by Richard Roll

After-Tax Investment Results from Long-Term vs. Short-Term Discount Coupon Bonds

The differential taxation of capital gains and ordinary income has important practical implications for the bond investor. With the capital gains tax rate being only 40 per cent of the ordinary rate, it pays to buy bonds whose pretax yields are mostly in the form of capital gains, everything else equal. The lower the coupon, the greater the fraction of total return comprised of capital gains.

Surprisingly, it does not always pay to buy long-term discount bonds in order to postpone the payment of capital gains taxes. Given the same pretax yields on long and short-maturity bonds, it is better, at most coupon levels, to roll over short-term bonds, even though the capital gains tax must be paid earlier.

This suggests a reason why pretax yields are higher on long-term bonds: They must compensate for the higher rate of effective taxation. Empirical evidence supports the hypothesis that the term structure is upward sloping at least partly because a smaller fraction of the total return on long-term bonds is in the form of capital gains.

CAPITAL GAINS are not taxed on an accrual basis. The tax is imposed only when the gain is "realized"—i.e., when the asset is converted into cash. This has led some to assert, and many to believe, that a major advantage of long-term investments is the postponement of taxes on the capital gains portion of the investment return. But tax postponement may not be a net advantage in all circumstances. Reliance upon the supposed benefits of tax postponement can lead to a suboptimal investment policy, a policy dominated by an alternative that involves paying the capital gains tax earlier.

Consider the case of an investor with a reasonably long horizon who buys coupon bonds selling at a discount from par. He may purchase a long-term, low-coupon bond and hold it until the horizon, reinvesting the coupons in the same bond. Alternatively, he may purchase a one-year, low-coupon bond, reinvest the coupon and maturity proceeds in another one-year bond at the end of the first year, and continue to "roll over" every additional year until the horizon. For the moment, ignore both transactions costs and risks. And, just to make things easy, assume that both the long and short-term bonds have the same coupons and the same pretax yields to maturity, which do not change between the current period and the investor's horizon.

A knowledgeable professional might advise this investor to purchase the long-term bond, in order to defer the capital gains tax payment until the horizon date. With the rollover strategy, the adviser might argue, the capital gains tax would have to be paid yearly; the tax paid on

Richard Roll is Professor of Finance at the Graduate School of Management of the University of California, Los Angeles. *He thanks George Constantinides, Jonathan Ingersoll, David Mayers and Stephen Schaefer for their constructive comments.*

coupon income is at the ordinary (higher) tax rate, but the two bonds have identical coupons. Thus, if yields are constant over time, the adviser might conclude that the long-term bond will generate a higher level of after-tax wealth at the investor's horizon.[1] This conclusion is often wrong.

After-Tax Proceeds on Long-Maturity vs. Rollover Strategies

Except in the case of original issue discounts, price appreciation due to amortization of discount is taxed at the capital gains rate (if the bond is held for at least one year), whereas coupons are taxed at the ordinary rate. Currently, the capital gains tax rate is 40 per cent of the ordinary rate. Even though purchase of a long-maturity bond postpones the capital gains tax, the fraction of total returns subject to favorable capital gains treatment declines, the longer the bond's original maturity. In the limiting case of a consol (a coupon bond with infinite maturity), for example, all income is taxed at the higher ordinary rate, provided the yield to maturity remains unchanged during the investment.

The consequences of the tax system can be seen in the following proposition: *Over a one-year holding period, the fraction of total return taxed as ordinary income increases with the bond's maturity* (holding the coupon constant). To demonstrate, let p_t be the market price at date t of a bond whose maturity value is one dollar. Let R be the pretax yield to maturity (assumed fixed) and let C be the coupon rate, which is assumed to be paid annually at the year's close. For the bond to sell at a discount, R would have to exceed C. If T is the bond's maturity, we have, by the definition of the yield:

$$p_t = \frac{C}{R} + \left(1 - \frac{C}{R}\right)b^T,$$

where

$$b \equiv \frac{1}{1 + R}.$$

With a constant yield, the total pretax return over a one-year holding period is R, the capital gain per dollar invested is $R - C/p_t$, and the ordinary income per dollar invested is C/p_t. With the increasing maturity, the bond's price, p_t, falls; so C/p_t rises and $R - C/p_t$ falls with increasing T.

1. Footnotes appear at end of article.

Thus, given a one-year horizon, the best policy is to buy a bond with a maturity of one year, which is just sufficient to qualify for capital gains treatment. The one-year bond has a lower effective tax rate than a longer-maturity bond, because a larger percentage of its total return is taxable at the lower capital gains rate. Figure A shows that the difference is not trivial.

Notice in Figure A that the biggest discrepancy between the long maturity and the rollover strategies occurs at intermediate coupon levels. As the coupon approaches the yield to maturity (12 per cent in Figure A), the fraction of total return taxed as ordinary income approaches 100 per cent for both strategies. As the coupon approaches zero, and the bonds become pure discount instruments, the fraction taxed as a capital gain approaches 100 per cent for both strategies. For the extremes, C = 0 and C = R, there is no tax difference between the strategies. It can be shown by a tedious but straightforward calculation that the maximum difference in after-tax proceeds between the rollover and long-maturity strategies over a one-year holding period occurs at a coupon level of:[2]

$$C_{MAX} = \frac{R}{1 + (1 + R)^{(T+1)/2}}.$$

With an investment horizon longer than one year, the relative merits of long and short maturities are complicated by the reinvestment of coupons and by the payment of capital gains taxes upon rolling over the short bond. Assume a capital gains tax rate of τ_k and an ordinary tax rate of τ_x. For the strategy of rolling over one-year bonds, the net after-tax proceeds at the horizon (t = H) per dollar invested (V_S) will be:

$$V_S = \left[\frac{1 - \tau_k(1 - p) + (1 - \tau_x)C}{p}\right]^H, \quad (1)$$

where p, the market price of a one-year bond, equals

$$p = \frac{1 + C}{1 + R},$$

and all after-tax proceeds are reinvested in another one-year bond at the end of each year.

The investor who purchases a long bond with a maturity at least equal to the horizon will receive net after-tax proceeds per dollar invested (V_L) of:

$$V_L = \frac{p_H - \tau_k(p_H - p_0)}{p_0} - \sum_{t=1}^{H} g_t \left[\frac{p_H - \tau_k(p_H - p_t)}{p_t} \right], \qquad (2)$$

where

$$g_t = \frac{C(1 - \tau_x)}{p_0} \left[1 + \frac{C(1 - \tau_x)}{p_1} \right] \cdots \left[1 + \frac{C(1 - \tau_x)}{p_{t-1}} \right], \text{ and} \qquad (3)$$

$$p_t = \frac{C}{R} + \left(1 - \frac{C}{R}\right)\left(\frac{1}{1 + R}\right)^{T - t}.$$

Figure A Fraction of Total Return Taxed as Ordinary Income (one-year holding period; yield to maturity = 12%)

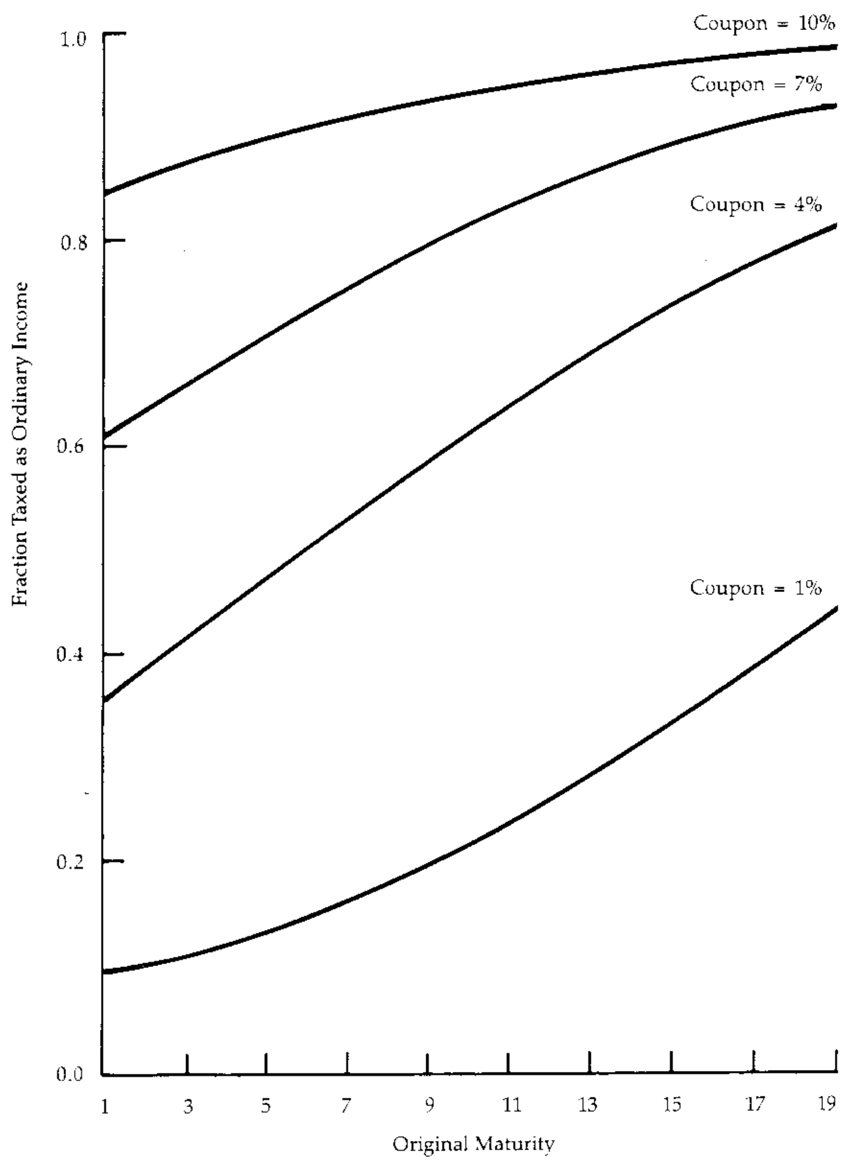
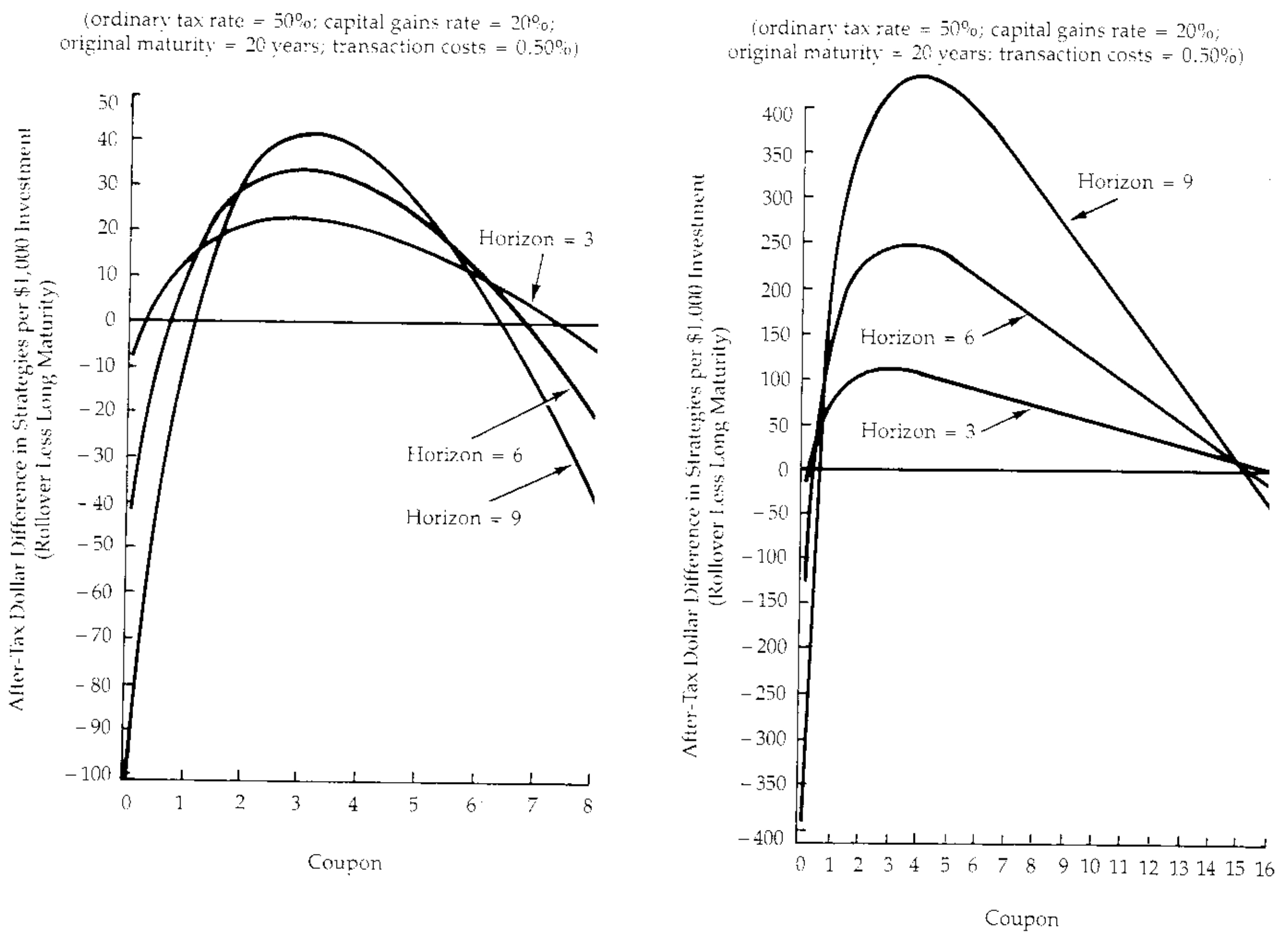


Figure B Rollover vs. Long Maturity

8 Per Cent Yield | 16 Per Cent Yield



Here T is the original maturity of the long bond. Coupons are assumed to be reinvested in the same bond. Total coupon receipts grow over time and are given by Equation (3) for period t.

The difference between the rollover strategy and the long-maturity strategy can be measured by the difference in after-tax proceeds per dollar of initial investment, $V_S - V_L$. There is no general dominance. A higher capital gains tax relative to the ordinary tax increases V_L relative to V_S, but the effects of coupon and yield to maturity are more ambiguous.

Figure B illustrates the effect of each parameter on the best investment strategy.[3] For investors who are currently in the highest marginal tax brackets (an ordinary tax rate of 50 per cent and a capital gains rate of 20 per cent), the rollover strategy is better for most coupon levels. The superiority of the rollover strategy can be substantial. At yields of 16 per cent, for example, it is well over $200 per $1,000 investment for some intermediate coupons and for horizons greater than five years.

Some Complicating Factors

Differences in coupons, yields, trading costs and risk complicate practical bond investment. It is relatively easy to incorporate coupon and yield differences into the previous framework of Equations (1) and (2). For a fixed horizon, simply insert the appropriate values.

Transaction Costs

Trading costs are likely to diminish the relative desirability of the rollover strategy. This is because the entire after-tax market value of the position is reinvested each year, whereas with the long bond, only the coupon payment is reinvested. However, the tax deductibility of trading costs (at the capital gains rate) mitigates this effect.[4]

If q is the percentage trading cost, the rollover strategy's terminal value per dollar invested becomes:

$$V_S^* = \left\{\frac{1 - \tau_k[1 - p(1 - q)] + (1 - \tau_x)C}{p(1 + q)}\right\}^H, \tag{1a}$$

corresponding to Equation (1) above. Note that trading costs are due only on the purchase of the new bond each year; they are not paid on the principal receipt from a maturing bond or on coupons.

With trading costs, the long-maturity strategy's terminal value per dollar invested becomes:

$$V_L^* = \frac{p_H(1 - q) - \tau_k[p_H - p_0 - q(p_H - p_0)]}{p_0(1 - q)}$$

$$+ g_H^* + \sum_{t=1}^{H-1} g_t^*$$

$$\left\{\frac{p_H(1 - q) - \tau_k[p_H - p_t - q(p_H + p_t)]}{p_t(1 + q)}\right\}, \tag{2a}$$

where

$$g_t^* = \frac{C(1 - \tau_x)}{p_0(1 + q)}$$

$$\left[1 + \frac{C(1 - \tau_x)}{p_1(1 + q)}\right] \cdots \left[1 + \frac{C(1 - \tau_x)}{p_{t-1}(1 + q)}\right]. \tag{3a}$$

If the original maturity of the long bond is just equal to the horizon (H = T), then, of course, no trading cost is paid on the principal or the coupon received at the horizon. Figure C illustrates the effect of trading costs on the after-tax difference between the rollover and long-maturity strategies.

Risk

A comprehensive examination of risk and return in bond investments is far beyond the scope of this article. The quantity and type of risk, and its compensating reward, is probably best determined by a portfolio-based asset pricing model. But consider some simple facts about the variability of returns from bond investments.

First, holding-period return variability increases with increasing maturity. The month-to-month standard deviation of long-term government bonds is several hundred times as large as the standard deviation of Treasury bill returns.[5] Second, long-term bonds have greater price sensitivity than short-term bonds to a *given* change in yields, but short-term yields have more variability over time.[6] The greater volatility of short yields does not, however, offset the greater price sensitivity of long-term bonds to yield changes: Long-term bonds have more volatile holding-period returns than short-term bonds.

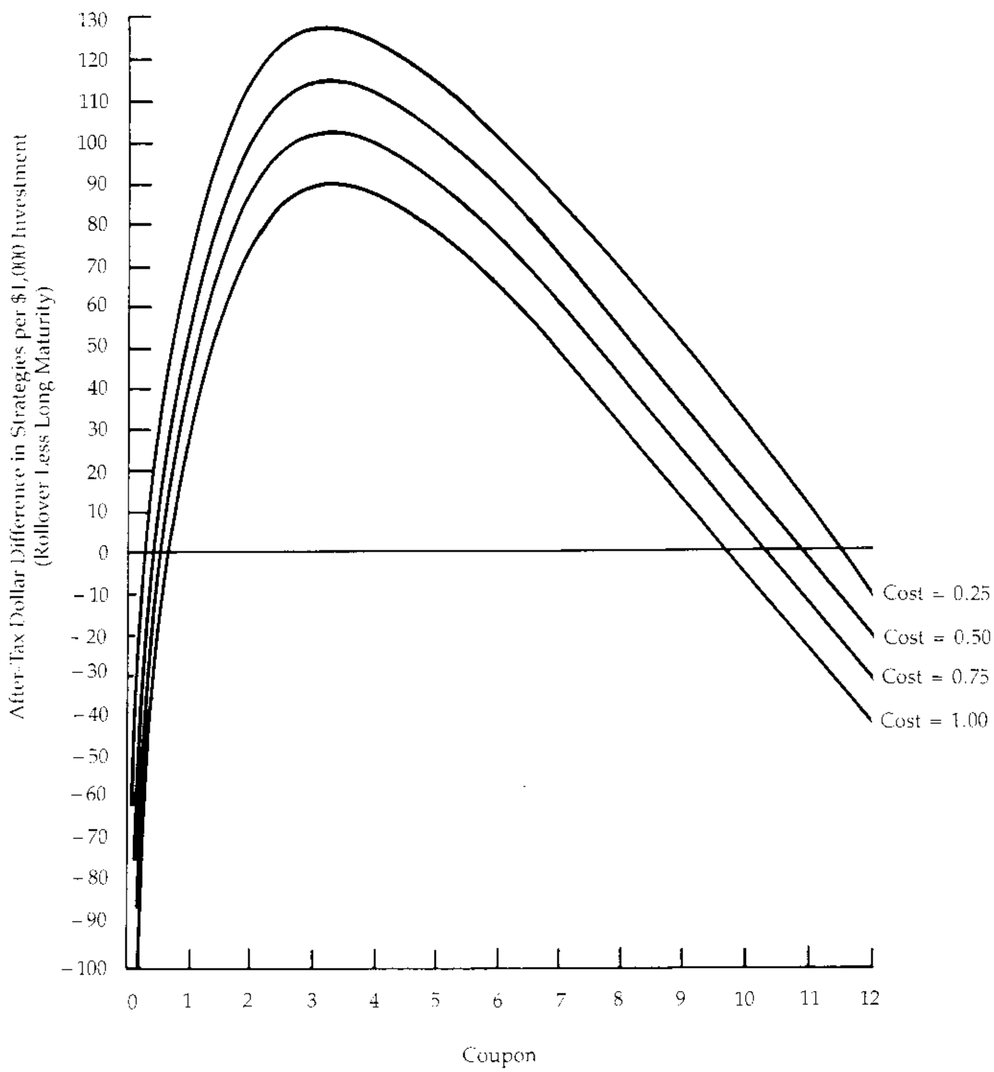
A bond investor choosing between rolling over a one-year position or buying and holding a long bond would find that the rollover strategy has less year-to-year variability in market value. This would no doubt be considered a substantial advantage if the investor's horizon were uncertain. In that case, closing out the position at an arbitrary future date could be accomplished with a lower *a priori* risk of loss.

Many individual bond investors are probably in just this position. For example, a high-net-worth individual who buys discounted bonds on margin in order to secure a capital gain and an interest deduction would consider the long-maturity strategy much riskier. For such an investor, the rollover policy not only offers higher after-tax expected benefits, but also a greater chance of preserving capital.

On the other hand, an investor with a certain horizon date—say, a taxed institutional investor, such as an insurance company—might have lower *a priori* risk with a long-maturity bond that is matched in the "immunization" sense to the horizon. With immunization, interest rate risk is minimized by selecting a bond, or a portfolio of bonds, whose income stream matches an anticipated stream of outflows (such as insurance claims).[7] For example, a zero-coupon bond would be immune to interest rate changes if the investment horizon were fixed by the date of a single liability payment that matched the bond's maturity.

With coupon bonds, perfect immunization against all possible changes in the term structure of yields is not feasible. Immunization strategies can nevertheless greatly reduce risk. For example, the returns on a coupon bond whose after-tax duration matched the investment horizon would be immunized against changes in yields over that horizon.[8] Perfect

Figure C Effect of Transaction Costs
(ordinary tax rate = 50%; capital gains rate = 20%;
original maturity = 20 years; horizon = 6 years; yield to maturity = 12%)



immunization would require that the yields of all maturities moved up or down by the same amount, however; if yields of different maturities did not move in lockstep, some risk would remain.

Inflation introduces another risk to bond management. Although bond yields reflect the *expected* rate of inflation, exposure to *unexpected* changes in inflation remains. The rollover strategy is less susceptible to this risk than the long-maturity strategy. Commitment to a given bond for an entire horizon implies complete exposure to unexpected inflation over the horizon. A rollover strategy, in contrast, is completely exposed to unexpected inflation only during the first period. At the beginning of the second period, when the first rollover occurs, the new yield on the short bond reflects a new level of expected inflation, which differs from the old level at least partly because of the unexpected inflation observed over the preceding period. If the variability of expected inflation is large relative to the variability of real interest rates, the real riskiness of the rollover strategy could be substantially less than that of the long-maturity strategy, despite the latter's ability to insure nominal promises *via* immunization.

Personal Taxes and Equilibrium Bond Yields

On average, the term structure of interest rates is upward sloping; long-term bonds have higher yields than short-term bonds.[9] This empirical regularity has been the subject of numerous studies endeavoring to explain it on grounds ranging from market imperfection to risk aversion. Another, very simple, explanation can be derived from the results presented above: The capital gains-ordinary tax differential alone is sufficient to produce an upward sloping average term structure. Risk aversion, liquidity preference and all the other paraphernalia behind complex term structure theories are not needed to explain observed term structures, provided the following two conditions are satisfied—(1) taxes are effective, and the effective capital gains tax rate is substantially below the effective ordinary rate, and (2) most outstanding bonds are selling at a discount from par.

If these conditions are met, a minimalist term structure hypothesis is sufficient to explain the empirical phenomena: *On average over time, after-tax yields are equal for all maturities.*[10] This implies that all maturities have the same riskiness, perhaps because volatility differences across maturities can be diversified away. The hypothesis implies that bond prices are set in the market so that every strategy, including those discussed above, produces identical after-tax expected wealth. To accomplish this result, bond prices would have to be set in market equilibrium to assure that pretax yields on long maturity bonds are higher than pretax yields on short bonds.

The plausibility of this term structure hypothesis depends upon whether conditions (1) and (2) are valid. What is the available empirical evidence about the differential effectiveness of capital gains and ordinary taxes? Do most outstanding bonds sell at a discount?

The second question is easier to answer, and the answer is yes. Most of the time, few outstanding bonds sell at premiums.[11] True, new coupon bonds are issued at or close to par, and because interest rates are stochastic, one might expect that many seasoned bonds with high coupons would be outstanding during periods of relatively low interest rates. But bonds are usually callable, which has two consequences. First, bonds are actually called when the issuer can refinance at lower prevailing interest rates. Second, even though they have not been called, the very threat of call inhibits high-coupon bonds from rising to substantial premiums.

Issuers of bonds also have a more subtle incentive to leave outstanding only those bonds selling at discounts. Discounted bonds have lower effective rates of taxation, hence the (pretax) cost of debt capital borne by the issuer will be correspondingly less.[12]

Effectiveness of Tax Rates

The effectiveness of tax rates is a more complex subject. It is probably safe to conclude that neither the full statute capital gains tax rate nor the ordinary tax rate applies to the typical bond investor. There are several reasons for this. First, the tax law confers the valuable options of realizing capital losses, deferring capital gains and establishing a short-term status as soon as possible. The net impact of these options is to reduce the effective tax rate.[13]

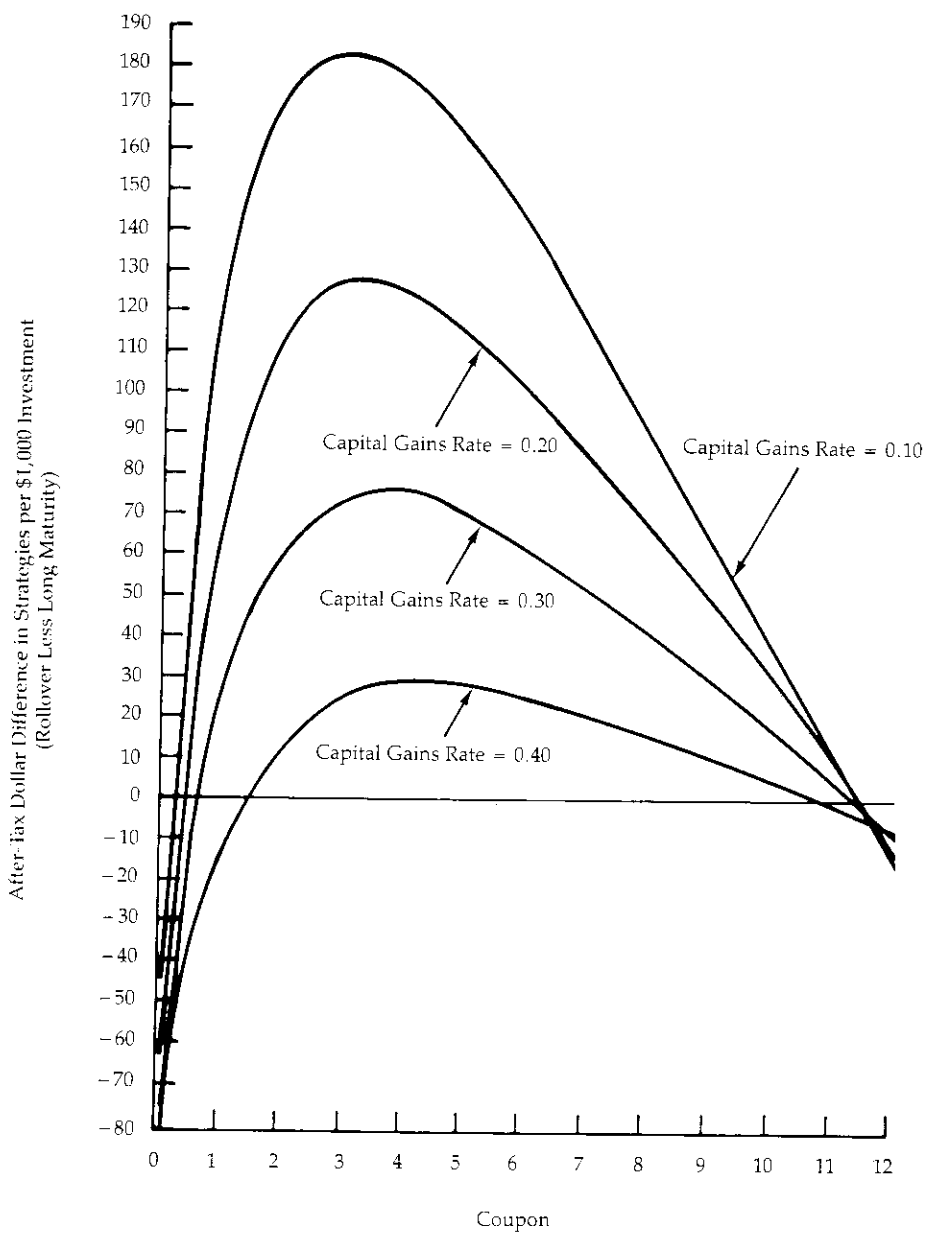
Second, the capital gains tax can sometimes be postponed by appropriate hedging strategies.[14] This increases the differential between the capital gains rate and the ordinary tax rate imposed on coupons. The desirability of the rollover strategy *vis-a-vis* the long-maturity strategy is thereby enhanced. As Figure D shows, the latter is better for relatively high capital gains tax rates.

Third, bond investment strategies can take advantage of tax timing options.[15] This involves such complexities of the law as tax asymmetry in the amortization of discounts and premiums and the ordinary taxation of "original issue" discounts. "Optimal" strategies under these conditions imply equilibrium term structures that differ considerably from the simpler ones in this article, which are, admittedly, based on strategies that may be suboptimal (though widely followed).

Finally, it may be possible to shelter *all* income not used for consumption purchases.[16] Capital gains taxes can be deferred, and ordinary taxes can be converted into capital gains taxes. If such options were fully utilized, there would be no tax difference between the rollover and long-maturity strategies. Furthermore, the general level of taxation would be quite low.

Nevertheless, there does seem to be evidence of general tax effectiveness in municipal bond yields. These federal tax-exempt issues sell at substantially lower pretax yields than even U.S.

Figure D Effect of Capital Gains Tax
(ordinary tax rate = 50%; original maturity = 20 years;
horizon = 6 years; yield to maturity = 12%)



government bonds (which certainly have a lower risk of default). The yield differential has persisted for many years, despite decreasing clientele on the demand side (tax-exempt institutions such as pension funds do not buy municipal bonds) and an arbitrage opportunity on the supply side (municipalities have increasingly taken over from private entities financing activities for hospitals, university dormitories, and even residential mortgages).

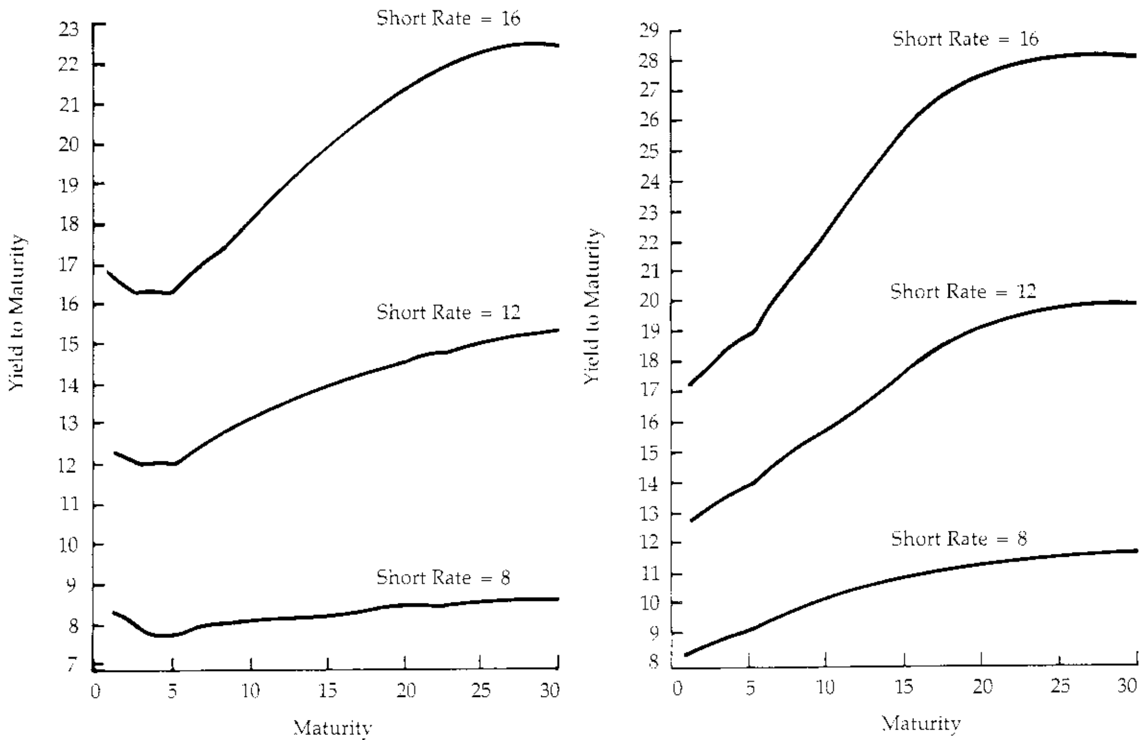
The apparent effectiveness of some kind of tax still leaves the possibility that arbitrage is sufficient to bring ordinary tax rates down to capital gains rates. Studies of dividend yields and equity returns give conflicting evidence about the effectiveness of such arbitrage.[17] Studies of bond yields have invariably found that high-coupon bonds have higher pretax yields

Figure E Pretax Yield Curves

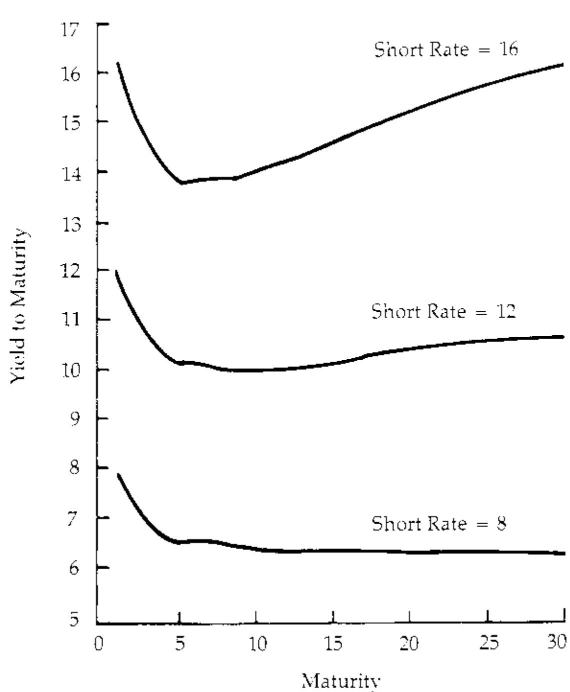
(ordinary tax rate = 50%; capital gains rate = 20%; coupon = 6%; horizon = 5 years; transaction costs = 0.50%)

1: Constant Expected One-Period Interest Rate Case

3: Expected One-Period Rate Increases Geometrically



2: Expected One-Period Rate Decreases Geometrically



than low-coupon bonds, presumably because a lower fraction of their total return benefits from the capital gains treatment.[18]

It would seem, then, that the minimalist term structure hypothesis has at least *prima facie* plausibility. The principle of Occam's Razor can be invoked against complications unless empirical evidence implies that a more complex theory is required.

There may very well be others, but I have been able to find just one compelling piece of evidence *against* taxes being the sole determinant of upward sloping term structures: Longer-term Treasury bills have higher yields, on average, than shorter-term bills.[19] The entire return on bills is taxed at the ordinary rate, and bills have no coupons anyway, so there is simply no tax reason for longer-term bills to have higher yields. Of course, it is still possible that taxes are an important influence on coupon bond term structure. The Treasury bill evidence suggests, however, that there are other factors at work, at least for short maturities.

For longer-maturity bonds, the simple tax hypothesis seems to work quite well, and is even supported by evidence in an article by McCulloch.[20] McCulloch, taking into account the differential taxation of coupons and capital gains, and using a tax rate that best explained the prices of bonds with different coupons, plotted after-tax yields on hypothetical zero-coupon bonds of varying maturities. The resulting after-tax yield curve was almost flat (less than one percentage point difference from two through 25 years to maturity), suggesting that taxes are the predominant influence on longer-maturity bond prices.[21]

Equilibrium Term Structures

Figures E1, E2 and E3 present some hypothetical pretax interest rate term structures based on the minimalist hypothesis that, after taxes and trading costs, expected yields are equal for all maturities.[22] Figure E1 shows yield curves for various tax rates when interest rates are expected to remain the same—i.e., the expected future one-period pretax interest rate equals the *current* one-period rate. Figures E2 and E3 illustrate cases of decreasing and increasing one-period interest rates, respectively. Underlying Figure E2 (E3) is the assumption that the one-period rate is expected eventually to decline (increase) to three-fourths (five-fourths) its current level, and that the change each period covers one-fourth of the remaining distance.[23]

The long-maturity bonds were priced so as to provide the same after-tax proceeds over the horizon.[24] In each graph of Figure E, the pretax one-period rate (labeled "short rate") is fixed at 8, 12, and 16 per cent in order to generate three separate term structures for three different general levels of rates. A comparison of these illustrations shows that tax-determined yield curves will be more steeply sloped, the higher the level of rates.

There are two circumstances under which pretax yield curves can be flat or downward sloping. The first is when one-period interest rates are expected to decline (Figure E2). The second is when the coupon rate is close to the yields (Figure E1, bottom curve). On average, the expected one-period rate should be flat with respect to maturity, so if coupons are substantially below the general level of interest rates, the pretax yield curve will have a positive slope for longer maturities. ■

Footnotes

1. My attention was directed to this problem because of a broker who suggested purchase of the long-term bond for the reason given (with unfortunate consequences).
2. The derivation is a straightforward problem to find the coupon that maximizes $V_S - V_L$ in Equations (1) and (2), given equal pretax yields (R) to maturity. A mimeographed formal derivation of this equation is available from the author on request.
3. Figure B incorporates two features that make the results differ slightly from Equations (1) and (2). Coupons are assumed to be paid semiannually (which is the case for most bonds) and transaction costs of 0.50 per cent (one per cent round trip) are included. Revised equations including transaction costs are given by Equations (1a) and (2a), given later in the text.
4. It is interesting to note that trading costs are tax deductible only because they are added to the "basis" of a purchase and deducted from sale proceeds. This amounts to a second-order tax distortion against high coupon bonds. A consol buyer, for instance, could never deduct his trading costs against coupon income even though he would not expect to receive a capital gain.
5. See Roger G. Ibbotson and Rex A. Sinquefield, *Stocks, Bonds, Bills and Inflation: The Past and the Future* (Charlottesville, VA: Financial Analysts Research Foundation, 1982).
6. See Burton G. Malkiel, *The Term Structure of Interest Rates: Expectations and Behavior Patterns* (Princeton: Princeton University Press, 1966), p. 54.
7. For an introduction to immunization, see H. Gifford Fong, *Bond Portfolio Analysis* (Charlottesville, VA: Financial Analysts Research Foundation, 1980).
8. The duration of a coupon bond is the weighted average of times until the bond's various coupons and principal payments, with each weight being proportional to the after-tax present value of the respective payment.
9. For the most recent evidence, see Edward J. Kane, "Nested Tests of Alternative Term-Structure Theories," *Review of Economics and Statistics* 65 (February 1983), pp. 115–123.
10. This should not be taken to imply that the term structure on every date is upward sloping. Because the long bond can be constructed as a portfolio of successive forward loans plus the shortest bond, long-term yields will be lower than short yields when short yields are expected to decline, and vice versa, aside from tax and risk effects.

    The hypothesis set forth in the text has been suggested by others. See, in particular, Alexander A. Robichek and W. David Niebuhr, "Tax-Induced Bias in Reported Treasury Yields," *Journal of Finance* 25 (December 1970), pp. 1081–1090. It has been intimated by many others—e.g., J. Huston McCulloch, "The Tax-Adjusted Yield Curve, *Journal of Finance* 30 (June 1975), pp. 811–830 and Kane, "Nested Tests," *op cit*. The interaction of taxes, coupons and maturity in Britain (where the capital gains tax rate is zero) has been analyzed by Stephen M. Schaefer, "Measuring a Tax-Specific Term Structure of Interest Rates in the Market for British Government Securities," *Economic Journal* 9 (June 1981), pp. 415–438. His results are qualitatively similar to those in this article.

11. The monthly *Treasury Bulletin* carries recent prices of all outstanding U.S. government coupon bonds and notes. In the January 1980 issue, when interest rates were relatively high (10 to 12 per cent), there were 125 different bonds and notes listed, of which seven were selling at premium and 118 at discount. In the January 1966 issue, when rates were relatively low (around 5 per cent), all of the 59 outstanding issues were selling at discount.
12. See George A. Racette and Wilbur G. Lewellen, "Corporate Debt Coupon Rate Strategies," *National Tax Journal* 29 (June 1976), pp. 165–177.
13. See George M. Constantinides, "Optimal Stock Trading with Personal Taxes: Implications for Prices and the January Seasonal" (Working paper no. 75, Graduate School of Business, University of Chicago, July 1982) and "Capital Market Equilibirum with Personal Tax," *Econometrica*, forthcoming.
14. See George M. Constantinides and Myron S. Scholes, "Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing," *Journal of Finance* 35 (May 1980), pp. 439–449.
15. See George M. Constantinides and Jonathan E. Ingersoll, Jr., "Optimal Bond Trading with Personal Taxes: Implications for Bond Prices and Estimated Tax Brackets and Yield Curves" (Working paper no. 70, Graduate School of Business, University of Chicago, April 1983).
16. See Merton H. Miller and Myron S. Scholes, "Dividends and Taxes," *Journal of Financial Economics* 6 (December 1982), pp. 333–364.
17. See, for instance, the recent exchange between Merton H. Miller and Myron S. Scholes ("Dividends and Taxes: Empirical Evidence, *Journal of Political Economy* 90 (December 1982), pp. 1118–1141) and Robert H. Litzenberger and Krishna Ramaswamy ("The Effects of Dividends on Common Stock Prices: Tax Effects or Information Effects?" *Journal of Finance* 37 (May 1982), pp. 429–443). A puzzling paper by John B. Long ("The Market Valuation of Cash Dividends: A Case to Consider," *Journal of Financial Economics* 6 (June/September 1978), pp. 235–264) finds that a high dividend yield stock, though presumably more heavily taxed, actually has a lower pretax return than an otherwise identical low dividend yield stock.
18. See John S. McCallum, "The Impact of the Capital Gains Tax on Bond Yields," *National Tax Journal* 26 (December 1973), pp. 573–583; McCulloch, "The Tax-Adjusted Yield Curve," *op cit.*, John G. McDonald, "Coupons versus Principal Return: Which is Worth More," In Peter L. Bernstein, ed., *The Theory and Practice of Bond Portfolio Management* (New York: Institutional Investor Books, 1977); John Caks, "The Coupon Effect on Yield to Maturity," *Journal of Finance* 32 (March 1977), pp. 103–115; and Stephen M. Schaefer, "Tax Induced Clientele Effects in the Market for British Government Securities: Placing Bounds on Security Values in an Incomplete Market," *Journal of Financal Economics* 10 (July 1982), pp. 121–159. Joseph W. Conard and Mark W. Frankena ("The Yield Spread Between New and Seasoned Corporate Bonds, 1952–63," in Jack M. Guttentag and Philip Cagan, eds., *Essays on Interest Rates: I* (New York: National Bureau of Economic Research, 1969)), and Robert J. Shiller and Franco Modigliani ("Coupon and Tax Effects on New and Seasoned Bond Yields and the Measurement of the Cost of Debt Capital," *Journal of Financial Economics* 7 (September 1979), pp. 297–318), noting that new issues generally have higher coupons and yields than seasoned issues, concluded that the effective tax rate on coupons is higher than the tax rate on capital gains.
19. See Richard Roll, *The Behavior of Interest Rates* (New York: Basic Books, 1970).
20. McCulloch, "The Tax-Adjusted Yield Curve," *op cit*.
21. Consistent with the disconfirming evidence from Treasury bills, McCulloch's after-tax yield curve was significantly upward sloping for maturities less than one year. It was downward sloping between one and two years (see his Figure 3, p. 824).
22. Equation (2a) does not apply to the case when the investment horizon is longer than the initial maturity of the long bond. For the region $H > T$ in Figure E, I assumed the following long-maturity strategy:
    (1) Buy a long bond with maturity T ($T < H$) and reinvest the coupons in the same bond.
    (2) After the initial bond matures at T, pursue a one-period rollover strategy between T and H.

    In other words, the total dollar return is the result of applying Equation (2a) for T periods and then applying Equation (1a) for $H - T$ periods.

    The exact expression is:

$$V_T^* = \left\{ \frac{1 - \tau_k[1 - p_0(1 + q)]}{p_0(1 + q)} + g_T^* - \sum_{t=1}^{T-1} g_t^* \left[\frac{1 - \tau_k[1 - p_t(1 + q)]}{p_t(1 + q)} \right] \right\} \left\{ \frac{1 - \tau_k[1 - p(1 + q)] + (1 - \tau_x)C}{p(1 + q)} \right\}^{H-T}$$

where g_t^* is given by Equation (3a) and p (without a subscript) is the one-year bond price.

23. In other words, if R_o is the (assumed) one-period rate at present, the expected one-period rate applicable to period t is $R_t = R_{t-1} + (0.75R_o - R_{t-1})/4$ $(t = 1, \ldots)$ for Figure E2 and $R_t = R_{t-1} + (1.25R_o - R_{t-1})/4$ $(t = 1, \ldots)$ for Figure E3. For the lowest yield level in E2, the lower bound on R_t was the coupon rate. This was done to insure that all (future) bonds were selling at discounts.
24. Actually, in the case of nonconstant future expected one-period rates, there is an infinite number of long-term interest rate structures consistent with equal after-tax proceeds between the rollover and long maturity strategies. For ease of illustration, Figures E2 and E3 assume that the forward rates applicable to a given long bond change in the same pattern as expected one-period rates. That is, the price of a T-period bond is given by the standard formula:

$$p_T = \frac{c}{1 + \rho_0} + \frac{c}{(1 + \rho_0)(1 + \rho_1)} + \cdots + \frac{1 + c}{(1 + \rho_0)(1 + \rho_1) \cdots (1 + \rho_{T-1})},$$

where ρ_0 is the implicit one-period rate on the long bond (which is different, because of taxes, from the one-period short bond rate), and $\rho_j (j \neq 0)$, the forward rate for period j, evolves as follows:

$$\rho_j = \rho_{j-1} + (k\rho_0 - \rho_{j-1})/4$$

where $k = 0.75$ in Figure E2 and $k = 1.25$ in Figure E3. The pretax yield (plotted in Figure E) is then computed as the internal rate of return equating p_T to the stream of coupons and the principal repayment.

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boom began to fall apart prior to the peak in interest rates. The following table summarizes the results (detailed results are available on request):

	Average Weight of Top 5 Groups (%)	*Average T-Bill Yield (%)*
1974:1–1975:1	40.9	8.3
1975:2–1978:2	36.5	5.5
1978:3–1981:3	38.5	11.3
1981:4–1983:1	37.0	10.5

As in my original analysis, the *levels* of the two series do not correlate closely, but the *movements* do.

The other set of evidence is more questionable in character but is still consistent with the hypothesis. It measures risk in terms of variability of monthly returns. Wells Fargo calculated the beta of the lowest quintile by market value of the S&P 500 against the full index for running 12-month periods from January 1972 through December 1981. If this beta rises, it would suggest that the index was becoming less variable relative to the small stock group; if this beta falls, it would suggest that the index was becoming more variable relative to the small stock group.

The results for the first two years were simply noise. After that, however, over four swings down and then back up again in interest rates, the small stock beta was higher at the troughs than it had been at the preceding peaks and vice versa. In other words, by this measure, the broader market measure was becoming more variable relative to small stocks as interest rates rose and was becoming less variable relative to small stocks as interest rates fell—precisely as I had hoped to find (detailed data available on request).

The difficulty with this measure goes beyond the random qualities of the relation in the first two years. The levels of interest rates and the small stock beta show even less consistency than in the other measure. Furthermore, the small stock beta seems to pursue cycles on its own that are not readily understandable to me but that appear to be independent of the interest rate cycles. The interest rate cycles just happened to catch the beta cycles in such a way as to produce the result described here.

—Peter L. Bernstein
President
Peter L. Bernstein, Inc.
New York