise price, our theorem would not hold. We thank Jon Ingersoll for this observation.

[2] Merton's approximation to the loss can be calculated easily for those cases in which only one dividend remains before the option's expiration. For example, in the case of D = \$2.00, $r = 0.20$ and

II. Variance "Biases"

Black and Scholes [2] tested their European call option pricing formula using over-the-counter options market data available before the CBOE commenced trading listed options. They made no adjustments for cash dividends because the OTC protection, although imperfect, was assumed adequate. In general their model performed well but a variance "bias" was detected. The model overvalued call options on high-variance securities and undervalued call options on low-variance securities. We demonstrate now that this variance bias should be expected when using the Black-Scholes European option pricing model to value OTC-protected American calls.

Using numerical methods, we found that the loss due to OTC dividend protection generally increases as the variance of the stock returns increases. (This is true for both puts and calls.) The larger bias for high-variance stocks would, by itself not explain the bias observed by Black and Scholes since they found that low-variance stocks had biases with the opposite sign. But recall the assumption that the stock price declines by the full amount of the dividend payout on the ex-dividend date. The Taylor series expansion Merton used to approximate the loss was based on this assumption, as were our earlier calculations. It is well known, however (cf. Elton and Gruber [3]), that the stock price often falls by less than the dividend on the ex-date because of market frictions such as taxes. The exact magnitude of the stock price decline is an empirical question, but reasonable estimates are in the range of 80 to 90% of the dividend. This implies that $S^- = S^+ - \alpha D$ where α is the percentage drop in the stock price at the ex-dividend date. The OTC protection ignores this empirical phenomenon and reduces the exercise price by more than the stock price drop. This results in a relative gain for call options and implies that there will be circumstances when the OTC protection provides a net gain instead of a loss in option value.

Table III depicts Black-Scholes and OTC-protected call option values for a range of variances and times to expiration when the stock price drops by only 90% of the 50 cent dividend at the ex-dividend date. All the tabulated values are for at-the-money options with an interest rate of 10%. Note that the Black-Scholes model appears to systematically undervalue options on low-variance stocks while overvaluing options on high-variance stocks. This is more noticeable for the longer maturity options. (i.e., $\tau = 4$ or 7 months). Note also that the Black-Scholes model tends to systematically undervalue near maturity options. Perhaps Black and Scholes did not detect this second "bias" because their study used options at the time of issue, when there were 6 months until expiration.

III. Conclusion and Summary

OTC-protected American options are not equivalent to straight European options. Thus, the Black-Scholes European option pricing model should exhibit

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$\tau = 3$ months in Table I, the exact loss is \$3.36 − \$3.20 = 16 cents. Merton's approximation is (2/40) \$3.36 = 16.8 cents. The approximation is less accurate for two or more dividends. For instance, for the same parameters when $\tau = 6$ months, the exact loss is \$5.27 − \$4.92 = 35 cents, while Merton's approximation depends upon what to use for D; using D = \$2 or \$4 for one and two dividends, respectively, gives a loss of 26 and 53 cents, respectively.

Table III

Variance Biases:
Black-Scholes vs. OTC-protected American Call Option Values

$S = X = \$40.00$, $D = \$0.50$, $r = 10\%$, $\tau_D = 0.5$, 3.5, 6.5 months, $\alpha = 0.9$

τ	σ	Black-Scholes (No Dividend)	OTC-protected* (50 cent dividend)
1	0.1	0.62	0.70
	0.5	2.46	2.47
	0.7	3.37	3.37
4	0.1	1.70	1.77
	0.5	5.21	5.18
	0.7	6.95	6.92
7	0.1	2.65	2.73
	0.5	7.09	7.05
	0.7	9.33	9.24

* The OTC protected values assume that the stock price falls by only 90% ($\alpha = 0.9$) of the dividend on the ex-dividend date.

systematic biases when used to value OTC options. We show that the Black-Scholes model will underprice near-maturity call options and calls on low-variance stocks and overprice calls on high-variance stocks. The variance "bias" was indeed found by Black and Scholes [2] in testing their model and can be explained by the following facts. When the stock price is assumed to drop by the full amount of the dividend at the ex-dividend date, the OTC dividend protection results in a loss in option value. For call options this loss increases with the variance. When market frictions such as taxes are considered so that stock prices decline by less than the full amount of the dividend, OTC protection can result in a relative gain in call option values.

The OTC protection does completely inhibit the early exercise of American call options (but not of put options). Thus, the bias in Black/Scholes valuation caused by early exercise of calls (cf. Whaley [6]) is not present with OTC-protected options.[3]

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[3] The proof that OTC protection inhibits early exercise was constructed in perfect markets. This result also holds in imperfect markets as long as the stock price drops by less than the full amount of the dividend.

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