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Throwing Good Money After Bad? Cash Infusions and Distressed Real Estate

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When a leveraged real estate project experience cash-flow problems, the owner must either inject additional cash or default on the mortgage. We show that it is not optimal for the owner to default as soon as net cash flow becomes negative. Surprisingly, the owner can expropriate some of the mortgage lender's wealth by injecting cash and continuing to pay interest. When the owner has cash constraints, outside investors may be able to extract significant economic rents by financing distressed real estate projects. These results have interesting implications for mortgage lending and the pattern of real estate transaction volume.

Consider the following problem. Because of a downturn in the real estate market, the owner of a major office building faces a situation in which the cash flow produced by the building is now less than the interest payment on the mortgage. The building is not divisible, so a fraction of it cannot be sold piecemeal to meet the mortgage payment. In addition, the drop in the value of the property has increased leverage to the point where further borrowing is no longer possible. As a result, the owner has only two choices: (1) make up any interest shortfall by injecting new equity into the project; or (2) walk away and give the building back to the lender.

This problem is not unique to real estate. Highly-leveraged enterprises often face the same tradeoff if assets are illiquid or if further borrowing is not feasible. Because real estate projects are usually highly leveraged, the analysis is particularly applicable to real estate.

The injection of new capital into real estate projects occurs frequently; from 1990 to 1993 dozens of private partnerships and closely-held real estate firms injected cash in order to retain control of their operations. High profile examples include numerous injections by Olympia and York into the Canary Wharf project before it finally collapsed. More recently, this scenario has been repeated by the Canadian real estate firm Trizec, which has been infusing cash into projects for the last three years.

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These examples raise questions regarding the circumstances under which it is optimal for owners to inject cash to retain control of a project and regarding the valuation effects of such infusions. The analysis developed here provides an answer to these questions in the context of the familiar contingent claims model of capital structure developed by Black and Scholes (1973), Merton (1974) and Black and Cox (1976). A number of interesting insights emerge from this analysis. We show that it is not optimal for the owner to default as soon as the net cash flow of the project becomes negative.[1] The owner has a strong incentive to inject capital and continuing meeting debt-service obligations until the cash drain reaches a critical value.

A surprising implication of this analysis is that if the owner follows the optimal strategy of injecting capital into the project and continuing to make mortgage payments, the mortgage lender can actually be worse off. The intuition for this striking result is that even though the mortgage lender may receive more interest payments when the owner injects cash, the lender is harmed because default is postponed until the value of the property is much less. Thus, mortgage lenders experience greater losses if default ultimately occurs.

An important implication of our analysis is that the benefits to the owner from following the optimal strategy can be many times larger than the present-value costs of the necessary cash injections. The intuition for this is that the cost of relaxing cash-flow constraints is not directly related to the corresponding transfer of wealth from the mortgage lender to the owner of the project. These results indicate that there are significant economic rents to be obtained from providing funds to distressed real estate ventures. This may explain why real estate organizations are often structured in a fashion that facilitates making cash infusions. Finally, we show that the optimal cash-injection policy may explain the well-known tendency for real estate transaction volume to decline when market prices fall, and to recover only after a significant drop in value.

Because our analysis is set in the context of rational option pricing, there are aspects of real estate restructuring that it cannot address. Most importantly, it does not take account of strategic and agency theoretic issues that arise when the entire financial structure of a distressed property is renegotiated under incomplete information. These problems are addressed in related papers including Benveniste, Capozza, Kormendi and Wilhelm (1994) and

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[1] Kau and Kim (1994) also find that it may be optimal for a value maximizing borrower to delay default.

Riddiough and Wyatt (1994). Similarly, our model cannot analyze potential wealth transfers between borrowers and lenders that may occur because of asymmetric information. Despite the fact all risks are priced fairly *ex-ante*, the option pricing framework does lead to interesting insights about how cash infusions can transfer wealth *ex-post*.

The remainder of this paper is organized as follows. Section 2 discusses the optimal cash-infusion strategy for a distressed project. Section 3 considers the implications for mortgage lenders. Section 4 examines the costs and benefits to outside investors from injecting capital into a distressed project. Section 5 discusses the implications of this analysis for transaction volume in the real estate market. Section 6 summarizes the results and concludes the paper.

The Optimal Strategy

Let V denote the value of the real estate project. This project is assumed to produce a constant stream of cash flows (rental income minus operating expenses). To allow for randomness in the stream of cash flows, we assume that cash flows are generated at rate δV.

The property is financed by equity and a first mortgage loan on the property. The face amount of the mortgage is designated F and m is the fixed mortgage interest rate. For simplicity, we assume that the mortgage is a consol. This assumption, however, could be relaxed since the maturity of the mortgage has little effect on the results of the analysis.[2]

Mortgage interest payments of $mF\,dt$ are payable continuously. In this simplified framework, we also assume that the mortgage cannot be prepaid unless the project is in foreclosure.

The owner of the project receives continuous cash inflows from the project of δV and has continuous debt-service obligations of mF. Let $B = mF/\delta$ be the value of V at which the net cash inflow $\delta V - mF$ equals zero. Clearly, when $V > B$ the net cash flows to the owner are positive and financial distress does not occur. When $V < B$, however, the net cash flow is negative. In this case, default is immediately triggered unless additional cash is injected into the project to cover the shortfall in cash flows. In particular, the mortgage

[2] When the maturity of the mortgage is finite, numerical analysis is required to solve the fundamental differential equation. Examination of the numerical solutions by the authors reveals that the general nature of the results is unchanged.

is a non-recourse loan and the mortgage lender cannot force the owner to inject additional cash to meet debt-service obligations.

Finally, we assume that the value of the project follows a standard geometric Brownian motion process. Since the project generates cash flows at a rate of δV, the dynamics for V are given by

$$dV = (\mu - \delta)V\,dt + \sigma V\,dZ, \tag{1}$$

where μ is the instantaneous expected return on the project (consisting of both appreciation and cash flow), σ is a constant and Z is a standard Wiener process. Note that although the property cannot be sold piecemeal, it is marketable as a nondivisible unit. Hence, the market value V is well defined and we assume that standard option-pricing techniques for valuing contingent claims on V are applicable.

We now consider how the choice of cash infusion strategy affects the value of the equity in the project. As a point of reference, we first examine the case where the owner defaults as soon as the cash flows from the project minus the debt-service cash flows reach zero. We designate this the zero-cash-flow case. We then derive the value of the equity when the owner follows the optimal cash-injection strategy and compare it directly to the zero cash-flow case.

The Zero Cash-Flow Case

If the owner chooses not to inject cash into the project, then the project defaults on its mortgage as soon as the cash flows δV equal the required debt-service cash flows mF. Thus, default occurs as soon as $V = B$.

Since we have assumed an infinite horizon, the value of a contingent claim on V does not depend on time. Let $G(V)$ denote the market value of the equity in the zero cash-flow case. In addition, let r denote the riskless interest rate, assumed constant. The value of the equity then solves the following ordinary differential equation

$$(\sigma^2V^2)/2)G'' + (r - \delta)VG' - r\,G + \delta V - mF = 0, \tag{2}$$

for $V \geq B$, subject to the boundary condition

$$G(B) = \max(0, B - F), \tag{3}$$

and the transversality condition $G(V) \leq V$ as $V \rightarrow \infty$.

The term $\delta V - mF$ in equation (2) reflects the cash flows to the owner after debt service, and is strictly positive in this case. The boundary condition reflects the fact that when default occurs, the mortgage lender has a claim of F on the property. When $B > F$, the project runs out of cash even though the value of the property is large enough to satisfy the mortgage lender's claim. In this situation, the mortgage lender never suffers a loss and the debt is riskless. When $B < F$, however, the project runs out of cash when the value of project is less than the mortgage amount and the mortgage lender suffers a loss in the event of a default.

Solving the differential equation gives the value of the equity. If $B > F$, the solution is simply,

$$G(V) = V - F. \tag{4}$$

In the more usual case where $B < F$,

$$G(V) = B^{\ \gamma}(F - B)V^{\gamma} + V - F, \tag{5}$$

where

$$\gamma = \frac{-(r - \delta - \sigma^2/2) - \sqrt{(r - \delta - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}.$$

The value of the equity is an explicit function of the value of the project and depends on the parameters r, δ, σ^2, m and F. $G(V)$ is an increasing function of V for all values of δ. In addition, when $B < F$, $G(V)$ converges to $V - F$ as $V \rightarrow \infty$.

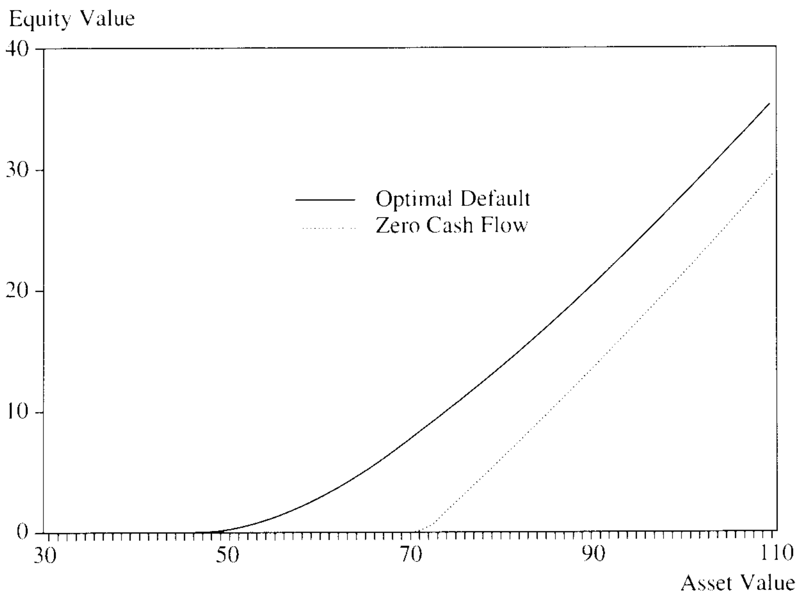
Numerical Example. Consider a real estate project that generates cash flows of δV, where $\delta = .07$. Let $r = .05$, $\sigma^2 = .04$, and assume that $F = 100$ and $m = .05$. Thus, the debt-service obligation is $mF = 5$. As long as V is greater than $B = 5/.07 = 71.43$, the cash flows from the project are greater than the debt-service obligations. At $V = 71.43$, however, net cash flow becomes zero and default is triggered. Figure 1 graphs the value of the equity in the project as a function of the value of the project. Note that $G(V)$ is slightly curved and concave upward.

The Optimal-Infusion Case

In the previous case, the owner defaults as soon as the cash flows from the project are insufficient to cover the mortgage obligations. This assumption

Figure 1 ■ The Value of the Equity.

The value of the equity graphed as a function of the value of the project for the zero cash-flow and optimal strategy cases. The parameter values used are $F = 100$, $m = .05$, $\delta = .07$, $r = .05$ and $\sigma^2 = .04$.



about default underlies many models for valuing corporate securities including Black and Scholes (1973) and Merton (1974). In general, however, this default condition may not represent an optimal choice for the owner. For example, when the value of the property is less than F, default implies that the owner loses all of his investment in the project. Hence, the owner may have an incentive to inject cash in order to keep the option on the equity of the project alive.

The problem of simultaneously determining the optimal cash-injection policy and the corresponding value of the equity is conceptually similar to determining the optimal exercise strategy and value for an American option. This is because the decision to default is analogous to deciding to exercise an American call option early. Let $S(V)$ denote the value of the equity under the optimal policy. Similarly, let A denote the value of V at which cash injections are optimally discontinued and default occurs. Following Merton (1974) and Black and Cox (1976), the optimal default value A can be found by solving for $S(V)$ as a function of A, and then solving for the value of A

that maximizes $S(V)$. The value of the equity $S(V)$ for a given default policy A satisfies the ordinary differential equation

$$(\sigma^2 V^2/2)S'' + (r - \delta)VS' - rS + \delta V - mF = 0, \tag{6}$$

for $V \geq A$, and where the term $\delta V - mF$ can now be positive or negative. This differential equation is solved subject to the following two boundary conditions,

$$S(A) = 0,$$

$$S'(A) = 0, \tag{7}$$

as well as the transversality condition.

Solving this problem leads to the result that the optimal default strategy is to default when V reaches the value

$$A = \frac{F}{(1 - 1/\gamma)}, \tag{8}$$

where γ is as given in equation (5).[3] This optimality condition has a number of important implications. First, A is strictly less than F. This means that when default occurs, the value of the project is insufficient to satisfy the claim of the mortgage lender. Thus, the mortgage debt is risky for all values of δ. Recall that in the zero cash-flow case, the mortgage is only risky when $B < F$. Second, A is also less than B. This means that when $A < V < B$ the owner always chooses to inject cash into the project to meet the debt-service obligation. Thus, it is optimal for the owner to keep the investment in the project alive even though it produces negative cash flows. Finally, observe that it is optimal to inject cash for some range of V irrespective of the value of μ. This means that the decision to tolerate negative cash flows is independent of the expected return on the property. This contrasts with the common wisdom that investments with negative cash flows should be held only if their expected return is sufficiently high.

The solution for the optimal default strategy also answers the question of whether it is ever optimal to default when the cash flow from a project is larger than the debt-service obligation. It is straightforward to show that the

[3] It can easily be shown that $\gamma < 0$.

default policy given above strictly dominates this policy for all values of the parameters. Thus, it is only optimal to default when the net cash flow is negative.

The value of the equity implied by the optimal default strategy is given by

$$S(V) = -\frac{1}{\gamma}A^{1-\gamma}V^{\gamma} + V - F. \tag{9}$$

As in the zero-cash-flow case, the equity is an increasing function of the value of the project and approaches $V - F$ as $V \rightarrow \infty$.

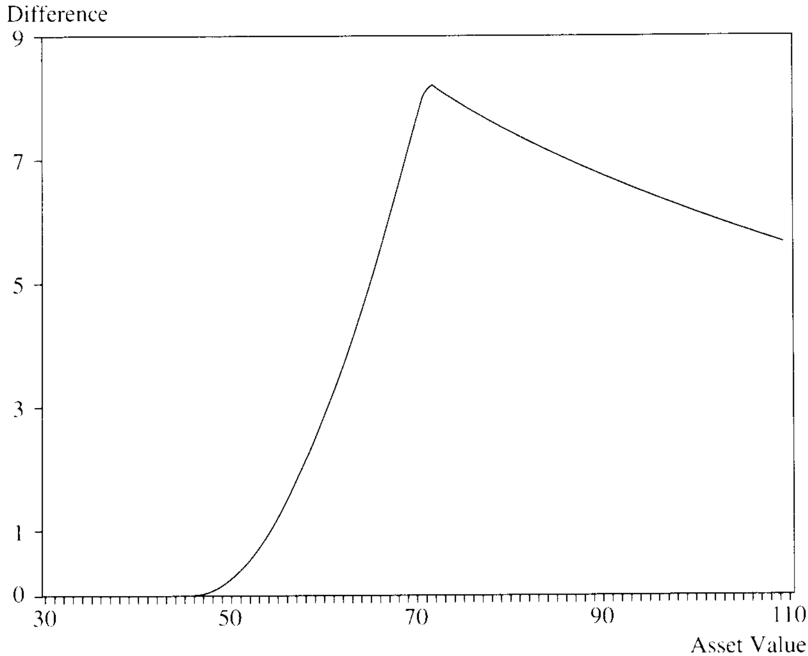
The increase in the value of the equity resulting from following the optimal strategy rather than the zero cash-flow default condition is found by subtracting $G(V)$ from $S(V)$. The maximum difference between the two values occurs at B. This is intuitive since, for example, when $B < F$, the value of the equity is zero at B under the zero cash-flow strategy. As $V \rightarrow \infty$ the difference between the two values decreases since it is less likely that cash injections will be necessary.

Numerical Example continued. The optimal value of A is 46.55. Thus, the owner should inject cash of $5 - .07 \times V$ when $46.55 < V \leq 71.43$. This means that the maximum negative cash flow the owner is willing to tolerate is $5 - .07 \times 46.55 = -1.74$. Figure 1 also graphs the value of $S(V)$ as a function of V. When $V = 71.43$, the value of the equity is zero in the zero-cash-flow case and 8.24 in the optimal-infusion case. Figure 2 graphs the difference between $S(V)$ and $G(V)$. The reason for the discontinuity in the slope of the difference at $V = 71.43$ is that the slope of $G(V)$ jumps to zero from some positive value at B. This jump occurs since $G(V)$ does not satisfy the high-contact or optimality condition at B.

Given the incentives that owners have to inject additional capital into distressed projects, this analysis suggests that there may be economic benefits to structuring real estate ventures in a way that facilitates capital infusions. For example, it may be less costly to inject additional capital into closely-held companies rather than publicly-traded companies. In addition, partnerships may allow capital infusions more readily than corporations. For example, real estate limited partnerships give the general partners effective control over the property until liquidation and often require limited partners to make added capital investments, at least within limits. This makes it easier for the general partner to follow an optimal cash infusion policy. While there are also tax-related reasons for the partnership form, our analysis suggests

Figure 2 ■ The Difference in the Values of the Equity.

The difference between the value of the equity in the optimal strategy case and the zero-cash-flow case graphed as a function of the value of the project. The parameter values used are $F = 100$, $m = .05$, $\delta = .07$, $r = .05$ and $\sigma^2 = .04$.



that facilitating capital injections may play a role in the choice of organization in real estate investments.

Implications for Mortgage Lenders

The previous section derived the optimal cash-injection policy and examined its effects on the value of the equity in the project. In this section, we examine the effects of owners following the optimal strategy on the value of the mortgage.

Let $D(V)$ denote the value of the mortgage in the zero cash-flow case. Using the same approach as in the previous section, we can solve for the value of $D(V)$. The value of the mortgage is

$$D(V) = V - G(V). \tag{10}$$

This is intuitive since the sum of the values of the debt and the equity must equal the total value of the project.

Recall that in the zero cash-flow case, when $B > F$, default occurs when V is greater than F. In this situation, the mortgage lender receives the full amount of the claim in foreclosure and the debt is riskless. In contrast, when $B < F$, the debt is risky. An interesting implication of this is that the nature of the real estate, which determines δ, has an important influence of the risk of the mortgage.

Now consider the value of the mortgage when the owner follows the optimal strategy. Let $P(V)$ denote the value of the mortgage in this case. It is now no longer clear that the value of the mortgage should equal the difference between the value of the project and the equity. This is because additional funds may be injected by the owner and become part of the project, thereby changing the total value of the project. Despite this complexity, however, it turns out that the value of the mortgage is still given by the difference between the current value of the property and
the equity

$$P(V) = V - S(V). \tag{11}$$

The intuition for this is that the value of the equity reflects optimal capital infusions by the owner.

Taking this analysis one step further leads to a number of surprising and important implications for mortgage lenders. In particular, since $S(V)$ is greater than $G(V)$, the value of the mortgage in the zero cash-flow case is higher than in the optimal-infusion case. This means that the mortgage lender would actually be better off if the owner defaulted as soon as the project were to run out of cash, rather than injecting new cash and continuing to meet the debt-service obligations. Note that the difference in the values of $P(V)$ and $D(V)$ equals the negative of the difference between $S(V)$ and $G(V)$, which is the gain to the owner from following the optimal strategy.

The intuition for this seemingly perverse result is best conveyed by considering the case when $B > F$. If the owner does not inject cash, the project defaults at a value of V greater than F and the mortgage is riskless. If the owner follows an optimal strategy, however, default occurs only when V is less than F. Hence, if there is a default, the mortgage lender loses, which means that the mortgage is no longer riskless and is worth less than in the zero cash-flow case. Although the owner may make more interest payments under the optimal strategy, the mortgage lender suffers greater losses in the event of a default. Optimality implies that the owner will only inject cash when the tradeoff is in his favor.

When designing mortgage contracts, it follows that lenders face a tradeoff between *ex-ante* pricing (increasing interest rates or points) and limiting the ability of borrowers to inject cash. To the extent that the ability of borrowers to inject cash can be reduced at the outset, interest rates and lending fees can be reduced. One way for mortgage lenders to mitigate the cash-injection problem, and thereby lower points or interest charges, is to broaden the criterion for default to include conditions other than the failure to meet the debt-service obligations. For example, the mortgage could include requirements that the cash flows or earnings generated by the ongoing operations of the project provide a minimum level of coverage for interest payments. Alternatively, the mortgage could place restrictions on the mortgage-loan-to-value ratio. Such types of conditions are not uncommon in commercial real estate loans.

The preceding analysis assumed that the property could be liquidated without cost. This means that any benefits that the owner achieves by injecting cash come at the expense of the lender. When transaction costs are introduced, however, the situation is no longer a zero-sum game. Both the owner and the lender can benefit if transaction costs can be postponed or reduced. In effect, the owner and lender benefit at the expense of the transactors—brokers, attorneys, and the like—if a sale can be avoided. This means that if the lender is not in a position to operate the property, and therefore, would sell if the owner defaulted, lenders may prefer that the owners inject cash.

The analysis of transactions costs is relatively straightforward because they do not affect the optimal decision by the owner. Recall that when the owner follows the optimal strategy, the owner receives nothing when default occurs. Therefore, the transactions costs are irrelevant to the owner. To analyze the impact on bondholders, we assume that transaction costs are a constant fraction k of the value of the property and that $B < F$. Given these assumptions, the value of the mortgage in the zero cash-flow case is

$$D(V) = F - B^{\gamma}(F - (1 - k)B)V^{\gamma}. \qquad (12)$$

Similarly, the value of the mortgage in the optimal-infusion case is

$$P(V) = F - A^{\gamma}(F - (1 - k)A)V^{\gamma}. \qquad (13)$$

Since $A < B < F$, it is straightforward to show that $P(V)$ can exceed $D(V)$

in some cases. The intuition for this is that the value of the mortgage is the sum of the present value of the stream of mortgage payments received until default, plus the present value of the amount recovered in foreclosure. In the optimal-infusion case, relatively less of the value of the mortgage is due to the amount recovered and more is due to the stream of payments than in the zero cash-flow case. Because the payment stream is not subject to transactions costs, an increase in k reduces the value of $D(V)$ more than it does $P(V)$. Consequently, for small k, $P(V) < D(V)$, but for k sufficiently large, this inequality can be reversed.

Numerical example continued. Recall that $A = 46.55$ and $B = 71.43$. Substituting these values in equations (12) and (13) shows that the mortgage lender is better off in the optimal-injection case than in the zero cash-flow case for any value of V when $k > .2134$.

Capital Infusions and Transfers of Ownership

In the previous sections, we assumed that the owner had the resources necessary to follow the optimal cash-injection strategy. In reality, however, owners with a substantial proportion of their wealth invested in a project may find that they are not in a position to make the required cash injections when the project is experiencing cash-flow problems. In this section, we examine the incentives that outside investors have to inject capital in exchange for an ownership interest in the project.

Earlier, we showed that by injecting cash, the owner was better off than if cash was not injected. Thus, the increase in the value of the equity from following the optimal strategy exceeds the present-value costs of the cash injection. The difference $S(V) - G(V)$ represents the increase in the value of the equity, net of the present-value costs of the cash injection. To compute the total or gross amount of the increase in the value of the equity, we first need to compute the present-value cost of the cash injection.

This calculation is complicated by the fact that the actual amount of cash injected is a random variable that depends on the value of V. The cash injection is zero when $V < A$, $mF - \delta V$ when $A < V < B$ and zero when $B < V$. The actual value of the cash that needs to injected during the remaining life of the project could be nearly zero, as in the case where V only reaches B once and then remains above thereafter, or it could be $F - A\delta/r$ as in the case where the value V remains a fraction above A during the entire life of the project. Because of the dependence of the cash flows on V, however, we can solve explicitly for the value of the contingent stream of cash injections. Let $I(V)$ be the value of the stream of cash injections. The value of $I(V)$ is determined as the solution of the ordinary differential equation

$$(\sigma^2 V^2/2)I'' + (r - \delta)VI' - rI + mF - \delta V = 0, \tag{14}$$

for $A < V \leq B$, and of

$$(\sigma^2 V^2)/2)I'' + (r - \delta)VI' - rI = 0, \tag{15}$$

for $V > B$. The appropriate boundary conditions for this contingent claim are

$$I(A) = 0,$$

$$I(\infty) = 0. \tag{16}$$

In addition, the continuity and high-contact conditions are required to hold at the interior boundary $V = B$.

The present value of the stream of cash injections is

$$I(V) = \alpha V^{\gamma} + \beta V^{\eta} + F - V, \tag{17}$$

for $A < V \leq B$, and

$$I(V) = (\alpha + \beta B^{\eta - \gamma} + (F - B)B^{-\gamma})V^{-\gamma}, \tag{18}$$

for $V > B$, where

$$\alpha = (A - F - \beta A^{\eta})A^{-\gamma},$$

$$\beta = \frac{B - \gamma(F - B)}{\gamma - \eta} B^{-\eta},$$

$$\eta = \frac{-(r - \delta - \sigma^2/2) + \sqrt{(r - \delta - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}.$$

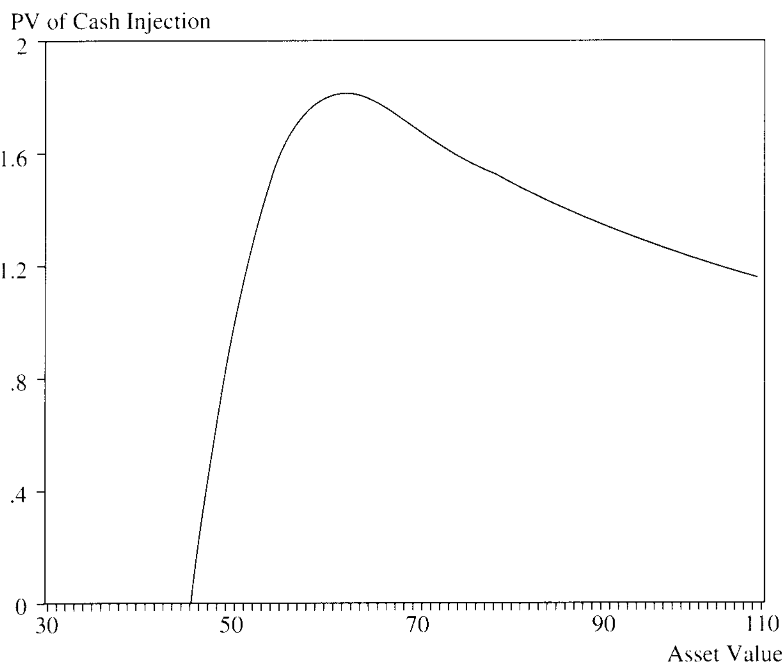
Using this expression for $I(V)$, we can now compute the total increase in value accruing to the owner from following the optimal cash injection policy. Since the increase in value net of injections is given by $S(V) - G(V)$, the total increase is $S(V) - G(V) + I(V)$.

Numerical example continued. When $V = B = 71.43$, the net increase in the value of the equity is 8.24 and the present value of the cash injections is 1.69. The gross increase in the value of the equity is 9.93. This implies that the benefit-cost ratio for optimal cash injections is $(8.24 + 1.69)/1.69 = 5.87$. This highly profitable ratio creates the potential for significant economic rents to current or prospective equity investors. The present value of the contingent cash injections is graphed as a function of V in Figure 3.

When the current owner does not have the funds needed to make cash injections, he may still benefit by selling some portion of the equity in the project to outside investors in return for cash. However, since the costs and benefits of following the optimal strategy are not equal, the percentage of the project that the outside investor acquires in the bailout is negotiable. For example, if the current owner is cash constrained and $V = B$, the value of the equity is zero if no bailout occurs. If there is only one prospective outside investor, it is possible that the outside investor could acquire nearly all of

Figure 3 ■ Present Value of the Contingent Cash Injections.

The present value of the contingent cash injections graphed as a function of the value of the project. The parameter values used $F = 100$, $m = .05$, $\delta = .07$, $r = .05$ and $\sigma^2 = .04$.



the equity in return for the bailout. In contrast, if there are many potential outsider investors, the current owner may be able to retain most of the net increase in the value of the equity arising from the bailout. Although the model does not permit analysis of strategic negotiation of the entire capital structure, it does suggest in which circumstances such negotiations are likely to occur. In particular, when the transaction costs of selling are large and when there are significant benefits to injecting cash, but equity holders do not have sufficient cash, strategic negotiation is likely.

The issue of the optimal timing of a bailout is also of interest. In particular, at what point is it optimal for cash-constrained owners to seek a partner who would agree to make cash injections when needed? Alternatively, for what value of V is the ratio of benefits to the present-value cost of the contingent cash injections maximized. Surprisingly, there is no optimal value of V. This is because the ratio of $S(V) - G(V) + I(V)$ to $I(V)$ is the same for all values of V, $V > B$. This means that there is no particular value of V at which the current owner might have more bargaining power relative to the costs and benefits of cash injections. This suggests that if the bargaining process is costly, it may be optimal for the current owner to postpone negotiations for a bailout until absolutely necessary.

From the perspective of an outside investor, these results simply that providing funds to real estate ventures nearing financial distress may be a highly profitable activity. An outside investor with significant bargaining power could extract several times the present value of his investment from the financially-distressed project by agreeing to make the optimal cash injections. It should be stressed that although these cash infusions expropriate lender wealth *ex-post*, they do not imply lender irrationality. As noted earlier, lenders will factor the possibility of expropriation into the price of the loan *ex-ante*.

Our analysis has focused on contingent pay-as-needed injections of capital rather than lump-sum capital infusions. The reason for this is that lump-sum infusions induce a path-dependent nature to the problem that complicates the analysis. Intuitively, however, it is clear that the basic results will be similar if infusions occur in lumps rather than flows. This is because these types of infusions provide liquid assets that allow the owner to avoid default until the value of the project is lower than B. This again has the effect of transferring wealth to the owner. As noted earlier, however, the option pricing approach, which assumes complete information, cannot be used to analyze the strategic negotiations that are likely to characterize lump sum infusions.

Real Estate Transaction Volume

The model also has implications for the volume of transactions in real estate markets. A common observation made by commercial real estate brokers is that when market prices fall, the volume of transactions also declines. Only after a significant drop in prices does volume begin to recover. This pattern is somewhat puzzling because there is no obvious reason why the volume of trading should be related to the level of prices in a competitive market. Studies of the stock and futures market have found no analogous effect.[4]

One hypothesis that some practitioners have advanced to explain the puzzle is that sellers set reservation prices which adjust slowly over time. When market prices drop sharply, the lagged adjustment of reservation prices leads to a decline in volume. Trading volume returns to normal only after the price decline has slowed and reservation prices have caught up with market prices. The problem with this explanation is that there is no reason why rational sellers would consistently behave in this fashion.

An alternative explanation is that the due-on-sale clauses contained in many real estate financing agreements lead to what can be called "mortgage lock-in." With a due-on-sale clause, the owner of a property must first exercise the option to purchase the property from the lender, by repaying the loan at face value, before the property can be sold. Because early exercise of the option is costly, even if interest rates have not changed, the owner will be unwilling to sell the property for his or her estimate of the market value, V. A potential buyer must bid a price enough in excess of the face amount of the debt to compensate the seller for the cost of exercising the option prematurely. The difference between the amount which must be bid by the potential buyer and the current value of V, which we term the premium, equals the sum of the face amount of the debt and the value of the equity, minus the current value of V. The greater this cost, relative to the market price, the fewer transactions that will occur.

To see how the mortgage lock-in phenomenon operates in the context of our model, assume again that the value of the underlying property is V and that it generates a cash flow δV. To simplify the exposition, assume further that $B < F$. Even without the possibility of cash infusions, mortgage lock-in will occur to some extent. This is illustrated in Figure 4 which graphs the premium as a function of V. Without cash infusions, however, the owner of the

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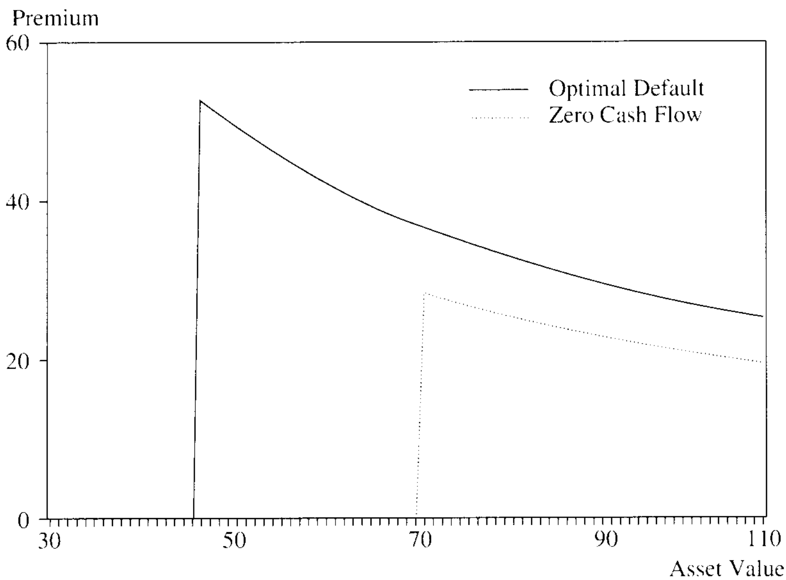
[4] See Karpoff (1986) for a review of the literature. The main finding is that volume is correlated with the volatility of prices, but there is little, if any, relation between volume and the level of prices.

Figure 4 ■ Mortgage Lock-in Cost.

The cost of selling the project as a function of the value of the project for the zero-cash-flow and optimal strategy cases. The cost of selling, or the mortgage lock-in cost, equals the value of the equity minus the difference between the value of the project and the face amount of the mortgage. The parameter values used $F = 100$, $m = .05$, $\delta = .07$, $r = .05$ and $\sigma^2 = .04$.



property will default as soon as $V = B$. At that point, the property will be put back to the bank and the mortgage will lock-in disappears.

When the possibility of cash infusions is added, the mortgage lock-in problem becomes more pronounced. As shown in Figure 4, owners will not walk away until the value of the property has fallen to the level A, which was shown earlier to be much less than the face amount of the debt, F. Consequently, the model predicts that when the prices of buildings fall below the face value of the debt, transactions will become almost nonexistent unless the due-on-sale clauses are relaxed.

As noted in the previous sections, lenders can protect themselves against expropriation of their wealth via capital infusions by writing added conditions into the loan agreement. When mortgage loans are made by depository institutions on real property, as opposed to private mortgage lenders, there are some added considerations. Because depository institutions, at least until recently, could carry a loan on their books at face value as long as it was performing and because they are subject to balance-sheet-based capital requirements, depository institutions have an incentive not to call a loan even when it appears to be a wealth-maximizing decision. Consider, for instance, the case in which the market value of a depository institution is negative, but a majority of its loans are still performing because of capital infusions by the borrowers. In other words, the values of the underlying properties are between A and F in Figure 4. Although calling the loans and liquidating the institution would be a value-maximizing decision, it is not in the interest of the institution's shareholders who themselves have an option to put the firm to the regulatory authorities. Consequently, lenders may not want to put strict clauses into lending agreements which force them to call performing loans and prefer instead to price the cost of possible future infusions, *ex-ante*. This is clearly an interesting area for future research.

Conclusion

When a levered real estate project encounters cash-flow problems, the owner must either inject additional cash or default on the mortgage. We examine the owner's optimal decision using the familiar contingent claims model of capital structure used by Black and Scholes (1973), Merton (1974) and Black and Cox (1976).

Three major results emerge from this analysis. First, we show that the owner always has an incentive to inject cash until the cash drain reaches a critical level. Second, we show that the mortgage lender can actually be worse off if the owner injects cash and continues to meet the debt-service obligations. This result, however, depends on the size of the transactions costs associated with a distressed sale of the real estate project. Finally, we show that the present value of the cash injections needed to assist a distressed real estate project may be far smaller than the present value gains to the owner of keeping the project alive. Thus, even if the owner does not have the necessary cash resources, the owner has considerable leeway with which to bargain with outside investors.

Capital infusions are important and frequent events which have major implications for the survival of real estate ventures and the risk of mortgage lending. The option pricing approach makes it possible to value the right to infuse cash, and thereby to assess when infusions and restructuring will occur. Because it assumes complete information, however, it does allow for realistic analysis of the restructuring process. In addition, the optimal strategy of continuing to hold real estate projects even when cash flows are negative provides an explanation for the dramatic drop in transaction volume that often accompanies a decline in real estate values.

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References

Benveniste, L.D., R. Capozza, R. Kormendi and W. Wilhelm. 1994. Contract Design for Problem Asset Disposition. *Journal of the American Real Estate and Urban Economics Association* 22: 149–168.

Black, F. and J.C. Cox. 1976. Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance* 31: 351–357.

Black, F. and M. Scholes. 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* 81: 637–654.

Karpoff, J.M. 1986. A Theory of Trading Volume. *Journal of Finance*: 41: 1060–1088.

Kau, J.B. and T. Kim. 1994. Waiting to Default: The Value of Delay. *Journal of the American Real Estate and Urban Economics Association* 22: 539–551.

Merton, R.C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29: 449–470.

Riddiough, T.J. and S.B. Wyatt. 1994. Strategic Default, Workout and Commercial Mortgage Valuation. *The Journal of Real Estate Finance and Economics* 9: 5–22.