Mortgage Securities Research

Adjustable Rate Mortgages: The Indexes

by Richard Roll

The Standard Indexes for Adjustable Rate Mortgages

The coupon rate of an adjustable rate mortgage (ARM) "adjusts" over time because it is linked to an index, a published number specified in the original contractual indenture of the mortgage.

Most ARMs are linked to constant maturity U.S. Treasury yield indexes or to a thrift institution Cost of Funds Index (COFI). The coupon of an ARM is calculated by adding a specified amount, the "margin," to the index, subject to limitations on the maximum and minimum coupon during the mortgage's life or to maximum and minimum changes during a given period.[1]

The ARM indexes differ considerably from each other. The Treasury indexes are selected at different points along the yield curve. The COFI

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This is the second in a series of four papers about adjustable rate mortgages by the same group of authors. The other papers are entitled "Adjustable Rate Mortgages: An Introduction," "Adjustable Rate Mortgages: Prepayment Behavior," and "Adjustable Rate Mortgages: Valuation." Copies of all papers in the series are available from a Goldman Sachs representative.

The collaboration of Lynn Bartholomew and Jonathan Berk, co-authors on several of the papers in this series, is gratefully acknowledged.

[1] Details about these limitations, or "caps," are given in our earlier paper, "Adjustable Rate Mortgages: An Introduction."

(2) Summer

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adds a further complication because it is partly a "book," or accounting, cost rather than a true market rate.

The behavior of the index is one of the principal influences on the actual cash flows paid by the mortgagor and on the market value of the mortgage. Thus, without a basic knowledge of the common indexes to which ARMs are linked, it is virtually impossible to analyze the relative merits of different ARM instruments.

The intention of the present paper is to provide empirical data about the indexes so that investors and issuers can make decisions based on better information.

How the Indexes Are Computed

The U.S. Treasury Security-Based Indexes

The H.15 Six-Month Treasury Bill Index. This index is the weekly auction average 6-month Treasury bill rate as reported by the Board of Governors of the Federal Reserve System in Federal Reserve Statistical Release H.15.[2]

Because T-bills are auctioned every week, there are approximately four possible values of this index reported each month. The value actually used for the index of an ARM must be specified in the ARM's indenture, so ARMs can and do differ by the reporting date of the H.15 series. The most common date corresponds to the auction on or immediately prior to the 15th of the month.

The H.15 6-month rate is a *weighted* average in the auction, the weights being proportional to the amounts sold at various rates. It is not the simple auction average. The H.15 average is quoted on a bond equivalent yield basis, whereas the simple auction average is quoted on a bank discount basis.

The H.15 One-Year Treasury Index. The 1-year Treasury index is published weekly by the Board of Governors of the Federal Reserve System in Federal Reserve Statistical Release H.15. The yield is for a "constant maturity" and is estimated from the Treasury's daily yield curve. The yield curve is fitted by hand to the median closing bid yields on actively traded Treasury securities in the over-the-counter market, as reported by five leading U.S. government securities dealers to the Federal Reserve Bank of New York.[3]

The index is then computed as a weekly average of the daily fitted values. As we shall see below, this averaging process introduces a

[2] The index can also be found in the *Federal Reserve Bulletin*, Table 1.35.

[3] We are told by authoritative sources at the Federal Reserve that the five dealers usually quote the same prices; so the median bid is usually the same as *every* bid.

characteristic pattern into the resulting index' behavior. Like the 6-month T-bill index, the H.15 1-year index has four or more values each month, and the value used depends on the particular ARM. Most often, it is the rate on or immediately prior to the 15th of the month, but there are some ARMs that use the last full week of the month and conceivably, of course, any other choice could be made by the originator.

The H.15 Five-Year Treasury Index. The 5-year Treasury index is calculated in the same manner as the 1-year index. It also is published weekly by the Board of Governors of the Federal Reserve System in Federal Reserve Statistical Release H.15. The appropriate rate to use for the month's index will, again, depend on the particular mortgage.

Cost of Funds Indexes

The Eleventh District Cost of Funds Index. The 11th district COFI represents the monthly weighted average interest cost for the liabilities of member institutions in the Federal Home Loan Bank's 11th district (Arizona, California, Nevada).

Interest paid on savings accounts, Federal Home Loan Bank (FHLB), advances, and other borrowing are included in total interest expense. This total is divided by the average of the two most recent month end balances of total savings capital, FHLB balances, and other borrowed funds. The final figure is annualized and adjusted for variation in the number of days in each month. The previous month's COFI is reported on or near the last working Friday of each month by the Federal Home Loan Bank of San Francisco. There is no document number.

The Monthly Median Cost of Funds for Federal Savings and Loan Insurance Corporation-Insured Institutions. This index includes interest on deposits, FHLB advances, and other borrowed money as a percentage of deposits and borrowed money at month end. Like the 11th district COFI, it is annualized and adjusted for variation in the number of days in each month.

The data for these calculations are obtained from the Federal Home Loan Bank Board's monthly financial report, which is required from all savings and loan institutions and savings banks that are Federal Savings and Loan Insurance Corporation (FSLIC)-insured.

The median COFI is reported monthly by the Federal Home Loan Bank Board (FHLBB), with the previous month's index rate available on or about the 11th day of the current month. The rate is included in the News release entitled "ARM Index Rates." There is no document number. Although the median COFI was introduced as an index in December 1982, the FHLBB publishes historical data back to January 1978 for this particular index.

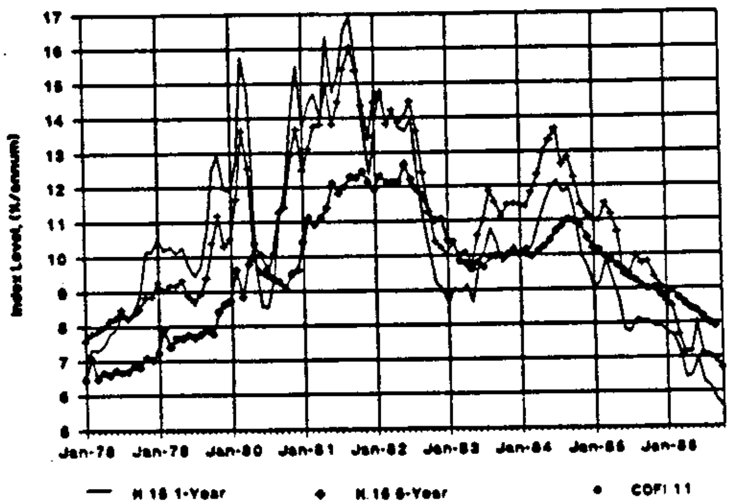

Figure 1. Arm indexes: H. 15 -1 and 5-year, 11th district COFI.

Behavior of the ARM Indexes

Figure 1 presents a plot since January 1978 of the three most commonly used ARM indexes, the 11th district COFI and the H.15 1- and 5-year indexes. Figure 1 reveals that the indexes are quite volatile and that they have experienced similar movements. Since the end of 1981, there has been a general downtrend. Over the 9 years, monthly percentage changes in the indexes have been as summarized in Table 1.

Over the entire period, there has been little net change in the index levels; the average percentage changes are all very close to zero. However, as Figure 1 shows, the fact that the indexes are now close to

Table 1. Percentage Rate of Change in Monthly Index

	Mean	Standard Deviation
	(%/Month)	
H.15 1 year	−.187	8.21
H.15 5 year	−.083	5.94
COFI 11	.190	3.43
	Correlations (in %) of Monthly Percentage Changes[a]	
	H.15 One-year	H.15 Five-year
H.15 5 year	92.4	
COFI 11	−1.39	1.74

[a] Correlation coefficient × 100.

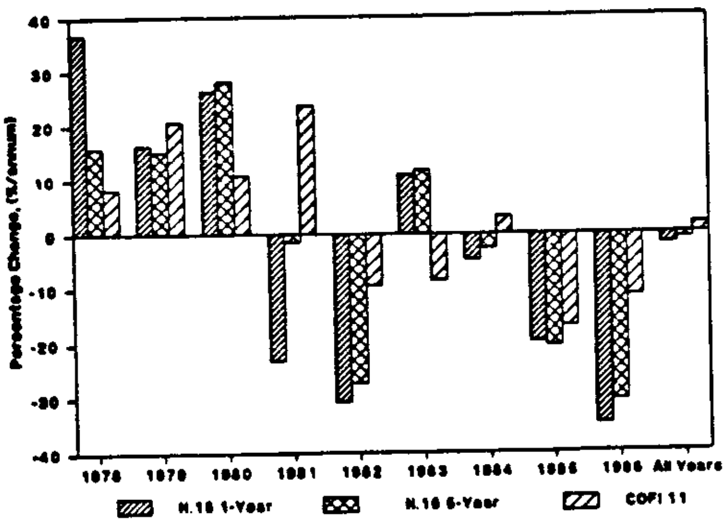

Figure 2. Arm indexes: percentage change by calender year.

where they were in January 1978 is fortuitous. There has been a major up and down movement over the sample period.

The standard deviations show that the COFI is the least volatile of the three indexes, by a wide margin, and that the 5-year index is considerably less volatile than the 1-year index.

Finally, the correlation coefficients indicate that the two H.15 indexes were highly correlated with each other but were virtually uncorrelated, from month to month during this sample period, with the COFI. The COFI may be slightly more correlated with the 5-year index than with the 1-year index but the difference is not statistically meaningful.

A visual impression of the comovements of the indexes is given in Figure 2, which shows the average monthly percentage changes during each calendar year from 1978 to the present. The two H.15 indexes moved in the same direction during every calendar year (i.e., the average percentage changes are *both* either positive or negative) while the COFI index moved in a different direction during several years and actually had a slightly different direction, positive rather than negative, over the entire period.

Figure 3 presents the monthly percentage change volatility by calendar year beginning in 1978.[4] Although it is commonly believed that

[4] There are 12 observations in each year except the first and last years. In 1978, there are only 11 observations, because the first month is lost in computing a percentage change between successive months and December 1977 was not available. Data for 1986 were available through September while this paper was being written.

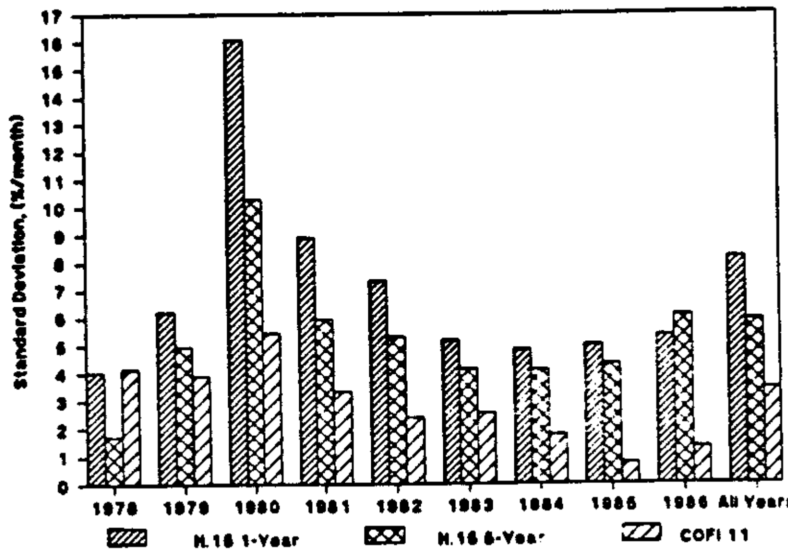

Figure 3. Arm indexes: volatility bt calendar year.

the volatility *of individual fixed-income securities* has increased during 1986, there does not appear to have been a long-term increase in the ARM indexes' volatilities over the past 8 years. The volatility of the COFI was greatest in the early years, and it has been relatively low and generally decreasing ever since.

The volatilities of the two H.15 indexes display some increase from 1985 to 1986, but they remain well below the levels experienced in 1980–1981. The record year for volatility was 1980, when the H.15 1-year index had a monthly standard deviation of percentage changes of 16.1%, twice as high as in any other year. The volatility of the H.15 5-year index was 10.3% in 1980, again twice as high as any year *except* 1986. (The volatility through 10 months of 1986 was 6.09%.)

One would expect, a priori, that the ARM indexes might display a significant degree of serial dependence over time and thus be susceptible of prediction. The Treasury-based indexes are weekly averages of daily data, and such averaging can introduce a particular form of dependence over time even when the basic numbers used in the average follow a random walk. The COFI indexes also should be dependent over time because they are constructed partly with accounting ("book") numbers, which are known to lag behind market-based numbers.

There is also reason to anticipate that the predictability of the indexes may have changed over time. The COFI index may have experienced a significant change in structure when thrift institutions were deregulated in the early 1980s. In the process of deregulation,

Table 2. Correlations (in %) of Monthly Percentage Changes January 1982–September 1986

	H.15 One-Year	H.15 Five-Year
H.15 5 year	90.4	—
COFI 11	40.2	37.2

thrifts became more susceptible to fluctuations in market interest rates and their costs of funds probably became more market dependent. Indeed, there is direct evidence in support of this view from the data themselves. Over the past 5 years, 1982 to the present, the correlations between the COFI and the H.15 indexes have been higher; see Table 2.

As we shall show later, the contemporaneous comovements reflected in correlation coefficients do not give a complete picture of the complexity of these indexes. There is still a significant difference in the behavior of the H.15 and the COFI indexes. To model and predict them, we have been obliged to develop distinctly different approaches.

The H.15 Indexes

The H.15 constant maturity Treasury yield indexes are intended to adhere closely to current market interest rates. We have compared the indexes with true market rates, for 1-year and 5-year Treasury securities, observed on a particular *day* every month, on or immediately prior to the fifteenth. The market rates are obtained from the Goldman Sachs data base and they have been carefully checked for errors and omissions. The data base is complete for 1-year market rates back to 1978, but the 5-year market rate is available only from March of 1980.

Figures 4 and 5 present a comparison of the H.15 indexes and the market rates for 1- and 5-year maturities, respectively. The top panels in both Figures 4 and 5 present a plot of the levels of the two series over time, and the bottom panels show how the monthly changes relate to each other.

This visual impression given by the series *levels* is an extremely close connection between the H.15 series and the market interest rates. Indeed, it is hard to distinguish the two series on many dates. Unfortunately, this visual impression is misleading. The scatter diagrams between the monthly percentage changes (lower panels of Figures 4 and 5) show correlations that are far from perfect.

The correlation coefficients between the H.15 and the market changes are given in Table 3. These correlations have a remarkable pattern: Even though they are for different maturities, the two H.15 series are highly correlated, and so are the two market series; both correlations are over 90%. However, the correlations between either of

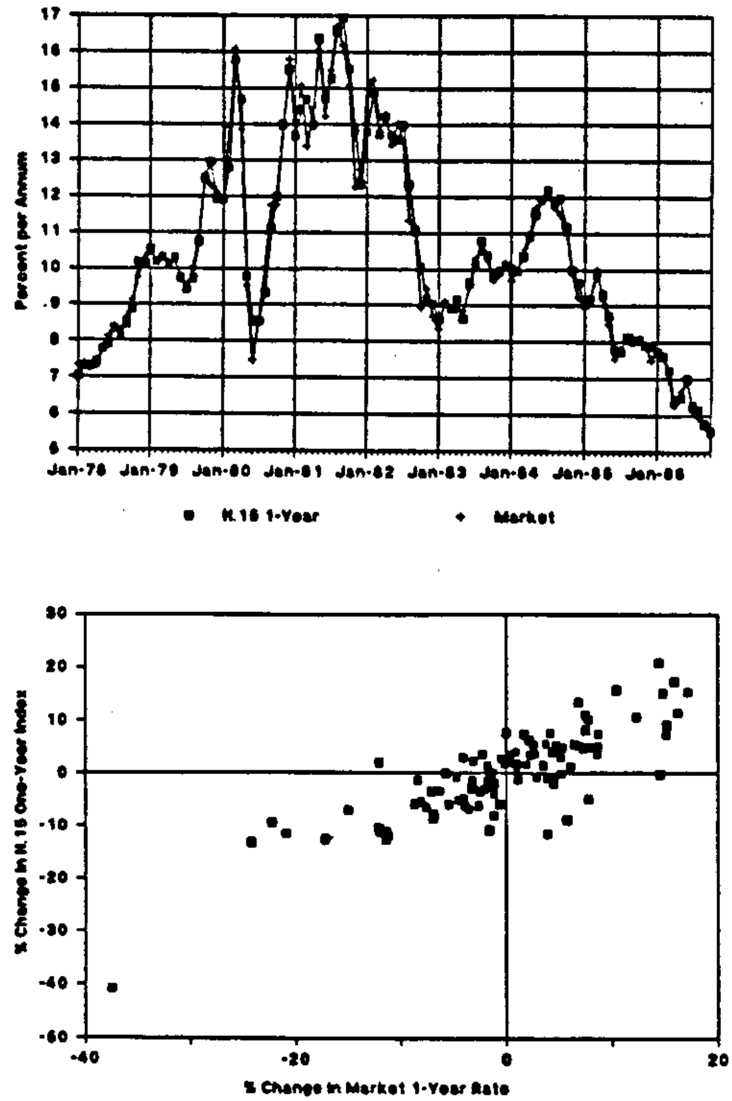

Figure 4. (a) One-year rates: H.15 versus market; and (b) monthly percentage changes: H.15 and market one-year rates.

the H.15 series and either market series are lower even when the maturities are the same. This strongly suggests that there is something factitious in the way the H.15 indexes are constructed. The method of construction induces a reduction in contemporaneous comovements with actual market rates and a corresponding increase in the comovements of the indexes themselves.

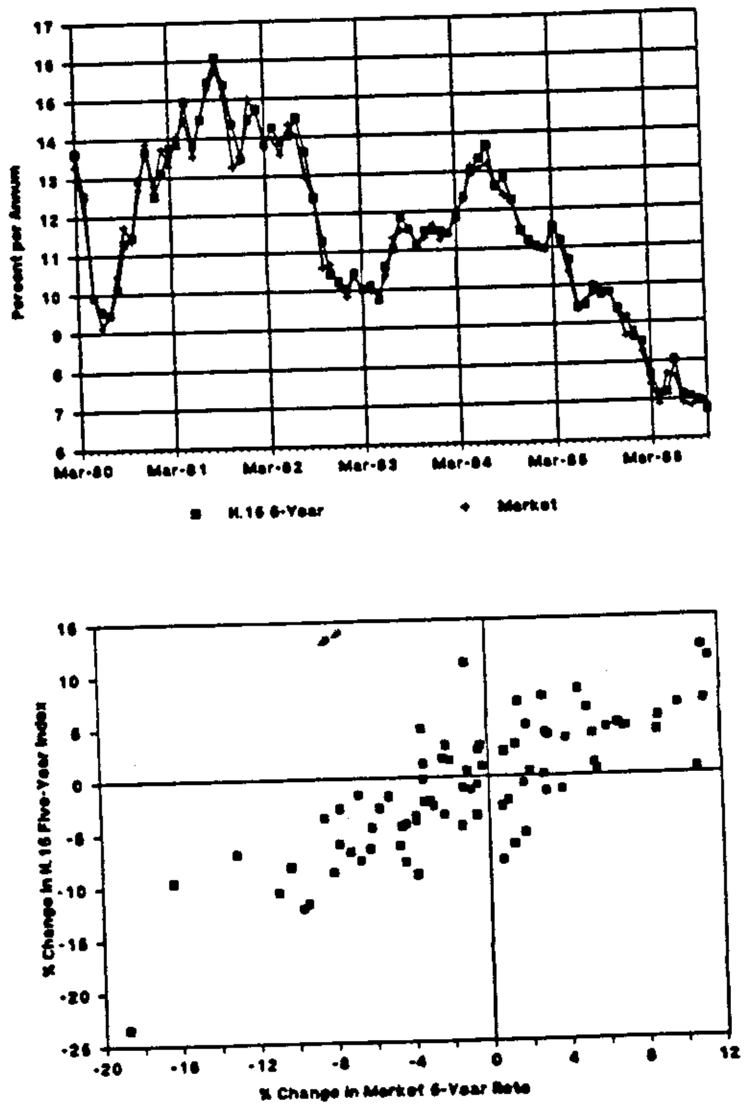

Figure 5. (a) Five-year rates: H.15 versus market; and (b) monthly percentage changes: H.15 and market 5-year rates.

Notice, however, that the correlations are slightly higher between market and H.15 series *for the same maturities*. It is 82.6% between the H.15 and the market 1-year rates and 80.1% between the H.15 and market 5-year rates. The cross-correlations, H.15 1-year versus market 5-year, and vice versa, are slightly lower, 76.0 and 76.8%, respectively.

Table 3. Correlations (in %) of Monthly Percentage Changes April 1980–September 1986[a]

	H.15 One-Year	H.15 Five-Year	Market One-Year
H.15 5 year	92.4		
Market 1 year	82.6	76.8	
Market 5 year	76.0	80.1	92.4

[a] Correlation coefficient × 100. This is the period over which all series had observations.

This suggests that the method of index construction may not have completely obviated the comovements between equal maturity series.

Another aspect of the H.15 indexes, an aspect not revealed by the contemporaneous correlation coefficients, is a lagged structure. The percentage change in the H.15 series in a given month is related to the percentage change in the previous month. During the past 48 months, for example, the H.15 1-year monthly percentage change has a correlation of 24% with the prior month's percentage change. The correlation is 15% for the H.15 5-year index.

In order to model both contemporaneous and lagged components of H.15 index behavior, we have employed a multiple regression model that contains both current and lagged values of market rates with the same maturity and lagged values of the H.15 index itself. The regression results fitted with data from 1982 through the present are presented in Table 4.

For both the H.15 1- and 5-year indexes, about 85% of the variability in monthly changes is explainable by current and lagged values of market interest rate changes plus lagged values of the index. The lagged terms are very important; if the changes in the single contemporaneous variable are excluded, about 30% of the monthly variability is still explained. This means that almost one-third of the monthly changes in the H.15 indexes can be forecast a month *in advance* with previous monthly changes in market rates combined with previous monthly changes in the index. About 80% of the monthly change can be forecast a few days in advance of the indexes' announcement.

We have examined the stability of this relation over time. All of the variables included in the regression were also statistically significant, and the coefficients were similar in magnitude with regressions calculated over previous time periods ending as far back as 1982.[5] It thus seems reasonable to deduce that the construction of the H.15 indexes

[5] The regression results probably were similar even earlier; but because our data sample begins in January 1978 and we include at least 48 months in each regression, the earliest prediction date possible was February 1982 for the 1-year index. The 5-year market rate was only available from March 1980, so the regressions for the 5-year index did not extend back as far.

Table 4. Multiple Regression Model for Monthly Percentage Changes in H.15 Constant Maturity Treasury Indexes (January 1982–September 1986)

Holding other Things Constant, a 1% Change in the	Would Cause a Change in the [H.15 One-Year Index Rate]$_t$ of[a]	
[Market 1-year rate]$_t$	.646%	(±.086%)[b]
[Market 1-year rate]$_{t-1}$	.809%	(±.172%)
[Market 1-year rate]$_{t-2}$	.343%	(±.195%)
[H.15 1-year rate]$_{t-1}$	−.730%	(±.240%)
[H.15 1-year rate]$_{t-2}$	−.179%	(±.170%)

Volatility explained [Adjusted R-Square] = 85.7%.

Holding other Things Constant, a 1% Change in the	Would Cause a Change in the [H.15 Five-Year Index Rate]$_t$ of	
[Market 5-year rate]$_t$	.698%	(±.107%)
[Market 5-year rate]$_{t-1}$	.976%	(±.187%)
[Market 5-year rate]$_{t-2}$	.491%	(±.233%)
[H.15 5-year rate]$_{t-1}$	−.899%	(±.244%)
[H.15 5-year rate]$_{t-2}$	−.261%	(±.191%)

Volatility explained [adjusted R square] = 84.2%.

[a] $[X]_t$ designates variable X in month t.
[b] The numbers in parentheses indicate a 95% confidence interval around the estimated value of the indicated response. A confidence interval measures the extent of uncertainty induced by statistical estimation. A confidence interval of 95% signifies that the odds are 20 to 1 that the true response is actually within the interval. Thus, .646% ± .086 implies a probability of 95% that the true response is between .560 and .732%. The "t-statistic" in this case would be 14.9.

has followed a similar method for a long time, *and* it is a method that permits changes in the indexes to be predicted successfully.

The Eleventh District Cost of Funds Index

The eleventh district COFI is composed partly of non-market-sensitive components. As we have already seen, this makes the COFI much less correlated with other indexes of market rates, somewhat less volatile over time, and probably quite predictable. Previous studies have found a slow adjustment of the COFI to a level dictated by market rates and we follow the same basic ideas in constructing a model.[6]

[6] See Michael R. Asay, "How ARMs Move," *Secondary Mortgage Markets* (Winter, 1985/86), pp. 7–13.

Think of the COFI as a portfolio that is partly driven by current market interest rates and partly driven by book costs of funds for thrift assets that are not "marked to market." The fraction composed of market-sensitive rates has probably changed over time and presumably has increased recently with deregulation. We can express returns on this portfolio in equation form as:

$$\mathrm{COFI}_t = x_t M_t + (1 - x_t)B_t. \tag{1}$$

where COFI_t is the observed value of the index in month t, M_t is the level of market interest rates in month t, B_t is the average non-market-sensitive book cost of funds for 11th district thrifts in month t, and x_t is the fraction in month t of 11th district thrift liabilities that are market sensitive. The COFI can be observed directly and M can be measured by current interest rates. Unfortunately, there are no existing *direct* measures of B or of x, so we must estimate them econometrically.

To accomplish this estimation, we can exploit the fact that any book cost of funds evolves over time. It seems sensible that 11th district book liability costs evolve toward market rates. As book liabilities mature and are paid off, they are replaced by other liabilities contracted at current interest rates. Thus, if the market interest rate were constant, the book costs of funds would eventually equal the (constant) market rate. Unfortunately, market rates are manifestly not constant, so book costs are continually trending toward a moving market target.

One way to depict this phenomenon is algebraically; for instance:

$$B_t = B_{t-1} + q(M_{t-1} - B_{t-1}). \tag{2}$$

In Eq. (2), the book cost of funds in a given month is equal to the book cost in the previous month plus an adjustment factor related to the previous difference between the market and book costs. The coefficient q can be considered the "speed of adjustment" of book costs to market costs.

By combining Eqs. (1) and (2), we can eliminate the unknown book cost of funds, B, and obtain a model in which x, the fraction of thrift costs that are market sensitive, can be estimated directly. If x changes over time, the resulting model is very complex. If, however, we assume that x is constant for relatively short periods (although it may change over longer periods), we obtain a formula that can be directly fit to data:

$$\mathrm{COFI}_t - \mathrm{COFI}_{t-1} = x(M_t - M_{t-1}) - q(\mathrm{COFI}_{t-1} - M_{t-1}). \tag{3}$$

This has the form of a regression model and its coefficients correspond to x, the market-sensitive fraction of thrift liabilities, and q,

Table 5. Portfolio Model (3) of the COFI: Five Years of Monthly Data Ending in September 1986

Market-sensitive fraction of thrift liabilities	x = .0890 (±.0621)[a]
Speed of adjustment of thrift book liabilities	q = .0768 (±.0334)
Volatility explained [Adjusted R-square] = 27.2%[b]	

[a] 95% confidence interval; see footnote b in Table 4.
[b] T-statistics for the coefficients were: for x, t = 2.84; for q, t = 4.56.

the speed of adjustment of thrift book asset costs. Estimates of the coefficients using the most recent data are given in Table 5.

The yield on 1-year Treasury securities was used as the measure of market rates. We also tried the 5-year Treasury yield and obtained very similar results.[7]

Figure 6 shows the results of fitting the same model at different times in the past. Each observation in Figure 6 gives the estimated market-sensitive fraction of eleventh district thrift liabilities and the estimated speed of adjustment of thrift book liabilities for a 5-year period indicated on the horizontal axis.

For example, the leftmost points in Figure 6 are estimates obtained from data covering calendar years 1978–1982 inclusive, i.e., ending in December 1982. As Figure 6 shows, the estimated fraction of 11th district thrift liabilities that were market rate sensitive was very close to zero prior to the 5-year period centered around the end of 1982. The estimated coefficient, x, was not statistically different from zero in earlier periods. Since then, however, there has been a noticeable increase in the estimated sensitivity of thrift liabilities to market rates, and *all* of the coefficients have been statistically significant.

The speed of adjustment of book assets toward market rates does not show an upward trend, and it even seems to have declined lately. Perhaps those book liabilities that formerly adjusted toward market rates more rapidly, such as shorter term sources of funds, have indeed been replaced by market-driven liabilities. Remaining book liabilities may thus display somewhat slower adjustment speeds, perhaps because they are longer term. However, the estimated adjustment speed (q) has been and remains statistically significant; the t-statistic was below 5 in only 2 of 45 fitted periods, and the minimum t-statistic was 4.3.

This portfolio model of the COFI seems to bring sensible empirical results, but it is not the whole story. Regression diagnostics indicate marginally significant misspecification, particularly in the early periods.[8] This may be due to the assumption we made in deriving the regression

[7] x = .0955 (t = 2.35); q = .0897 (t = 4.31).
[8] First-order negative serial dependence is present in the regression residuals.

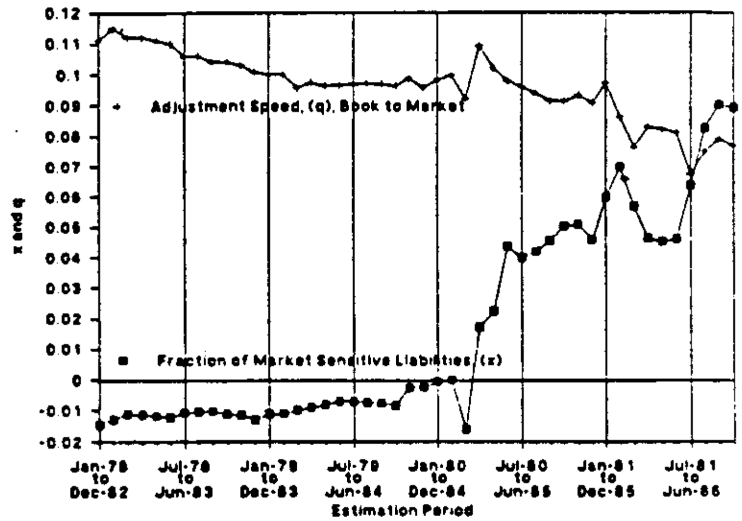

Figure 6. Portfolio model for cost of funds index.

model, viz., that the fraction of market-sensitive liabilities is constant over short periods. Thus, a more complex time series forecasting approach is a desirable supplement to the insights provided by the simple version of the portfolio model with nonvarying parameters.

With variation over time in the fraction of market-sensitive thrift assets and in the speed of adjustment of thrift book assets to market rates, an elaborated version of the portfolio model implies that the currently observed change in the COFI is related to lagged values of market rates extending back over several periods and, possibly, to lagged values of the COFI itself. In addition, there is some justification for including a seasonal effect. The seasonal pattern of COFI percentage changes is shown in Figure 7, which plots the COFI's percentage changes by calendar month from 1978 to the present. Figure 7 reveals that the COFI has declined in *every* month of March during this period. In addition, the COFI has increased in almost every month of February. Other calendar months do not display any obvious tendency for the COFI to decrease or increase.

We have incorporated both the seasonal effect and lagged variables in a composite predictive model, which is reported in Table 6.

All of the "explanatory" variables in Table 6 can be observed 1 month in advance of the COFI index change to be predicted. Adding the latest market rate a few days in advance of the COFI announcement improves the prediction slightly.

We have fit the same model for a rolling window of 5 years for as far

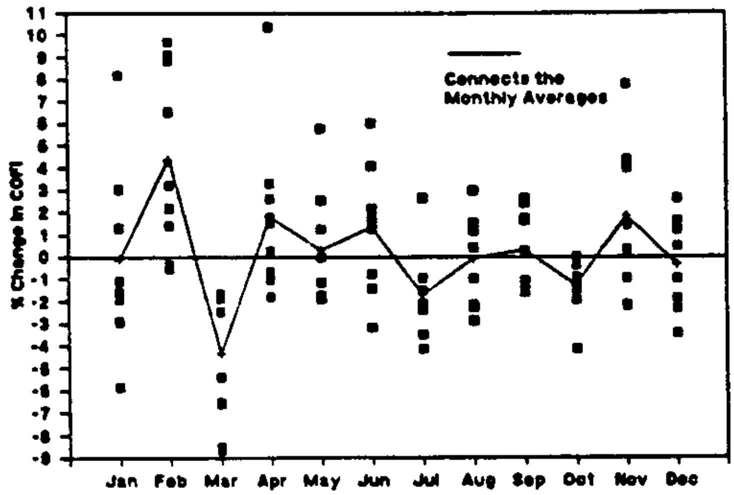

Figure 7. Monthly percentage changes in COFI by calendar month.

back as the data are available. The structure remains similar, but in the earlier periods the effect of the month of March was more significant as was the effect of the 6 month lagged market rate. There has been a small but persistent tendency for the shorter lags to become more statistically significant over time and for the longer lags to become less significant.

Table 6. Multiple Regression Model for Monthly Percentage Changes in the Eleventh District Cost of Funds Index (January 1982–September 1986)

Holding other Things Constant, a 1% Change in the	Would Cause a Percentage Change in the $[\text{COFI 11 Index}]_t$ of[a]	
$[\text{COFI 11 index}]_{t-1}$	−.363%	(±.280%)[b]
$[\text{Market 1-year rate}]_{t-1}$	.118%	(±.0588%)
$[\text{Market 1-year rate}]_{t-2}$	.114%	(±.0558%)
$[\text{Market 1-year rate}]_{t-3}$	.109%	(±.0557%)
$[\text{Market 1-year rate}]_{t-4}$	.0837%	(±.0568%)
$[\text{Market 1-year rate}]_{t-5}$	.136%	(±.0549%)
$[\text{Market 1-year rate}]_{t-6}$	.0467%	(±.0615%)
And in the Month of	There would be an Additional Change of	
February	2.10%	(±1.39%)
March	−.544%	(±1.51%)
Volatility explained [adjusted R-square] = 56.6%.		

[a] $[X]_t$ designates variable X in month t.
[b] 95% confidence interval; see footnote b in Table 4.

Summary and Conclusion

The ARM indexes do not display simple behavior. To model them, one is obliged to employ econometric methods that are standard in scientific practice but are, perhaps, less familiar to many investors. Unfortunately, there is no simple way to forecast a complex phenomenon adequately. One would not expect it to be easy to forecast the perihelion of Jupiter, for instance, and ARM indexes may be even more complicated, because their behavior changes over time.

Perhaps the most surprising result in the paper concerns the H.15 constant maturity Treasury indexes. Many investors might consider them tantamount to market rates; however, we have found that the H.15 indexes are serially dependent and differ significantly from true market rates obtained in actual trading. This fact makes it possible to develop a successful prediction model for changes in the H.15 indexes; the model presented predicts a significant proportion of the indexes' monthly change.

The 11th district COFI is widely known to be poorly correlated with market rates. The index' level can be regarded as the return on a portfolio consisting of both market-sensitive and nonmarket-sensitive liabilities. We have derived a portfolio model that allows the direct estimation of the fraction of 11th district thrift liabilities which are market sensitive and the speed of adjustment of 11th district thrift book costs as they evolve toward market rates.

The COFI can be predicted by a model containing lagged values of market rates and lagged values of the COFI itself, combined with seasonal factors. The model presented here predicts, a month in advance, over 50% of the monthly percentage change.