MARKETING SCIENCE
Vol. 7, No. 4, Fall 1988
Printed in U.S.A.

MODELING ASYMMETRIC COMPETITION

GREGORY S. CARPENTER, LEE G. COOPER,
DOMINIQUE M. HANSSENS AND DAVID F. MIDGLEY
Columbia University
University of California, Los Angeles
University of California, Los Angeles
Australian Graduate School of Management

The effects of the marketing actions of one brand can be distributed among its competitors' market shares in a complex manner. This paper presents and illustrates methods for modeling brand competition and brand strategies in markets where competitive effects can be differentially and asymmetrically distributed. We discuss the empirical specification, parameter estimation and competitive—strategy implications of the models proposed. Price and advertising competition among eleven brands of an Australian household product is used to illustrate the application of these procedures.
(Market Shares; Competition; Market-Response Models)

1. Introduction

When the effects of a brand's marketing actions are distributed among its competitors out of proportion to their market shares, competition is called *asymmetric*. For example, in the cola market advertising competition between Pepsi and Royal Crown Cola would be asymmetric if a 10% increase in advertising by Pepsi affected Royal Crown much more than it did a local brand with the same market share as Royal Crown. Similar asymmetries in competition may exist for all elements of the marketing mix, in other markets for consumer packaged goods (e.g. coffee, detergent, disposable diapers), and for industrial products and services (e.g. overnight package-delivery services).

Asymmetries in competition arise from two main sources. First, some brands may have unique features of their strategy which either shield them from competitors' marketing actions, or which make them particularly vulnerable to such actions. Comparative advertising, such as the "Pepsi Challenge", may make one brand's share disproportionately sensitive to another brand's efforts. Unique distribution, reputation or an especially valuable brand name can also produce asymmetries.

Second, asymmetries can arise from period-to-period variation in marketing-mix elements such as relative advertising spending or prices. For example, Pepsi Cola may be less sensitive than Coca-Cola to increases in advertising spending by Royal Crown—simply because Pepsi may at times outspend Coke in advertising to ensure that its message is heard through the din of competitive messages. These differences in strategy may be only temporary, such as the temporary distinctiveness created by advertising pulsing,

0732-2399/88/0704/0393$01.25

but a brand which uses such 'distinctiveness' as a part of a longer-term strategy may also establish a differentiated position in the marketplace.

In addition, brands that compete asymmetrically must contend with the practical reality that some have more effective marketing strategies, or more effective organizations implementing those strategies, than others. In the cola market, a large competitive effect of Pepsi on Royal Crown could reflect a superior advertising campaign. The dynamics of consumer response may also contribute to the effectiveness of marketing actions, as in the case of advertising carryover. Dominant brands, with extensive advertising capital, may have more effective advertising simply because of the carryover from past advertising messages. Not having that benefit, smaller brands might need to spend more to have the same impact.

The combination of these sources of asymmetries and differential effectiveness helps define the competitive structure of the market and the strategic opportunities each firm faces. In our cola-market example, Coke's competitive position clearly differs from Royal Crown's and as a result their strategies may be substantially different. Furthermore, highly price-sensitive brands, perhaps C & C, may pursue different competitive strategies than brands with greater advertising capital and less price-sensitive buyers.

Methods exist for modeling these sources of asymmetries and for exploring their strategic implications. Bultez and Naert (1975), for example, propose an extension of the attraction (multiplicative competitive interaction, or MCI) model of market-share formation suggested by Nakanishi and Cooper (1974) to include "differential effects" (unique, brand-specific parameters) and "cross-effects" (idiosyncratic effects between brands). More recently, Batsell and Polking (1985) suggest a class of market-share models to estimate specific cross-effects using experimental data. Competitive dynamics and strategic implications can be explored using concepts from time-series analysis, game theory and numerical methods. Hanssens (1980) analyzes competitive dynamics in the airline industry using time-series analysis. Karnani (1983) uses the MCI model to analyze the minimum share needed to be profitable in an industry. His analysis is theoretical, unlike Ghosh and Craig (1983) who use the same model to analyze retail site location numerically.

However, applying these methods to the managerial problem of understanding competition, and making resource-allocation decisions based on those models, is a challenge. Market-share models like the one proposed by Bultez and Naert can be difficult to estimate because of the large number of parameters involved and problems with collinearity (Bultez and Naert 1975, Cooper and Nakanishi 1988). Other models, such as Batsell and Polking's (1985), have been developed on experimental data, and have not been adapted to the large data bases now becoming increasingly common. Moreover, these models are typically static; exactly how one can extend them into dynamic models with theoretically sound and empirically tested methods has so far not been clarified. Finally, deriving resource-allocation implications from such a model would be complex even if one could construct, specify and estimate one. Concepts from game theory are largely descriptive, not normative, and appropriate allocations are hard to derive even numerically because of uncertainty about how to incorporate competitors' actions.

In this paper, we describe approaches to overcoming these problems and develop implementable methods to model asymmetric competition and to explore the strategic implications of those models in terms of brand profits. Our strategy is to integrate market-share models, time-series analysis, and methods from game theory and optimization. Our starting point is the extended attraction model allowing differential effects as well as specific cross-effects. We present simple methods to test for and incorporate differences in the effectiveness of brands' strategies, and unusually strong competitive relationships between brands. We also extend these models to include the effect of period-to-period variation in relative marketing efforts, and the dynamics of brand attraction using modern

time-series analysis. This integration yields a dynamic, asymmetric market-share model, for which we present simple least-squares estimation procedures free of some of the problems that plagued earlier models. Finally, we show how these models can be used to allocate resources. We construct profit scenarios, based on the market-share models, and show how brand strategies can be derived and computed under various assumptions about competitive behavior.

This integration produces three valuable results. First, the methods we present can provide managers with an implementable technology for understanding markets in which competition is complex, and for making competitive decisions in that environment. Second, the models provide a basis for future work on asymmetric competition and strategy. For example, the market-share models we construct and estimate imply a set of cross-elasticities that capture elements of competition between brands. The market structure implied by these elasticities can be spatially represented, as Cooper (1988) shows, drawing further insight from the models. Third, the models can also be used as a basis for evaluating short-run marketing strategies. Long-run strategies may be examined as well if methods can be developed for optimizing multi-period games under realistic competitive assumptions. These we leave as important future work.

Our paper is structured as a continuous mix of theory and application. We begin the discussion with the generalized attraction model and we present some details about the Australian household-product market to which this model will be applied. In subsequent sections we discuss and illustrate the specification, estimation, cross validation and strategic uses of our asymmetric market-share model and we outline limitations and directions for future research.

2. An Asymmetric Attraction Model

The asymmetric market-share models we propose are attraction models, based on the simple hypothesis that a brand's market share is equal to its *attraction* relative to all others. More formally, the market share M_{it} for brand i ($i, j = 1, 2, \ldots, N$) in time period, region or segment t ($t = 1, 2, \ldots, T$) is its attraction A_{it} relative to the total attraction of all brands:[1]

$$M_{it} = \frac{A_{it}}{\sum_{j=1}^{N} A_{jt}}. \tag{1}$$

To allow for differential effects, temporal distinctiveness of a brand's marketing actions, the dynamics of brand attraction, and unusually strong competitive relationships, we chose an extremely general and flexible specification for A_{it}. We let

$$A_{it} = \exp(\alpha_i) \prod_{k=1}^{K} [f_t(Y_{kit})]^{\beta_{ki}} \prod_{(k^*j^*) \in C_i} [f_t(Y_{k^*j^*t})]^{\beta_{k^*ij^*}} \tag{2}$$

where α_i is brand i's constant component of attraction, β_{ki} is brand i's market-response parameter for the kth marketing-mix element, $f_t(Y_{kit})$ is a dynamically-weighted measure of brand i's relative competitive position on the kth marketing-mix element, C_i is the set of unusually strong competitive relationships that we will call "cross-competitive effects" for brand i, and $\beta_{k^*ij^*}$ is the influence of brand j^*'s k^*th marketing-mix element on brand i's market share.

The market-share model implied by equations (1) and (2) captures four important dimensions of competition. First, the effectiveness of marketing actions varies across

[1] The error term is omitted from the attraction specification for ease of exposition. The discussion of specification errors and sampling errors may be found in Cooper and Nakanishi (1988, Chapter 5).

brands and across elements of the marketing mix as reflected in each brand i's set of unique parameters α_i, β_{ki}, $k = 1, 2, \ldots, K$.

Second, the model incorporates cross-competitive effects. This is a very important step in the development of an asymmetric attraction model. The asymmetries they imply can not be represented in a simple Luce-type choice model because of the IIA (independence of irrelevant alternatives) property that they embody (cf. Luce 1959, Currim 1982, Kamakura and Srivastava 1986). Rather than include all possible cross effects, many of which might be insignificant, we include only a subset for each brand, denoted by C_i. This greatly reduces the number of parameters to be estimated but still ensures that the model can reflect stable asymmetric effects.

Third, the current and past marketing actions of a brand are incorporated by creating dynamically weighted attraction variables. Let price be the first marketing-mix variable ($k = 1$) and advertising expenditures be the second ($k = 2$). Suppose, for example, that advertising for brand i, X_{2it}, affects its market share for three periods with declining influence, and suppose that we know the structure of the lagged effects. The current *effective* advertising of brand i might, therefore, consist of half of its current outlay and a declining portion of past expenditures. This implies its effective advertising at time t, Y_{2it} is

$$Y_{2it} = 0.5 \times X_{2it} + 0.3 \times X_{2i(t-1)} + 0.2 \times X_{2i(t-2)}. \tag{3}$$

A separate, dynamically weighted, attraction component can be constructed for each marketing variable for each brand.

Fourth, to capture the temporal distinctiveness of a brand's marketing mix, we employ transformations of these dynamically-weighted attraction components. All the allowable transformations must be positive and ratio-scale, i.e. $f_t(\cdot) \geq 0$. Cooper and Nakanishi (1983, 1988) discuss two allowable transformations, exp(z-scores) and zeta-scores. More formally, let

$$z_{kit} = \frac{Y_{kit} - \bar{Y}_{k \cdot t}}{S_{Y_{k \cdot t}}} \tag{4}$$

where $\bar{Y}_{k \cdot t}$ is the mean for instrument k over all firms in period t, and $s_{Y_{k \cdot t}}$ is the sample standard deviation over all firms in each period. Then $f_t(Y_{kit}) = \exp(z_{kit})$ is the exp(z-score) transformation, and zeta-scores are defined by:

$$\zeta_{kit} = \begin{cases} (1 + z_{kit}^2)^{1/2} & \text{for} \quad z_{kit} \geq 0, \\ (1 + z_{kit}^2)^{-1/2} & \text{for} \quad z_{kit} \leq 0. \end{cases} \tag{5}$$

While exp(z-score) is justifiable as a standardization on statistical grounds, zeta-scores are derived both from a theory of distinctiveness and from the physics of competitive forces. The distinctiveness of brand i is represented by the moment of inertia of all brands computed about brand i, relative to the moment of inertia of all brands computed from their centroid. The main advantage of zeta-scores viz-a-viz raw scores lies in separating the underlying importance of a marketing-mix element from the particular pattern in any given market context. If Coke and Pepsi are both on feature in a store they do not each get the same boost in market share as if they were featured alone. By specifically modeling such contextual effects we overcome the limitations imposed by the IIA (context-free) assumption of Luce choice models.

The elasticities implied by the model in equations (1) and (2) illustrate these asymmetries. The cross-elasticities from the generalized attraction model with respect to a percent change in a raw variable X_{kjt} are:

$$e_{ijt}^{(k)} = \overbrace{\beta_{ki} s_{ijt}^{(k)} - \sum_{i'=1}^{N} M_{i't} \beta_{ki'} s_{i'jt}^{(k)}}^{\text{Direct Effect}} + \overbrace{\sum_{(kj^*)\in C_i} \beta_{kij^*} s_{j^*jt}^{(k)} - \sum_{i'=1}^{N} M_{i't} \sum_{(kj')\in C_{i'}} \beta_{ki'j'} s_{j'jt}^{(k)}}^{\text{Cross Effect}} \quad (6)$$

where each term is weighted by the appropriate slope factor, e.g. $s_{ijt}^{(k)}$ is the percent change in $f_t(Y_{kit})$ relative to a percent change in X_{kjt}:

$$s_{jt}^{(k)} = \frac{\partial f_t(Y_{kit})}{\partial X_{kjt}} \times \frac{X_{kjt}}{f_t(Y_{kit})}. \quad (7)$$

These cross elasticities consist of two separate competitive effects. The *direct effect* reflects how the strength of brand i's own effect influences the impact of brand j's marketing-mix changes on brand i's market share. This strength is expressed as the deviation from average strength (weighted by market share) of each competitor's own effect. The *cross-effect* is the combined influence of all the competitors j^* which have a specific influence on brand i. Again, this cross-effect is expressed as the deviation from the average strength of the cross effects of all competitors. While it may not be obvious from equation (6), $e_{ijt}^{(k)}$ is asymmetric, that is to say, $e_{ijt}^{(k)} \neq e_{i'jt}^{(k)}$ (for $i' \neq i$) in general. Therefore, the impact of brand j's kth marketing mix-element can be asymmetrically distributed across the remaining brands. In contrast, a special case of equation (2) in which $f_t(Y_{kit}) = X_{kit}$ and $\beta_{kij} = 0$, the so-called "raw-score differential-effects" model implies the following elasticities:

$$e_{ijt}^{(k)} = -\beta_{kj} M_{jt}^{(k)} \quad (8)$$

which are symmetric. The actions of brand j are symmetrically distributed across all the remaining brands in the raw-score differential-effects model. A complete review of these and related market-share models may be found in Cooper and Nakanishi (1988).

We illustrate the specification, parameter estimation and strategic implications of asymmetric market-share models in price and advertising in an eleven-brand market. First we present methods for specifying the dynamics of brand attraction. Second we develop methods to test for and incorporate differential effects and cross-competitive effects. Third we illustrate OLS and GLS estimation of the model with cross validation. Finally we present techniques for exploring the strategic implications of our findings.

3. Data

Data from an Australian household-product market are used for our illustration. The market consists of 11 brands of a mature, nonseasonal product, purchased once per month, on average, by most households. These brands are marketed by two multinational corporations (1 and 2), or by an independent domestic producer (3), or they are amalgams of minor brands (4). Competition is fierce, with market shares of individual brands displaying high variance, but little overall gain or loss during the 26 months recorded in the database. Competition is primarily by price manipulation and advertising expenditure with some product differentiation noticeable.

Two distinct forms ("D"—dry and "W"—wet) exist, and within these some major brands also claim unique physical characteristics (denoted "T" for *tangible benefit*). There are three premium *tangible-benefit* brands (TD1, TD2, TW3) which typically have higher shelf prices. The remaining major brands are categorized as premium *image* brands, of which there are two (ID1 and ID2), and *economy* brands of which there are three (ED1, ED2, ED3). The premium *image* brands are the market leaders, have higher prices, and compete mainly through advertising. Economy brands compete mainly on price. In the price-sensitive segment there are also a large number of minor and generic brands, which are aggregated into AD4, for the dry versions, AW4 for the wet versions

TABLE 1
Australian Household-Products Descriptions

	Brand Symbol	Description	Average Market Share	Average Price	Average Advertising
1	ED1	Economy dry	5.4	$1.48	$8,100
2	ID1	Image dry	10.0	1.79	34,800
3	TD1	Tangible dry	4.5	2.14	25,600
4	AO1	All other co. 1	9.5	1.82	37,300
5	ID2	Image dry	4.9	1.87	42,400
6	ED2	Economy dry	2.6	1.51	0
7	TD2	Tangible dry	4.5	1.87	29,400
8	TW2	Tangible wet	7.8	1.81	50,000
9	ED3	Economy dry	7.1	1.59	14,300
10	AD4	All other dry	26.2	1.34	2,300
11	AW4	All other wet	17.5	1.00	39,500

and AO1 for all other minor brands from company 1. A glossary of brands, average prices, advertising expenditures and market shares is provided in Table 1.

Volume-based market shares and weighted price indices are measured monthly by

TABLE 2
Sales-Volume Variation for the Estimation and Validation Samples[a]

Time Period	Estimation State	Validation State
1	82	72
2	78	75
3	84	73
4	107	99
5	97	98
6	89	82
7	97	111
8	94	95
9	102	89
10	106	107
11	110	96
12	99	104
13	103	99
14	99	102
15	104	110
16	111	106
17	99	110
18	116	112
19	113	122
20	106	107
21	107	110
22	104	105
23	105	112
24	101	105
25	92	96
26	97	103
Coefficient of variation	9%	13%

[a] To preserve confidentiality volumes are indexed to a mean of 100.

means of a consumer panel of about 3000 families nationally. Price indices are simple monthly averages of weekly store prices, and are computed by the panel agency as part of standard reporting practice. Advertising expenditures are also provided monthly, but by a separate agency. Advertising is predominantly channeled through television, and copy content is focused on image and product-benefit issues, rather than pricing. Generally speaking the major brands have similar distribution levels, and so that issue is ignored here.

One strength of this database is that all variables are available on a monthly basis for four distinct geographical regions (states). Thus we are able to build share models on one state (market 1) and cross-validate them on another (market 2). Given the usual risks of capitalization on chance, the ability to cross-validate is essential in the testing procedures advocated here.

Total category volume for this mature household product is stable, as one might expect from a several-decades-old product. Brand managers also consider it mature, and compete principally for market share. Total sales of the category are shown in Table 2 (indexed to preserve confidentiality). As can be seen, some variability in month-to-month volume exists, but this is minor and decreasing. Furthermore, category sales are slightly increasing (by 8% and 16% per annum), but these gains are thought to come from greater usage, rather than additional household penetration. In all, the picture is one of a mature category.

Finally, cost data for profit calculation are the estimates provided from one major company. These estimates are certainly reasonable for purposes of illustration, but have been difficult to verify. Only variable costs are used here, because of the difficulty of finding a sensible basis for allocating fixed costs, and because a marginal analysis is more relevant to our purpose.

4. Specification and Estimation

The specification of the asymmetric market-share model involves three steps. First, we determine the extent to which marketing-mix variables have a dynamic effect on shares, using time-series analysis to identify the lag structure for each marketing variable for each brand. Second, we test the market-response parameters for equality across brands to determine whether or not each brand has a different brand-specific intercept or unique response to its marketing mix. Finally, we investigate the residuals from this tentatively specified model to see if cross-competitive effects can explain what is left. If so, we add unique pair-wise competitive effects to the basic model in a final stage.

We discuss specification starting from the linear form of the attraction model in equation (2). Assuming for the moment that $f(Y) = X$, the raw-score form of these models, and that no unique cross-competitive effects exist, we obtain a differential-effects attraction specification

$$A_{it} = \exp(\alpha_i) \prod_{k=1}^{K} [X_{kit}]^{\beta_{ki}}. \tag{9}$$

Nakanishi and Cooper (1982) showed that the resulting market-share model is equivalent to a linear specification in logarithms with period- and brand-specific intercepts added:

$$\log M_{it} = \alpha_0 + \sum_{i'=1}^{N-1} \alpha_i d_{i'} + \sum_{t'=1}^{T-1} \gamma_t c_{t'} + \sum_{k=1}^{K} \beta_{ki} \log (X_{kit}) + u_{it} \tag{10}$$

where α_i is a brand-specific intercept, $d_{i'}$ are dummy variables for brands with $d_{i'} = 0$ if $i' \neq i$, $c_{t'}$ is a time-period dummy variable with $c_{t'} = 0$ if $t' \neq t$, γ_t is a time-period coefficient,[2] α_0 is an overall intercept adjusting for the influence of the last brand and

[2] The denominators in equation (1) are constants in each time period; taking the log of each side of the equation makes these denominators linear terms whose role is conveyed by these parameters.

the last time period, and u_{it} is a stochastic disturbance which combines specification error and sampling error.

4.1. *Determining Attraction Dynamics*

First we determine the duration of the effects of marketing efforts such as advertising expenditures on market shares. These durations may differ across competitors. For example, if aggregate advertising expenditures are used, differences in media allocations may imply that the advertising for some brands lasts longer than for others. Thus it is desirable to have brand-specific dynamic attraction components in the general model. The extension of the differential-effects model in equation (10) to include dynamic attraction components is

$$\log M_{it} = \alpha_0 + \sum_{i'=1}^{N-1} \alpha_i d_{i'} + \sum_{t'=1}^{T-1} \gamma_t c_{t'} + \sum_{k=1}^{K} \beta_{ki} \log (Y_{kit}) + u_{it} \tag{11}$$

where $Y_{kit} = \exp[\omega_{ki}(L) \log X_{kit}]$, is a weighted combination of X_{kit} over t, ω_{ki} refers to the weights mentioned in equation (2),[3] $\omega_{ki}(L) = \omega_{ki0} + \omega_{ki1} L + \cdots + \omega_{kiD_{ki}} L^{D_{ki}}$, L is the lag operator (e.g. $L^p X_{kit} = X_{ki(t-p)}$), and D_{ki} is the maximum duration for marketing-mix element k for brand i. For example, in the previous illustration in equation (3), the log of attraction component 2 has the following dynamics: $\omega_{2i0} = 0.5$, $\omega_{2i1} = 0.3$, $\omega_{2i2} = 0.2$.

Determining the dynamics of attraction requires two tasks. First we establish the maximum duration D_{ki} of the impact of each marketing-mix element on market share for each brand. Second, we establish the dynamic attraction weights associated with each time period. If 30 or more time-series observations are available it is possible to determine the maximum lag length D_{ki} by following a least-squares specification procedure proposed by Liu and Hanssens (1982). The method was developed for the more general case of a transfer function (where D_{ki} is allowed to be ∞) and involves direct-lag OLS estimation of the model

$$\log M_{it} = c_i + \sum_{k=1}^{K} v_{ki}(L) \log (X_{kit}) + u'_{it} \tag{12}$$

where c_i is the intercept for the brand-by-brand specification, and $v_{ki}(L) = v_{ki0} + v_{ki1} L + \cdots + v_{kiD_{ki}} L^{D_{ki}}$ for each brand separately. The maximum duration D_{ki} is found by estimating the model with a sufficiently large number of lags and observing at which lag the OLS parameters $\{v_{ki}\}$ become insignificant. Next, we transform the OLS parameters into dynamic weights by normalizing them.[4] For example, the OLS parameters $v_{110} = 4$, $v_{111} = 2$, $v_{112} = 2$, $v_{113} = 1$, for a dynamic attraction component with duration $D_{11} = 3$ would result in weights $\omega_{110} = 0.8$, $\omega_{111} = 0.4$, $\omega_{112} = 0.4$, $\omega_{113} = 0.2$.

The direct-lag specification method may encounter some practical problems. For example, the OLS parameters may be unstable due to collinearity among the lagged explanatory variables. Liu and Hanssens (1982) demonstrated that this problem is related to an autoregressive spending pattern in X and proposed to filter the data by a common autoregressive filter (e.g. $\log M_{it} - 0.8 \times \log M_{i(t-1)}$), only for the purpose of establishing D_{ki} and the attraction weights. Further, if the data are nonstationary it may be necessary to specify a model in changes, which is not a problem for direct-lag specification, but would create new challenges for estimation of the final model. Fortunately the natural range constraints in the data (market shares and standardized marketing efforts) make

[3] We are using geometric dynamic weights in order to be consistent with the multiplicative nature of the market-share model. Using additive weights provides the same substantive findings in this application.

[4] In the Australian household-product illustration the sum of the weights is set to 1.0, but it may be more generally acceptable to normalize so that the sum of squares of the weights is 1.0.

nonstationarity unlikely in this context. Finally, if too few time-series observations are available, the specification search must be drastically simplified, for example by assuming, in the extreme case, that all attraction effects are expended within one period. Details on these methods can be found in Hanssens, Parsons and Schultz (1988).

The specification of the dynamic structure on the Australian household-product data revealed, first, that several time series are highly autoregressive. Seven price series, one advertising and one market-share series may be represented by an AR(1) process. These series are transformed using the filter $(1 - 0.7L)$, where 0.7 is the highest AR parameter found in the univariate analysis. Next, we specify transfer functions relating market shares to advertising and prices, using the Liu-Hanssens (1982) least-squares method. The results are remarkably simple: prices have only contemporaneous effects on market shares and advertising expenditures have zero- and first-order dynamic effects, with weights that vary significantly across brands. Zero-order price effects have been observed before and may be due to value- or bargain-shopping with very little brand loyalty, or due to intense price-matching behavior which precludes a competitor from developing a long-term price differential, or they may in part be an artifact of data aggregation (i.e. segment-specific analysis might discover different lag structures across segments). The short lag structure in advertising is also consistent with previous research. For the purpose of attraction modeling, we will use an effective-advertising variable with brand-specific weights for current and last-period expenditures.

4.2. *Specifying Differential and Cross-Competitive Effects*

We specify differential effects using a series of hypothesis tests against the null of a simple-effects model. The simple-effects model is hierarchically-nested within equation (2). Using a standard Chow test, differential brand-specific intercepts are tested against the hypothesis that $\alpha_i = \alpha \ \forall i$, and differential slopes are tested for each marketing-mix variable against the hypothesis that $\beta_{ki} = \beta_k \ \forall i$. As several test sequences are possible, we use the following marketing-intuitive strategy: first assume homogeneous slopes and test for heterogeneous base-share levels, as they are the most likely to exist when attraction is composed of only a few marketing variables, as in our illustration. Next, test for differential slopes on each variable in descending order of expected market-share effects (e.g. price followed by advertising in the illustration). If desired these tests could also be applied to subgroups of equal parameters, for example testing for equal price sensitivity within premium brands.

A practical way to find cross-competitive effects is to collect the residual market shares from the final differential-effects specification for each brand and cross-correlate these with the $K(N-1)$ attraction components for the other brands at all relevant lags, similar to the Box-Jenkins method for transfer function specification. Significant correlations provide strong evidence that differential-effects underspecify the competitive interactions. The resulting set of correlation matrices is only a specification instrument, though, and is subject to confirmation. We incorporate the significant cross-competitive effects into the attraction model and re-estimate all the parameters.

As a result of specifying dynamics, temporal distinctiveness and cross-competitive effects the estimation equation in (11) changes. If we expand the members of the set C_i to include i if there is a unique intercept for brand i, k if there is a differential effect for marketing-mix element k, as well as members k^*j^* if there is the corresponding cross-competitive influence on brand i, then the parameter estimates for equation (2) come from:

$$\log M_{it} = \alpha_0 + \sum_{i \in C_i} \alpha_i d_{it} + \sum_{t'=1}^{T-1} \gamma_{t'} c_{t'} + \sum_{k \in C_i} \beta_{ki} \log f_t(Y_{kit}) + \sum_{k^*j^* \in C_i} \beta_{k^*ij^*} \log f_t(Y_{k^*j^*t}) + u_{it}. \quad (13)$$

The test of differential effects on the dynamically weighted attraction components for the Australian household product revealed that brand-specific intercepts and slopes are needed to represent the market: all parameter restrictions on base shares and/or price and advertising slopes are statistically significant ($p < 0.01$ or $p < 0.05$). Therefore, the market is characterized by competitors whose attractions have unique constant levels and whose market shares respond differently to changes in their marketing mix.

Cross-competitive effects were identified by taking the residuals from the differential-effects model and cross-correlating them, brand-by-brand with the marketing instruments for the other competitors for lags from 0 to 4. Among the 209 possible contemporaneous cross-effects, 17 were statistically significant at $p < 0.05$, and 8 at $p < 0.01$. However, none of these effects were confirmed in cross-calibration on market 2.[5] Furthermore, no significant lagged cross-competitive effects were observed in either market. Thus we conclude that the market forces in this case are sufficiently accounted for by dynamic advertising variables in the differential-effects specification, and the asymmetries implied by the temporal distinctiveness of competitive positions.

4.3. *Estimating and Cross-Validating the Model*

Equation (13) is a logically consistent system of N seemingly unrelated regressions. Ordinary least-squares estimates of the parameters are unbiased, but if the covariance matrix of errors is nondiagonal, then GLS estimates will be more efficient. The GLS procedures for this model are reported in Cooper and Nakanishi (1988).

[5] Cross-calibration involves estimating the parameters of the model specified on market 1 to the data from market 2.

TABLE 3

GLS Parameter Estimates for Zeta-Score Model

Brand Symbol	Brand Intercept	Price Parameter	Advertising Parameter
ED1	−0.85	−2.26	0.82
(Std. err.)	(0.13)[a]	(0.47)[a]	(0.16)[a]
ID1	0.07	−1.36	0.22
	(0.12)	(0.57)[a]	(0.16)
TD1	−0.26	−0.95	0.11
	(0.29)	(0.45)[a]	(0.23)
AO1	—[c]	−0.67	0.23
		(0.67)	(0.18)
ID2	0.55	−1.14	0.14
	(0.13)[a]	(0.34)[a]	(0.11)
ED2	−1.69	−2.17	—[c]
	(0.12)[a]	(0.40)[a]	
TD2	−0.61	−1.26	0.52
	(0.13)[a]	(0.40)[a]	(0.20)[a]
TW2	−0.12	−0.67	−0.11
	(0.15)	(0.31)[a]	(0.18)
ED3	−0.42	−1.38	0.58
	(0.11)[a]	(0.80)[b]	(0.15)[a]
AD4	1.13	0.18	0.38
	(0.18)[a]	(0.38)	(0.37)
AW4	−0.21	−0.95	−0.19
	(0.45)	(0.52)[b]	(0.15)

[a] $p < 0.01$, one-tailed test.
[b] $p < 0.05$, one-tailed test.
[c] Parameter not estimated.

As a result of not finding stable cross-competitive effects, an attraction model with a full complement of differential effects and $(N-1)$ brand-specific intercepts was estimated. The OLS estimation of the model using $f(X) = \zeta$ fits very well ($R^2 = 0.906$, $p < 0.0001$) and no residual autocorrelation is found. The OLS residuals are used as the first step in the simplified GLS procedure whose results are presented in Table 3. Because the raw-score model is used in the optimization which follows, the GLS parameters for this model are reported in Table 4 for comparison. The fit of this model is virtually the same as for the specification using zeta-scores.

Whenever possible, we cross-validate the results on a fresh data set by forming the market shares implied by the estimated parameters and noting the correlation or congruence with the actual market shares. Having used the developmental data to estimate the dynamic weights for the advertising effects and to help in preliminary specification of differential effects, we validate the model on 26 months from the same time period from another large state. The cross-validation model, which contains a single composite variable and no intercept term, has an R^2 of 0.942. Although R^2 for calibration and cross-validation models are not strictly comparable, this has to be considered an excellent result. The parameter estimate is 1.018. With a standard error of 0.016 there is no significant departure from a one-to-one relation. The root-mean-squared forecast error (excluding the aggregate brands) is 0.019, for market shares expressed as proportions, and the first-order autocorrelation is significant ($p = 0.51$, $p < 0.0001$). This suggests that, while the model does hold up under cross-validation, there may be autocorrelated missing variables which are specific to each market. State-specific effects may be manifest here. The retail structure differs across markets, the television channels are different, and brand managers talk about their brands being stronger or weaker in one state versus another. A cross-validation of the raw-score model led to the same conclusion.

TABLE 4

GLS Parameter Estimates for Raw-Score Model

Brand Symbol	Brand Intercept	Price Parameter	Advertising Parameter
ED1	6.07	−2.70	0.22
(Std. err.)	(3.72)	(0.58)[a]	(0.04)[a]
ID1	0.93	−1.49	0.07
	(3.23)	(0.50)[a]	(0.03)[b]
TD1	−0.35	−1.35	0.07
	(4.42)	(0.74)[b]	(0.04)
AO1	—[c]	−1.31	0.06
		(0.51)[a]	(0.03)[b]
ID2	4.20	−2.21	0.02
	(3.93)	(0.64)[a]	(0.03)
ED2	10.02	−3.60	—[c]
	(3.36)[a]	(0.48)[a]	
TD2	−2.11	−1.08	0.11
	(3.65)	(0.57)[b]	(0.06)[b]
TW2	0.56	−1.43	0.00
	(2.87)	(0.32)[a]	(0.04)
ED3	6.05	−2.59	0.07
	(3.15)[b]	(0.46)[a]	(0.02)[a]
AD4	−2.04	−0.74	0.03
	(3.09)	(0.44)[b]	(0.06)
AW4	−4.60	−0.22	−0.12
	(3.43)	(0.56)	(0.07)

[a] $p < 0.01$, one-tailed test.
[b] $p < 0.05$, one-tailed test.
[c] Parameter not estimated.

5. Strategic Implications

Asymmetric market-share models may be used to compute elasticities and cross elasticities to help understand patterns of competition, to simulate the impact on brand profits of a brand's strategy, a competitor's strategy, or a competitive reaction, and to derive optimal brand strategies under differing assumptions about competitive behavior.

5.1. *Patterns of Marketing Effectiveness and Competition*

The results of the validated models presented in Tables 3 and 4 provide insights into the differences in the effectiveness of brands' marketing actions. First, there is large variation in price sensitivity. Excluding the aggregate brands, the three economy brands have the highest (in absolute value) price coefficients (ED1, ED2 and ED3). Among the premium brands, those with tangible benefits to the consumer command lower price sensitivity (TD1, TD2 and TW2). The premium-image brands (ID1 and ID2) have coefficients in between the economy and the tangible-benefit brands. Second, the advertising effects also substantially differ across brands, with two economy brands (ED1 and ED3) having the highest advertising parameters,[6] followed by two dry premium brands, ID1 and TD2.

Similar patterns of brand price and advertising sensitivity are found in the matrices of elasticities given in Tables 5 and 6, based on average prices, market shares and advertising expenditures. The diagonal elements in the tables are the own-brand price elasticities and the off-diagonal elements are cross elasticities that provide insights into patterns of competition between brands. The columns of both tables give the impact of a change in the marketing mix of that brand across all competitors.

The cross-price elasticities in Table 5 show how price changes are asymmetrically distributed among competitors. Some features of these estimates are noteworthy.[7] First, the differences in cross-elasticities can be relatively large. If ED1 cuts its price in an average time period, the other most price-sensitive brand (ED2) is affected more than twice as much as any other competitor. A 10% price cut by ID1 has almost twice as much impact on TD2 (a 6.3% drop in share) as it does on TW2 (a 3.6% share loss).

Secondly, cross-price elasticities suggest distinct groupings of brands. The economy brands ED1 and ED2, for instance, are vulnerable to each other's price cuts, but, inter-

[6] The third economy brand (ED2) never advertises.

[7] We will exclude the aggregate brands AO1, AD4 and AW4 from the discussion which follows.

TABLE 5

Price Elasticities for Zeta-Score Model[a]

	ED1	ID1	TD1	AO1	ID2	ED2	TD2	TW2	ED3	AD4	AW4
ED1	−4.47	0.57	−0.06	0.39	0.33	0.60	0.33	0.36	0.55	0.47	0.87
ID1	0.30	−2.76	0.62	0.39	0.43	0.15	0.43	0.35	0.19	−0.16	0.07
TD1	0.12	0.59	−2.04	0.49	0.57	−0.01	0.58	0.43	0.09	−0.42	−0.30
AO1	0.21	0.37	0.37	−1.62	0.26	0.06	0.26	0.19	0.09	−0.22	0.05
ID2	0.26	0.58	0.82	0.46	−3.07	0.12	0.52	0.42	0.19	−0.22	−0.04
ED2	0.67	0.54	0.05	0.37	0.33	−4.18	0.33	0.34	0.46	0.33	0.70
TD2	0.28	0.63	0.91	0.51	0.57	0.14	−3.40	0.46	0.21	−0.22	−0.06
TW2	0.21	0.36	0.34	0.22	0.24	0.06	0.25	−1.60	0.08	−0.21	0.06
ED3	0.29	0.32	0.10	0.16	0.16	0.13	0.16	0.13	−1.58	−0.10	0.23
AD4	0.05	0.14	0.03	−0.01	−0.00	−0.11	0.00	−0.05	−0.10	0.07	−0.02
AW4	0.52	0.15	−0.72	−0.06	−0.14	0.33	−0.14	−0.08	0.23	0.27	−0.43

[a] Based on average prices and market shares.

TABLE 6

Effective-Advertising Elasticities for Zeta-Score Model[a]

	ED1	ID1	TD1	AO1	ID2	ED2	TD2	TW2	ED3	AD4	AW4
ED1	0.14	0.04	0.02	0.06	0.10	0.00	0.02	0.17	−0.02	−0.00	0.11
ID1	−0.01	0.11	−0.03	−0.07	−0.08	0.00	−0.05	−0.09	−0.02	−0.00	−0.04
TD1	−0.01	−0.03	0.06	−0.03	−0.03	0.00	−0.03	−0.02	−0.02	−0.00	0.01
AO1	−0.00	−0.07	−0.03	0.11	−0.08	0.00	−0.06	−0.10	−0.02	−0.00	−0.04
ID2	−0.01	−0.04	−0.02	−0.05	0.07	0.00	−0.04	−0.05	−0.02	−0.00	−0.01
ED2	−0.01	−0.00	0.01	−0.00	0.01	0.00	−0.00	0.03	−0.01	0.00	0.05
TD2	−0.00	−0.15	−0.08	−0.16	−0.19	0.00	0.26	−0.24	−0.04	−0.00	−0.14
TW2	−0.01	0.03	0.03	0.04	0.06	0.00	0.02	0.00	−0.00	0.00	0.09
ED3	−0.01	−0.10	−0.05	−0.11	−0.12	0.00	−0.08	−0.14	0.17	−0.00	−0.07
AD4	−0.01	−0.00	0.01	−0.00	0.01	0.00	−0.01	0.03	−0.01	0.00	0.04
AW4	−0.01	0.06	0.04	0.06	0.10	0.00	0.04	0.14	0.00	0.00	−0.04

[a] Based on average effective advertising expenditures and market shares.

estingly, have little influence on other brands. The other economy brand, ED3, is less vulnerable perhaps because its normally large advertising expenditures are more effective in isolating it from price competition. The image brands are in the middle in the sense that they never are pressured either the most or the least by other competitors, and the tangible-benefit brands are apparently affected least by price competition from economy brands.

The advertising cross-elasticities in Table 6 show analogous asymmetries. The economy brands ED1 and ED3 are sensitive to their own advertising, but exert very little pressure on their competitors. Image brands influence both the economy brand ED3 and the tangible-benefit brand TD2. The tangible-benefit brands compete less with the economy brands ED1 and ED2 than with the other premium brands.

5.2. *Profit Simulations*

Simulating the impact of various brands' or competitors' strategies on brands' profits is straightforward once the market-share model is specified and estimated. Profits for brand i in period t are

$$\Pi_{it} = Q_t M_{it}(X_{1it} - c_i) - \sum_{k=2}^{K} X_{kit} \tag{14}$$

where Π_{it} is profit, Q_t is market size in units and c_i are unit costs. These profit functions may be used to evaluate the impact of various brand strategies.

Simulating the profit surfaces illustrates how differential effects and asymmetries affect brand profits in the Australian household-product market. Figure 1 presents the market-share and profit surfaces for three brands (ED1, TD1 and ID2), based on the zeta-score parameters in Table 3. To generate these figures, price is varied from the unit cost for each brand up to the maximum price in the data set ($2.42) and the advertising spending is varied from zero to the maximum single-period outlay ($161,000). Competitor's prices and advertising levels are fixed at their actual values for a period selected as the real month which contained prices and advertising levels closest to the average.

The market-share surfaces for three brands in Figure 1 are recognizably similar in shape, but important differences exist. The highest value marked on the abscissa shows the greatly differing market-share potentials for these brands. ED1 ranges up to 42% of the market, while TD1 and ID2 never exceed 16% during simulations in the stated ranges. All three surfaces show the flattening near the mean which characterizes zeta-score models. The mean is the least distinct location, and zeta-scores reflect the benefits

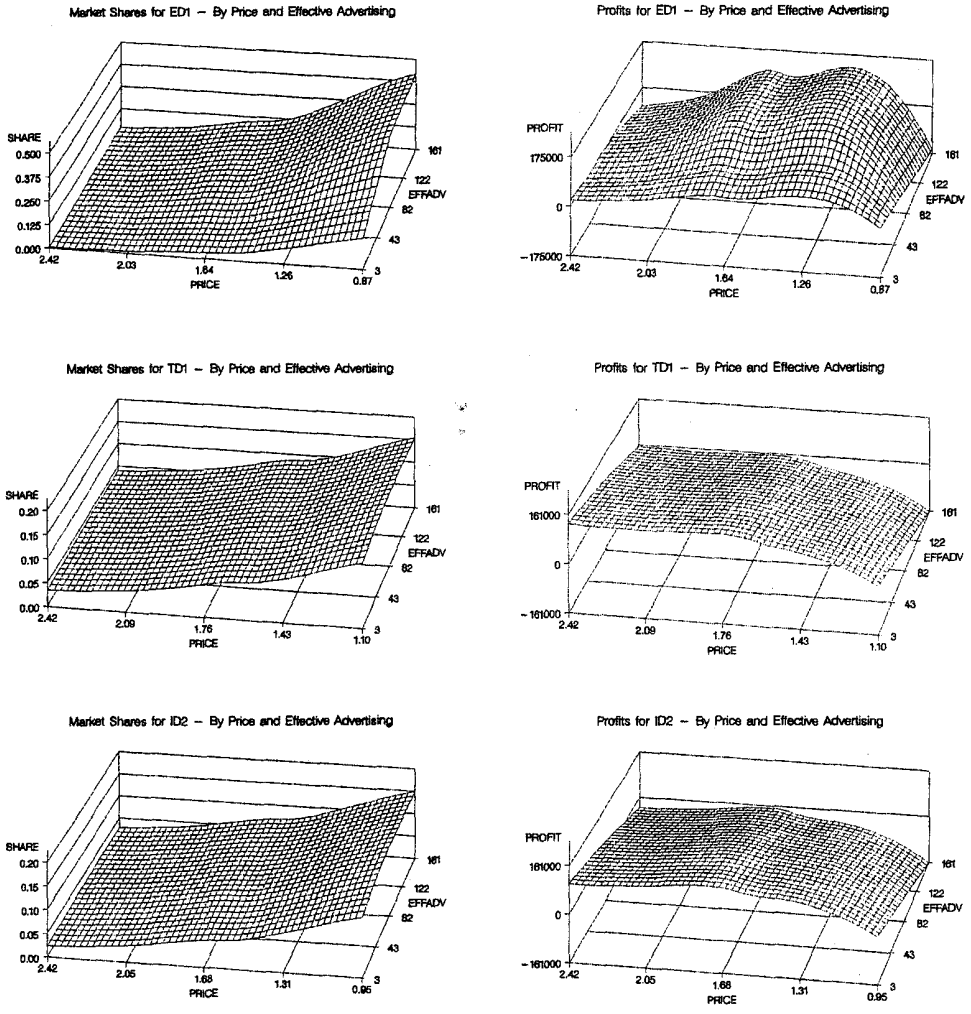

FIGURE 1

and penalties associated with such an undifferentiated position.[8] The economy brand ED1 loses almost all of its market-share potential by the time price rises to the mean level in the market, while premium brands can maintain their average market share (between 4% and 5%) at prices above the mean of the market.

Some differences in profit sensitivity between the economy and premium brands are evident in Figure 1. The economy brand ED1 is much more sensitive to price and advertising expenditures than are the premium brands. ED1's profit is greatest when it is priced substantially below the mean.[9] The profit surfaces for premium brands are relatively insensitive to price increases after some point. For both ID2 and TD1, being underpriced builds share, but hurts profits; being priced higher hurts share substantially, but not profits.

[8] This indistinct region is similar to what DeSarbo, Rao, Steckel, Wind and Colombo (1987) are representing with their friction-pricing model and what Gurumurthy and Little (1986) discuss in their pricing model based on Helson's adaptation-level theory.

[9] Simulating the profit consequences if an economy brand raises its price above the mean of the market,

5.3. *Brand Strategies*

To use equation (14) to solve for strategies, we need to make additional assumptions about one important feature of competition not captured in the market-share model: competitive actions. Equation (14) is a system of N profit functions driven by the strategies of all brands. Incorporating competitive actions is therefore essential, but doing so is troublesome because many different types of competitive actions are possible. For instance, competitors may not react at all, because they consider rivals too small and inconsequential, or because they lack sufficiently sensitive information systems to detect a change in a competitor's marketing mix. Or competitors may react much more aggressively because they are very well informed about both their rivals' actions and the responsiveness of buyers to those marketing actions.

Our strategy for dealing with this difficult issue is to solve equation (14) using two different assumptions about competitors. First, we assume that competitor's strategies are fixed or that we can forecast, with reasonable accuracy based on past behavior, how they will change (e.g. a 10% increase in advertising expenditure). In this case, we are implicitly assuming competitors will not react to our strategy, but we are responding to theirs. We refer to these as "optimal response strategies." Second, we assume that all brands simultaneously maximize profits, and are in equilibrium. We refer to these strategies as "equilibrium strategies."

5.3.1. *Optimal response strategies.* If we let $\mathbf{X}_{it}$ be a vector of brand i's marketing mix in period t, then brand i's optimal marketing mix is given by the solution to

$$\max_{(\mathbf{X}_{it})} \Pi_{it} = Q_t M_{it}(X_{1it} - c_i) - \sum_{k=2}^{K} X_{kit} \tag{15}$$

given our estimate of competitors' strategies, $\mathbf{X}_{jt}\ \forall j \neq i$, which simply requires that

$$\frac{\partial \Pi_{it}}{\partial X_{kit}} = 0. \tag{16}$$

A solution to equation (15) will exist so long as the market-share model is smooth and well behaved, and hence the profit function is also smooth and concave. This will be true generally for asymmetric market-share models using raw scores ($f(Y) = X$), or exp(z-scores), but zeta-scores have to be restricted in ranges for this to be true.[10] Assuming these conditions are satisfied, the solution to equation (15) is a system of K optimal marketing-mix conditions:

$$X^*_{1it} = c_i\left(\frac{1}{1 + (1/e^{(1)}_{iit})}\right). \tag{17}$$

$$X^*_{kit} = Q_t M_{it}(X^*_{1it} - c_i)e^{(k)}_{iit} \qquad (k = 2, 3, \ldots, K), \tag{18}$$

for fixed $\mathbf{X}_{jt}$ ($\forall j \neq i$), where $e^{(1)}_{iit}$ is brand i's price elasticity and $e^{(k)}_{iit}$ ($k = 2, 3, \ldots, K$) is the elasticity for each of the remaining ($K - 1$) marketing-mix elements.

Equations (17) and (18) are also useful for simulating effective competitive reactions or simulating the impact of a change in a competitor's marketing mix. Formally, these

---

reveals when common sense must be used. The second hump in profits, just above the mean price, is due to the relatively flat portion of the market-share function pictured just to the left in Figure 1. In this region the brand is not losing market share very rapidly, because price variations near the mean are indistinct, but its profit margins are growing. One must be very wary, however, of what it means to try to sustain an economy brand at such a high price. An image change might result which would alter the nature of ED1's business.

[10] As depicted in Figure 1, price-sensitive economy brands have to have prices restricted to below the average price in the market for the profit function to remain concave. While this is a very sensible restriction, it does make optimization based on zeta-scores much more difficult than desired in this illustration.

equations define the optimal marketing mix for brand i, $\mathbf{X}_{it}^*$, as a function of $\mathbf{X}_{jt}$, so we can write $\mathbf{X}_{it}^* = \mathbf{X}_{it}^*(\mathbf{X}_{jt})$. Therefore, different competitor's strategies imply different $\mathbf{X}_{it}^*$ and simulating various competitors' strategies can suggest effective competitive responses.

5.3.2. *Equilibrium strategies.* Nash equilibrium strategies[11] are given by the solution to

$$\max_{(\mathbf{X}_{it})} \Pi_{it} = Q_t M_{it}(X_{1it} - c_i) - \sum_{k=2}^{K} X_{kit}; \qquad i = 1, 2, \ldots, N, \tag{19}$$

which requires that

$$\frac{\partial \Pi_{it}}{\partial X_{kit}} = 0, \qquad i = 1, 2, \ldots, N; \qquad k = 1, 2, \ldots, K, \tag{20}$$

that is, all brands maximizing profits simultaneously.

Equation (19) implies that $K \times N$ nonlinear equations must be satisfied simultaneously, i.e. one condition per marketing-mix variable, K in all, for each of N brands. A solution, denoted $\mathbf{X}_i^\circ$, will exist so long as the profits of each brand are strictly quasi-concave in each marketing-mix variable (Friedman 1977).[12] Assuming these conditions are met, the optimal competitive strategies that satisfy equation (19) are

$$X_{1it}^\circ = c_i\left(\frac{1}{1 + (1/e_{iit}^{(1)})}\right), \tag{21}$$

$$X_{kit}^\circ = Q_t M_{it}(X_{1it}^\circ - c_i)e_{iit}^{(k)}; \qquad k = 2, 3, \ldots, K; \qquad i = 1, 2, \ldots, N, \tag{22}$$

for all N brands simultaneously, where X_{1it}° denotes the Nash-equilibrium price for brand i, X_{kit}° ($k = 2, 3, \ldots, K$) denotes the Nash-equilibrium expenditures on marketing-mix element k, and $e_{iit}^{(k)}$ are the elasticities described earlier.

5.3.3. *Application.* Using the time period chosen for the simulations in Figure 1, we computed profit-maximizing prices and advertising levels for each brand under the two assumptions concerning competitive reactions discussed above. The necessity of having concave profit functions makes using zeta-score models impractical for this optimization, and methods for doing so must await further research. Instead, we use the raw-score model whose parameters are shown in Table 4.

For the "optimal response" case, prices and advertising levels are computed, brand by brand (holding competitors' strategies fixed at actual levels for the period chosen), as described in equation (15). This involves solving 11 systems of two nonlinear equations[13] in prices and advertising levels using an algorithm based on Brown (1960).[14] These results appear in Table 7 under the "Optimal Response" heading.

We also computed Nash-equilibrium strategies as in equation (19). This requires solving one system of 22 nonlinear equations simultaneously, but we reduced the problem by eliminating three price conditions and five advertising conditions,[15] leaving a system of 14 equations, which was solved using the same algorithm. To decrease the chances of

[11] See Friedman (1977) and Moorthy (1985) for discussions of this and related equilibrium concepts.

[12] This is much easier to guarantee with raw-score models than with models using zeta-scores.

[13] The problem was slightly reduced by eliminating two price conditions (for the aggregates AD4 and AW4) and four advertising conditions (for the aggregates AD4 and AW4, for ED2 which never advertises and for TW2 which has an advertising parameter of 0.0), and fixing these brands' strategies at observed levels.

[14] A sensitivity analysis showed these optima to be remarkably robust.

[15] The price and advertising conditions for the other aggregate AO1 were eliminated in addition to the constraints already mentioned.

TABLE 7

Current and Optimal Prices, Advertising Expenditures, Market Shares and Profits by Brand

	Prices[a]			Advertising[b]			Market Share			Profit[b]		
Brand	Actual	Optimal Response	Nash Equilibrium	Actual	Optimal Response	Nash Equilibrium	Actual	Optimal Response	Nash Equilibrium	Actual	Optimal Response	Nash Equilibrium
ED1	$1.53	$1.47	$1.47	$0.00	$35.70	$35.60	1.0	10.3	10.2	$82.85	$146.00	$146.00
ID1	1.89	2.73	2.71	0.00	19.80	18.06	5.2	5.7	5.2	203.00	268.00	242.00
TD1	2.22	3.20	3.17	41.20	14.10	12.40	5.4	2.9	2.5	143.93	167.00	145.00
AO1	1.88	4.83	1.88[c]	50.90	70.84	50.90[c]	10.9	NA	9.7	220.51	1019.05	220.51[c]
ID2	2.02	1.65	1.64	55.40	2.80	2.66	4.7	7.3	6.8	87.16	151.00	140.00
ED2	1.65	1.19	1.20	0.00	0.00[c]	0.00[c]	1.9	NA	5.8	41.58	16.58	57.35
TD2	1.90	4.62	4.58	21.20	19.80	17.00	5.1	1.6	1.4	125.49	157.00	135.00
TW2	1.98	3.03	2.81	67.70	67.70[c]	67.70[c]	7.8	NA	4.1	230.88	426.65	162.00
ED3	1.67	1.70	1.70	0.10	9.30	8.50	5.4	7.0	6.5	94.76	129.00	118.05
AD4	1.38	1.38[c]	1.38[c]	1.90	1.90[c]	1.90[c]	31.3	NA	28.5	516.50	516.50[c]	455.30[d]
AW4	1.02	1.02[c]	1.02[c]	29.80	29.80[c]	29.80[c]	21.3	NA	19.3	234.80	234.80[c]	206.45[d]

[a] Prices in dollars (Australian).

[b] In thousands of dollars (Australian).

[c] Constrained to be the same as the actual value.

[d] Estimated using actual prices and advertising expenditures and competitive market shares.

NA Not applicable.

obtaining local maxima, we conducted a grid search of 900,000 points in the 14-dimensional space. From this search, candidate solution regions were identified and a variety of starting values were used to produce a solution. The final results proved to be quite stable and are therefore unlikely to be local optima. They appear in Table 7 under the "Nash equilibrium" heading.

The results shown in Table 7 provide insights on how some brands can increase profits. Excluding the three aggregate categories, 7 of 8 brands are earning less than maximum profit given competitors' strategies (namely ED1, ID1, TD1, ID2, TD2, TW2 and ED3). All these brands, except for TW2, are also earning less than their Nash-equilibrium profit levels. Comparing actual strategies, optimal responses, and Nash-equilibrium strategies in Table 7 suggests how these brands can increase profits by altering their strategies. ED1 and ID2 are especially interesting cases which demonstrate our analysis has normative uses. The economy brand ED1 is spending virtually nothing on advertising in the period chosen for analysis, but could increase profits substantially by raising advertising outlays. Spending at either optimal-response or Nash-equilibrium levels (roughly $36,000) would increase ED1's profits by 76%. This profit gain is consistent with ED1's Nash-equilibrium strategy. In contrast, ID2 appears over-priced and over-advertised. In the period examined, dropping price by 18% and cutting deeply into advertising would, according to our estimates, increase market share by roughly half and nearly double profits, if competitors' strategies are the actual ones shown in Table 7. If the Nash-equilibrium strategies prevail, price and advertising cutting remain optimal for ID2 and profits should increase by a substantial 61%.

6. Discussion and Conclusion

Asymmetric competition can arise because of differences in the vulnerability of one brand to the efforts of others, and the temporal distinctiveness of brands' marketing efforts. These asymmetries are compounded by the differential effectiveness which with brands execute their marketing strategies and by marketing dynamics, including cumulative advertising spending. So the outcome of any brand strategy fundamentally depends on the diversity of competitive patterns that exist. We have presented implementable methods for modeling asymmetries and differential effectiveness, and we have used the resulting asymmetric market-share models normatively. Our approach to this is, however, limited in at least three ways.

First, we have focused on *static* estimates of brand strategies, even though in many markets *dynamics* are important. Our method essentially constructs the payoff matrix for a single-period game among N players, and examines various equilibrium points within that matrix, particularly the Nash equilibrium. Recent developments in game theory analyze strategies for brands playing a single-period game a number of times over some horizon (e.g. Friedman 1977). Analyzing strategies for so-called "super games" will provide insight into *dynamic* strategies such as ad pulsing or competitive pricing strategies such as "Tit for Tat" for markets in which competition is asymmetric. This is an important direction for future work.

Second, we have outlined the strategic implications of asymmetric market-share models for two important cases, yet others clearly exist. For example, a market may be dominated by one brand which *leads* price changes much the way General Motors initiated annual price increases in the auto industry, followed shortly thereafter by Ford, Chrysler and others. Extending our analysis to account for other reaction schemes could be insightful.

Third, our optimization results are based on a raw-score model. While representing temporal distinctiveness has intuitively appealing advantages for modeling asymmetries, profit functions based on zeta-scores have to be bounded before they are sufficiently

smooth to be useful for optimization. Investigation of other transformations to reflect competitive position could prove quite useful for incorporating these effects into optimization.

Two other directions for future work seem potentially fruitful. First, the model could be extended to the segment level to integrate other types of information. Our efforts have focused on market-level data and excluded consumer-level measurements of attitudes or perceptions. Clearly, segment-level models can provide additional insight, as can integrating additional variables. Furthermore, data transformations such as zeta-scores transform interval-scale consumer measurements into the ratio-scaled variables required by these models, so that integrating consumer-level data can be straightforward.

Second is the development of competitive maps based on the own-brand and cross-elasticities produced by asymmetric market-share models. As we discussed, elasticities provide important insights into patterns of brand competition. But they can be burdensome to interpret. In our example we interpreted a single table of elasticities based on averages. But for each marketing-mix element there is, in reality, a separate table of such elasticities which could be computed in each of the 26 monthly periods. Cooper (1988) provides methods to signal when there are important changes in these tables, and to map the elasticities corresponding to each marketing-mix element. It is hoped that easing the interpretive burden will facilitate empirical investigations of competition using asymmetric market-share models.

In conclusion, brands are often differentially affected by their own actions and those of their competitors. We describe a diversity of specification alternatives (cross-competitive effects, dynamic attraction components, and was to represent the distinctiveness of competitive position), as well as estimation methods to model asymmetric competition. We develop and illustrate methods exploiting the value of these models in devising effective competitive brand strategies. In an era characterized by the enormous expansion of data for marketing information systems, we expect that these methods for modeling asymmetric competition will find widespread application.[16]

[16] This paper was received in October 1984 and has been with the authors 20 months for 4 revisions.

References

Batsell, Richard R. and John C. Polking (1985), "A New Class of Market Share Models," *Marketing Science*, 4, 3 (Summer), 177–98.

Brown, K. M. (1969), "A Quadratically Convergent Newton-Like Method Based on Gaussian Elimination," *SIAM Journal on Numerical Analysis*, 4, 1–40.

Bultez, Alain V. and Philippe A. Naert (1975), "Consistent Sum-Constrained Models," *Journal of the American Statistical Association*, 70, 351 (September), 529–535.

Cooper, Lee G. (1988), "Competitive Maps: The Structure Underlying Asymmetric Cross Elasticities," *Management Science*, 34, 6 (June), 707–723.

——— and Masao Nakanishi (1983), "Standardizing Variables in Multiplicative Choice Models," *Journal of Consumer Research*, 10 (June), 96–108.

——— and ——— (1988), *Market-Share Analysis: Evaluating Competitive Marketing Effectiveness*. Boston: Kluwer Academic Publishers, in press.

Currim, Imran (1982), "Predictive Testing of Consumer Choice Models Not Subject to Independence of Irrelevant Alternatives," *Journal of Marketing Research*, 19 (May), 208–222.

DeSarbo, Wayne S., Vithala R. Rao, Joel H. Steckel, Jerry Wind and Richard Colombo (1987), "A Friction Model for Describing and Forecasting Price Changes," *Marketing Science*, 6, 4 (Fall), p. 299–319.

Friedman, James W. (1977), *Oligopoly and the Theory of Games*. Amsterdam: North Holland.

Ghosh, Avijit and C. Samuel Craig (1983), "Formulating Retail Location Strategy in a Changing Environment," *Journal of Marketing*, 47 (Summer), 56–68.

Gurumurthy, K. and John D. C. Little (1986), "A Pricing Model Based on Perception Theories and Its Testing on Scanner Panel Data," Massachusetts Institute Of Technology, Working Paper, May.

Hanssens, Dominique M. (1980), "Market Response, Competitive Behavior and Time Series Analysis," *Journal of Marketing Research*, 17 (November), 470–485.

———, Leonard J. Parsons and Randall L. Schultz (1988), *Market Response Models: Econometric and Time-Series Analysis*. Boston: Kluwer Academic Publishers, in press.

Kamakura, Wagner A. and Rajendra K. Srivastava (1986), "An Ideal-Point Probabilistic Choice Model for Heterogeneous Preferences," *Marketing Science*, 5, 3 (Summer), 199–218.

Karnani, Aneel (1983), "Minimum Market Share," *Marketing Science*, 2 (Winter), 75–93.

Liu, Lon-Mu and Dominique M. Hanssens (1982), "Identification of Multiple-Input Transfer Function Models," *Communications in Statistics—Theory and Methods*, 11, 297–314.

Luce, R. Duncan (1959), *Individual Choice Behavior*. New York: John Wiley & Sons.

Moorthy, K. Sridhar (1985), "Using Game Theory to Model Competition," *Journal of Marketing Research*, 22 (August), 262–282.

Nakanishi, Masao and Lee G. Cooper (1974), "Parameter Estimation for A Multiplicative Competitive Interaction Model—Least Squares Approach," *Journal of Marketing Research*, 11 (August), 303–311.

——— and ——— (1982), "Simplified Estimation Procedures for MCI Models," *Marketing Science*, 1, 3 (Summer), 314–322.