Cursed Beliefs in Search Markets*

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March 2019

Abstract

We study a search market in the spirit of Diamond (1971) in which a fraction of consumers is cursed and searches as if they underestimate the correlation between price and quality (distributions) in the market. In equilibrium, cursed consumers have unrealistically high expectations, inducing them to search excessively. This stimulates competition and leads to equilibria that are novel in the search literature where cursed expectations are partially self-fulfilling. The associated positive externality that cursed consumer exert on all other consumers is the strongest for an intermediate degree of cursedness when also standard and cursed consumer welfare is maximal.

Keywords: Consumer search, Bounded Rationality, Cursed Beliefs.

*We would like to thank Paul Heidhues as well as seminar audiences at Humboldt University Berlin, Bonn University, Vienna University, the Berlin IO Day and the Search and Matching Workshop in Bristol for helpful comments and suggestions.

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1 Introduction

When a consumer searches for a product, she has to form a belief about what the market supplies so as to decide whether to accept a current offer or continue to search. Forming correct beliefs about market conditions is difficult, in particular when a product is purchased rarely or even for the first time.

Taking this observation as point of departure[1], we study implications for consumer search markets when the belief of some consumers about what the market supplies is cursedly misspecified. This (small) departure from rational expectations captures that cursed consumers only have correct beliefs about the marginal distribution of product attributes such as price and quality, but partially neglect the correlation of product attributes when searching.[2] Our first observation is that the presence of cursed consumers enhances competition among firms because they are overly optimistic about the benefits of search, thus triggering excessive search and enhanced price competition. The equilibria that arise with cursed consumers are consistent with well-known equilibria in the search literature such as in Diamond (1971) or Stahl (1989), but we also show that new equilibria types arise that are incompatible with the idea that consumers have rational expectations. In these equilibria, firms offer "penny sales", that is, discontinuously low prices at the bottom of the price spectrum. Second, we show that the presence of cursed consumers can improve the welfare of all consumers in the market, standard and cursed. More specifically, we show that cursed consumers exert a positive externality on all other consumers which is the strongest if cursed consumers are only moderately cursed.

We establish our insights in a model of a search market in the spirit of Diamond (1971) in which there are multiple firms of different qualities which display the same surplus from trade. Firms set prices at the outset, and a consumer can engage in sequential search to discover a firm's price and quality by paying a search cost. A fraction of consumers is cursed, meaning they search as if they underestimate the correlation between price and quality. In the absence of cursed consumers, the unique equilibrium outcome is the seminal Diamond paradox in which all firms charge the monopoly price and all consumers buy from the first firm they visit.

In section 3, we show that the Diamond paradox breaks down if there are cursed consumers and search costs are sufficiently small. The intuition behind this finding is that a cursed consumer's belief about the distribution of utility that is offered by firms is always a mean preserving spread of the true distribution of utility that is offered. As a consequence, a cursed consumer believes that there is utility dispersion in the market even if all firms offer zero utility, as in the Diamond

[1] The prevailing assumption is that consumers have rational expectations and thus know what the market supplies.

[2] For example, in housing markets, a consumer might have a good idea about the distribution of apartment sizes and prices but might be too optimistic about the apartment size that she can afford given her budget.

paradox. Intuitively, if search costs are sufficiently small, even if all firms offer zero utility, a cursed consumer therefore continues her search when she encounters a firm which offers zero utility, because she falsely believes that there are golden eggs out there, product which offer more utility than any product that is actually offered in equilibrium, in example, zero utility. This causes the Diamond paradox to break down.

In section 4, we characterize the set of equilibrium types that can emerge instead, and in section 5, we characterize in terms of the primitives of the model which equilibrium type is actually an equilibrium. We establish that only three types of equilibrium may exist and that there always is a unique equilibrium. If the degree of cursedness is small so that cursed consumers' beliefs are just slightly misspecified, then the Diamond paradox obtains, and, in this sense, there is a continuous transition from the situation where cursedness is small to the situation where all consumers have rational expectations.[3] On the contrary, when cursedness is large, and search costs are sufficiently small, then an equilibrium outcome emerges which is reminiscent of the equilibrium outcome in Stahl (1989). Because cursedness is large, cursed consumers believe that with a high probability they can find a golden egg which offers more utility than any product that is actually offered in equilibrium. As a consequence, if search costs are sufficiently small, they never stop searching for a golden egg until they have visited all firms in the market. In this sense, they behave like shoppers in Stahl (1989) which have zero search costs. And indeed, the equilibrium outcome is behaviorally indistinguishable from Stahl (1989), because exactly the same market structure arises and cursed consumer behave just like shoppers. However, the normative implications are rather different. While a shopper in Stahl loves shopping and has zero search costs, a cursed consumer is doomed to incur the search costs for visiting all firms due to her unrealistic expectations.

For an intermediate level of cursedness, there is a novel type of equilibrium in which firms offer penny sales. More specifically, the equilibrium utility distribution features a mass point at the top which is separated from the rest of the distribution.[4] In this case, we say that a firm offers a penny sale when it offers the maximal utility, because a penny sale offers discontinuously more utility than any other offer in the market. The intuition is that cursed consumers have unrealistic expectations about what the market supplies, and in order to induce a visiting cursed consumer to stop her search, a firm must offer a penny sales, as otherwise, a cursed consumer falsely believes that it pays off to continue her search. In other words, the unreasonable belief that there are golden eggs out there, which supply more utility than any product that is actually offered in equilibrium, becomes partially self-fulfilling in that it causes firms to make extremely

[3] On the contrary, an arbitrarily small fraction of cursed consumers can cause the Diamond Paradox to break down.

[4] To the best of our knowledge, such an equilibrium utility distribution is novel in the literature on consumer search. We explain in the main text, why in our framework, a mass point at the top is incompatible with all consumers having rational expectations.

good offers. Perversely, although firms know that a cursed consumer will never actually find a firm that offers a golden egg, it cannot even offer slightly less utility than the utility a penny sale offers, because it knows that with positive probability the consumer will encounter another firm which offers a penny sale, and in this case, she would not return.

In section 6, we examine the welfare implication of the presence of cursed consumers as well as of changes in the degree of consumer cursedness. As argued before, cursed consumers engage in excessive search. Intuitively, this enhances competition among firms in the market and improves the welfare of all consumers. Perhaps surprisingly, this positive externality that cursed consumer exert on all other consumers in the market is non-monotonic in the degree of cursedness. Recall that for intermediate level of cursedness, firms offer penny sales so to meet the high expectations of cursed consumers. However, if cursed consumers are too cursed, then their expectation are just absurdly high from the point of view of firms, and as a consequence, firm gradually begin to refrain from offering penny sales as cursedness increases. As a result, at some point, the quality of offers in the market becomes worse as cursedness increases. This explains why an intermediate level of cursedness maximizes the welfare of standard consumers. As to cursed consumers, they suffer in addition from the fact that they engage in too much costly search when they are too cursed. Intuitively, as a result, an intermediate level of cursedness is also optimal for them. This means that the presence of moderately cursed consumers improves the welfare of all consumers in the market.

In section 7, we generalize our results. We study a search market with many firms which all generate the same surplus from trade and decide only how much utility to offer to consumers. We consider general naive consumers who are endowed with a belief function which governs how a naive consumer forms her misspecified belief about the distribution of utility that is offered given the true utility distribution of offers in the market. For example, the belief function of a standard consumers is the identity function. For this general case, we show first that for any arbitrary belief function the only equilibrium types that can emerge in equilibrium are those that we characterized before for the case of cursed consumer beliefs. Second, we derive sufficient conditions in terms of properties of the belief function which characterize the set of equilibrium types that actually are an equilibrium for some parameter values. We identify two decisive properties of the belief function which broadly speaking capture the notion of optimism and pessimism. If the belief function is optimistic, then for any equilibrium type there are parameter values such that this type is an equilibrium. On the contrary, if the belief function is pessimistic, then the only equilibrium outcome is the Diamond paradox. Consistent with the rational consumer benchmark, the identity function which is the belief function of a consumer with rational expectations is pessimistic.

2 Model

The market consists of N firms indexed by $n = 1, \ldots, N$. Each firm produces a product of either a high quality q_H or low quality q_L at respective marginal costs c_H and c_L where $q_H > q_L$, $c_H > c_L$.[5] Let

$$\Delta q = q_H - q_L, \quad \Delta c = c_H - c_L. \tag{1}$$

Quality is independently and identically distributed across firms with λ being the probability of high quality.

To isolate the effects of unrealistic consumer beliefs on market outcomes, we make the assumption that no firm has a competitive advantage, which means that all products generate the same surplus:

$$\omega \equiv q_H - c_H = q_L - c_L. \tag{2}$$

Below, we will offer alternative interpretations of this assumption. We will also argue that most of our results are robust to the introduction of efficiency differences among firms.

At the outset, each firm observes its quality and sets a price p_n. We restrict attention to the case that a firm's price depends on its quality only (but not on its identity n). Let F_θ be the pricing cdf of a firm with quality q_θ, $\theta \in \{H, L\}$.

There is a unit mass of consumers, each with unit demand. A consumer's utility from consuming quality q at price p is $q - p$. Consumers do not know qualities or prices ex ante, but can engage in costly, sequential search, undirected and with recall, to find out about quality and price. In addition, there is an outside option which supplies zero utility to the consumer. Search (except of the first search which is free)[6] entails a marginal search cost $s > 0$. For technical reasons, we assume that these search costs are sufficiently small and Δq sufficiently large so that $\Delta q > s$ and $\Delta q > \omega$.

We are interested in how market outcomes are affected when some consumers have limited knowledge of market conditions and, as a result, may entertain wrong beliefs about what the market supplies. We focus on the case that some consumers have inconsistent beliefs about the connection between quality and price. This may be the case for several reasons. It could be a manifestation of the consumers' well documented tendency to neglect correlations,[7] or capture

[5] The restriction to two quality levels is to facilitate tractability.

[6] This common assumption in the search literature ensures that there are equilibria with trade. For a discussion, see Stiglitz (1979).

[7] For empirical evidence on correlation neglect see for example Enke and Zimmermann (2019).

a form of wishful thinking, for example, that there are cheap, high quality products "out there". Alternatively, it could reflect the way that the consumer obtains information about the market prior to his search. For example, a flat-hunter in a new city, having seen flats of friends living there in the past, may have some ideas about the (marginal) distributions of flat qualities in the market, but, as a newcomer to the city who has limited knowledge about neighbourhoods, may fail to understand that quality and price is correlated through a what is for him latent neighbourhood factor.

To capture this formally, we assume that a fraction γ of consumers is cursed in the sense of Eyster and Rabin (2005). In our setting, this means that while the true price distribution of a firm with quality q_θ is F_θ, a cursed consumer believes that the firm's price distribution is a convex combination of the true price distribution and the price distribution that would occur if quality and price were independent:

$$F_\theta^\chi \equiv (1-\chi)F_\theta + \chi[\lambda F_H + (1-\lambda)F_L], \quad \chi \in (0,1]. \tag{3}$$

The cursedness parameter $\chi \in (0,1]$ captures the extent to which a consumer fails to make the correct connection between quality and price (or suffers from correlation neglect). For example, if a consumer is fully cursed, that is, $\chi = 1$, she mistakenly believes that quality and price are independent of one another.

Alternatively, a cursed consumer believes that a high quality firm prices according to F_H with probability $(1-\chi)+\chi\lambda$, and according to F_L with probability $\chi(1-\lambda)$. Similarly, she believes a low quality firm to price according to F_L with probability $(1-\chi)+\chi(1-\lambda)$, and according to F_H with probability $\chi\lambda$.

While a cursed consumer neglects correlation, she has correct beliefs about the (unconditional) marginal price distribution given by

$$\lambda F_H^\chi + (1-\lambda)F_L^\chi = \lambda F_H + (1-\lambda)F_L. \tag{4}$$

In addition, there is a fraction $1-\gamma$ of standard consumers who have correct beliefs.

3 Example

To illustrate the mechanics behind and the effects of cursed beliefs, it is useful to discuss the benchmark with only standard consumers as studied in Diamond (1971). In this case, the only equilibrium outcome is the Diamond paradox where each firm charges the consumer's willingness to pay with probability 1: $p_\theta = q_\theta$. Anticipating (correctly) that all firms supply a utility of zero, a standard consumer does then not search but buys at the first firm she visits.

We shall now argue that the Diamond paradox breaks down when there are cursed consumers, and search costs are not too large. In particular, we argue that in this case, cursed consumers have incentives to visit more than one firm. This entails that $p_\theta = q_\theta$ is not an equilibrium strategy for firms anymore, as firms profit from marginally undercutting their competitors' prices.

To see that consumers have incentives to visit more than one firm, assume a Diamond outcome, $p_\theta = q_\theta$, and consider a cursed consumer's decision to buy at the first firm or search for a better one. By assumption, the first firm supplies the consumer with zero utility. On the other hand, the consumer could adopt the (not necessarily optimal) search strategy to visit one more firm and buy from that firm among the two which offers more utility. To determine the utility from this strategy, notice that a cursed consumer believes that a high quality firm charges the price q_H with probability $(1-\chi)+\chi\lambda$, and the price q_L with probability $\chi(1-\lambda)$. In particular, she (mistakenly) believes that there is a chance of

$$\lambda\chi(1-\lambda) \tag{5}$$

that she finds a high quality firm which charges the low price p_L and thus supplies the utility $q_H - p_L = \Delta q$. Therefore, if she searches, she believes that visiting a single additional firm yields the additional utility

$$\lambda\chi(1-\lambda)\Delta q - s. \tag{6}$$

Hence, if s is sufficiently small, visiting more than one firm is better than buying at the first firm, and so the Diamond outcome breaks down.

In the remainder of the paper, we shall characterize precisely when the Diamond paradox does or does not break down and derive the equilibrium outcomes that arise instead.

4 Equilibrium definition and equilibrium structure

To analyse equilibria, it will prove useful to work in utility- rather than quality-price-space. A firm with quality q_θ and price q_θ supplies the (true) utility $u_\theta = q_\theta - p_\theta$, which is distributed according to the cdf

$$\kappa_\theta(u) \equiv 1 - F_\theta(q_\theta - u). \tag{7}$$

We next derive the distribution of utility that a cursed consumer believes to receive from a high quality firm. Recall that with probability $(1-\chi)+\chi\lambda$, a cursed consumer believes that the high quality firm prices according to F_H, resulting in the perceived utility distribution

$$1 - F_H(q_H - u) = \kappa_H(u). \tag{8}$$

With probability $\chi(1-\lambda)$, she believes that the high quality firm prices according to F_L, resulting in the perceived utility distribution

$$1-F_L(q_H-u)=1-F_L(q_L-(u-\Delta q))=\kappa_L(u-\Delta q). \tag{9}$$

Hence, the "cursedly perceived" utility distribution, conditional on high quality, is given by

$$\kappa_H^\chi(u) \equiv [(1-\chi)+\chi\lambda]\kappa_H(u)+\chi(1-\lambda)\kappa_L(u-\Delta q). \tag{10}$$

Similarly, the cursedly perceived utility distribution, conditional on low quality, is given by

$$\kappa_L^\chi(u) \equiv [(1-\chi)+\chi(1-\lambda)]\kappa_L(u)+\chi\lambda\kappa_H(u+\Delta q). \tag{11}$$

A cursed consumer thus believes that she samples iid draws from the utility distribution

$$\kappa^\chi \equiv \lambda\cdot\kappa_H^\chi+(1-\lambda)\cdot\kappa_L^\chi. \tag{12}$$

Our analysis focuses on symmetric equilibria that are symmetric in the sense that the (true) utility a firm offers is independent of its quality:

$$\kappa_H=\kappa_L\equiv\kappa. \tag{13}$$

Focussing on symmetric equilibria reflects that the market is competitive in the sense that no firm has an intrinsic efficiency advantage and hence faces the same optimization problem. In this case, inserting (10) and (11) in (12), yields that the consumer's belief is given by

$$\kappa^\tau(u)=[1-2\tau\lambda(1-\lambda)]\cdot\kappa(u) \tag{14}$$

$$+\tau\lambda(1-\lambda)\cdot\kappa(u-\Delta q)+\tau\lambda(1-\lambda)\cdot\kappa(u+\Delta q). \tag{15}$$

where $\tau=0$ stands for a standard, and $\tau=\chi$ for a cursed consumer, and $\kappa^0=\kappa$ follows from the fact that a standard consumer's belief is consistent.

Both a standard and cursed consumer's optimal search rule is myopic and fully characterized by a reservation utility because both consumers believe that they are in an iid environment so that Kohn and Shavell (1974) applies. This reservation utility is the minimal utility that a consumer's best option must supply so that she stops her search and takes this option (possibly her outside option of 0). Otherwise she continues her search. If the consumer has visited all firms, she returns to her best option. Let a consumer's reservation utility be denoted by U_τ.

Lemma 1 *A consumer's reservation utility U_τ is given as the unique solution to the equation*

$$U_\tau=R_\tau(U_\tau)\equiv -s+[1-2\tau\lambda(1-\lambda)]\int\max\{u,U_\tau\}\,d\kappa(u) \tag{16}$$

$$+\tau\lambda(1-\lambda)\int\max\{\Delta q+u,U_\tau\}\,d\kappa(u)+\tau\lambda(1-\lambda)\int\max\{-\Delta q+u,U_\tau\}\,d\kappa(u).$$

Moreover, U_τ increases in τ.

Expression (16) follows straightforwardly from the fact that the optimal search rule is myopic, which means that if the consumer's best offer supplies U_τ, then she is indifferent between purchasing this product and visiting a single additional firm. The fact that U_τ is strictly increasing entails that a cursed consumer's reservation utility exceeds a standard consumer's. Intuitively, a cursed consumer is unrealistically optimistic to find a favorable deal (high quality at a low price), and is therefore more reluctant to accept current offers than a standard consumer.

A cursed equilibrium can then be defined as follows:[8,9]

Definition 1 *A (symmetric) cursed equilibrium is a triple* $(\kappa^*, U_0^*, U_\chi^*)$ *such that*
(i) κ^* *maximizes a firm's profit, given that all other firms adopt* κ^* *and consumers search according to* U_0^* *and* U_χ^**;*
(ii) U_0^* *and* U_χ^* *are optimal reservation values, given firms supply utility according to* κ^**.*

Equilibrium structure

We provide two structural lemmata which show that only three types of symmetric equilibria can arise. Below, we then characterize which equilibrium arises in terms of the primitives. To state the lemmata, we denote the bounds of the support of a utility distribution κ by

$$\underline{u} = \inf[Supp(\kappa)], \quad \overline{u} = \sup[Supp(\kappa)]. \tag{17}$$

We indicate the bounds by a star if they belong to an equilibrium utility distribution. Moreover, to simplify the exposition of our results in the main text, we assume from now on that[10]

$$\frac{s}{\chi\lambda(1-\lambda)} \neq \Delta q. \tag{18}$$

[8]Strictly, speaking, Eyster and Rabin (2005) define Cursed Equilibrium only for static Bayesian games. Our equilibrium definition is a straightforward extension to dynamic games, and is essentially Perfect Bayesian Equilibrium with cursed beliefs. Similarly, for the case with fully cursed consumer, Ettinger and Jehiel (2010) define sequential analogy-based expectations equilibrium for dynamic games. Our equilibrium definition entails that a consumer's belief is passive and does not change when she observes an offer which is inconsistent with what she believes to be the true distribution of offers. Note, however, that by definition a cursed consumer attaches positive probability to all offers that she encounters in equilibrium. See Janssen and Shelegia (2019) for a recent paper that departs from the common assumption of passive beliefs.

[9]To derive and characterize equilibrium, we assume that a consumer buys if she is indifferent between buying and continuing her search, and that she never chooses her outside option when she is indifferent. The first assumption is actually a property of most equilibria but does not hold in any equilibrium, as Footnote 11 illustrates. The second assumption rules out equilibria without trade in which consumers do not even visit a single firm although a first search is for free. Under this second assumption, trade occurs in equilibrium and firms make strictly positive profits. We omit the details.

[10]As we show in Lemma 7 in the appendix, the non-generic case that $\frac{s}{\chi\lambda(1-\lambda)} = \Delta q$ is special in that there exists a continuum of equilibria with $\overline{u}^* = \underline{u}^* = U_\chi^*$.

Lemma 2 *In any symmetric cursed equilibrium, we have:*
(i) $\underline{u}^* = \max\{0, U_0^*\}$*;*
(ii) $\overline{u}^* \leq \max\{0, U_\chi^*\}$.

To understand the lemma, note that, conditionally on visiting a firm, a consumer stops and buys its product if and only if the firm offers more utility than both her outside option ($u \geq 0$) and reservation utility ($u \geq U_\tau^*$). The reason is that only when $u \geq U_\tau^*$, she stops. Moreover, all other firms that she might have visited before must supply less utility than U_τ^*, as she otherwise would have stopped before. Therefore, if the firm offers, in addition, more utility than her outside option, then its products is her best offer and she buys it upon stopping.

With this in mind, part (i) shows that the lowest utility $\underline{u}^*$ offered in equilibrium is the minimal utility that induces a standard consumer to stop and buy: $\max\{0, U_0^*\}$. Intuitively, this is optimal because a firm that offers $\underline{u}^*$ is not competing for returning consumers. On the other hand, if it offered less utility and $\underline{u}^* < \max\{0, U_0^*\}$, then it would derive no demand, because no consumer would stop or return, because the firm offers the lowest utility in the market. In other words, part (i) implies that in any equilibrium, standard consumers stop and buy at the first firm they visit.

The intuition for part (ii) is that, as argued above, each visiting consumer buys from a firm if it offers positive utility and more utility than each consumer's reservation utility, that is, if $u \geq \max\{0, U_\chi^*\}$, as $U_\chi^* > U_0^*$. Therefore, $u > \max\{0, U_\chi^*\}$ is never profit maximizing, because the firm could offer marginally less utility so as to increase its mark-up without losing demand.

Building on Lemma 2, the next lemma shows that there are only three types of candidates for symmetric equilibria.

Lemma 3 *In a symmetric cursed equilibrium, only the following three constellations can arise.*
(i) The equilibrium features monopoly pricing without price dispersion:

$$\underline{u}^* = \overline{u}^* = 0, \quad U_0^* \leq U_\chi^* \leq 0. \tag{19}$$

(ii) The equilibrium features "smooth" price dispersion, and cursed consumers visit all firms, and then return to the best offer:

$$Supp(\kappa^*) = [\underline{u}^*, \overline{u}^*], \quad U_0^* \leq \underline{u}^* < \overline{u}^* \leq U_\chi^*. \tag{20}$$

(iii) The equilibrium features price dispersion with "penny sales", and cursed consumers stop searching only when offered a penny sale $\overline{u}^*$*, and otherwise, visit all firms and then return to the best offer:*

$$Supp(\kappa^*) = [\underline{u}^*, u_\dagger^*] \cup \{\overline{u}^*\}, \quad U_0^* \leq \underline{u}^* < u_\dagger^* < \overline{u}^* = U_\chi^*. \tag{21}$$

In all cases, standard consumers buy from the first firm they visit.

To see the intuition behind the lemma, consider first the case that a cursed consumer stops and buys when she encounters a firm which offers the maximal utility $\overline{u}^*$, that is, $\overline{u}^* > U^*_\chi$. In this case, an equilibrium arises as described in (i). The reason is that then $\overline{u}^* = 0$ by part (ii) of Lemma 2. Together with part (i) of Lemma 2 (and since $\underline{u}^* \leq \overline{u}^*$), this implies $\overline{u}^* = \underline{u}^* = 0$. This means, however, that firms leave consumers zero utility by charging the monopoly price $p^*_\theta = q^*_\theta$ with probability 1. We refer to this type of equilibrium hence as a Diamond-type equilibrium.

In the case that $\overline{u}^* < U^*_\chi$, we have an equilibrium as described in (ii). Recall that $\overline{u}^* < U^*_\chi$ means that cursed consumers never encounter an offer that induces them to stop searching. Hence, they visit all firms and buy the best offer. Moreover, from (i) in Lemma 2, standard consumers buy from the first firm they visit. Effectively, consumers behave like the consumers in Stahl (1989): cursed consumers behave like "shoppers" with zero search costs in Stahl, and standard consumers behave like consumers with positive search costs in Stahl. Intuitively, both models feature therefore the same equilibrium structure with smooth price dispersion. We refer to this type of equilibrium hence as a Stahl-type equilibrium.

Finally, consider the case that $\overline{u}^* = U^*_\chi$. In this case, a penny sales equilibrium and a mass point at $\overline{u}^*$ as described in (iii) may occur.[11] To understand why a mass point can occur in equilibrium, note that when a firm offers a penny sale $\overline{u}^*$, a visiting cursed consumer stops his search and buys the firm's product. Hence, on the equilibrium path, a cursed consumer never visits two firms that both offer penny sales. Hence, unlike in a Stahl-type equilibrium with shoppers, these firms do not compete with each other for returning consumers so that offering marginally more utility does not attract discontinuously more consumers, despite the mass point. On the other hand, it is not profitable for a firm to offer marginally less utility, as otherwise a cursed consumer would not stop anymore, but rather continue his search and not return if she encounters a firm with penny sales (which occurs with positive probability).

To understand why the equilibrium utility distribution in (iii) also features an interval $[\underline{u}^*, u^*_\dagger]$, corresponding to smooth price dispersion, note that with positive probability no firm in the market offers a penny sale $\overline{u}^*$. In this case, a cursed consumer behaves like a shopper, visits all firms and returns to the one which offers the most utility. Intuitively, apart from penny sales, the price distribution can hence have no gaps and mass points, and features a price regime with

[11]Notice that also a Stahl-type equilibrium with $\overline{u}^* = U^*_\chi$ may occur, in which case there is no mass point at $\overline{u}^*$. Interestingly, in such an equilibrium, it is irrelevant whether a cursed consumer stops or continues his search when she encounters a firm which offers $\overline{u}^* = U^*_\chi$ (and is indifferent about what to do), because in both cases, she ends up purchasing its product (with probability one). On the contrary, in a Diamond-type equilibrium with $\overline{u}^* = U^*_\chi$ as well as in a penny sales equilibrium, it is an equilibrium property that the consumer stops his search at $\overline{u}^*$ (with probability one), since otherwise, firms that offer $\overline{u}^*$ would marginally increase the utility they offer.

smooth price dispersion as described in (iii). We refer to this type of equilibrium as a penny sales equilibrium.

5 Equilibrium characterization

In this section, we characterize in terms of the primitives of the model when any of the three candidates described in Lemma 3 actually is an equilibrium. We will show that the space of search costs can be partitioned in three regions in each of which exactly one equilibrium type occurs. In other words, the three equilibrium types are mutually exclusive and ordered according to search cost size.

The basic task is to identify for each equilibrium type when there are a firm strategy κ^* and reservation values U_0^* and U_χ^* which are mutual best replies and consistent with the restrictions placed on them in Lemma 3. Since this is a largely algebraic exercise, we relegate the details of the derivations to the appendix.

Diamond-type equilibrium

We begin by characterizing a Diamond-type equilibrium. The next lemma shows that there is Diamond-type equilibrium if and only if search costs are sufficiently large.

Lemma 4 (Diamond-type equilibrium) *There is a Diamond-type equilibrium if and only if*

$$s > s_1 \equiv \Delta q \chi \lambda (1-\lambda) \tag{22}$$

Intuitively, large enough search costs prevent even unrealistically optimistic cursed consumers to engage in any search beyond the first firm. This grants firms full monopoly power, and the Diamond outcome obtains. This is reflected by condition (22) which identifies the smallest search cost s_1 such that a cursed consumer stops and buys at the monopoly price when all firms charge it. Formally, U_χ is negative when all firms supply zero utility if and only if $s > s_1$.

Lemma 4 implies that if search costs are smaller than $\hat{s}$, then the Diamond-type equilibrium breaks down, because unrealistic optimistic cursed consumers begin to search although there is no (true) utility dispersion in the market. Our example in Section 3 illustrated this. As we show next, if search costs are sufficiently small, then Stahl-type equilibria emerge.

Stahl-type equilibrium

To characterize a Stahl-type equilibrium, we introduce some notation. Define the auxiliary parameters

$$\rho_0 = \frac{1-\gamma}{N\gamma + 1 - \gamma}, \quad \mu_0 = 1 - \frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0} \right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\rho_0}} (\nu - 1)^{-\frac{N-2}{N-1}} \cdot \frac{1}{\nu} \, d\nu. \tag{23}$$

Note that $\rho_0 \in [0,1]$. Moreover, Lemma 8 in the appendix establishes that $\mu_0 > 0$ and $\rho_0 + \mu_0 < 1$. For this reason, the following function Λ_0 is strictly decreasing in s:

$$\Lambda_0(s) \equiv \frac{s}{\chi\lambda(1-\lambda)} + (1-\rho_0-\mu_0)\min\left\{\omega, \frac{s}{\mu_0}\right\} - \Delta q. \tag{24}$$

and we define $\tilde{s}$ as its (unique) root.[12]

With these preparations, the next lemma characterizes a Stahl-type equilbrium.

Lemma 5 (Stahl-type equilibrium) *There is a Stahl-type equilibrium if and only if $s \leq s_0$. Moreover, in a Stahl-type equilibrium, a firm's strategy is given by*

$$\kappa^*(u) = \left[\frac{\rho_0}{1-\rho_0} \cdot \left(\frac{\omega - \underline{u}^*}{\omega - u} - 1\right)\right]^{\frac{1}{N-1}} \quad \textit{with } Supp(\kappa^*) = [\underline{u}^*, \overline{u}^*], \tag{26}$$

where $\underline{u}^*$ and $\overline{u}^*$ are given by

$$\underline{u}^* = \max\left\{0, \omega - \frac{s}{\mu_0}\right\} \quad \textit{and} \quad \overline{u}^* = (1-\rho_0)\omega + \rho_0 \underline{u}^*. \tag{27}$$

The intuition behind Lemma 5 is that a cursed consumer wrongly believes that there are golden eggs out there. Now, if search costs are sufficiently small, then a cursed consumer is not willing to settle for anything less than a golden egg, and because in reality there are none, she will never stop searching before she has visited all firms in the market.[13] That is, she behaves like a shopper in Stahl. Now, recall that a standard consumer always buys at the first firm she visits in equilibrium. Hence, intuitively, there is a Stahl-type equilibrium when search costs are sufficiently small. The critical search costs s_0 are the largest search costs such that a cursed consumer still behaves like a shopper when firms charge prices as in Stahl. That is, given (26) and (27), we have that $U_\chi^* > \overline{u}^*$ if and only if $s < s_0$.

Notice that while a Stahl-type equilibrium and the equilibrium in Stahl (1989) are behaviorally indistinguishable, because exactly the same market structure arises and cursed consumer behave just like shoppers, the normative implications are rather different. While a shopper in Stahl loves

[12] $\tilde{s}$ can be explicitly written as

$$s_0 \equiv \max\left\{\chi\lambda(1-\lambda)\cdot[\Delta q - (1-\rho_0-\mu_0)\omega]\,,\ \frac{\Delta q}{\frac{1-\rho_0-\mu_0}{\mu_0} + \frac{1}{\chi\lambda(1-\lambda)}}\right\}. \tag{25}$$

[13] More formally, for any κ^*, let $\overline{u}_\chi^* \equiv \Delta q + \overline{u}^*$ denote the highest utility which a cursed consumer believes to be offered in equilibrium (compare (10)). Intuitively, as search costs vanish, $U_\chi^* \to \overline{u}_\chi^*$, because visiting another firm becomes costless. Thus, for sufficiently small search costs, $U_\chi^* > \overline{u}^*$, and a cursed consumer never encounters an offer that induces her to stop so that she visits all firms.

shopping and has zero search costs, a cursed consumer is doomed to incur the search costs for visiting all firms due to her unrealistic expectations.

Our findings so far show that for $s > s_1$, there are only Diamond-type equilibria, while for $s \leq s_0$, there are only Stahl-type equilibria. Because $s_0 < s_1$, there are neither Diamond- or Stahl-type equilibria for an intermediate level of search costs. As we show next, if (and only if) search costs are in this regime, then there is a penny sales equilibrium.

Penny sales equilibrium

The next lemma states that whenever there is no Diamond- or Stahl-type equilibrium, then there is a penny sales equilibrium.

Lemma 6 (Penny sales equilibrium) *There is a penny sales equilibrium if and only if*

$$s_0 < s < s_1. \tag{28}$$

Moreover, there are strictly increasing functions $\rho, \phi, \mu : (0,1) \to [0,1]$ *with*[14] $\rho(0^+) = \phi(0^+) = \rho_0$, $\mu(0^+) = \mu_0$ *and* $\mu(1^-) + \rho(1^-) = \phi(1^-) = 1$ *so that in a penny sales equilibrium, a firm's strategy is given by*

$$\kappa^*(u) = \left[\frac{\rho_0}{1-\rho_0} \cdot \left(\frac{\omega - \underline{u}^*}{\omega - u} - 1\right)\right]^{\frac{1}{N-1}} \quad \text{for } u \in [\underline{u}^*, u_\dagger^*], \quad \text{and a mass point of mass } \zeta^* \in (0,1) \text{ at } \overline{u}^*, \tag{29}$$

where $\underline{u}^*, u_\dagger^*$ *and* $\overline{u}^*$ *are given by*

$$\underline{u}^* = \max\left\{0, \omega - \frac{s}{\mu(\zeta^*)}\right\}, \quad u_\dagger^* = (1-\phi(\zeta^*))\omega + \phi(\zeta^*)\underline{u}^*, \quad \text{and} \quad \overline{u}^* = (1-\rho(\zeta^*))\omega + \rho(\zeta^*)\underline{u}^*, \tag{30}$$

and $\zeta^* \in (0,1)$ *solves* $\Lambda(\zeta^*, s) = 0$ *with*

$$\Lambda(\zeta, s) \equiv \frac{s}{\chi\lambda(1-\lambda)} + (1-\rho(\zeta)-\mu(\zeta)) \cdot \min\left\{\omega, \frac{s}{\mu(\zeta)}\right\} - \Delta q. \tag{31}$$

Before we explain the intuition behind the lemma, we discuss the equilibrium outcome which is shown on the right panel of Figure 5. The figure plots the equilibrium reservation values U_0^* and U_χ^* (dashed lines) as well as the support of the equilibrium utility distribution (gray area) as functions of s.

As described in Lemma 5, for $s \leq s_0$, there is a Stahl-type equilibrium in which cursed consumers behave like shoppers. Accordingly, their reservation utility U_χ^* exceeds $\overline{u}^*$. Moreover,

[14] For a function $\varphi(\cdot)$, we denote by $\varphi(x^+)$ (resp. $\varphi(x^-)$) the right limit $\lim_{y\downarrow x}\varphi(y)$ (resp. the left limit $\lim_{y\uparrow x}\varphi(y)$) if it exists. In Lemma 8, we provide closed form expressions for $\rho(\cdot)$, $\phi(\cdot)$ and $\mu(\cdot)$.

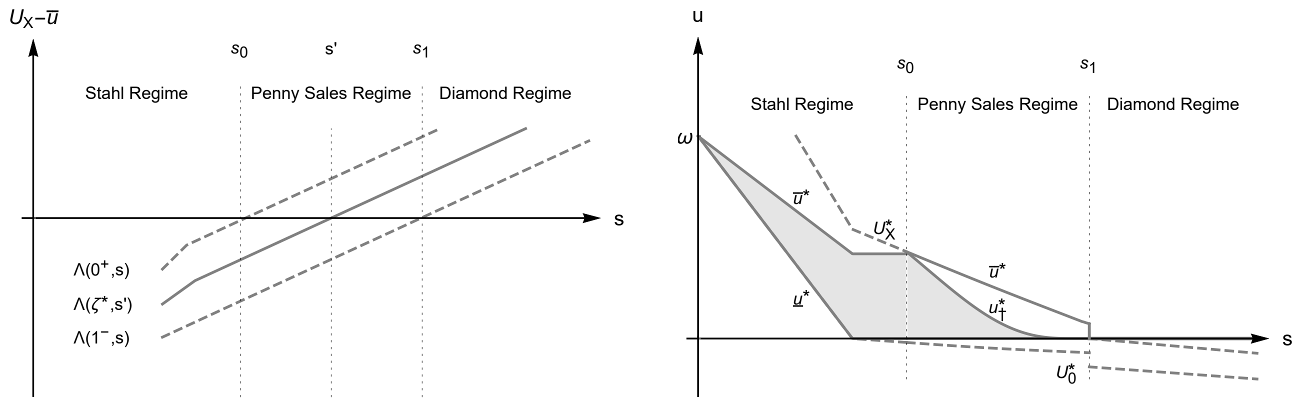

Figure 1: *The left panel plots $\Lambda(\zeta,s)$ for $\zeta \in \{0^+,\zeta^*,1^-\}$ as a function of s. It illustrates that the critical search costs s_0 and s_1 are the (unique) root of $\Lambda(0^+,s)$ and $\Lambda(1^-,s)$, and in the penny sales regime, for $s' \in (s_0,s_1)$, the equilibrium probability that a firm offers a penny sale ζ^* is given as the (unique) roof of $\Lambda(\zeta,s')$. The right panel plots the equilibrium reservation values U_0^* and U_χ^* ("dashed lines") as well as the support of the equilibrium utility distribution κ^* ("gray area") as functions of s. In the penny sales regime, κ^* features a mass point of size ζ^* at $\overline{u}^*$ and its support is not connected.*

there is utility dispersion which vanishes as search costs vanish, capturing that in the limit as the market becomes frictionless the Bertrand outcome obtains, and all firms offer the entire surplus from trade to consumers. Notice also that $\underline{u}^* = \max\{0, U_0^*\}$ as required by Lemma 2. From (27), U_0^* (and thus $\underline{u}^*$) decreases in s in this regime, capturing that as search costs increase, standard consumers become less selective. Intuitively, firms offer in response less utility to consumers so that, as the figure shows, the support of the utility distribution shifts downwards as search costs increase. The transition to the penny sales regime occurs when U_χ^* is equal to $\overline{u}^*$. At this point, all else equal, U_χ^* would drop below $\overline{u}^*$ as search costs increase and cursed consumers would not behave like shoppers anymore, causing the Stahl-type equilibrium to break down.

For large search costs, as described in Lemma 4, there is a Diamond-type equilibrium in which all firms offer zero utility. Observe that $U_\chi^* \leq 0$ as well as $U_0^* \leq 0$, reflecting that cursed and standard consumers are willing to accept an offer which supplies zero utility. Indeed, the Diamond-type equilibrium breaks down when s drops below s_1, because then, all else equal, U_χ^* would exceed 0 so that cursed consumers would not accept such an offer anymore.

For intermediate search costs, there is a penny sales equilibrium with utility dispersion in which all firms offer a penny sale with positive probability (notice the gap in the support). In this regime, $U_\chi^* = \overline{u}^*$, meaning that cursed consumers are indifferent about whether to buy at a penny sale or continue search.

To shed light on why the penny sales regime perfectly spans the gap in search costs between the

Diamond and Stahl regime, observe that the utility distributions in a Stahl- and a Diamond-type equilibrium can be viewed as the boundaries of the utility distribution in a penny sales equilibrium. More precisely, consider κ^* in (29) as a function of ζ^* where $\underline{u}^*$, $u^*_\dagger$ and $\overline{u}^*$, as functions of ζ^*, are given by (30). Notice that (29) differs from the utility distribution in a Stahl-type equilibrium, (26), only in that it is truncated at $u^*_\dagger$, and the remaining mass ζ^* is shifted to $\overline{u}^*$. Consequently, (29) converges to (26) as $\zeta^* \to 0$, (because, in addition, (30) converges to (27) as $\zeta^* \to 0$).[15] On the other hand, as $\zeta^* \to 1$, that is, in the limit when all firms offer penny sales, then (29) becomes similar to the utility distribution in a Diamond-type equilibrium in that all firms offer the same utility level $\overline{u}^*$ (which however is distinct from zero).

To see whether a particular equilibrium exists, it thus suffices to consider (29) and verify whether the utility distribution for the associated value of ζ^* induces the desired search behaviour of cursed consumers. Specifically, there is a Stahl-type equilibrium if, and only if, given (29) with $\zeta^* = 0$, cursed consumers behave like shoppers, that is $U^*_\chi > \overline{u}^*$. Similarly, there is a Diamond-type equilibrium if, and only if, given (29) with $\zeta^* = 1$, we have $U^*_\chi < \overline{u}^*$. The reason is that only then cursed consumers refrain from searching when all firms offer the same utility. Finally, there is a penny sales equilibrium if, and only if, there is $\zeta^* \in (0,1)$ such that given (29), we have $U^*_\chi = \overline{u}^*$.

In other words, all that matters for the cursed consumer's search behaviour (and hence for which equilbrium type obtains) is the sign of $U^*_\chi - \overline{u}^*$, given (29) and ζ^*. As shown in the proof of Lemma 6, this is formally captured by the function $\Lambda(\zeta^*, s)$ which is proportional to $U^*_\chi - \overline{u}^*$. Hence, the previous paragraph implies that there is a penny sales equilibrium if and only if there is $\zeta^* \in (0,1)$ with $\Lambda(\zeta^*, s) = 0$; That there is a Diamond-type equilibrium if and only if $\Lambda(1^-, s)$ is negative; And that there is a Stahl-type equilibrium if and only if $\Lambda(0^+, s)$ is positive. The left panel of Figure 5 depicts Λ as a function of s, given ζ^*. Because Λ is monotone in ζ^* and s (as shown in the proof of Lemma 6), the critical search costs s_0 and s_1 are the (unique) roots of $\Lambda(0^+, s)$ and $\Lambda(1^-, s)$, and the three equilibrium regimes span the entire space of search costs and do not overlap.

Lemma 6 makes clear that completely novel equilibrium types may emerge when some consumers are cursed. More specifically, a penny sales equilibrium, that is, an equilibrium with a mass point at $\overline{u}^*$, is a unique feature of our environment with misspecified beliefs in the following sense: In general, utility dispersion with a mass point at the top of the utility distribution may

[15] The underlying reason is that in both equilibria, κ^* is pinned down by the requirement that each utility level offered in equilibrium yields the same profit and for $u < \overline{u}^*$ the equilibrium profit functions in the two equilibria coincide, because in both equilibria, a firm that offers $u < \overline{u}^*$ sells it product to each standard consumer who visits it first and each cursed consumer if (and only if) it supplies the most utility.

only occur if there is a consumer of type τ out there who satisfies $U^*_\tau = \overline{u}^*$.[16] Now, a consumer's reservation utility is bounded from above by $\overline{u}^*_\tau - s$ where $\overline{u}^*_\tau$ denotes the highest utility that she expects firms to offer. The reason for this is that the best that a searching consumer can hope for is that the next firm offers $\overline{u}^*_\tau$ (with probability one) so that she is always willing to accept any current offer which supplies more than $\overline{u}^*_\tau - s$, as visiting another firm entails search costs $s > 0$. Therefore, $U^*_\tau = \overline{u}^*$ can only occur if there are consumers who attach positive probability to the event that firms offer more than $\overline{u}^*$, meaning their belief must be wrong. In section 7, we examine this idea in greater detail and identify a class of misspecified consumer beliefs which guarantee the existence of penny sales equilibria.

6 Comparative statics results

7 General Belief Functions

[16] To see this, if the reservation utility of all consumers is below $\overline{u}^*$, then a firm which offers $\overline{u}^*$ could offer marginally less utility so as to increase its mark-up without loosing demand. On the other hand, if $U^*_\tau > \overline{u}^*$ for some consumers, then these consumers visit all firms and return to the best offer, meaning that there may not be any mass point at the top as otherwise, a firm could offer marginally less utility and discontinuously increase its market share.

Appendix

Proof of Lemma 1 From Kohn and Shavell (1974), the consumer's optimal search rule is myopic and a consumer who is offered her reservation utility is, given her (cursed) belief, indifferent about whether to buy the current product or conduct a single additional search. Her reservation utility is therefore implicitly given by

$$U_\tau = -s + \int \max\{u, U_\tau\}\, d\kappa^\tau(u) \tag{32}$$

Inserting κ^τ from (14) then yields (16).

To see that U_τ increases in τ, note that (16) can be rewritten as

$$\begin{aligned} U_\tau = &-s + \int \max\{u, U_\tau\}\, d\kappa(u) \\ &+ \tau\lambda(1-\lambda) \int \big[\max\{\Delta q + u, U_\tau\} + \max\{-\Delta q + u, U_\tau\} - 2\max\{u, U_\tau\}\big]\, d\kappa(u). \end{aligned} \tag{33}$$

Then, U_τ increases in τ, because for any U_τ and u, we have

$$\max\{\Delta q + u, U_\tau\} + \max\{-\Delta q + u, U_\tau\} - 2\max\{u, U_\tau\} \geq 0, \tag{34}$$

which holds with strict inequality for some $u \in Supp(\kappa)$, because $U_\tau \in (\underline{u} - \Delta q, \overline{u} + \Delta q)$.[17] ■

Lemma 7 *(i) If $\underline{u}^* = \overline{u}^*$, then $U^*_\chi = \underline{u}^*$ if, and only if, $\frac{s}{\chi\lambda(1-\lambda)} = \Delta q$.*
(ii) If $\frac{s}{\chi\lambda(1-\lambda)} \neq \Delta q$, then $\underline{u}^* \neq U^*_\chi$.
(iii) If $\frac{s}{\chi\lambda(1-\lambda)} = \Delta q$, then there is a continuum of equilibria with $\underline{u}^* = \overline{u}^* = U^*_\chi$ and $\underline{u}^* \in \left[0, \max\left\{\gamma\omega, \omega - \frac{1-\gamma}{\gamma}s\right\}\right]$.

Proof of Lemma 7 As to (i), from (16), if $\underline{u}^* = \overline{u}^*$, then

$$\begin{aligned} U^*_\chi = &-s + \big[(1-\chi) + \chi(\lambda^2 + (1-\lambda)^2\big]\max\{\underline{u}^*, U^*_\chi\} \\ &+ \chi\lambda(1-\lambda)\max\{\Delta q + \underline{u}^*, U^*_\chi\} + \chi\lambda(1-\lambda)\max\{-\Delta q + \underline{u}^*, U^*_\chi\}. \end{aligned} \tag{35}$$

Hence, $\underline{u}^* = U^*_\chi$ if and only if

$$0 = -s + \chi\lambda(1-\lambda)\Delta q, \tag{36}$$

as desired.

[17] $U_\tau \leq \overline{u} + \Delta q$ follows straightforwardly from (33). To see $U_\tau \geq \underline{u} - \Delta q$, note that otherwise, from (33), $U_\tau = \mathbf{E}(u) - s$, and thus, $U_\tau \geq \underline{u} - s$. Together, $U_\tau < \underline{u} - \Delta q$ and $U_\tau \geq \underline{u} - s$ contradict that $\Delta q \geq s$.

As to part (ii), suppose to the contrary that $\underline{u}^* = U^*_\chi$ and $\frac{s}{\chi\lambda(1-\lambda)} \neq \Delta q$. Then, together with part (i), $\overline{u}^* > \underline{u}^* = U^*_\chi$, and it follows from (ii) in Lemma 2 that $\overline{u}^* = 0$.[18] Together, $\overline{u}^* > \underline{u}^*$ and $\overline{u}^* = 0$ imply $\underline{u}^* < 0$, contradicting that firms make positive profits in equilibrium, because a firm which offers $\underline{u}^*$ derives no demand, as it offers less utility than the consumer's outside option.

As to (iii), suppose $\frac{s}{\chi\lambda(1-\lambda)} = \Delta q$ and consider the firm's candidate equilibrium strategy described in (iii) with $\underline{u}^* = \overline{u}^*$. In this case, from (i), $U^*_\chi = \underline{u}^*$. Moreover, $U^*_0 < \underline{u}^*$ because $U_\chi > U_0$ from Lemma 1. Hence, because $\underline{u}^* = U^*_\chi$ and $\underline{u}^* > U^*_0$, all consumers buy from the first firm they visit in equilibrium if $\underline{u}^* \geq 0$. Therefore, offering more utility than $\underline{u}^*$ is not a profitable deviation for a firm, as it does not generate any additional demand. On the other hand, if it offers less utility than U^*_χ, then it loses the entire demand from cursed consumers. Therefore, conditional on offering less than U^*_χ, offering $u^{dev} = \max\{0, U^*_0\}$ is optimal, because this is the lowest level of utility that induces a standard consumer to stop and buy. We conclude that the firm's strategy described in (iii) is indeed an equilibrium strategy if and only if offering u^{dev} is not a profitable deviation for firms.

Now, because $\underline{u}^* = \overline{u}^*$, (16) implies that $U^*_0 = \mathbf{E}(u) - s = \underline{u}^* - s$, and hence $u^{dev} = \max\{0, \underline{u}^* - s\}$. Therefore, u^{dev} is not a profitable deviation if and only if

$$\pi(\underline{u}^*) \geq \pi(\max\{0, \underline{u}^* - s\}) \tag{37}$$

$$\Leftrightarrow \quad \frac{1}{N}(\omega - \underline{u}^*) \geq \frac{1-\gamma}{N}(\omega - \max\{0, \underline{u}^* - s\}). \tag{38}$$

To understand (38), recall that $\omega - u$ is the firm's mark-up if it offers u. Its market share when it offers $\underline{u}^* = U^*_\chi$ is $1/N$, because in this case, the demand of all consumers is split equally among all firms. If it offers $u^{dev} = \max\{0, U^*_0\}$, then only standard consumers buy from the firm, leading to a market share of $(1-\gamma)/N)$. To show (38), there are two cases to consider. First, if $\underline{u}^* \geq s$, then (38) is true if, and only if,

$$\underline{u}^* \leq \omega - \frac{1-\gamma}{\gamma}s. \tag{39}$$

Second, if $\underline{u}^* \in [0, s)$, then (38) holds if, and only if,

$$\underline{u}^* \leq \gamma\omega. \tag{40}$$

Now, observe that

$$\omega - \frac{1-\gamma}{\gamma}s \geq s \quad \Leftrightarrow \quad \gamma\omega \geq s. \tag{41}$$

[18]Notice that (ii) in Lemma does not depend on the assumption that $\frac{s}{\chi\lambda(1-\lambda)} \neq \Delta q$.

Together with (39) and (40), this implies that (38) is true if, and only if,

$$\underline{u}^* \in \left[0, \max\left\{\omega - \frac{1-\gamma}{\gamma}s, \gamma\omega\right\}\right]. \tag{42}$$

This completes the proof. ■

Proof of Lemma 2 First, $\underline{u}^* \geq \max\{0, U_0^*\}$, because otherwise, a firm that offers $\underline{u}^*$ derives no demand, contradicting that firms make strictly positive profits. To see this, if a firm offers $\underline{u}^* < 0$, then it clearly derives no demand. On the other hand, if $\underline{u}^* < U_0^*$, then a firm derives no demand because (a) no standard or cursed consumer stops when visiting the firm because $U_0^* < U_\chi^*$, and (b) no consumer returns. To see (b), note that there are only consumers who return to a firm that offers $\underline{u}^*$ if there is a mass point at $\underline{u}^*$ which satisfies $\underline{u}^* \geq 0$ and $\underline{u}^* < U_\chi^*$ (otherwise, each consumer stops when visiting the first firm so that no consumer returns). This can, however, not be the case in equilibrium, as otherwise, the firm could offer slightly more utility than $\underline{u}^*$ (slightly reduce the price) and discontinuously increase its demand (recall that a firm charges prices strictly above costs in equilibrium as it makes strictly positive profits in equilibrium.)

To show $\underline{u}^* \leq \max\{0, U_0^*\}$, assume to the contrary first that $\underline{u}^* > U_\chi^*$: Then all consumers purchase from the first firm they visit, and any firm could offer slightly less utility (slightly increase the price) without losing any demand because $\underline{u}^* > U_\chi^* \geq U_0^*$. Second, if $\underline{u}^* \in (\max\{0, U_0^*\}, U_\chi^*)$, then a firm which offers $\underline{u}^*$ only derives demand from those standard consumers who visit it first, because, as argued above, no consumer returns when it offers the least utility. Thus, as $\underline{u}^* > U_0^*$, it could offer slightly less utility without losing any demand from standard consumers. Finally, $\underline{u}^* \neq U_\chi^*$ because of assumption (18) and part (ii) in Lemma 7.

The argument for (ii) is given in the text. ■

Proof of Lemma 3 The arguments for the cases $\overline{u}^* > U_\chi^*$ and $\overline{u}^* < U_\chi^*$ are given in the main text. For $\overline{u}^* = U_\chi^*$, let ζ denote the mass of a (potential) mass point at $\overline{u}^*$. If $\zeta = 0$, then a cursed consumer never encounters an offer that stops her. She visits all firms and returns to the best offer. From standard arguments, κ^* has thus no mass points and its support has no gaps, as described in part (ii) of Lemma 3. Next, if $\zeta = 1$, then all firms offer the same utility and $\underline{u}^* = \overline{u}^*$ with $U_0^* \leq U_\chi^* \leq 0$, as described in part (i) of Lemma 3. Finally, if $\zeta \in (0,1)$, then with probability $(1-\zeta)^N$ no firm offers $\overline{u}^*$ so that a cursed consumer visits all firms and returns to best offer. Then, standard arguments imply that on $Supp(\kappa^*) \cap [\underline{u}^*, \overline{u}^*)$,[19] κ^* has no mass points and its support no gaps in the sense that $Supp(\kappa^*) \cap [\underline{u}^*, \overline{u}^*)$ is connected, as described in part (iii) of Lemma 3. To complete the proof, we show that the support has a "gap between" $Supp(\kappa^*) \cap [\underline{u}^*, \overline{u}^*)$ and $\overline{u}^*$, that is, $\{Supp(\kappa^*) \cap [\underline{u}^*, \overline{u}^*)\} \cup \overline{u}^*$ is not connected. The reason for this is that it cannot be profit

[19]The restriction to $Supp(\kappa^*) \cap [\underline{u}^*, \overline{u}^*)$ reflects that there are only consumers that return if no firm offers $\overline{u}^*$.

maximizing for a firm to offer marginally less utility than $\overline{u}^*$ when there is a mass point at $\overline{u}^*$ and $\overline{u}^* = U^*_\chi$. This follows from the fact that offering marginally less utility than $\overline{u}^*$ increases the firm's mark-up only marginally whereas its loss in demand increases discontinuously, because a cursed consumer does not stop anymore and only returns if she does not encounter a firm that offers $\overline{u}^*$ (which occurs with positive probability because there is a mass point at $\overline{u}^*$ by assumption). ■

Proof of Lemma 4 We show first that there is a Diamond-type equilibrium if and only if $U^*_\chi \leq 0$ when $\underline{u}^* = \overline{u}^* = 0$.

"⇒": From Lemma 3, in a Diamond-type equilibrium $U^*_\chi \leq 0$, and $\underline{u}^* = \overline{u}^* = 0$.

"⇐": $U^*_\chi \leq 0$ implies $U^*_0 \leq 0$, as required by Lemma 3. Then, it is indeed optimal for firms to offer zero utility (if all other firms do so): Because $U^*_0 \leq 0$ and $U^*_\chi \leq 0$, offering more utility than zero does not generate any additional demand, as all consumers buy from the first they visit that supplies positive utility, but it reduces the firm's mark-up. On the hand, offering less utility than zero is never optimal as it generates no demand.

Now, recall from (i) in Lemma 7, if $\underline{u}^* = \overline{u}^*$, then $U^*_\chi = \underline{u}^*$ if and only if $\frac{s}{\chi\lambda(1-\lambda)} = \Delta q$. Moreover, from (16), for any given utility distribution κ^*, U^*_χ decreases in s. Together, this implies that if $\underline{u}^* = \overline{u}^* = 0$, then $U^*_\chi \leq 0$ if and only if $\frac{s}{\chi\lambda(1-\lambda)} \geq \Delta q$ which completes the proof. ■

Proof of Lemma 5 The proof is identical to the proof of Lemma 6 for the special case that $\zeta^* = 0$ and where instead of the condition $\overline{u}^* = U^*_\chi$ in (44), we impose the condition $\overline{u}^* \leq U^*_\chi$. Analogously to Claim 5 in the proof of Lemma 6, one then verifies that $\overline{u}^* \leq U^*_\chi$ by showing that $\Lambda(0^+, s) \geq 0$ if and only if $s \leq s_0$. Because $\Lambda(0^+, s) = \Lambda_0(s)$ by Lemma 8, this establishes the claim.
■

Proof of Lemma 6 The result follows from the following five claims.

Claim 1: $(U^*_0, U^*_\chi, \kappa^*)$ is a penny sales equilibrium if and only if it satisfies (43), (44), (45) as well as (46) and (47).

To see Claim 1, note that by Lemma 2 and 3, in a penny sales equilibrium

$$\underline{u}^* = \max\{0, U^*_0\}, \tag{43}$$
$$\overline{u}^* = U^*_\chi, \tag{44}$$
$$Supp(\kappa^*) = [\underline{u}^*, u^*_\dagger] \cup \{\overline{u}^*\}, \quad \text{with a mass point of size } \zeta^* \text{ at } \overline{u}^*. \tag{45}$$

For $(U^*_0, U^*_\chi, \kappa^*)$ to be an actual equilibrium, in addition, κ^* needs to be profit maximizing, given all other firms adopt κ^* and (43), (44) and (45). Note first that, given (43), (44) and (45), $u \notin [\underline{u}^*, u^*_\dagger] \cup \{\overline{u}^*\}$ is never profit maximizing. Offering less than $\underline{u}^*$ generates no demand, as no consumer stops or returns, and offering more than $\overline{u}^*$ does not generate any additional demand in comparison to offering $\overline{u}^*$ but reduces the firm's margin (in either case any visiting consumer buys). Similarly, offering $u \in (u^*_\dagger, \overline{u}^*)$ does not generate any additional demand in comparison to offering $u^*_\dagger$ but reduces the firm's margin. It induces no cursed consumer to stop, as $u < \overline{u}^* = U^*_\chi$. Moreover, no cursed consumer returns who would not do so if the firm offered just $u^*_\dagger$. Therefore, κ^* is profit maximizing if, and only if, each $u \in Supp(\kappa^*)$ generates the same profit, that is,

$$\pi^* = (\omega - u) \cdot \left[\frac{1-\gamma}{N} + \gamma \kappa^*(u)^{N-1} \right] \quad \text{for all } u \in [\underline{u}^*, u^*_\dagger], \quad \text{and} \tag{46}$$

$$\pi^* = \left[\omega - \overline{u}^* \right] \cdot \left[\frac{(1-\gamma)}{N} + \gamma \cdot \eta(\zeta^*) \right], \tag{47}$$

where the function $\eta : (0,1) \to [0,1]$ is defined by

$$\eta(\zeta) \equiv \sum_{t=1}^{N} \binom{N-1}{t-1} \frac{(\zeta)^{t-1}(1-\zeta)^{N-t}}{t}. \tag{48}$$

To see that Condition (46) describes a firm's profit when it supplies $u \in [\underline{u}^*, u^*_\dagger]$, note that given (43) and (45), a standard consumer buys from any firm with probability $1/N$, as with equal probability each firm is the first one she visits and buys from. Moreover, given (44) and (45), a cursed consumer never stops at the firm since $u < \overline{u}^*$ but buys from it if it supplies the highest utility in the market (in which case the cursed consumer visits all firms and returns to the firm) which occurs with probability $\kappa^*(u)^{N-1}$.

To understand (47), if a firm offers $\overline{u}^*$, then its margin is $\omega - \overline{u}^*$ and it receives its share $\frac{1-\gamma}{N}$ of standard consumers. In addition, if there are $t-1$ many other firms that offer $\overline{u}^*$, then a cursed consumer buys from this firm with probability $1/t$, as this is the probability that she visits it first among the t many firms that offer $\overline{u}^*$. Furthermore, the probability that among its $n-1$ many competitors exactly $t-1$ offer $\overline{u}^*$ is $\binom{N-1}{t-1}(\zeta^*)^{t-1}(1-\zeta^*)^{N-t}$. Therefore, the probability that a cursed consumer buys from a firm that offers $\overline{u}^*$ is $\eta(\zeta^*)$, and (47) follows.

Claim 2: Given $\underline{u}^*$ and ζ^*, κ^* satisfies (45), (46), and (47) if and only if κ^* is given by (29), and $\overline{u}^*$ and $u^*_\dagger$ are given by (30).

Indeed, observe first, because $\kappa^*(\underline{u}^*) = 0$, (46) implies that

$$\pi^* = \pi(\underline{u}^*) = (\omega - \underline{u}^*) \cdot \frac{1-\gamma}{N}. \tag{49}$$

From this, it follows straightforwardly that (46) holds if and only if κ^* is given by (29).

Second, (45) holds if and only if $u_\dagger^*$ satisfies $\kappa^*(u_\dagger^*) = 1-\zeta^*$. Together with κ^* from (29),

$$\kappa^*(u_\dagger^*) = 1-\zeta^* \quad \Leftrightarrow \quad \frac{\rho_0}{1-\rho_0}^{\frac{1}{N-1}} \left(\frac{\omega-\underline{u}^*}{\omega-u_\dagger^*}-1\right)^{\frac{1}{N-1}} = 1-\zeta^* \tag{50}$$

$$\Leftrightarrow \quad \frac{\omega-\underline{u}^*}{\omega-u_\dagger^*}-1 = (1-\zeta^*)^{N-1}\cdot\frac{1-\rho_0}{\rho_0}. \tag{51}$$

By defining

$$\phi(\zeta) \equiv \frac{\rho_0}{\rho_0+(1-\rho_0)(1-\zeta)^{N-1}}, \tag{52}$$

and re-arranging terms, we obtain $u_\dagger^*$ as given by (30).

Third, given (46), condition (47) holds if and only if

$$\pi(\underline{u}^*) = \pi(\overline{u}^*) \quad \Leftrightarrow \quad (\omega-\underline{u}^*)\cdot\frac{\rho_0}{1-\rho_0}\gamma = (\omega-\overline{u}^*)\cdot\left(\frac{\rho_0}{1-\rho_0}\gamma+\gamma\eta(\zeta^*)\right) \tag{53}$$

$$\Leftrightarrow \quad \frac{\omega-\underline{u}^*}{\omega-\overline{u}^*}-1 = \eta(\zeta^*)\frac{1-\rho_0}{\rho_0}. \tag{54}$$

By defining

$$\rho(\zeta) \equiv \frac{\rho_0}{\rho_0+(1-\rho_0)\eta(\zeta)}, \tag{55}$$

and re-arranging terms, we obtain $\overline{u}^*$ as given by (30).

Finally, we verify that $u_\dagger^* < \overline{u}^*$, as required by (45). From (30), because $\underline{u}^* < \omega$, we have

$$u_\dagger^* < \overline{u}^* \quad \Leftrightarrow \quad \phi(\zeta^*) > \rho(\zeta^*). \tag{56}$$

We establish the right hand side in part (iii) of Lemma 8.

Claim 3: $\underline{u}^*$ satisfies (43) if and only if $\underline{u}^* = \max\left\{0, \omega - \frac{s}{\mu(\zeta^*)}\right\}$, where the function $\mu : (0,1) \to [0,1]$ is defined by

$$\mu(\zeta) \equiv 1 - \frac{1}{N-1}\cdot\left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\phi(\zeta)}} (v-1)^{-\frac{N-2}{N-1}}\cdot\frac{1}{v}\,dv - \zeta\rho(\zeta). \tag{57}$$

To see Claim 3, observe that $\kappa^*(u)$ as defined in the proof of Claim 2 is pinned down as a function of $\underline{u}^*$ and ζ^*. Taking ζ^* as given and treating $\underline{u}^*$ as a variable, let $U_0^*(\underline{u}^*)$ be the standard consumer's reservation value, that is,

$$U_0^*(\underline{u}^*) = R_0(U_0^*(\underline{u}^*)) = -s + \int \max\{u, U_0^*(\underline{u}^*)\}\,d\kappa^*(u). \tag{58}$$

Hence $\underline{u}^*$ satisfies (43) if and only if $\underline{u}^*$ is a solution to the fixed point equation

$$\underline{u} \;=\; \max\{0, U_0^*(\underline{u})\}. \tag{59}$$

(a) We first show that $\underline{u}^* = 0$ is a solution to (59) if and only if $\omega - \frac{s}{\mu(\zeta)} \leq 0$. Indeed, $\underline{u}^* = 0$ is a solution if and only if $0 \geq U_0^*(0)$. Now, because the function R_0 defined in (16) intersects the diagonal exactly once from above,[20] we have for arbitrary $\underline{u}$ that

$$0 \geq U_0^*(\underline{u}) \quad \Leftrightarrow \quad 0 \geq R_0(0) = -s + \int \max\{u, 0\}\, d\kappa^*(u) = -s + \int u\, d\kappa^*(u). \tag{60}$$

Below, we show that for all $\underline{u}$ and ζ^*, we have

$$\int u\, d\kappa^*(u) = \mu(\zeta^*)\omega + (1 - \mu(\zeta^*))\underline{u}. \tag{61}$$

Hence, by inserting $\underline{u} = 0$ in (60), we obtain that $0 \geq U_0^*(0)$ if and only if

$$0 \geq -s + \mu(\zeta^*)\omega, \tag{62}$$

which is equivalent to $\omega - \frac{s}{\mu(\zeta^*)} \leq 0$, as desired.

(b) We next show that there is a solution $\underline{u}^* > 0$ to (59) if and only if $\omega - \frac{s}{\mu(\zeta^*)} > 0$, and in this case, $\underline{u}^* = \omega - \frac{s}{\mu(\zeta^*)}$. Indeed, $\underline{u}^* > 0$ is a solution to (59) if and only if $\underline{u}^* = U_0^*(\underline{u}^*)$. By definition of R_0, this is equivalent to $\underline{u}^* = R_0(\underline{u}^*)$, where

$$R_0(\underline{u}^*) = -s + \int \max\{u, \underline{u}^*\}\, d\kappa^*(u) = -s + \int u\, d\kappa^*(u) = -s + \mu(\zeta^*)\omega + (1 - \mu(\zeta^*))\underline{u}^*, \tag{63}$$

where the final equality follows from (61). Hence, $\underline{u}^* = U_0^*(\underline{u}^*)$ is equivalent to

$$\underline{u}^* = \omega - \frac{s}{\mu(\zeta^*)}, \tag{64}$$

and this implies (b).

Together, (a) and (b) imply that there is a unique solution to (59) which is $\underline{u}^* = \max\left\{0, \omega - \frac{s}{1-\sigma}\right\}$, as desired.

Claim 4: Given Claims 2 and 3, (44) holds if and only if ζ^* is implicitly given by (**??**).

Indeed, by Lemma 1, (44) is equivalent to $\overline{u}^* = R_\chi(\overline{u}^*)$. To show this, we make use of the following property:

$$\Delta q \geq \overline{u}^* - \underline{u}^*. \tag{65}$$

To see (65), because firms offer positive utility and charge prices above marginal costs, $\underline{u}^* \geq 0$ and $\overline{u}^* \leq \omega$. Therefore, $\omega \geq \overline{u}^* - \underline{u}^*$, and (65) follows from $\Delta q > \omega$ by assumption.

[20] From standard arguments, the reservation utility is unique, that is, there is a single crossing. Moreover, R_χ intersects the diagonal from above, because from its definition, R_χ is strictly increasing with $R'_\chi \leq 1$.

With (65), we obtain by (16) that

$$R_\chi(\overline{u}^*) = -s + [1-\chi\lambda(1-\lambda)]\,\overline{u}^* + \chi\lambda(1-\lambda)\int \Delta q + u\, d\kappa^*(u). \tag{66}$$

It now follows from tedious but straightforward algebra that if we insert $\mathbf{E}(u)$ from (61), and $\overline{u}^*$ and $\underline{u}^*$ from (30), then $\overline{u}^* = R_\chi(\overline{u}^*)$ (and hence, $U^*_\chi = \overline{u}^*$), if and only if $\Lambda(\zeta^*, s) = 0$.[21]

Claim 5: There is a penny sales equilibrium, which is unique, if and only if $s_0 < s < s_1$.

Equivalently, we show that there is a unique solution $\zeta^* \in (0,1)$ to $\Lambda(\zeta^*, s) = 0$ if and only if $s_0 < s < s_1$. To see this, note that since $1-\rho(\zeta^*)-\mu(\zeta^*) \geq 0$ and $\mu(\cdot)$ and $\rho(\cdot)$ are strictly increasing by Lemma 8, $\Lambda(\zeta, s)$ is strictly decreasing in ζ. Hence, because $\Lambda(\zeta, s)$ is continuous in ζ, there is indeed a (unique) solution $\zeta^* \in (0,1)$ to $\Lambda(\zeta^*, s) = 0$ if and only if

$$\Lambda(0^+, s) > 0 \quad \text{and} \quad \Lambda(1^-, s) < 0. \tag{71}$$

Now, Lemma 8 implies that

$$\Lambda(0^+, s) = \frac{s}{\chi\lambda(1-\lambda)} + (1-\rho_0-\mu_0)\cdot \min\left\{\omega, \frac{s}{\mu_0}\right\} - \Delta q, \quad \text{and} \quad \Lambda(1^-, s) = \frac{s}{\chi\lambda(1-\lambda)} - \Delta q, \tag{72}$$

and straightforward algebra yields that $\Lambda(0^+, s) > 0$ is equivalent to $s < s_0$, and $\Lambda(1^-, s) < 0$ is equivalent to $s > s_1$, as desired.

This completes the (main part of the) proof of Lemma 6. Lemma 8 below establishes that ρ, ϕ, μ are indeed strictly increasing functions with $\rho(0^+) = \phi(0^+) = \rho_0$, $\mu(0^+) = \mu_0$ and $\mu(1^-) + \rho(1^-) = \phi(1^-) = 1$. It only remains to show (61). To see this, note that

$$\int_{\underline{u}}^{\overline{u}} (\omega - u)\, d\kappa^*(u) = \int_{\underline{u}}^{u^\dagger} (\omega - u)(\kappa^*)'(u)\, du + \zeta^*(\omega - \overline{u}), \tag{73}$$

[21] To see this, inserting (61) for $\mathbf{E}(u)$ in (66) yields that

$$R_\chi(\overline{u}^*) = \overline{u}^* \quad \Leftrightarrow \quad -\frac{s}{\chi\lambda(1-\lambda)} + \mu(\zeta^*)\omega + (1-\mu(\zeta^*))\underline{u}^* + \Delta q = \overline{u}^* \tag{67}$$

$$\Leftrightarrow \quad -\frac{s}{\chi\lambda(1-\lambda)} + \mu(\zeta^*)\cdot(\omega - \underline{u}^*) + \Delta q = \overline{u}^* - \underline{u}^*. \tag{68}$$

Inserting $\overline{u}^* - \underline{u}^* = (1-\rho(\zeta^*))(\omega - \underline{u}^*)$ from (30) yields

$$R_\chi(\overline{u}^*) = \overline{u}^* \quad \Leftrightarrow \quad -\frac{s}{\chi\lambda(1-\lambda)} + (\mu(\zeta^*) - (1-\rho(\zeta^*)))\cdot(\omega - \underline{u}^*) + \Delta q = 0 \tag{69}$$

$$\Leftrightarrow \quad \frac{s}{\chi\lambda(1-\lambda)} + (1-\rho(\zeta^*)-\mu(\zeta^*))\cdot(\omega - \underline{u}^*) = \Delta q. \tag{70}$$

Finally, inserting $\underline{u}^* = \max\left\{0, \omega - \frac{s}{\mu(\zeta^*)}\right\}$ establishes the desired result that $R_\chi(\overline{u}^*) = \overline{u}^*$ if and only if $\Lambda(\zeta^*, s) = 0$.

and inserting κ^* from (29) and some manipulations yield

$$\int_{\underline{u}}^{\overline{u}} (\omega - u)\, d\kappa^*(u) = \frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_{\underline{u}}^{u^\dagger} \left(\frac{\omega - \underline{u}}{\omega - u} - 1\right)^{-\frac{N-2}{N-1}} \frac{\omega - \underline{u}}{\omega - u}\, du + \zeta^*(\omega - \overline{u}). \tag{74}$$

A change of variables and inserting $(\omega - \overline{u}) = \rho(\zeta^*)(\omega - \underline{u})$ from (30) yield[22]

$$\int_{\underline{u}}^{\overline{u}} (\omega - u)\, d\kappa^*(u) = \left[\frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\phi(\zeta^*)}} (v-1)^{-\frac{N-2}{N-1}} \cdot \frac{1}{v}\, dv + \zeta^* \rho(\zeta^*)\right] (\omega - \underline{u}) \tag{75}$$

$$= (1 - \mu(\zeta^*)) \cdot (\omega - \underline{u}). \tag{76}$$

Hence, (61) follows from the fact that $\mathbf{E}(u) = \omega - \mathbf{E}(\omega - u)$. ■

[22]Let $v(u) = \frac{\omega - u}{\omega - u}$, then $du = \frac{(\omega - u)^2}{\omega - \underline{u}} dv = \frac{\omega - u}{v+1}\, dv$, $v(\underline{u}) = 1$, and $v(u_\dagger) = 1/\phi(\zeta^*)$ from (50).

8 Appendix B

Lemma 8 *We have:*
(i) $\eta(\cdot)$ *is strictly decreasing with* $\eta(0^+) = 1$ *and* $\eta(1^-) = \frac{1}{N}$.
(ii) $\rho(\cdot)$ *is strictly increasing with* $\rho(0^+) = \rho_0$ *and* $\rho(1^-) = 1-\gamma$.
(iii) $\phi(\cdot)$ *is strictly increasing with* $\phi(0^+) = \rho_0$ *and* $\phi(1^-) = 1$. *Moreover,* $\phi(\zeta) > \rho(\zeta)$.
(iv) $\mu(\cdot)$ *is strictly increasing with* $\mu(0^+) = \mu_0 > 0$ *and* $\mu(1^-) = \gamma$.
(v) $1-\rho(\zeta)-\mu(\zeta)$ *is positive for all* $\zeta \in (0,1)$ *and strictly decreasing with* $1-\rho(1^-)-\mu(1^-) = 0$.

Proof of Lemma 8 (i) Recall from definition (48) that

$$\eta(\zeta) = \sum_{t=1}^{N} \binom{N-1}{t-1} \frac{\zeta^{t-1}(1-\zeta)^{N-t}}{t}, \tag{77}$$

and observe that $\eta(0^+) = 1$ and $\eta(1^-) = \frac{1}{N}$. Moreover, differentiating (77) yields

$$\eta'(\zeta) = \sum_{t=2}^{N} \binom{N-1}{t-1} \frac{t-1}{t} \zeta^{t-2}(1-\zeta)^{N-t} - \sum_{t=1}^{N-1} \binom{N-1}{t-1} \frac{N-t}{t} \zeta^{t-1}(1-\zeta)^{N-t-1}. \tag{78}$$

By an index change, the second term on the right hand side can be written as[23]

$$\sum_{t=2}^{N} \binom{N-1}{t-2} \frac{N-t+1}{t-1} \zeta^{t-2}(1-\zeta)^{N-t} = \sum_{t=2}^{N} \binom{N-1}{t-1} \zeta^{t-2}(1-\zeta)^{N-t}, \tag{80}$$

Therefore,

$$\eta'(\zeta) = \sum_{t=2}^{N} \binom{N-1}{t-1} \left[\frac{t-1}{t} - 1\right] \zeta^{t-2}(1-\zeta)^{N-t} \tag{81}$$

$$= -\frac{1}{\zeta} \sum_{t=2}^{N} \binom{N-1}{t-1} \cdot \frac{1}{t} \cdot \zeta^{t-1}(1-\zeta)^{N-t} \tag{82}$$

$$= -\frac{1}{\zeta} \left[\eta(\zeta) - (1-\zeta)^{N-1}\right] \tag{83}$$

$$= -\frac{1}{\zeta} \frac{\rho_0}{1-\rho_0} \left[\frac{1}{\rho(\zeta)} - \frac{1}{\phi(\zeta)}\right] < 0, \tag{84}$$

where the third line follows from the definition of $\eta(\cdot)$ in (77) and the last line from the definition of ρ_0 in (23), $\phi(\cdot)$ in (90) and $\rho(\cdot)$ in (85). Finally, $\eta(\cdot)$ strictly decreasing follows from (82).

[23] Note that

$$\binom{N-1}{t-2} = \frac{(N-1)!}{(t-2)!(N-t+1)!} = \frac{(N-1)!}{(t-1)!(N-t)!} \frac{t-1}{N-t+1} = \binom{N-1}{t-1} \frac{t-1}{N-t+1}. \tag{79}$$

(ii) Recall from definition (55) that

$$\rho(\zeta) = \frac{\rho_0}{\rho_0 + (1-\rho_0)\,\eta(\zeta)}. \tag{85}$$

Because $\eta(0^+) = 1$ and $\eta(1^-) = \frac{1}{N}$ from part (i), we have

$$\rho(0^+) = \rho_0 \quad \text{and} \quad \rho(1^-) = 1-\gamma, \tag{86}$$

where the last equality follows straightforwardly from the definition of ρ_0 in (23) and re-arranging terms. Moreover, differentiating (85) yields

$$\rho'(\zeta) = -\frac{\rho_0}{(\rho_0 + (1-\rho_0)\,\eta(\zeta))^2} \cdot (1-\rho_0)\,\eta'(\zeta) \tag{87}$$

$$= -\frac{1-\rho_0}{\rho_0} \cdot \rho(\zeta)^2 \cdot \eta'(\zeta) \tag{88}$$

$$= \frac{\rho(\zeta)^2}{\zeta} \cdot \left[\frac{1}{\rho(\zeta)} - \frac{1}{\phi(\zeta)}\right], \tag{89}$$

where the last equality follows from inserting η' from (84). Moreover, $\rho(\cdot)$ strictly increasing follows from (88) and $\eta(\cdot)$ strictly decreasing by part (i).

(iii) Recall from definition (52) that

$$\phi(\zeta) = \frac{\rho_0}{\rho_0 + (1-\rho_0)(1-\zeta)^{N-1}}, \tag{90}$$

and observe that $\phi(0^+) = \rho_0$ and $\phi(1^-) = 1$. Moreover, differentiating (90) yields

$$\phi'(\zeta) = \frac{1-\rho_0}{\rho_0} \cdot \phi(\zeta)^2 \cdot (N-1)(1-\zeta)^{N-2} > 0, \tag{91}$$

as desired. Finally, $\phi(\zeta) > \rho(\zeta)$ follows immediately from equation (84) in the proof of part (i).

(iv) Recall from definition (57) that

$$\mu(\zeta) \equiv 1 - \frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\phi(\zeta)}} \frac{(v-1)^{-\frac{N-2}{N-1}}}{v}\, dv - \zeta\rho(\zeta), \tag{92}$$

and observe that $\mu(1^-) = 1 - \rho(1^-) = \gamma$ and $\mu(0^+) = \mu_0$, as $\phi(0^+) = \rho_0$ and $\phi(1^-) = 1$ from part (iii). Differentiating (92) yields

$$\mu'(\zeta) = -\frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \cdot \left(-\frac{\phi'(\zeta)}{\phi(\zeta)^2}\right) \cdot \phi(\zeta)\left(\frac{1}{\phi(\zeta)} - 1\right)^{\frac{1}{N-1}-1} - \rho(\zeta) - \zeta\rho'(\zeta)$$

$$= \left\{\left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}-1} (1-\zeta)^{N-2} \left(\frac{(1-\rho_0)(1-\zeta)^{N-1}}{\rho_0}\right)^{\frac{1}{N-1}-1}\right\} \cdot \phi(\zeta) - \rho(\zeta) - \rho(\zeta)^2 \cdot \left[\frac{1}{\rho(\zeta)} - \frac{1}{\phi(\zeta)}\right]$$

$$= \frac{(\phi(\zeta) - \rho(\zeta))^2}{\phi(\zeta)} > 0, \tag{93}$$

where the second line follows from inserting ρ' from (89) and ϕ' from (91), and the third line from the fact that the term in curly brackets in the second line is equal to one. The inequality in the third line, in turn, follows from $\phi(\zeta) > \rho(\zeta)$ by part (iii). Finally, $\mu_0 > 0$ follows from

$$\mu_0 = 1 - \frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\rho_0}} \frac{(\nu-1)^{-\frac{N-2}{N-1}}}{\nu}\, d\nu \tag{94}$$

$$> 1 - \frac{1}{N-1} \cdot \left(\frac{\rho_0}{1-\rho_0}\right)^{\frac{1}{N-1}} \int_1^{\frac{1}{\rho_0}} \frac{(\nu-1)^{-\frac{N-2}{N-1}}}{1}\, d\nu = 0, \tag{95}$$

where the second line follows from $1/\nu < 1$ for $\nu \in [1, \frac{1}{\rho_0}]$, and the final equality from a straightforward calculation.

(v) Part (v) is an immediate consequence of part (ii) and (iv). ■

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