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# Adjustable Rate Mortgages: Valuation

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*Abstract*

A simulation method is employed to value Adustable Rate Mortgages, (ARMS). It is used to price two typical instruments: an ARM linked to a Treasury interest rate and an ARM linked to a "Cost of Funds" Index. Contractual provisions such as the margin over the index, caps and floors on the ARM's rate or on the monthly prepayment, reset frequency, and the "teaser" rate are examined for their influence on value. The effects of interest rate trend and volatility are also analysed.

## Introduction

The market value of an adjustable rate mortgage (ARM), depends on the mortgage's contractual provisions, the characteristics of the index to which the ARM is linked, the behavior of prepayments, and the market environment. The *market environment* refers to current interest rates and, more important, to *all future* paths of interest rates that are reasonable to contemplate as possibilities on the valuation date. Statisticians and economists might refer to this concept of environment as the *stochastic process* (of interest rates), or as the *probability distribution* of future rates. The current value of an ARM depends on what investors believe future cash flows will be and when they will be received. Because of the complexity of ARM indentures, the timing and amount of cash flows are highly uncertain and are materially affected by the actual course of interest rates during the ARM's lifetime.

The actual path of future interest rates will influence ARM cash flows in at least four ways. First, the ARM's coupon is linked to an index which is related partially to market interest rates; thus, the *scheduled* interest payments from the ARM respond to the level of rates. Second, due to the interaction of interest rate

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movements and ARM contractual provisions (such as caps), scheduled principal payments depend also on the actual course of rates. Third, prepayments of principal respond to movements in rates. Finally, the holding period return from an ARM investment is affected by future rate movements, because (a) cash flows received during the holding period must be reinvested, and (b) should the ARM be sold prior to maturity, its price will depend upon interest rate levels on the sale date.

The ex post outcome of any particular investment in an ARM will depend on just one of the many possible interest rate paths that could conceivably occur a priori. But the *ex ante* market value depends on a consideration of *all* possible paths, properly weighted by their probability of occurrence. In a single number, the market value encapsulates an infinite number of possible investment results. The task of a valuation model is to mimic the aggregation accomplished by the market pricing mechanism. The statistical technique of *simulation* is employed here for this purpose.

### The ARM Simulation Model

There are three steps involved in performing an ARM valuation with the Goldman Sachs model. First, the ARM's contractual provisions must be specified. This simply means that all the relevant parameters concerning the valuing of the particular ARM must be determined and provided as inputs to the model. Second, the market environment must be specified. Estimates, guesses, or deliberate opinions must be provided about the future course of interest rates, the characteristics of the ARM index, and perhaps the cost of funds for the investor. Third, the results must be summarized. This last step can be provided in various ways according to the choice of the evaluator. For instance, the evaluator can specify present and future funding costs, and the model will provide a present market value using the funding costs as discount rates.[1] Alternatively, the evaluator can specify a price, and the model will provide a profit margin or spread over funding costs.

Perhaps the easiest way to understand the simulation model is to conduct an actual evaluation session. Figure 1 shows the input screen on a computer terminal as the evaluator begins the session. The major input parameters have been entered. The ARM to be valued is linked to the 11th District Cost of Funds Index[2] (COFI), and it has a margin of 125 basis points over the Index. The ARM has an initial teaser rate of 7.15%, which expires in seven months. The annual and lifetime caps and other pertinent information also have been specified as shown on the input screen in Figure 1.

The market environment is entered on tertiary screens, illustrated in Figure 2. The current value and recent history of the COFI is automatically retrieved by the model from the Goldman Sachs data base, but the evaluator can override this information and insert different values.[3] The evaluator can also choose from among a number of different models for the ARM Index.[4] The choice that has been made in Figure 2A, top panel, choice #2, is to use the COFI model developed in our

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MORTGAGE SECURITIES RESEARCH                                  ARMS PRICING MODEL

                                Name   cofi
                              Screen   1

   1: Characteristics         3: Discount Methods        5: Price/Margin
+--------------------------------------------------------------------------+
|                        C H A R A C T E R I S T I C S      Next Screen    |
|                                                                          |
|                 Time                                   Caps              |
|      Final Maturity 360 months                Lifetime Cap  13.15 %      |
|     Rate Reset Freq   1 months                       Floor   3.15 %      |
|  Payment Reset Freq  12 months        Periodic PAYMENT Cap   7.50 %      |
|    First Rate Reset   7 months                       Floor   7.50 %      |
| First Payment Reset   7 months       Neg Am Cap (0 = no cap)  0.00 %      |
|    Payment Catch Up  60 months                                            |
|                                                                          |
|                   Coupon                           Other                 |
|              Teaser Rate   7.15 %                    Servicing 0.00 %     |
|                   Margin    125 bp        Negative Am Allowed YES        |
|     Rounded to Nearest 1/8  NO                                           |
|return = submit                ? = help            \ = back to main screen |
+--------------------------------------------------------------------------+
```

*Fig. 1.* Mortgage securities research arms pricing model—characteristics.

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MORTGAGE SECURITIES RESEARCH                                  ARMS PRICING MODEL

                                Name   cofi
                              Screen   2

   1: Characteristics         3: Discount Methods        5: Price/Margin
+--------------------------------------------------------------------------+
|                        Index Scenario   2                  Next Screen   |
|                                                                          |
|                      1: H15 Model                                        |
|                      2: Cofi Model                                       |
|                      3: Moving Average                                   |
|                      4: User Specified Model                             |
|                                                                          |
|  Initial Rate          6.50 %    Initial Value of COFI          7.220  %  |
|  of 1Yr H15                                                              |
|                                                                          |
|                                                                          |
|                                                                          |
|   return = submit model      ? = help        \ = back to main screen     |
+--------------------------------------------------------------------------+
return = submit program              ? = help                        \ = quit
```

*Fig. 2A.* Mortgage securities research arms pricing model—index scenario.

paper on the Indexes.[5] This model is partly based on predicting the COFI with lagged values of the H.15 one-year Treasury Index; as a result, the current value of the H.15 Index appears on the screen.

The simulated market environment of the future is determined by the stochastic process of the rate used for discounting cash flows. In this illustration, the discount rate follows a path determined by the observed Treasury yield curve on the valuation date. The expected path of the discount rate is the Treasury forward rate curve. The volatility of the discount rate is specified by the evaluator to be 20% per annum (standard deviation of relative changes).[6]

The current value of the refinancing rate into fixed-rate mortgages is 10%, (see Figure 2B). Its volatility is also 10%, and it has a correlation of 0.8 with the discount rate; (the correlation coefficient is determined within the model). This specification is only one of many possible simulated market environments from which the evaluator can choose. Alternatively, for instance, the discount rate trend could be chosen to follow the COFI itself, or to be related to the H.15 Index, while the refinancing rate into FRMs could be specified with larger or smaller volatility and with more or less correlation to funding cost.

The discount rates for funding costs used in the Goldman Sachs ARMs model can be chosen in quite a variety of ways, and the spread over funding costs must be interpreted in accordance with the choice. This should be contrasted with the common practice of gauging an ARM's value by an *effective margin* relative to the ARM *Index*, i.e., by a *profit* margin *relative to the Index* after taking account of the caps and other restrictions on ARM cash flows.

Although the effective margin gives some indication of an ARM's investment value, it is inadequate because it does not necessarily measure the investment's

```
MORTGAGE SECURITIES RESEARCH                                 ARMS PRICING MODEL

                                  Name   cofi
                                Screen   3

   1: Characteristics       3: Discount Methods       5: Price/Margin
+--------------------------------------------------------------------------+
|                   Discount Method   1                  Next Screen        |
|                                                                           |
|   1: Short Rate    DRIFT: IMPLIED FORWARD RATES                           |
|   2: COFI Model                                                           |
|   3: Index                                                                |
|                   Volatility of Short Rate                 20.00  %       |
|                                                                           |
|                                                                           |
|                   Date (  /  /   means today's date)      7/28/87         |
|                                                                           |
|                   Current Fixed Mortgage Rate             10.0     %      |
|                   Volatility of Fixed Mortgage Rate       10.0     %      |
|                                                                           |
|  return = submit model        ? = help        \ = back to main screen    |
+--------------------------------------------------------------------------+
return = submit program                  ? = help                  \ = quit
```

*Fig. 2B.* Mortgages securities research arms pricing model—discount method.

true profit over the investor's *own* funding costs. It also makes inter-ARM comparisons difficult. For example, if the effective margin of a COFI-linked ARM is 100 basis points while a Treasury-linked ARM's effective margin is 150 basis points, the Treasury ARM is not necessarily a better investment. The COFI could have an expected value more than 50 basis points above the H.15 Treasury Index. Perhaps more important, the COFI ARM could conceivably have a higher and more stable profit margin over the investor's funding costs, because those costs could be more closely related to the Cost of Funds Index than to the Treasury H.15 Index. The opposite could conceivably apply to other investors whose funding costs were more closely related to Treasury rates.

If the evaluator believes that the actual funding costs are highly correlated with an ARM index, the index itself (plus or minus a spread) can be specified as the discount rate in our valuation model. However, the model also permits the evaluator to specify other discount rates which could possibly be mandated by the relation of funding costs to other market interest rates, and/or to the ARM indexes.

After all of the required inputs are entered, the ARMs model begins to generate random numbers and replications for the simulation. In a few moments (the time depends on the number of replications), the results are ready; the computer has undertaken a large number of calculations during this interval.

*For each replication*, it must perform the following series of computations:

1. The market environment is generated for the replication. This means that (at a minimum) a Treasury interest rate, a fixed-rate mortgage refinancing rate, a funding cost (or discount rate), and the ARM Index are produced for each remaining month of the ARM's life. The ARM Index is produced according to the specified predictive model as a function of the generated interest rates.
2. The ARM's cash flows are produced for each month of the replication. This means that the value of the Index, the caps, the reset period, the teaser, etc., all interact to generate the *required* cash flow from the ARM in each month. In addition, the Goldman Sachs ARM prepayment model[7] is employed to produce a prepayment rate estimate for each month. Prepayments depend not only on the environment in *that* month but on the entire path of interest rates since the origination of the ARM.
3. ARM cash flows are discounted at the funding costs prevailing in different months, plus any premium or spread, to obtain a present value. If a price has been specified and a spread is to be determined, an iterative solution is necessary; the spread over the funding costs is calculated so that the resulting discounted present value of the cash flows equals the specified price.

Depending on the volatility of interest rates, somewhere in the neighborhood of 10,000 replications are required in order to produce a value accurate to within .125%, ($4/32 per $100 dollars.) For each of these replications, all of the above steps

*Table 1.* Adjustable Rate Mortgage Model, Date: 7/28/87

| Price/Margin Table | |
|---|---|
| *Arm Characteristics* | |
| Current balance on mortgage | $100.000 |
| Final maturity | 360 months |
| Servicing percentage | 0.000%/annum |
| Days to first payment | 44 |
| *Terms and Caps* | |
| Teaser rate | 7.150%/annum |
| Teaser termination in | 7th month |
| Upper life cap (including margin) | 13.150%/annum |
| Lower life cap | 3.150%/annum |
| Margin | 1.250%/annum |
| First coupon reset in | 7th month |
| Coupon reset on anniversary every | 1 month(s) |
| Upper PAYMENT cap | 7.500%/adjustment period |
| Lower PAYMENT cap | 7.500%/adjustment period |
| PAYMENT reset on anniversary every | 12 months |
| PAYMENT cap release, anniversary every | 60 months |
| Negative amortization allowed? | yes |
| *Scenarios* | |
| Prepayment assumption | Goldman Sachs model |
| Index | COFI: Goldman Sachs model |
| Initial level of one year T-Bill | 6.500%/annum |
| Initial level of COFI Index | 7.220%/annum |
| Number of replications in simulation | 10000 |
| *Interest Rate Parameters* | |
| Implied forward rates in current yield curve used as drift. | |
| Standard deviation of short rate | 20.000%/annum |
| Standard deviation of long rate | 10.000%/annum |
| *Results* | |
| Confidence limits around a margin of | 104.018%* |
| Price | 102.25 |
| Price standard deviation | 6.19 |
| Price 95% confidence interval | +/−0.1213 |

*% of the discount rate

must be independently repeated. For our example ARM, the output appears as shown in Table 1.

In this illustration, the model computed a spread relative to funding costs *given* an initial price. This might be the typical approach for an investor who observes a quoted price for an ARM in the secondary market and is attempting to ascertain whether the expected rate of return over costs is adequate compensation for the risk. The ARM could be purchased for $102 8/32. The spread over costs produced by the model is 104.018%.

This is a relative spread.[8] If the average future funding costs were, say, 10%, the average return from investing in this ARM, given its purchase price, its contractual features, and its assumed market environment, would be 10.4018% or 40.18 basis points higher than funding costs. However, the initial Treasury rate is actually only 6.50%, but the current yield curve is rather steeply upward sloping, so the rates used to discount cash flows increase significantly over the ARM's life.[9]

In each replication, the discount rate for each future month was 1.04018 times the simulated stochastic Treasury rate in that month. Some replications might have displayed strong upward movements in rates; for example, if the simulated Treasury rate were 20% in a given future month, the discount rate used for ARM cash flows *in that month* would have been 20(1.04018)=20.8036, an absolute spread of 80.36 basis points. Similarly, if rates trended downward during a particular replication, the *multiplicative* nature of the margin over costs implies a smaller *basis point* spread. If the Treasury rate happened to be, say 4%, the discount rate would have been 4(1.04018) = 4.16072, a 16.072 basis point spread.

The cash flow that the ARM produced in month $t$ of replication $i$ was discounted to the present by the cumulative factor

$$1/[(1 + mR_{i,t})(1 + mR_{i,t-1}) \cdots (1 + mR_{i,1})],$$

where $m$ is the margin multiple (e.g., 1.04018), and $R_{i,t}$ is the Treasury interest rate for month $t$ during replication $i$. Note that $m$ is a constant over all time periods and replications. The model solves for the value of $m$ such that the average discounted present value of all cash flows over all replications equals the initial market price.

Because the simulation used only 10,000 replications, there is some possibility of error from the true value. A guage of this possible error is the 95% confidence interval, ±0.1213, which means that for a margin of 104.018% the true price is between \$102.3713 and \$102.1287 with probability .95.

## Comparative analysis

The ARMs model can be used to evaluate any particular security of interest, but it can also be employed in a more analytical mode to examine the extent of the many influences on ARM values. In this section, we present a comparative analysis of ARMs, using as illustrations ARMs linked to the most common indexes. The idea of comparative analysis is to isolate the influence of different attributes by changing only one of them at a time, holding all others constant, and observing the effect on value.

For instance, suppose one wished to ascertain the impact of the margin (over the index) on ARM value while holding constant all other contractual features and maintaining the same assumptions about the market environment. This is a very practical problem because there are many ARMs available in the primary and secondary markets which differ almost exclusively in their margins and in nothing else. Of course, they sell for different prices and the investment question is whether an ARM with higher margin and a higher price returns more or less than an ARM with a lower margin and a lower price.

We first present results for two base cases with common attributes, a Treasury Index-Linked ARM and a COFI ARM, and then we separately examine the impact on value of the following attributes, holding other features the same:

A. Market Environment
   1. Interest Rate Drift
   2. Interest Rate Volatility
B. Contractual Provisions
   1. Margin
   2. Lifetime Coupon Caps
   3. Level and Reset Frequency of Periodic Caps
   4. Teaser Rate

*The Base Cases, a Treasury Indexed ARM and a COFI ARM*

Table 2 summarizes the parameters and results for the base cases. Actually, the base case for the COFI-linked ARM was already used in the discussion of section II, so its parameters are familiar. Notice that there are some minor differences between the two ARMs, in addition to the major difference that they are linked to distinct indexes. These parameters were chosen because they are commonly observed in the market place.

The teaser rate is slightly higher for the COFI ARM, which makes its lifetime ceiling correspondingly higher, 13.15% vs. 12.5%. The margin is less for the COFI ARM, 125 basis points versus 175 basis points for the Treasury ARM. However, this is more than offset, and can be expected to remain at least partially offset, by the higher level of the COFI. Adding the 175 basis point margin to the current one-year Treasury rate gives a current ARM coupon of 8.25%, while the corresponding calculation for the COFI ARM gives a coupon of 8.40%. This is one reason that the COFI ARM has a higher current market price. Another factor lowering the Treasury ARM's value relative to the COFI ARM is the longer period until teaser expiration.

The value comparison is complicated, however, by other differences. For example, the COFI ARM has a periodic payment cap, while the periodic cap of the Treasury ARM is directly on the coupon. Finally, negative amortization is allowed on the COFI ARM, and it also has a catch-up date every five years on which any previously-imposed payment cap is removed.

There is sometimes a suspicion that the market has slightly mispriced either one security or the other. The COFI's basic rate or return over funding costs is about 104%, while the Treasury's is about 106%. This would seem to imply that the Treasury ARM is a relatively good value at its price of $97 20/32. If the dollar-weighted funding cost over time is 7.5%, the advantage of the Treasury ARM is about 15 basis points.

This may or may not be a compensation for risk. The H.15 Treasury Index is more closely linked than the COFI to market rates so that the Treasury ARM might be somewhat *less* exposed to interest rate risk. However, the fact that the

*Table 2.* ARM Valuation Examples: A COFI Arm and a Treasury ARM

| INPUTS | COFI | TREASURY |
|---|---|---|
| *Arm Characteristics* | | |
| Current balance on mortgage | $100.000 | $100.000 |
| Final maturity (months) | 360 | 360 |
| Servicing percentage (%/annum) | 0.000 | 0.000 |
| Days to first payment | 44 | 74 |
| *Terms and Caps* | | |
| Teaser rate (%/annum) | 7.150 | 6.500 |
| Teaser termination in month | 7 | 13 |
| Upper lifetime cap, including margin (%/annum) | 13.150 | 12.500 |
| Lower lifetime cap (%/annum) | 3.150 | 0.500 |
| Margin (%/annum) | 1.250 | 1.750 |
| First coupon reset in month | 7 | 13 |
| Period between coupon resets (months) | 1 | 12 |
| Upper COUPON Cap (%/adjustment period) | N/A | 2.000 |
| Lower COUPON Cap (%/adjustment period) | N/A | 2.000 |
| Upper PAYMENT Cap (%/adjustment period) | 7.500 | N/A |
| Lower PAYMENT Cap (%/adjustment period) | 7.500 | N/A |
| COUPON reset on anniversary every (months) | N/A | 12 |
| COUPON cap release, anniversary every (months) | N/A | 999 |
| PAYMENT reset on anniversary every (months) | 12 | 12 |
| PAYMENT cap release, anniversary every (months) | 60 | N/A |
| Negative amortization allowed? | yes | no |
| *Scenarios* | | |
| Prepayment assumption: Goldman Sachs model. | | |
| Initial level of one year T-Bill (%/annum) | 6.500 | 6.500 |
| Initial level of COFI index (%/annum) | 7.220 | N/A |
| Number of replications in simulation | 10,000 | 10,000 |
| *Interest Rate Parameters* | | |
| Drift set equal to implied forward rates | yes | yes |
| Volatility of short rate (%/annum) | 20.0 | 20.0 |
| Volatility of long rate. (%/annum) | 10.0 | 10.0 |
| *Price* | $102.25 | $97.625 |
| RESULTS | | |
| PROFIT MARGIN AFTER FUNDING COSTS | 104.018% | 106.014% |

COFI ARM's coupon adjusts more frequently offsets this advantage and makes it difficult to assess precisely the relative riskiness of the two ARMs.

*The Impact of Market Environment on Value.*

***The trend in market interest rates.*** The most attractive feature of ARMs for interest-sensitive investors is that their coupons are adjustable. This provides a degree of protection against interest rate movements. However, because of caps and other restrictions, the linkage of ARM coupons to market interest rates is not perfect, particularly in the short run. Over longer periods, ARMs should provide a degree of protection against secular movements in rates; indeed, their market values should depend on the *anticipated* drift, or long-term trend, in rates.

Figure 3 shows the effectiveness of ARMs in protecting against trends in interest rates, holding the volatility of rates constant (at 15% per annum). This figure, and all the figures that follow, show several different plots. The *Normal* ARMs have contractual provisions more or less corresponding to actual ARMs.[10] For comparison, we also show two companion mortgages that are identical to the Normal ARMs except that one companion has a fixed coupon and the other is completely unencumbered by the various restrictions of the typical ARM indenture.

The illustrated COFI Normal ARM has a lifetime cap of 14% and a floor of 4%, an initial COFI index of 7%, a margin of 200 basis points, and monthly coupon adjustment frequency. The *COFI Bound* ARM has exactly the same provisions except that its lifetime caps are set equal to the *current* ARM coupon. This has the effect of converting the ARM into a fixed-rate mortgage. The *COFI Unbound* ARM also has the same provisions as the COFI Normal ARM except that it has no lifetime caps, no periodic caps, and adjusts monthly. It is a fully index-linked security, a risk-averse investor's dream.[11] The illustrated Treasury-linked ARM has similar provisions are listed in the figure. Thus, at least initially, both ARMs have identical contractual characteristics. The Treasury also has *Bound* and *unbound* companions.

As the figure shows, an anticipation of increased upward rate drift would greatly decrease the current market value of the *Bound* ARMs, both the COFI and Treasury versions; while anticipation of declines in rates would increase their values. The impact on the *Unbound* ARMs is smaller; indeed, one may wonder

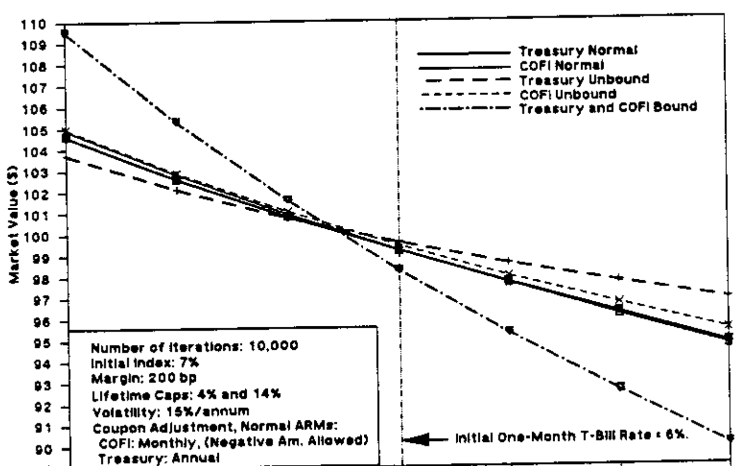


*Fig. 3. Market value vs. interest rate trend.*

why there is any effect at all of rate trend on a completely uncapped and unrestricted floating instrument. The explanation is that the ARM indexes differ materially from the short-term interest rate used as the funding costs. In these illustrations, we have assumed that this discount rate is equal to the *one-month* Treasury Bill rate.

The Treasury ARM is linked to the H.15 *one-year Treasury Indexes*, not to the one month T-Bill. One-year Teasuries are not perfectly related to one-month Treasuries and one-year rates do not move up (or down) with the same amplitude as do one-month rates. In addition, the H.15 Index is not actually a one-year market rate.[12]

The situation is exacerbated for the COFI-linked ARMs. The COFI is related even less than the H.15 Index to the one-month T-bill rate; therefore the negative impact of short rate increases is more pronounced for a COFI ARM.

The impact of anticipated rate drift on actual ARMs is closer to the Unbound than to the Bound counterparts. This seems to imply that, despite the various restrictions on adjustability that are typically part of an ARM indenture, there is still a substantial degree of risk protection conferred by the variable coupon. As an aside, the risk protection of ARMs is not just a benefit to lenders, it is also a benefit to borrowers. True, borrowers must make higher payments when interest rates increase, but they benefit from lower payments in rate declines because they are not required to incur the two major incremental costs of fixed rate mortgages (refinancing costs and higher initial coupon).[13]

Comparing the COFI and Treasury Indexed ARMs in Figure 3, we observe that the COFI ARM's caps appear to have a relatively small influence on value; i.e., the COFI Unbound curve is very close to the actual ARM or *Normal* curve; also, the Unbound price is above the Normal price for all interest rate trends, both positive and negative. In contrast, the Treasury Unbound curve is relatively far from the Normal Treasury ARM curve, and the Unbound price falls below the Normal price for negative interest rate trends.

The relative positions of the COFI and Treasury prices can be explained by the probability of encountering a lifetime cap, which is lower for the COFI because its volatility is lower. The probability of hitting a lower lifetime cap (or floor) is small enough in the case of the COFI that, combined with greater prepayments in lower rate environments, the price of the Unbound COFI ARM never falls below the price of the Normal COFI ARM.

***The volatility of market rates.*** Figure 4 shows the influence of interest rate volatility on ARM value.

Interest rate volatility has two distinct influences on the value difference between an ARM and a FRM. First, like any floating rate security, an ARM will be more valuable than a fixed rate instrument in a volatile interest rate environment. To understand this point, consider zero volatility. In this case , there would be no risk reducing advantage to an investment in ARMs, because the fact that the ARM's coupon is *allowed* to vary is immaterial. Thus, an ARM with the same

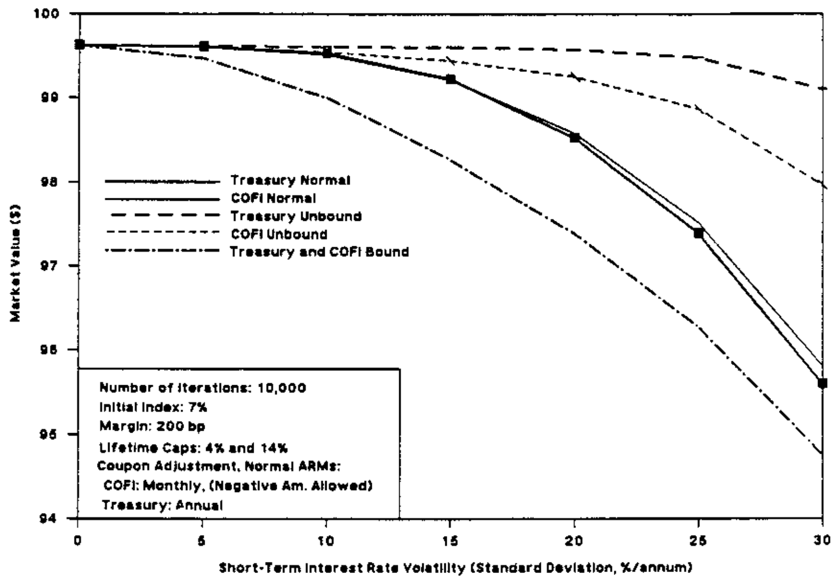


*Fig. 4.* Market value vs. interest rate volatility.

coupon as a FRM ought to have the same value. The model does produce this result; in Figure 4, all of the illustrated securities have the same price at zero volatility.[14]

Second, the volatility of interest rates has a substantial influence on the value of the option to prepay. For fixed rate mortgages, increased rate volatility will make prices decline because the option becomes more valuable and is more likely to be exercised in a low rate environment during the lifetime of the mortgage. To a lesser but still significant extent, the prepayment option also influences the value of ARMs. When the ARM is at a cap, for instance, it behaves much like a fixed rate mortgage. Thus, increased volatility should also decrease the value of a capped ARM. With increased volatility, not only is the ARM coupon more likely to encounter a cap, it is more likely to be capped out for longer periods. An ARM is actually a hybrid between a purely floating and a purely fixed rate security, and increased volatility slants the hybrid toward the fixed component.

As Figure 4 shows, there is indeed a very substantial price decline of the *Bound*, i.e., fixed-rate, mortgage as volatility increases. For example, the Bound mortgage falls in value from \$99.63 to \$96.28 as the annual standard deviation of interest rates rises from zero to 25%. In contrast, there is a relatively minor impact of volatility on perfectly floating rate mortgages, the COFI and Treasury Unbound ARMs.[15] The Treasury Unbound ARM falls by only 15 cents for the same increase in volatility, and the COFI Unbound ARM falls by 77 cents.

Actual ARMs are intermediate cases, but their precise intermediate position is a function of the level of volatility. Notice in Figure 4 that the Normal ARMs are priced quite close to the Unbound ARMs when volatility is 10% or less. However, as volatility increases, the Normal ARM values fall away from the Unbound values and converge on the Bound curves. At a volatility of 30%, for example the Normal ARMs are valued only 70–75 cents above the Bound ARMs but several dollars below the Unbound ARMs. Volatility is the gene that determines the proportions of its life the ARM hybrid passes in the floating state and in the fixed state.

Interest rate volatility has a greater impact on Treasury than on COFI ARMs. This is attributable to a higher correlation between the one-year (H.15) index and the short-term interest rate relative to the correlation between the COFI and the one-year rate.

*The Impact of Contractual Provisions on Value.*

***The margin.*** The margin of the ARMs coupon over the index has an obvious and direct influence on value, but it also increases the probability of prepayment. Figure 5 shows the results. The base case is a margin of 200 basis points; the price is approximately par (less a discount for payment delay) at this margin. A change in the margin of plus or minus 25 basis points induces a change in price of approximately 85 cents. The market values of the Unbound ARMs are between 10 and 60

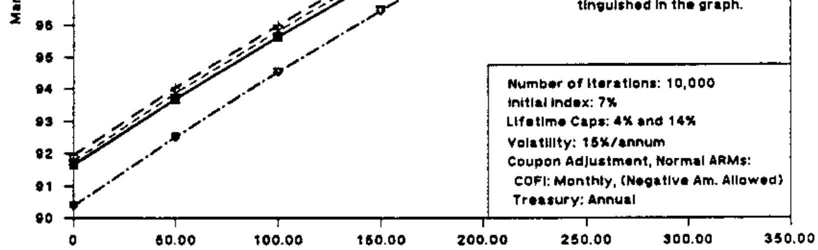


*Fig. 5.* Market value vs. margin.

cents above the Normal ARM. Again, there is less of a difference for the COFI ARM, because the caps are less likely to be encountered.

The influence of margin on prepayments is surprisingly minor; it is responsible for the slight downward curvature of the prices as margin increases and the slight widening between the Unbound and the Normal cases.

***The lifetime caps.*** Removing (both) the lifetime caps increases the value of the ARM. This is because the lower cap, or floor, is less materially important. The floor has a smaller influence than the ceiling for three reasons.

First, the caps are usually set relative to the teaser rate, which is below the Index+Margin on the origination date of the ARM. This means that the coupon will probably be closer to the upper cap after the teaser period has ended. Second, the caps are usually expressed as plus and minus the same number of basis points, e.g., ±500, bp from the teaser. But the chances of interest rates dropping 500 basis points are much less than the chances of their increasing by 500 basis points. The stochastic process of rates gives about equal probability to *relative* increases and decreases. Thus, if the initial rate is 9%, there is about an equal chance of an increase to 14% and a decline to 5.79%, i.e., plus 500 and minus 321 basis points in absolute amount, but equal changes of ±55.6%[16] relative to the beginning index level of 9%. Third, prepayments increase as interest rates fall. By the time the Index + Margin falls close to the floor, most of the original mortgages will have prepaid, thereby mitigating the floor's influence on original market value.

Figure 6 shows the impact on ARM value of changing *both* caps, the ceiling and

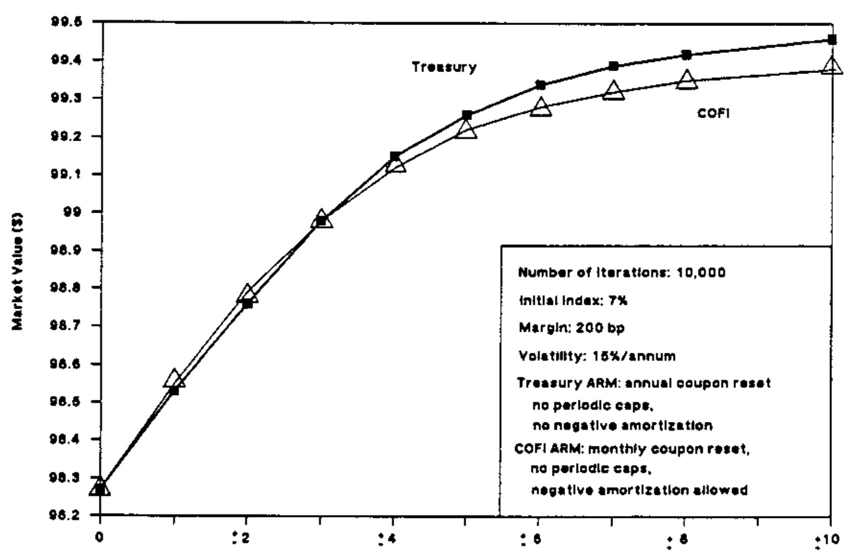


*Fig. 6.* Market value vs. lifetime caps.

the floor. The reference case is ±0 basis points, i.e., a completely capped mortgage. The graph is calibrated to make the Treasury and the COFI ARM equal in price at this point.

Increasing the caps from ±0 at first has a slightly more beneficial impact on the COFI ARM than on the Treasury ARM. For instance, at ±200 basis points, there has been a price increase of 52 cents for the COFI and 49 cents for the Treasury ARM. This is explained by the relative likelihood of each index hitting a relatively *tight* lifetime cap (the chances are considerably less for the COFI because it is less volatile). The beneficial effect of widening the caps becomes relatively greater for the Treasury after about ±300 basis points. Beyond this level, the greater volatility of the Treasury Index implies that any widening of the caps is more materially important.

Since the COFI has a relatively low volatility, it encounters caps as wide as ±500 basis points infrequently, so there is little additional increase in value from widening the caps further. There is a similar decreasing marginal benefit of widening the caps for the Treasury ARM, but it occurs for wider cap levels because of the Treasury Index's greater volatility. Increasing the caps from ±0 to ±500 basis points increases the COFI ARM's value by 95 cents and the Treasury ARM's value by 99 cents. Removing the caps entirely increases value by another 22 cents in the case of the COFI and by 34 cents in the case of the Treasury.

For both the COFI and the Treasury ARM, removing the lower cap alone, while retaining an upper cap 500 basis points above the origional coupon, has only an imperceptible, negative impact on market value.

***The periodic caps and their reset frequency.*** There are two common types of periodic caps, one type limiting the periodic movement of the ARM coupon and the other type limiting the movement of the monthly payment. Typically, the payment cap is associated with *negative amortization*; that is, shortfalls in the payment relative to the coupon are simply added to the loan balance. With negative amortization, payment caps ought to have a lower impact on market value than rate caps, because interest actually *is* earned on the loan at a current rate, albeit in the form of principal accrual rather than in the form of cash.

First examine the impact on value of a periodic cap on the coupon rate, which is typical for H.15—indexed (Treasury) ARMs. Figure 7 shows market value versus periodic rate caps for four different reset adjustment frequencies: six months, and one, three, and five years. A periodic cap of zero means that no change is permitted in the coupon; in other words, this is a completely Bound ARM. (Its price, $98.27, can also be found on the *Bound* curve at the 15% volatility level in Figure 4).

Increasing the periodic rate cap from zero to ±200 basis points has a significant effect on value. If the rate is reset every year, the ARM's value increases almost a full point, from $98.27 to $99.22. Less frequent resetting mitigates this benefit; a five-year reset of ±200 basis points, for instance, induces an increase in value of only 34 cents.

Much of the potential total benefit of widening the periodic caps is received by ±200 basis points, especially for the shorter reset frequencies. For example, with a

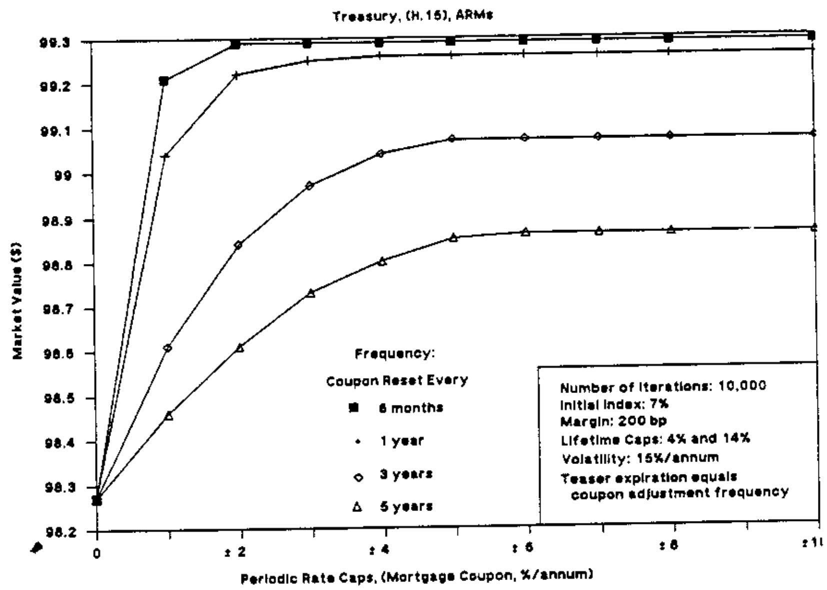


*Fig. 7.* Market value vs. periodic rate caps and their reset frequency.

one-year reset frequency, widening the rate cap from ±200 to ±1000 basis points increases market value by only four cents.

Although COFI ARMs usually have payment caps, not rate caps, we have priced the hypothetical effect of a rate-capped COFI ARM in order to contrast it with a Treasury ARM (see Figure 8.) The relative impact of the periodic cap on COFI vs. Treasury ARMs is similar to the relative impact of the lifetime caps. Relaxing the caps at first has a greater benefit for a COFI ARM, but further relaxation ultimately has the most benefit for a Treasury ARM.

Periodic rate caps can decrease market value, even when they may seem wide enough to be innocuous. For example, given 15% annual volatility, a semi-annual rate adjusting Treasury ARM with a ±500 basis point lifetime cap but no periodic cap is worth $99.29. The same ARM which also has a ±500 basis point periodic cap with a three-year reset frequency is worth only $99.07. Even though the periodic cap is set at the same level as the lifetime cap, there is an impact on market value. This is attributable to the periodic cap's preclusion of *any* rate adjustment between reset dates. The lifetime cap carries no proscription against rate movements *within* a reset period, merely an absolute limit at a given rate level.

The situation with respect to periodic *payment* caps is illustrated in Figure 9, which presents results for both a COFI ARM and a Treasury ARM.[17] Both ARMs are shown with and without negative amortization. There is also a *catch-up* provision that resets the payment to a fully amortizing level every five years on the anniversary date even though such a reset violates the payment cap.

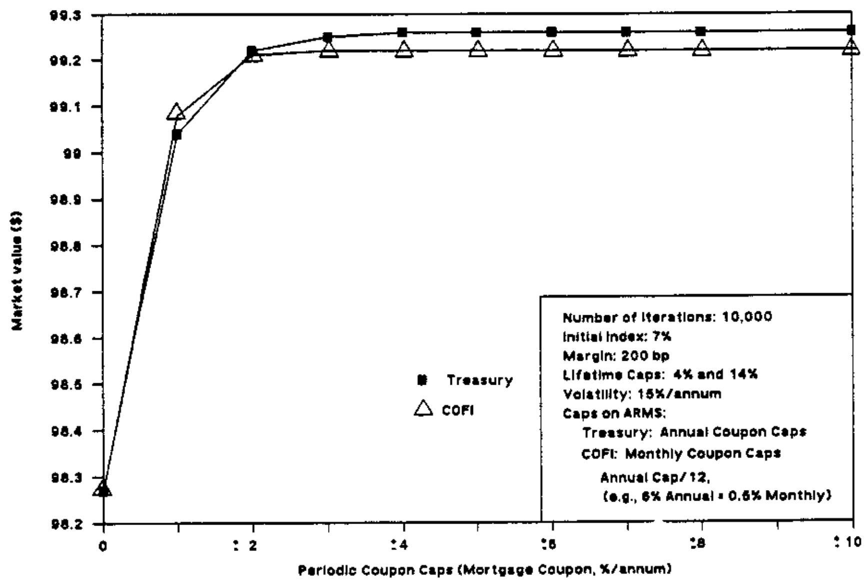


*Fig. 8.* Market value vs. periodic rate caps, COFI vs. Treasury

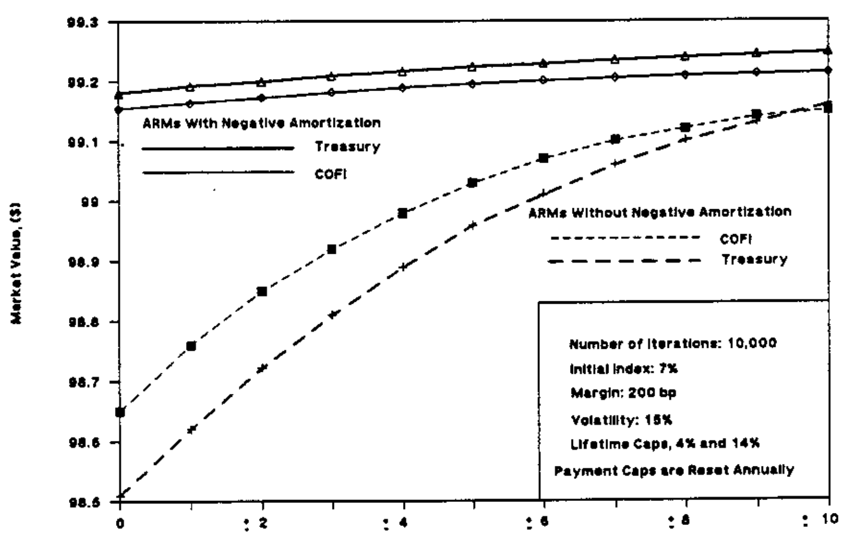


*Fig. 9.* Market value vs. periodic payment caps (with and without negative amortization).

When negative amortization is permitted, periodic payment caps have a very modest influence on value. The Treasury ARM is slightly more valuable in this case because of the closer correlation between the Treasury Index and the funding costs (the one-month T-Bill rate in this illustration).

Without negative amortization, there is a very significant negative impact on value attributable to the periodic payment caps. The Treasury ARM is more affected by tighter caps in this situation because it is more likely to hit the caps. Thus, as the caps tighten from $\pm 10\%$ to zero, the Treasury ARM's value decreases by 65 cents, while the COFI ARM's value declines by 50 cents.

The COFI ARM is also influenced to a minor extent by the possibility of *involuntary prepayment*. Whether or not negative amortization is permitted when payments have been capped on the up side, payments capped on the down side are still applied to reducing the mortgage principal balance. Furthermore, to the extent that the payments exceed the amount required to amortize principal on schedule at the current (uncapped) coupon, they are prepayments.

Notice in Figure 9 that the Treasury ARM's value (without negative amortization permitted) first exceeds the COFI ARM's value, at the far right side of the graph, as the payment caps are relaxed from $\pm 9\%$ to $\pm 10\%$. For caps wider than $\pm 10\%$, the effect of the higher correlation between the Treasury Index and the discount rate, a positive factor of the Treasury ARM, outweighs the effect of the higher probability of hitting the cap (a negative factor). As the periodic payment caps widen further, the relevance of negative amortization decreases, because the likelihood of encountering a cap becomes quite small. Thus, the two Treasury curves in Figure 9 eventually converge to the same value for cap levels off the graph to the right. The two COFI curves also converge, but to a lower value than the Treasury curves.

The majority of payment capped ARMs (with negative amortization) are in the $\pm 7.5\%$ area. Yet it would appear that originators could tighten the payment caps substantially without any appreciable impact on market value. Perhaps their reluctance to do so can be explained by another consideration: a borrower's loan to value ratio is increased by negative amortization, which could conceivably increase the incentive to default, particularly during periods of housing price decline. This represents a potentially negative influence on value that is not explicitly incorporated in our valuation model.

For Agency-guaranteed ARMs, such as the FNMA securitized ARM pools used in our study of prepayments, this effect *is* incorporated into our results, because defaults appear as extra prepayments. Such *prepayments* may be more likely in upward month. The completely Bound case, meaning that the rate is *never* reset during the ARMs life, displays the greatest impact on value. The ARM declines in price by \$12.60 as the teaser falls from the market level (9%) to 6%. This is similar to the price decline that one would observe across fixed-rate mortgages with coupons ranging from 9% to 6%. The *Normal* ARM curves also display reductions in value, \$3.85 for the Treasury ARM and \$2.88 for the COFI ARM as the teaser falls to 6% from the market level of 9'%.

The difference between the COFI and the Treasury can be explained by the fact that the COFI is periodic payment capped (at ±7.5%), but *with negative amortization*, while the Treasury is periodic rate capped (at ±2%). At the end of the teaser period, the payment or the rate can move up by these amounts at most. A teaser of 8% allows a full reset to the market rate of 9%.[18] However, teasers below about 8% induce greater cash flow for the COFI for two reasons: (a) negative amortization permits shortfalls from payment caps to be recaptured through principal accretion, and (b) even if the rate does not hit the cap in the first reset period (as it would not if the teaser were 7.5%), periodic caps in subsequent caps in subsequent periods are established relative to the index level at the end of the first reset period. The rate-capped Arms cash flows in later periods are therefore impacted to a greater extent than are the payment-capped ARM's cash flows, again because the latter enjoys negative amortization.

Figure 10 is extended to show the impact of teasers *above* the current level of Index+Margin (9%). We are not aware of any ARMs that have been issued with excess teasers, but such issuance is conceivable, and could even be advisable under some trending rate environments when payments have been capped and amortization is negative (relative to prepayments that would be observed from ARMs without payment caps). For non-insured ARMs, however, the wary investor would want to reduce the estimated value by the likelihood of negative amortization-induced default.

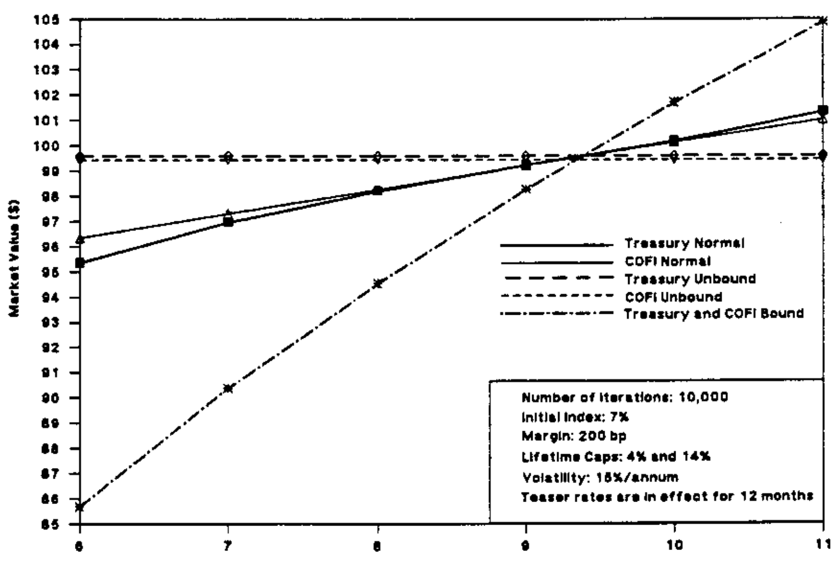


*Fig. 10.* Market value vs. teaser rate.

***The teaser rate.*** The teaser rate, usually set below the initial level of Index+Margin, clearly has a substantial negative influence on value. The lender receives below market interest for the duration of the teaser period. Even more important, the lifetime caps are often set relative to the teaser, and the initial periodic cap is imposed relative to either the teaser rate or to the payment during the teaser period.

The separate influence of the lifetime caps has already been examined. Thus, Figure 10 holds them constant and examines the teaser's impact on value that stems from its delaying the receipt of market interest and its interaction with the periodic reset cap at the termination of the teaser period.

The base case in Figure 10 is a teaser level of %, which is actually the same as the initial Index+Margin. Thus, 9% is essentially *no* teaser. Decreasing the teaser to a level below 9% causes the ARM's market value to decline. In the completely Unbound case, the decline is trivial, since the rate is reset to the Index+Margin in the first circumstances. For instance, an excess teaser could allow the originator to eliminate points while recuperating the loss in a form that is fully tax-deductible to the borrower and is payable over a series of months, the teaser period, rather than as a lump sum deduction from the loan proceeds at origination.

**Summary and conclusions.**

Adjustable Rate Mortgages are among the most complex of existing assets. Their contractual features are intricate and their cash flows depend, often in unexpected ways, on the market environment and on the psychological propensities of borrowers. ARM valuation presents a real challenge and thus a real opportunity to the astute investor.

The Goldman Sachs simulation model can value most existing ARMs. It was used here to illustrate the valuation of a typical Treasury-linked ARM and a typical Cost of Funding (COFI)-linked ARM. It was also used to investigate the influence on value of the market environment, i.e., the trend and volatility of interest rates, and the influence on value of various contractual features such as the margin, the caps, the reset frequency, and the teaser rate.

Comparative analytics offer the following conclusions about ARM values:

1. Because of the lifetime and periodic caps, ARM values generally (but not invariably) lie between the values of fixed-rate mortgages (FRMs) and fully variable-rate mortgages with otherwise similar features.
2. Upward trends in interest rates decrease ARM values, and vice versa. However, compared to fixed-rate mortgages, ARM values are less affected by rate trends. In their characteristic response to rate trends, ARMs are closer to purely variable-rate mortgages than to FRMs.
3. Interest rate volatility decreases ARM values, but not as much as it decreases the value of fixed-rate mortgages. Volatility has a complex interaction with

ARM caps and with ARM prepayments. At low levels of volatility, ARM values are close to values of uncapped floating rate instruments. However, as volatility increases, ARM values move closer to FRM values and there is an increasing gap between ARM values and uncapped floating-rate mortgage values.
4. An ARM's margin over its index has a significant and positive impact on market value. There is surprisingly little difference between Treasury-linked ARMs and Cost-of-Funds-linked ARMs in their responses to the size of the margin, holding other factors equal.
5. The lifetime ceiling and floor on the ARM's coupon have a significant influence on value, but the influence is predominantly a result of the ceiling. Widening both caps increases the ARM's value. For instance, widening the caps from zero, i.e., completely capped, to ±500 basis points, increases an ARM's value by approximately $1 per $100 face amount.
6. The effect of periodic caps depends on whether they are rate caps or payment caps, on the frequency of reset, and on whether negative amortization is allowed in the case of payment caps. Cap reset frequency is important. For periodic rate caps of ±200 basis points, a six-month reset frequency creates extra value of 70 cents relative to a five-year reset frequency.[19] For payment-capped ARMs, value is hardly affected by the tightness of the caps when negative amortization is permitted. Without negative amortization, however, there is a significant increase in value when the caps are widened.
7. The teaser rate has a negative influence on market value, i.e., the lower the teaser relative to prevailing rates and the longer it remains in effect, the lower the ARM's value. There is a slightly greater impact of the teaser on Treasury-indexed ARMs than on COFI ARMs because the reset at the end of the teaser period is limited by the periodic cap, which is generally a payment cap with negative amortization for the COFI, while it is usually a rate cap for the Treasury.

The market value of every existing ARM depends on the aggregate influence of all these factors. The Goldman Sachs Valuation Model is a useful tool for keeping track of the various influences and assessing relative values of existing ARMs.

### Notes

1. The funding cost specification can be very general. Examples, (not exhaustive), are: (a) the funding cost is correlated, perfectly or imperfectly, with the ARM index; (b) the funding cost is a set of fixed discount rates; (c) it is the simulated future Treasury rate.
2. The index is the aggregate book interest cost of liabilities for member institutions in the Federal Home Loan System's eleventh district, (Arizona, California, and Nevada).
3. This might be done, for instance, when conducting a scenario analysis, which requires the evaluation of an ARM at a horizon date in a hypothetical future rate environment.
4. These are models adopted from the existing literature or developed by Goldman Sachs. For instance, the "Asay Model" appeared in Asay, 1985/86.
5. See Roll, 1986.

6. Throughout the paper, interest rate volatility refers to the annual relative percentage change in the level of interest rates. For example, a change in the rate from 5% to 6%, or a change from 7% to 8.4%, would both be a 20% relative increase. If $R_t$ is the one-month interest rate at the end of month $t$, $\ln_e[R_t/R_0]$ is distributed normally with a mean $u = -(t/2)s^2$, and a variance of $ts^2$, where $s$ is the standard deviation of the relative change *per month*. The negative mean is required to assure zero drift, because the expected value of $R_t$ is $R_0e^{(u+ts^2/2)}$.

7. Described in Bartholomew, Berk, and Roll, 1987.

8. The model is capable of producing either a relative spread or an absolute basis point spread. The absolute spread is probably more familiar, but we feel the relative spread is a better indication of value.

9. Distant forward rates are well above 9%.

10. In most graphs, the contractual provisions are altered from what might be considered a typical ARM in order to isolate the impact of the variable being analyzed. For instance, since Figure 3 focuses on the impact of the trend in rates, the margins are set equal (to 200 bp) for both the Treasury and COFI ARMs.

11. However, the ARM is still linked to the Cost of Funds Index, which is not perfectly related to market conditions.

12. Roll (1986, pp. 7–12) discusses the empirical differences.

13. For a detailed explanation see Bartholomew, Berk, and Roll, 1986, pp. 2–3.

14. The price at zero volatility was calibrated to be par less a correction for an assumed 44-day payment delay.

15. The behavior of the yield curve explains why there is *any* impact. The one-year Treasury index and the COFI index do not fluctuate to the same extent as the short-term interest rate.

16. I.e., 14/9=9/5.7857=1.556.

17. Payment capped ARMs are more commonly COFI ARMs.

18. Assuming that the market rate is still 9% at the end of the teaser period.

19. Given 15% annual relative volatility of short-term interest rates.

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