Hedging Executive Compensation Risk Through Investment Banks

Ronald A. Dye and Sri S. Sridhar

February 22, 2016

Abstract

Allowing CEOs to hedge the risk in the compensation contracts their firms give them has been controversial because such hedging permits the executives to undo some of the incentive effects of those contracts; it also results in a divergence between the compensation firms *pay* their senior executives and the compensation those executives effectively *receive*. We analyze the personal hedging activities of CEOs and identify when firms may gain or lose by allowing or prohibiting such hedging. We also describe variations in CEOs' demands for various compensation hedges, and how firms will restructure their CEOs' compensation contracts in anticipation the CEOs will engage in such hedging.

1 Introduction

Hedging some of their compensation risk by engaging in transactions with investment banks has become increasingly popular among CEOs. Bloomberg Business Week, in an article titled "Some CEOs Are Selling Their Companies Short, With a Little Help from Their Bankers...,"[1] stated that in 2009, 107 companies reported to the SEC that their executives had hedged compensation risk, up from 48 in 2007. This article also asserted that "Investment banks ... rushed to provide hedge services" to senior managers of firms, and it quotes an attorney as saying "I don't know of a bank that doesn't have a department doing this."[2] Related, Dash [2011] reported that more than a quarter of the partners at Goldman Sachs hedged their compensation risk over the years 2007 to 2010.

At the same time it has become increasingly popular, compensation hedging also has become increasingly controversial. Opposition to such hedging has extended all the way to the U.S. Senate: U.S. senators Menendez, Merkley, and Lautenberg, in a letter to the FDIC, opined:

> "We strongly believe that hedging strategies used by highly-paid executives on their own incentive-based compensation should be prohibited. Quite simply, the use of hedging takes the "incentive" out of incentive-based compensation."[3]

One of the conditions of federal aid for those financial institutions that received aid during the recent financial crisis was that the financial institutions prohibit their executives from engaging in such hedging.[4] Also, many well known public companies explicitly prohibit such hedging: e.g., both Procter and Gamble and Kellogg impose such prohibitions.[5]

In this article, we try to account for both the popularity of and controversy surrounding CEO compensation hedging by developing a model that explains why some companies benefit from allowing compensation hedging, while other companies do not. While we show that there are several economic considerations at work simultaneously that influence how compensation hedging affects firm value, we believe there is one fundamental factor, namely how compensation hedging allows managers to transform their compensation contracts into effectively different contracts, that is responsible both for the

[1] February 25, 2010 issue, Jane Sasseen byline.

[2] Bettis, Bizjack and Kalpathy [2010] document extensive personal hedging by firms' senior managers.

[3] The letter can be found at: http://www.sec.gov/comments/s7-12-11/s71211-4.pdf

[4] For example, Crimson Exploration corporate policy states: "You [CEO] may not engage in certain hedging transactions with respect to Company securities. Certain forms of hedging transactions, such as zero-cost collars and forward sale contracts, allow a stockholder to lock in the value of his or her stock holdings, often in exchange for all or a portion of any future appreciation in the stock. The stockholder is then no longer exposed to the full risks of stock ownership and may no longer have the same objectives as the Company's other stockholders. Therefore, such hedging transactions are prohibited under this policy." From: http://crimsonexploration.com/default/Insider_Trading_Policy_Preclearance_3_1_2010.pdf

[5] http://www.sec.gov/comments/s7-12-11/s71211-4.pdf

popularity of, and opposition to, CEO compensation hedging. In short, we contend that opposition to such hedging derives principally from firms losing some control over their CEOs' incentives when they allow the CEOs to transform their compensation contracts via hedging, whereas the popularity of such hedging derives principally from the potential such hedging has to improve the firm-CEO relationship by reducing the variance in CEOs' compensation.

Expanding on these two points, allowing CEOs to transform their original compensation contracts through hedges can expand the agency problem firms face with their CEOs from the usual (and extensively studied) "operating action" agency problem firms always face with their CEOs to include an agency problem with respect to the CEOs' contract transformation choices. Further, since CEOs' operating action choices are determined by maximizing their expected utility under the transformed contracts, and not under the original contracts, the "operating action" agency problem firms face potentially can be exacerbated because of the CEOs' opportunities to transform their original contracts. But, if firms anticipate the contract transformation choices their CEOs make when they are allowed to hedge, these extra agency problems sometimes can be more than offset to the extent that investment banks (IBs) can construct compensation variance-reducing hedging instruments that the firms and their CEOs could not construct on their own. Exactly when and how such improvements are possible is the main focus of this article.

Being aware that CEOs can and do transform their compensation contracts by hedging has implications not only for firms' market values, but also for the proper interpretation of virtually any CEO compensation data, hypothesis, or claim, because it forces one to distinguish between the pay packages firms *give* their executives and the (net) pay packages the executives actually *receive*. In particular, this distinction between pay packages given and pay packages received forces one to recognize that *stated* pay-for-performance sensitivities (PFPS) of executives' compensation contracts can exceed their *effective* PFPS once CEOs' hedging is taken into account. We show that an unwary outsider (researcher, journalist, etc.,) can be further misled in their perceptions of a firm's CEO's contract's effective PFPS when the firm employing the CEO correctly anticipates the CEO's propensity to hedge his compensation risk, because such a firm will "boost" its CEO's contract's stated PFPS in anticipation that the CEO's subsequent hedging will "knock down" the contract's effective PFPS through his transactions with IBs.

We also study how a CEO's demand for hedges varies with the hedging instrument. We consider two distinct types of hedges, a "fixed-for-variable" swap ("swap") in which a CEO receives a fixed payment from an IB in return for giving the IB a fraction of the variable portion of the CEO's compensation, and what we call a "performance hedge" (PH), a financial instrument whose cash flows have nonzero covariance with the performance measure(s) underlying the variable portion of the CEO's compensation. We produce an array of comparative statics related to CEOs' demand curves for these two hedging instruments and firms' optimal contract design in anticipation of such CEOs' demand for hedging instruments.

While there is a vast literature studying the incentives firms have to hedge at the corporate level (e.g., Froot, Scharfstein and Stein, 1993 and 1994), our article is not closely connected to that literature, since we are concerned with the effects of hedging on the design of CEO compensation contracts and, hence, are interested in the incentives of managers to hedge at the personal level. Much of the analytical literature on hedging in economics and finance has emphasized financial investment issues rather than the contracting and PFPS issues that are central to this article.[6] Allowing CEOs to hedge some of the risk in their compensation contracts by hedging through investment banks has some similarities to allowing executives to unwind the equity stakes in their firms. Just as we observe that allowing compensation hedging can dampen the incentive effects of CEO's contracts with their firms, Bar-Gill and Bebchuk (2003) and Bebchuk and Fried (2004) (among others) have observed that allowing equity unwinding can have negative incentive effects. We note however that, at least over short horizons, compensation hedging may be allowed while unwinding equity stakes is prohibited, since firms typically have vesting provisions for their equity grants that specifically prohibit their CEOs from selling the shares granted to them during the vesting period at the same time they may not forbid their CEOs from transacting with IBs. Laux (2010) has demonstrated that granting a CEO this opportunity can be optimal because it makes the CEOs' ex post incentives for project abandonment align more closely with the firm's shareholders's incentives. In contrast, in our model, the benefit of compensation hedging comes through the improved risk-sharing afforded by transacting with IBs. Since, as we show, the amount and kind of hedging a CEO does is affected by his original contract's PFPS, this article is also related to Baker, Gibbons, and Murphy [1994] who showed that, when subjective performance measures are available, it may be necessary to adjust the PFPS placed on objective performance measures to prevent the objective measures from "crowding out" the use of subjective measures. None of the extant literature we are familiar with has studied how a manager's ability to reconfigure the contract his firm offers him by engaging in transactions with IBs to hedge his compensation risk affects either the general design of the manager's contract or the manager's firm's

[6] As examples: Demarzo and Duffie [1995] and Sapra [2002] establish how the disclosure or non-disclosure of hedging information affects the incentives of firms to hedge; and Jorgensen [1999] and Melumad, Weyns, and Ziv [1999] study hedge accounting's effects on the hedging firm's information acquisition decisions or preferred hedge positions.

value.[7]

This article is part of the large literature on induced moral hazard and multi-task agency problems, as studied by, among others, Holmstrom and Ricarti-Costa [1986], Holmstrom and Milgrom [1991], Feltham and Xie [1994], Datar, Kulp and Lambert [2001], and Demski and Sappington [1987], since when a CEO transforms his original contract into a revised contract by transacting with IBs, his ultimate goal is to affect both what personally costly effort choice he eventually adopts and what consumption he eventually receives. The article is also related to the literatures on relative performance evaluation (see, e.g., Lazear and Rosen [1981], Antle and Smith [1986]) and earnings management (see, e.g., Dye [1988], Demski [1998], Arya, Glover, and Sunder [1998]). Relative performance evaluation (RPE) entails having a CEO's compensation tied to an index of the performance of other firms in the same industry in which the CEO's firm operates or having his firm's performance judged against some market index. This is another implicit means of hedging the CEO's compensation risk because it factors out noise in the performance measure common to firms in an industry or common to firms exposed to the same market factors. Also, the ability of a CEO to transform his contract into effectively a different contract is related to the ability of a CEO to engage in earnings management or, more generally, to alter the distribution of the measure used to judge the CEO's performance. In both cases, when CEOs engage in the activity, the compensation they receive is altered from the original intentions of the firm that employs the CEO.[8]

We conclude this section by outlining how the article proceeds and also by summarizing the article's principal results section by section. In Section 2, we introduce our base model, which posits that the firm and its CEO have common information concerning the effectiveness of the hedging instruments offered by the IB in reducing the compensation risk the CEO is subject to. Section 3 contains some preliminary results designed to develop the reader's intuition about the effects of an IB offering to sell a firm's CEO a hedging instrument in a simple setting where the IB offers only one type of hedging instrument - either just swaps or just PH - for sale. Section 4 expands upon the analysis of Section 3 by examining the CEO's demand for hedging instruments when the IB offers both swaps and PH for sale. In that section, we show that: swaps and PH are always substitutes, the CEO's demand

[7] But, as a prelude to his empirical analysis, Gao [2010] developed a simple agency model that also demonstrates that a firm which anticipates that its CEO will go to an IB to off-load some of his compensation risk will boost the PFPS of the CEO's contract. However, his analysis is much less focused on the contract transformation process and the interaction between IBs and CEOs than our analysis. As examples, his analysis is silent on the issue of how the prices IBs charge for the hedges affects CEOs' demands for the hedges, as well as how the availability of multiple hedging instruments from IBs interact to affect CEOs' demand for hedges.

[8] Of course, a fundamental difference between earnings management and hedging compensation risk is that, over time, the opportunistic discretionary accruals CEOs have their firms "book" under earnings management eventually have to be reversed, whereas there need be no corresponding reversals associated with the contract transformation effects of hedging risk through investment banks.

for both hedging instruments increases in the CEO's contract's PFPS, and the CEO's demand for PH increases in each of: the PH's effectiveness (measured by the correlation between the hedge's payoffs and the CEO's compensation), the marginal productivity of the CEO's effort, and a measure of the CEO's effort aversion, whereas the CEO's demand for swaps decreases in both of these last two variables (the marginal productivity of his effort and the measure of effort aversion), but increases in the variance of the performance measure underlying the CEO's compensation.

In Section 5, we study comparative statics, i.e., how the optimal design of the CEO's PFPS and how the maximal value of the firm changes with exogenous parameters of the model. We show that provided the prices the IB charges for both of the hedging instruments are not too large, the optimal contract's PFPS strictly increases in the price of the PH and strictly decreases in the price of the swap, whereas the firm's maximal value moves in the opposite direction: the maximal value of the firm strictly decreases in the price of the PH and strictly increases in the price of the swap.

In section 6, we determine which of: allowing the CEO to transact directly with the IB or prohibiting the CEO from transacting with the IB but having the firm itself transact with the IB yields maximum firm value. We show that if the IB charges actuarially fair prices for both hedges, maximal firm value is the same regardless of whether the firm or its CEO transacts with the IB, but if the IB charges above actuarially fair prices for at least one of the two hedges, firm value is higher by prohibiting the CEO from transacting with the IB and instead transacting itself with the IB. We also show that, regardless of the PH's effectiveness, if the price of the PH is sufficiently high and the price of the swap is any amount above its actuarially fair price, then firm value is highest when neither the CEO nor the firm itself transacts with the IB. We also show that when the CEO is allowed to hedge through the IB, the stated (resp., effective - i.e., after taking into account the effects of transacting with the IB) value of the optimal contract's PFPS always weakly exceeds (resp., is always weakly less than) the PFPS of the optimal contract were no IB available for hedging.

In Section 7 we study an extension of the base model where the CEO has better knowledge about the effectiveness of PH hedges than does the firm; there, we identify conditions under which the firm may be better off or worse off delegating the decision to transact with the IB to the CEO. We also find conditions under which the firm is better off prohibiting the CEO from transacting with the IB at the same time the firm finds it undesirable to hedge the CEO's compensation risk on its own.

Section 8 concludes the article. The Appendix contains proofs of most results stated in the text, with omitted proofs available from the authors.

2 Base Model Setup

We study two principal classes of compensation hedges: swaps and PHs.[9] In a swap, a risk-neutral IB offers to pay the CEO a fixed amount in return for receiving an (endogenously determined) fraction of the variable portion of the CEO's compensation. In a PH, an IB supplies the CEO with a financial instrument whose cash flows we posit negatively covary with the performance measure used to determine the variable portion of the CEO's compensation.[10] Without loss of generality, we presume the expected present value of the cash flows generated per unit of the PH is zero. These two instruments cover the principal means by which IBs can assist CEOs in reducing the risk in their compensation contracts: either an IB can reduce the risk in a CEO's compensation contract directly by absorbing some of that risk (in the form of the fixed-for-variable swap) or indirectly by helping the CEO hedge the variability in the performance measure underlying the CEO's compensation (in the form of the PH). These two risk reduction methods are broadly consistent with, and span a significant portion of the space of, hedging instruments commonly used in practice under names such as variable prepaid forward contracts, costless collars, etc.[11]

In the following analysis, we take the price $\$q$ IBs charge for each unit of the PH, along with the "price"/haircut l IBs charge for the swap (as described further below), as exogenously given. We do not assume that these prices are actuarially fair to allow for the possibility that IBs have market power.[12]

[9] We do not study "real" hedges, i.e., those that reduce the variability of the firm's performance directly, such as locating some operations overseas as a means of hedging foreign exchange risk or exploiting the diversification advantages of conglomerates. Though such real hedges are potentially important alternative risk management devices, we do not study them here for three reasons: first, unlike financial hedges, real hedges do not appear to be the subject of outrage in Congress or concern in the financial press; second, to shorten this article; and third, we study "real" hedges in a separate working paper.

[10] Obviously, whether the covariance between the PH and the performance measure is positive or negative is irrelevant to the economic value of the PH and, hence, to our analysis. We choose the covariance to be negative, so that hedging the performance measure will entail having the CEO going "long" the hedge.

It is useful at this point also to address a question initially raised by a referee as to why an IB can construct hedges that an operating firm cannot construct on its own. On a corporate level, it is well known that investment banks market and sell hedges that are specifically tailored to their clients' needs. For example, consider the following excerpt of an advertisement by the Australian investment bank ANZ: "*ANZ Investment Bank is a market leader in the Australian and international Capital Markets and has the capacity to tailor specific solutions to the needs of Natural Resources clients by providing a comprehensive range of products and services such as: investigating the hedging needs of your company; the ability to model proposed hedging outcomes based on ANZ Investment Bank's proprietary hedging models; designing interest rate solutions that accommodate your risk profile and corporate strategy.*" As the above excerpt asserts, these hedging services often exploit proprietary models that no operating company can easily duplicate. Moreover, even if an operating company could duplicate some of the hedging services offered by an IB, the operating company inevitably has a cost disadvantage relative to an IB in doing so, since not only do IBs employ people specifically to provide such functions, IBs can sell the same or similar services to multiple clients, and so, exploit economies of scale that an individual operating company cannot. Related, several empirical papers in the literature document that IBs provide tailored hedges for executive compensation. See, e.g., Bettis, Bizjak, and Lemmon [2001] and Bettis, Bizjak, and Kalpathy [2010].

[11] See, e.g, Jagolinzer, Matsunaga, and Yeung [2007], IRS [2007], or Sasseen [BloombergBusinessweek, 2010].

[12] There is concern about IB's market power: the Financial Conduct Authority (FCA) in the United Kingdom initiated a study in July 2014 to investigate the competitiveness of the investment banking market. Details are available at: http://www.osborneclarke.com/connected-insights/blog/fca-review-competition-investment-

The assumed sequence of events in the model is as follows. First, the firm gives the CEO a contract. The contract must meet the CEO's opportunity cost of working for the firm, i.e., satisfy the CEO's individual rationality (IR) constraint. Second, the CEO decides how much of the PH to buy and what size swap to engage in with the IB.[13] When the CEO decides to buy either hedge, he pays for the hedge. Third, the CEO privately selects his (operating) effort choice. Finally, all random variables realize their values, the CEO is paid, and the IB and the CEO settle up with each other. The sequence of events is detailed in Figure 1 below after more features of the model are specified.

The random variable $\tilde{v}$ denotes the performance measure the firm uses to compensate the CEO. $\tilde{v}$ could be the firm's operating cash flows, net earnings, etc. The random variable $\tilde{y}$ denotes the cash flows per unit of the PH the IB offers to sell the CEO. We take the compensation contract the firm gives the CEO to be linear, i.e., $s_p(\tilde{v}) = c_p + b_p\tilde{v}$, where c_p is the contract's base salary and b_p is the contract's PFPS.[14] We assume this contract is observable to the IB. If the CEO subsequently "lays off" or swaps the quantity $\beta\tilde{v}$ of the variable portion of his compensation contract in return for receiving the fixed payment $c_{IB} = c_{IB}(\beta|b_p)$ from the IB (how c_{IB} is determined is described below) and, at the same time, the CEO purchases $\hat{\theta}$ units of the PH, then the CEO's net payoff when $\tilde{v} = v$ and $\tilde{y} = y$ is given by

$$T(v, y|c_{IB}, \beta, \hat{\theta}, s_p) \equiv c_{IB} + (b_p - \beta)v + c_p + \hat{\theta}y - \hat{\theta}q. \tag{1}$$

$T(v, y|c_{IB}, s_p, \beta, \hat{\theta})$ is the CEO's transformed contract: the PFPS of the original contract reduces from b_p to $b_p - \beta$; the base pay increases from c_p to $c_p + c_{IB}$; and the set of random variables affecting the CEO's compensation expands from $\tilde{v}$ to $\tilde{v}$ and $\tilde{y}$. We refer to

$$b_p - \beta \tag{2}$$

(or sometimes the realized variable pay, $(b_p - \beta)v$, depending on context) as the CEO's *retained residual interest* (RRI) in the variable portion of the firm's original contract.

Given this background information, we can describe the primary questions we seek to address as: first, for a given contract $s_p(v)$ the firm offers the CEO, how does the CEO optimally transform this

banking-and-corporate-banking. Also, researchers have documented that profits accrue to banks due to vendor lock-in effects: once an investment bank establishes a relationship with a client, it exploits that relationship in setting fees for other services it offers to that client. See, e.g., Schenone [2010] and Bharath, Dahiya, Saunders, and Srinivasan [2007]. Finally, there is evidence that IBs charge customized fees for the services they offer (see, e.g., http://www.crossbordermanagement.com/en/guides/mergers-a-acquisitions-in-the-us/investment-bankers/investment-bankers-fees).

[13] Implicitly, we assume in the following that the CEO cannot precommit not to transact with the IB. Later in the paper, when we consider the possibility that the firm transacts directly with the IB rather than letting the CEO transact with the IB, we further assume that the firm can preclude the CEO from directly transacting with the IB if it so chooses.

[14] Bushman and Indjejikian [1993], Datar, Lambert, and Kulp [2001], and Lambert [2001] among others, have shown how the use of linear contracts can help generate a variety of useful predictions.

contract into the revised contract $T(v, y|c_{IB}, \beta, \hat{\theta}, s_p)$; second, and related, how do the CEO's optimal demands for the two risk-reducing instruments, the PH and the swap, vary with the contract the firm gives the CEO; third, how does the firm's anticipation of the CEO's subsequent transactions with the IB affect the firm's initial design of the CEO's contract; and finally, when is the expected value of the firm higher if the firm allows the CEO to transact with the IB than if the firm prohibits the CEO from transacting with the IB?[15]

Before continuing, we note some differences between adding the PH $\tilde{y}$ as a contracting variable derived from the CEO transacting with the IB and incorporating another variable into a contract in a standard principal-agent (P-A) contracting relationship. In the standard P-A relationship, the desirability of incorporating another variable into the contract is determined by Holmstrom's [1979] "informativeness criterion." The conditions under which the agency relationship improves here by allowing the CEO to transact with the IB should not be expected to coincide with the "informativeness criterion" condition because the "informativeness criterion" determines when the agency relationship improves by including an extra variable in the contract only when that variable: (a) is observable to both the principal and agent, (b) is contractible, (c) is incorporated into the contract at the time the contract is written, and (d) is costless to acquire and incorporate into the contract. In contrast, in the present analysis, we study when the expected value of a firm increases by allowing its manager to acquire a hedge from an IB under circumstances where the hedge: (a) may not be observable to the firm; (b) is not contracted upon by the firm; (c) is obtained after the contract between the CEO and the firm is written; and (d) is acquired from the IB for a positive fee.[16]

[15] We wish to thank an anonymous referee for the observation that the CEO can incur risk not simply due to fluctuations in the value of the CEO's compensation, but also due to fluctuations in the value of his portfolio of assets, which could include holdings in the firm he works for. While we do not incorporate the effects of fluctuations in the value of the CEO's portfolio in our formal analysis, our analysis could be extended to incorporate such effects. In this footnote, we briefly sketch what we believe to be the major effect of introducing these random wealth effects into our analysis. Let $\tilde{W}$ denote the random variable describing the CEO's end-of-period wealth not deriving from the CEO's compensation, so that if the CEO were given the compensation contract $c_p + b_p\tilde{v}$, then the CEO's total end of period wealth is $\tilde{W} + c_p + b_p\tilde{v}$. Assume that $\tilde{W}$ and $\tilde{v}$ are jointly normally distributed. Then, as is well known, the conditional expectation $E[\tilde{W}|v]$ is linear and given by, say, $c + bv$, and the residual $\tilde{\varepsilon} \equiv \tilde{W} - E[\tilde{W}|\tilde{v}]$ is independent of $\tilde{v}$. Let σ_ε^2 denote the variance of $\tilde{\varepsilon}$. It follows that the variance in the CEO's end of period wealth is given by: $var(b_p\tilde{v} + \tilde{W}) = var(b_p\tilde{v} + b\tilde{v} + \tilde{\varepsilon}) = (b_p + b)^2\sigma_v^2 + \sigma_\varepsilon^2$, and so, if the CEO selected effort a and did not transact with the IB, then his certainty equivalent would be given by: $E[\tilde{W}] + c_p + c + (b_p + b)wa - .5a^2/k - .5r((b_p + b)^2\sigma_v^2 + \sigma_\varepsilon^2)$. It is clear, upon comparing this expression for the CEO's certainty equivalent with the corresponding expression in the text, that the main consequence of introducing the CEO's non-compensation related wealth into the analysis is to increase the contract's effective PFPS by the amount b. While introducing this exogenous increase in the contract's PFPS will affect some of our computations, we believe it is unlikely that the introduction of this extra term fundamentally alters our analysis.

[16] Another significant difference between our analysis and Holmstrom [1979] is that the "informativeness criterion" determines when to include an additional variable in a contract only when the principal can choose publicly and optimally how much weight to place on the extra variable in the contract. The "informativeness criterion" says nothing about the desirability of including an extra variable in a contract when the weight assigned to that variable in the contract is some quantity other than the optimal weight derived in Holmstrom's [1979] analysis, When a CEO is deciding what quantity of a PH to purchase from an IB, he is not constrained in his choice of the quantity of the PH to purchase, and there is nothing to suggest that his preferred quantity will match up with the firm's optimal weighting of that hedging

We next describe the distributions of the random variables $\tilde{v}$ and $\tilde{y}$, and also the terms of the swap, i.e., the relationship between the fixed payment c_{IB} and the stochastic quantity $\beta\tilde{v}$ of variable portion of the CEO's compensation the CEO transfers to the IB. We suppose that if the CEO takes unobservable effort $a \geq 0$, then the distribution of performance measure $\tilde{v}$ has mean wa, where $w > 0$ is a constant representing the CEO's marginal productivity of effort, and variance σ_v^2. As noted above, we take the mean $E[\tilde{y}]$ of the PH to be zero and the covariance $\sigma_{vy} = cov(\tilde{y}, \tilde{v})$ to be negative (and hence the associated correlation ρ also to be negative). We denote the variance of $\tilde{y}$ by σ_y^2. Initially, we take these characteristics of the distributions of these random variables to be common knowledge. In Section 7 below, we consider an extension where the CEO has better information about the effectiveness of the PH $\tilde{y}$ in hedging $\tilde{v}$ than does the firm, i.e., better information about the covariance σ_{vy}.

As was mentioned above, we suppose that the IB imposes a "haircut" on the CEO in executing the fixed-for-variable swap. That is, we assume there is a constant fraction $l \in [0, 1)$ such that the fixed payment $c_{IB} = c_{IB}(\beta|b_p)$ the IB gives the CEO in return for receiving the random payment $\beta\tilde{v}$ is only $1 - l$ times the expected value of $\beta\tilde{v}$. We also suppose there are constants $k > 0$ and $r > 0$ such that if the CEO consumes the random quantity $\tilde{t}$ after selecting effort $a \geq 0$, then the CEO's certainty equivalent is given by $CE(\tilde{t}, a) \equiv E[\tilde{t}|a] - .5rVar(\tilde{t}) - .5a^2/k$. That is, the CEO has preferences that are additively separable in consumption and effort, the CEO has linear mean-variance preferences for consumption, and the CEO's disutility from exerting effort a is given by $.5a^2/k$. Thus, the higher r, the more risk-averse the CEO, and the higher k, the less effort-averse the CEO. This is consistent with preferences as employed in the so-called LEN contracting framework (see, e.g., Lambert [2001]). It follows that the CEO's certainty equivalent from exerting effort a and consuming $\tilde{T} = T(\tilde{v}, \tilde{y}|c_{IB}, \beta, \hat{\theta}, s_p)$, where $T(\tilde{v}, \tilde{y}|c_{IB}, \beta, \hat{\theta}, s_p)$ is as defined in (1) above, is given by:

$$CE(\tilde{T}, a) = c_{IB} + c_p + (b_p - \beta)wa - \hat{\theta}q - .5rVar((b_p - \beta)\tilde{v} + \hat{\theta}\tilde{y}) - .5a^2/k \qquad (3)$$

(the preceding uses the facts that $E[\tilde{v}|a] = wa$ and $E[\tilde{y}] = 0$). Consequently, given the transformed contract $T(\tilde{v}, \tilde{y}|c_{IB}, \beta, \hat{\theta}, s_p)$, it is clear that the CEO's preferred action choice a, denoted $a^*(b_p, \beta)$, is given by

$$a^*(b_p, \beta) \equiv (b_p - \beta)wk. \qquad (4)$$

The CEO's optimal effort is increasing in each of: his RRI (recall (2)) in the original compensation contract, the marginal productivity of his effort, and the extent to which he is not effort averse. It

variable.

follows from (4) that the CEO's certainty equivalent (3) evaluated at this optimal action is given by

$$CE^*(\tilde{T}) \equiv c_{IB} + c_p + .5(b_p - \beta)^2 w^2 k - \hat{\theta} q - .5rVar((b_p - \beta)\tilde{v} + \hat{\theta}\tilde{y}). \tag{5}$$

(4) implies that the expected value of the variable portion of the fixed-for-variable swap $\beta\tilde{v}$ is given by $\beta E[\tilde{v}|a^*(b_p, \beta)] = \beta(b_p - \beta)w^2 k$, and so, given the haircut l, the relationship between the variable portion $\beta\tilde{v}$ of the contract transferred to the IB and the fixed payment c_{IB} received by the CEO in the fixed-for-variable swap is given by:[17]

$$c_{IB}(\beta|b_p) = (1 - l)\beta E[\tilde{v}|a^*(b_p, \beta)] = (1 - l)\beta(b_p - \beta)w^2 k. \tag{6}$$

The analysis proceeds by maintaining the following bounds on the haircut l:

$$l \in [0, \min\{1, \frac{1}{2} + \frac{r\sigma_v^2(1 - \rho^2)}{2w^2 k}, \frac{r\sigma_v^2(1 - \rho^2)}{w^2 k}\}). \tag{7}$$

The requirement that the haircut l be less that 1 needs no explanation. The requirement that $l < \frac{1}{2} + \frac{r\sigma_v^2(1-\rho^2)}{2w^2k}$, which is equivalent to the requirement that $(1 - 2l)w^2 k + r\sigma_v^2(1 - \rho^2) > 0$, is needed to ensure satisfaction of the second-order condition for the problem of maximizing the CEO's certainty equivalent with respect to β.[18] The condition that $l < \frac{r\sigma_v^2(1-\rho^2)}{w^2k}$ is necessary to ensure that the CEO's interaction with the IB is sensible, in so far as the CEO uses the IB to off-load compensation risk, rather than to acquire additional compensation risk, by having access to the swap.[19]

The sequence of events is summarized in Figure 1.[20]

The firm offers the CEO the contract $s_p(\tilde{v}) = c_p + b_p\tilde{v}$.	The CEO buys $\hat{\theta}$ units of the PH $\tilde{y}$ and swaps fraction β/b_p of the variable portion of his contract in return for receiving the fixed amount c_{IB}.	The CEO privately takes action a.	The realized value of the firm's cash flows occur, and all contracts are settled. The CEO consumes.

Figure 1 : Time line

A foresightful firm who anticipates the CEO's optimally transacting with the IB will solve the following program. In this program, the quantity $\beta^*(l|b_p, q)$ denotes the CEO's demand for the swap

[17] Even though the IB does not observe the CEO's action choice, the IB can infer the CEO's optimal action choice $a^*(b_p, \beta)$, since: 1. the firm's original contract is observable to the IB (in particular, the IB sees b_p); 2. as a party to the swap, the IB sees β; and 3. since the IB knows the entire information set of the CEO's decision problem, the IB can compute $a^*(b_p, \beta)$.

[18] See the appendix at (42) and notice that the coefficient of β in RHS(42), which can be rewritten as $(1 - 2l)w^2 k + r\sigma_v^2(1 - \rho^2)$, equals the *negative* of the second-order condition for a maximum for optimizing the CEO's certainty equivalent.

[19] See (43) in the appendix, and note that $l < \frac{r\sigma_v^2(1-\rho^2)}{w^2k}$ is necessary for $\beta_+^*(b_p|q, l)$ to be positive when $q = 0$.

[20] The order in which the CEO purchases the PH and swap from the IB is irrelevant as long as both hedges are purchased before the CEO selects his operating action choice.

given the haircut l (which is the "price" of the swap), the PFPS b_p of the original contract, and the price q per unit of the PH. The quantity $\hat{\theta}^*(q|b_p, l)$ denotes the CEO's demand for the PH as a function of these same three variables.

Program 1 (Optimal contracting when the CEO transacts directly with the IB)

$$\max_{s_p(\tilde{v})=c_p+b_p\tilde{v}, a^*(b_p,\beta),\ \beta^*(l|b_p,q),\ \hat{\theta}^*(q|b_p,l)} E[\tilde{v} - s_p(\tilde{v})|a^*(b_p, \beta^*(l|b_p, q))]$$

subject to

$$\text{for each } b_p \text{ and } \beta,\ a^*(b_p, \beta) \text{ is as in (4) above;} \tag{8}$$

$$\text{for each } b_p \text{ and } \beta,\ c_{IB}(\beta|b_p) \text{ is as in (6) above;} \tag{9}$$

$$\beta^*(l|b_p, q),\ \hat{\theta}^*(q|b_p, l) \in \arg \max_{b_p \geq \beta \geq 0,\ \hat{\theta} \geq 0} CE(T(\tilde{v}, \tilde{y}|c_{IB}(\beta|b_p), \beta, \hat{\theta}, s_p), a^*(b_p, \beta)) \tag{10}$$

$$CE(T(\tilde{v}, \tilde{y}|c_{IB}(\beta^*(l|b_p, q)), \beta^*(l|b_p, q), \hat{\theta}^*(q|b_p, l), s_p), a^*(b_p, \beta^*(l|b_p, q)) \geq \bar{U} \tag{11}$$

Constraint (8) ensures that the CEO's optimal action choice is determined correctly, given whatever RRI the CEO keeps in the original contract. Constraint (9) ensures that the terms of the swap are defined consistently with the IB charging the CEO a haircut of l. Constraint (10) ensures that the two functions $\beta^*(l|b_p, q)$ and $\hat{\theta}^*(q|b_p, l)$ represent the CEO's demand curves for the swap and for the PH, respectively. We impose the restriction $\hat{\theta} \geq 0$ since we (reasonably) posit that the CEO cannot be a source or supplier of the PH,[21] and we impose the restrictions $b_p \geq \beta$ and $\beta \geq 0$, since (a) a nonnegative "haircut" only makes sense for $\beta \geq 0$[22] and (b) we also (reasonably) suppose that the CEO cannot be a source or supplier for swaps. Constraint (11) is the CEO's individual rationality constraint when the CEO's opportunity cost of working for the firm is $\bar{U}$. In the following, without loss of generality, we set $\bar{U} = 0$.

[21] Expanding on this last point, it is theoretically possible (absent this constraint) for $\hat{\theta} < 0$ to occur, in which case the CEO would be using the PH not to hedge but to increase the variance in his compensation contract. We do not study this possibility for two reasons: first, because, as far as we know, this theoretical possibility never arises in practice. Second, since we want to reflect the IB's potential market power by letting the IB charge the CEO a price $\$q$/unit of the PH that is potentially above its actuarially fair price (zero), no market-power exploiting IB would agree to **pay** the CEO $\$q$/unit if the CEO could somehow synthesize the PH on his own and the CEO sought to sell the synthesized PH to the IB. (If such an odd situation were somehow to arise, the IB - in exercising his market power - would agree to purchase PH from the CEO only by buying the PH at a less than actuarially fair price (i.e., the IB would insist on getting paid to receive PHs synthesized by the CEO).) Thus, to properly reflect the IB's potential market power, the price of $\$q$/unit of the PH is applicable only when the CEO's demand for the PH is nonnegative.

[22] Expanding on this point, we assert that *positive* haircuts (i.e., $l > 0$) make economic sense only when β is nonnegative. No IB would agree to pay the CEO a positive haircut when $\beta < 0$, as a positive haircut when $\beta < 0$ would imply that the IB receives a *lower* payment $-c_{IB}$ (>0) than he would with a zero haircut, as:

$$0 < -c_{IB} = -(1 - l)(b_p - \beta)\beta w^2 k < -(b_p - \beta)\beta w^2 k.$$

For economically sensible interpretations of haircuts in the following, we confine attention to those parameter values for which the optimal β is nonnegative.

Notice that while Program 1 is written assuming the IB can sell the CEO *both* PH and swaps, this program can be easily modified to cover the special cases where the IB is limited to selling the CEO just swaps (by adding the additional constraint $\hat{\theta} \equiv 0$) or just performance hedges (by adding the constraint $\beta^* \equiv 0$). We exploit this observation when stating and deriving Theorems 1 and 2 in the next section.

Before concluding this section, we note that throughout the rest of the article, we maintain the assumption that $w^2k - r\sigma_v^2 > 0$. We refer to this as the "typical" situation. The typical situation holds whenever a CEO is better off if the bonus coefficient b_p of the contract $s_p(v) = c_p + b_p v$ is increased, while holding the contract's base salary c_p fixed, in the standard contracting setting where no IB is present and no hedging possibilities exist.[23]

3 When the IB offers only one type of hedge for sale

Before describing the solution to the general problem in Program 1 where the IB offers to sell both hedging devices to the CEO, in this section we consider each of the two special cases where the IB offers only one type of hedge, either just swaps or just performance hedges. We do this as a way of developing intuition for the solution to the general problem. In the first theorem below, we consider the situation where the IB offers to sell only swaps to the CEO.[24]

Theorem 1 *When the CEO is allowed to transact with the IB, but the IB only offers to sell swaps, then:*

(a) if the firm gives the CEO a contract with stated PFPS $b_p > 0$, *then the optimal variable portion of the contract the CEO sells the IB is:* $\beta^*(l|b_p) \equiv b_p \times \frac{r\sigma_v^2 - lw^2k}{(1-2l)w^2k + r\sigma_v^2}$ *and the CEO's optimal retained residual interest in this contract is:* $b_p - \beta^*(l|b_p) = b_p \times \frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2}$;

(b) the optimal contract's stated PFPS is: $b_p^*(l) = \frac{1}{1+l}$;

(c) the effective PFPS of the optimal contract is: $b_p^* \times \frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2} = \frac{1}{1+l}\frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2}$; *and*

(d) the value of the firm evaluated at the optimal contract is: $V_A(l) \equiv \frac{1}{2}\left(\frac{(w^2k)^2(1-l)}{(1+l)((1-2l)w^2k + r\sigma_v^2)}\right)$.

Regarding part (a), notice that for all l in the interval (7), $\frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2} < 1$. So, if the CEO is given a contract with stated PFPS $b_p > 0$, then part (a) shows that the net effect of allowing the CEO to transact with the IB is to "knock down" the PFPS of the contract to $b_p \times \frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2} < b_p$. This

[23] To see this, observe that the CEO when given this contract will choose action $a = b_p wk$, and hence, will obtain certainty equivalent from the contract of $CE = E[c_p + b_p\tilde{v}|a] - .5a^2/k - .5r\sigma_v^2 b_p^2 = c_p + b_p^2 w^2 k - .5b_p^2 w^2 k - .5r\sigma_v^2 b_p^2 = c_p + .5b_p^2(w^2k - r\sigma_v^2)$. Hence, $\frac{\partial CE}{\partial b_p} > 0$ if and only if $w^2k - r\sigma_v^2 > 0$.

[24] We do not present the proof of Theorem 1 in the appendix as a self-contained result, since the solution to Theorem 1 is a special case of the general solution to Program 1 when the price q for the PH is set above the CEO's reservation price.

shows the incentive compromising effects of allowing the CEO to transact with the IB, and hence that some of the concerns voiced by senators and others cited in the Introduction regarding hedging by CEOs are at least sometimes justified.

Perhaps somewhat unexpectedly, part (a) shows that this effect occurs *regardless of* the size of the initial PFPS of the firm's original contract, as long as that PFPS is positive. That is, the CEO *always* knocks down the incentive effects of the original contract.[25] This last result is in fact extremely general;[26] it is a consequence of the observation that any risk-averse person subject to moral hazard in any insurance-related setting will buy a positive quantity of insurance provided the insurance is "not too actuarially unfair;" in our setting, "not too actuarially unfair" entails that the haircut l belong to the interval (7).[27] An additional implication of part (a) is that were an unforesightful firm to fail to anticipate that the CEO will transact with the IB, then the CEO will work less hard than what the firm expected him to (as can be seen by observing that (4) is increasing in the CEO's RRI in the original contract).

Part (b) reports that the optimal value of the *stated* PFPS $b_p^*(l)$ the firm gives the CEO is specified by $b_p^*(l) = \frac{1}{1+l}$.[28] The optimal stated PFPS is always decreasing in the haircut l. This is intuitive because the firm ultimately bears the cost of the haircut, in so far as the firm has to compensate the CEO well enough so that, inclusive of the haircut the CEO pays the IB, the CEO gets high enough expected utility so as to be willing to work for the firm. As the haircut increases, the firm reduces the PFPS he offers the CEO so as to economize on these costs.

The case where the IB charges the CEO no haircut ($l = 0$) is of special interest. In that case, in view of part (b), $b_p^* = 1$. If the CEO did not subsequently transact with the IB, i.e., if we

[25] Part (a) follows directly from (16) below, specialized to the case where $q \geq q^{res}(b_p|l)$.

[26] It requires only that the CEO be risk averse and that the contract $s_p(v)$ be strictly increasing; it does not require any assumptions such as CARA preferences, linear contracts, or normal distributions. It also holds when the IB is posited to offer both PH and swaps for sale. (The proof of this general claim is omitted from the text, but available from the authors.)

[27] To see this, substitute (6) into (5), and evaluate the result at $q = \hat{\theta} = 0$ (which is appropriate presently where the IB is presumed to only offers swaps for sale), to find the CEO's certainty equivalent in the context of Theorem 1 as equal to:

$$c_p + (1-l)\beta(b_p - \beta)w^2k + .5(b_p - \beta)^2(w^2k - .5r\sigma_v^2).$$

The derivative of this certainty equivalent with respect to β is in general given by:

$$(1-l)(b_p - \beta)w^2k - (1-l)\beta w^2k - (b_p - \beta)(w^2k - .5r\sigma_v^2).$$

Evaluating this derivative at $\beta = 0$ we obtain

$$b_p((1-l)w^2k - (w^2k - .5r\sigma_v^2)).$$

Thus, for all $b_p > 0$, we see that this derivative is positive, i.e., the CEO has an incentive to purchase a positive amount of the swap from the IB, provided $(1-l)w^2k - (w^2k - .5r\sigma_v^2) = -lw^2k + .5r\sigma_v^2 > 0$. This last inequality holds for all haircuts l less than $\frac{r\sigma_v^2}{w^2k}$, and in particular the inequality holds for all l in (7).

[28] Part (b) follows from (22) below, specialized to the case where $q \geq q^{*res}(l)$. Part (c) follows by combining this last result with part (a).

were considering a standard principal-agent contracting problem where the CEO was restricted to consuming what the firm paid him, then a contract with $b_p^* = 1$ would be interpreted as "selling (or renting) the entire firm to the CEO" for the price of the (negative of) base salary $-c_p$. When the firm is risk-neutral and the CEO is risk averse, such a contract is undesirable, because it imposes too much risk on the CEO. It is easy to check that the solution to the standard contracting problem in this case (when attention is paid to linear contracts) is to give the CEO a contract of the form $c_p + b_p v$, where

$$b_p = \frac{w^2 k}{w^2 k + r\sigma_v^2}. \tag{12}$$

This contract optimally trades off the incentive effects of CEO's compensation against both the risk the contract imposes on the CEO's consumption and the CEO's cost in exerting effort. Notice that in the contracting problem we are presently concerned with (where the CEO can lay off some or all of the risk in his compensation contract by transacting with the IB) when $l = 0$, the firm sets the CEO's contract at $b_p^* = 1$, so it follows that, as part (c) reports, the CEO's effective PFPS is given by $b_p^* \times \frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2} = 1 \times \frac{w^2k}{w^2k + r\sigma_v^2}$; hence, the CEO's effective PFPS is identical to the PFPS of the contract the CEO would have been given had no IB, or no transactions with the IB, been available, as we reported in (12).

More generally, part (c) shows, by combining the results of parts (a) and (b), that the optimal effective PFPS is the stated PFPS of the optimal contract times the optimal fraction of the stated PFPS the CEO retains, i.e., is $\frac{1}{1+l} \frac{(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2}$. From this last expression, it is easy to verify that regardless of the size of the haircut l (subject only to the haircut belonging to the interval (7)), that the CEO's effective PFPS associated with the optimal contract is always (weakly) below the effective PFPS of the optimal contract reported in (12) had the CEO been forbidden from transacting with the IB.

Part (d) reports that the firm's value is given by $V_A(l) \equiv \frac{1}{2}\left(\frac{(w^2k)^2(1-l)}{(1+l)((1-2l)w^2k + r\sigma_v^2)}\right)$.[29] (The subscript A indicates that the firm is allowing the CEO to transact with the IB.) It is easy to check that $V_A(l)$ is declining in l for all $l \in [0, \frac{r\sigma_v^2}{w^2k})$ and hence for all l in (7). The firm is worse off the higher the value of the haircut, and the firm's value is highest when the haircut is zero, in which case the firm's value is $V_A(0) = \frac{1}{2}\frac{(w^2k)^2}{w^2k + r\sigma_v^2}$. From the discussion of parts (b) and (c), one might conjecture that when the haircut is zero, the value of the firm is the same when the CEO is allowed to transact with the IB as when there is no IB (and hence, the CEO consumes exactly what the firm paid him.) Using the expression $V_A(l)$ for the firm's value along with (12), this conjecture can be verified to be true. The

[29] Part (d) follows from (23) below, specialized to the case where $q \geq q^{*res}(l)$.

preceding discussion shows that, whenever the haircut on the swaps is strictly positive, firm value is strictly lower when the CEO is allowed to buy swaps from the IB than when no swaps are available.

In summary, when the only hedge available through an IB is a swap and the CEO is allowed to transact with the IB, then while the CEO will always take advantage of the opportunity to engage in a swap (for all haircuts in the interval (7)), the firm is always weakly worse off as a consequence of this opportunity; when the firm anticipates that the CEO will engage in the swap, the firm will boost the PFPS of the contract he offers the CEO (relative to what he would have offered had the CEO not had access to the IB), but notwithstanding this "boosting," the CEO's effective PFPS is always (at least weakly) below what it would have been had the CEO been forbidden from transacting with the IB.

In the next theorem below, we consider the situation where the IB only offers to sell the CEO PH. To present this theorem, we make use of three additional constructs: the *first-best* quantity θ^{FB} of the PH and the CEO's two "reservation prices" $q^{res}(b_p)$ and q^{*res} for the PH. θ^{FB} is that θ that minimizes $var(\tilde{v}+\theta\tilde{y})$. It is well known (see, e.g., Hull [2007]) that

$$\theta^{FB} = -\frac{cov(\tilde{v},\tilde{y})}{\sigma_y^2} \text{ and } var(\tilde{v}+\theta^{FB}\tilde{y}) = \sigma_v^2(1-\rho^2). \tag{13}$$

For a given PFPS b_p, the reservation price $q^{res}(b_p)$ is the least upper bound of all prices q for the PH at which the CEO's demand for the PH is positive. The reservation price q^{*res} is the least upper bound on the prices q at which the CEO's demand for the PH is positive when evaluated at the optimal contract that solves the version of Program 1 where the IB is restricted to selling only PH to the CEO.

Theorem 2 *When the CEO is allowed to transact with the IB, but the IB only offers to sell the PH, then:*

(a) if the firm gives the CEO a contract with stated PFPS $b_p > 0$, then the CEO's optimal demand for the PH is:

$$\hat{\theta}^*(q|b_p) = \begin{cases} \frac{b_p w^2 k - \theta^{FB} q}{w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q}{r\sigma_y^2}, \text{ if } q < q^{res}(b_p) \equiv -\sigma_{vy} r b_p; \text{ and} \\ 0, \text{ if } q \geq q^{res}(b_p), \end{cases} \tag{14}$$

(b) the optimal contract's stated (and also, in this case, effective) PFPS is:

$$b_p^*(q) = \begin{cases} \frac{w^2k - \theta^{FB} q}{w^2k + r\sigma_v^2(1-\rho^2)}, \text{ if } q < q^{*res} \equiv \frac{-\sigma_{vy} r w^2 k}{w^2k + r\sigma_v^2}; \text{ and} \\ \frac{w^2k}{w^2k + r\sigma_v^2}, \text{ if } q \geq q^{*res}, \text{ and} \end{cases}$$

(c) the value of the firm evaluated at the optimal contract is:

$$V_A(q) \equiv \begin{cases} \frac{1}{2}\frac{(w^2k - \theta^{FB} q)^2}{w^2k + r\sigma_v^2(1-\rho^2)} + \frac{q^2}{2r\sigma_y^2}, \text{ if } q < q^{*res}; \text{ and} \\ \frac{1}{2}\frac{(w^2k)^2}{w^2k + r\sigma_v^2}, \text{ if } q \geq q^{*res}. \end{cases}$$

Part (a) of Theorem 2 shows that the amount of PH the CEO buys is increasing in the PFPS b_p of the contract the firm offers him. This makes sense, since the CEO is subject to more risk the bigger b_p is. Part (a) also shows that the CEO's demand curve for the PH is downward sloping: as q rises, the CEO's demand for the PH falls. This is as one would naturally expect. Part (b) shows that if the price of PH is so high that the CEO does not purchase any PH from the IB, then the effective (which in the context of Theorem 2 is the same as the stated) PFPS of the optimal contract, $\frac{w^2k}{w^2k+r\sigma_v^2}$, is the same as the effective PFPS of the optimal contract under the terms of Theorem 1 when the haircut l there is zero. At those high prices for the PH, the contracting environment in the present case where only PH is available through the IB and in the contracting environment where only the swap is available through the IB are the same, and so it is not surprising that the firm offers the CEO a contract with the same effective PFPS in both environments. Part (b) also shows that for those prices q of the PH below its reservation price, as q rises the PFPS of the optimal contract declines. This is a natural result in view of part (a), since part (a) shows that as the PFPS of the contract the firm gives the CEO increases, the CEO hedges more. It follows naturally that, as the cost of hedging rises, the firm will induce the CEO to buy less PH by subjecting the CEO to less risk by lowering b_p.

Part (c) displays the value of the firm when evaluated at the optimal contract as a function of the price q per unit of the PH. Comparing this value to its counterpart in Theorem 1(d), we see that if the IB charges actuarially fair prices in the case when he just sells swaps (at $l = 0$) and also when he just sells PH (at $q = 0$), then it is apparent that the firm value in Theorem 2(c) is strictly higher by allowing the CEO to buy only the PH from the IB than by allowing the CEO to buy only the swap from the IB. That is, $V_A(q = 0) = \frac{1}{2}\frac{(w^2k)^2}{w^2k+r\sigma_v^2(1-\rho^2)} > V_A(l = 0) \equiv \frac{1}{2}\left(\frac{(w^2k)^2}{(w^2k+r\sigma_v^2)}\right)$, where the latter value follows from Theorem 1(d). This is intuitive: the availability of the swap at actuarially fair rates does nothing to improve the principal-agent contracting relationship that the firm could not do on its own, whereas the availability of the PH at actuarially fair rates permits the firm to offer the CEO a contract with strictly improved risk-sharing properties. Also, this expression for firm value can be shown to be strictly decreasing in q for $q < q^{*res}$, and is continuous at $q = q^{*res}$. Since firm value at $q > q^{*res}$ equals what the value of the firm would be were no hedges available, we can conclude from this discussion that by allowing the CEO to transact with the IB when the IB only offers PH for sale, firm value is strictly higher than what firm value would have been were no hedges available, provided $q < q^{*res}$.

4 CEOs' demand curves for the PH and the swap when both hedging instruments are available through the IB

In this section, we take the first step toward solving Program 1 in the general case where the IB offers both the PH and the swap to the CEO concurrently by explicitly calculating the CEO's joint demand curves for both the PH and the swap. This section also includes a variety of comparative statics related to those two demand curves.

To begin, suppose the CEO, upon being given the contract $s_p(v) = c_p + b_p v$, purchases quantity $\hat{\theta}$ of the PH, swaps the variable amount $\beta\tilde{v}$ in return for receiving the fixed payment $c_{IB}(\beta|b_p)$ as specified in (6), and then chooses his action optimally in accordance with (4). Throughout the remainder of the article, we write $\hat{\theta}$ as the product: $\hat{\theta} = (b_p - \beta) \times \theta$, i.e., θ is the quantity of the PH per unit of the agent's RRI. Substituting this expression for $\hat{\theta}$ into (5), we see that the CEO's certainty equivalent can be written as:

$$CE^*(\tilde{T}) = (1-l)(b_p - \beta)\beta w^2 k + c_p + .5(b_p - \beta)^2 w^2 k - (b_p - \beta)\theta q - \frac{r}{2}(b_p - \beta)^2 var(\tilde{v} + \theta\tilde{y}). \quad (15)$$

Maximizing this certainty equivalent jointly with respect to the choice of both β and θ yields the CEO's demand curves for both the PH and the swap as reported in the following theorem.[30]

Theorem 3 *When the CEO is allowed to transact with the IB, the IB offers to sell the CEO both the PH and the swap, and the firm gives the CEO the contract $s_p(v) = c_p + b_p v$ with $b_p > 0$, then:*

(a) the CEO's demand curve for the fixed-for-variable swap is:

$$\beta^*(l|b_p, q) = \begin{cases} \frac{b_p(r\sigma_v^2(1-\rho^2) - lw^2k) + \theta^{FB} q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}, & \text{if } q < q^{res}(b_p|l) \equiv \frac{b_p(1-l)w^2k\theta^{FB} r\sigma_y^2}{(1-2l)w^2k + r\sigma_v^2}; \text{ and} \\ \frac{b_p(r\sigma_v^2 - lw^2k)}{(1-2l)w^2k + r\sigma_v^2}, & \text{if } q \geq q^{res}(b_p|l), \text{ and} \end{cases} \quad (16)$$

(b) the CEO's demand curve for the PH is:

$$\hat{\theta}^*(q|b_p, l) = \begin{cases} \frac{b_p(1-l)w^2k - \theta^{FB} q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q}{r\sigma_y^2}, & \text{if } q < q^{res}(b_p|l); \text{ and} \\ 0, & \text{if } q \geq q^{res}(b_p|l). \end{cases} \quad (17)$$

While the demand curves in Theorem 3 appear similar to those that appear in Theorems 1 and 2, they are distinct. It is easy to confirm that the demand for the swap $\beta^*(l|b_p, q)$ as it appears in (16) is identical to the demand for the swap $\beta^*(l|b_p)$ as it appears in Theorem 1 when, in the former case, the correlation ρ between the performance measure $\tilde{v}$ and the PH $\tilde{y}$ is set equal to zero and the price q for the PH is taken to be above the reservation price for the PH. Similarly, the demand for the PH $\hat{\theta}^*(q|b_p, l)$ as it appears in (17) is identical to the demand for the PH $\hat{\theta}^*(q|b_p)$ as it appears in Theorem

[30] Alternatively, given the demand function $\hat{\theta}^*(q|b_p, l)$ described in (17), $q^{res}(b_p|l)$ is the smallest q satisfying $\hat{\theta}^*(q|b_p, l) = 0$.

2 when in the former case the haircut l is set equal to 0. The explanation for these similarities is obvious: when only one hedging instrument is available from the IB, the price of the "other" hedging instrument does not appear in the CEO's demand curve for the hedging instrument that is available, and the non-price effects of the "other" variable on the CEO's demand curve for available hedging instrument (the effect of improved risk sharing in the case of the PH; and the effect of the haircut on transforming the stated PFPS of the contract the firm gives the CEO into the contract's effective PFPS in the case of the swap) do not appear either.

The first corollary presented below contains comparative statics related to the CEO's demand for the PH when swaps are also available through the IB.

Corollary 1 [31] *(Comparative statics of the CEO's demand curve for the PH)*

When the CEO is allowed to transact with the IB, the IB offers to sell the CEO both the PH and the swap, and the firm gives the CEO the contract $s_p(v) = c_p + b_p v$ *with* $b_p > 0$*, then for all* $q < q^{res}(b_p|l)$ *and all* l *in (7), the CEO's demand curve for the PH is:*

(a) strictly decreasing in q;

(b) strictly increasing in the haircut l*;*

(c) strictly increasing in the PFPS b_p;

(d) strictly increasing in the absolute value of the correlation $|\rho|$, *for all* $\rho < 0$*; and*

(e) strictly increasing in both the CEO's marginal productivity of effort w *and the parameter* k *determining the CEO's disutility of effort.*

We confine discussion here of the corollary to those parts of it that were not discussed in related remarks following Theorem 1, i.e., to parts (b), (d), and (e). Corollary 1(b) demonstrates that the swap and the PH are always substitutes: as the haircut on the swap becomes bigger, the CEO buys a larger quantity of the PH. Intuitively, if both the swap and the PH are offered for sale, as the swap becomes more expensive, i.e., as the haircut gets larger, the CEO will make greater use of the PH. To explain Corollary 1(d), note that as the absolute value of the correlation $|\rho|$ increases, the PH $\tilde{y}$ becomes more effective as a hedge of the CEO's compensation risk. Holding its price fixed, it is natural that the CEO purchases more of the PH as it becomes more effective. Corollary 1(e) shows that the demand for the PH increases in the CEO's productivity w and also increases as the CEO's disutility of effort declines. This is explained by examining the CEO's optimal RRI in the original

[31] The proofs of this corollary and all other comparative statics results below are omitted, but are avaliable from the authors.

contract:

$$b_p - \beta^*(l|b_p, q) = \frac{b_p(1-l)w^2k - \theta^{FB}q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}, \text{ for } q < q^{res}(b_p|l). \tag{18}$$

One can confirm that the CEO's optimal RRI in his original contract increases in both w and k. This is intuitive: as the CEO becomes more productive, or as the CEO experiences lower disutility of effort, the CEO naturally retains a higher fraction of the risky part of the firm's original contract. Having retained more risk (in the form of RRI of the original contract), it makes sense for the CEO to buy more insurance (in the form of the PH). This explains Corollary 1(e).

The next corollary presents comparative statics related to the CEO's demand for the swap when PH is also available through the IB.

Corollary 2 *(Comparative statics regarding the CEO's demand curve for swaps)*

When the CEO is allowed to transact with the IB, the IB offers to sell the CEO both the PH and the swap, and the firm gives the CEO the contract $s_p(v) = c_p + b_p v$ *with* $b_p > 0$*, then the CEO's demand curve* $\beta^*(l|b_p, q)$ *for swaps is:*

(a) strictly decreasing in the haircut l *for* l *in* (7);

(b) increasing in q*, strictly so for* $q < q^{res}(b_p|l)$;

(c) strictly increasing in b_p*;*

(d) strictly increasing in σ_v^2*; and*

(e) strictly decreasing in w *and* k*.*

As in the case of the previous corollary, we discuss here only those parts of the corollary that were not discussed in related remarks following Theorem 2, i.e., parts (b), (d), and (e). Corollary 2(b) reinforces the conclusion of Corollary 1(b) that the swap and the PH are substitutes: as the price of the PH increases, the CEO demands a greater quantity of the swap. Corollary 2(d) shows that as the variance in the firm's performance measure increases, the CEO's demand for swaps increases. Corollary 2(e) reports that the CEO engages in a smaller swap as either the marginal productivity of the CEO's effort increases or as his disutility of effort declines. The economic forces that explained Corollary 1(e) above also explain Corollary 2(e): since the CEO's preferred RRI in the original contract is increasing as the marginal productivity of the CEO's effort increases or as his disutility of effort declines, the CEO's preferred RRI in the original contract, $b_p - \beta^*(l|b_p, q)$, moves in the opposite direction of the CEO's preferred quantity of the fixed-for-variable swap, $\beta^*(l|b_p, q)$. The effect of an increase in k is explained similarly.

5 Solution to the firm's optimal contracting problem

In this section, we employ the results of the preceding section to determine the solution to the firm's optimal contracting problem, as described in Program 1 above, where the firm correctly anticipates how the CEO will transform optimally whatever contract it gives him by transacting with the IB.

We start by taking a generic contract $s_p(v) = c_p + b_p v$, with $b_p > 0$, and substitute the CEO's demands for the PH and the swap given this contract into the CEO's certainty equivalent (15). By making these substitutions, we obtain expression (19) in the next lemma as the CEO's certainty equivalent.

Lemma 1 *Given the contract $s_p(v) = c_p + b_p v$, with $b_p > 0$, if the CEO chooses his demands for the PH and the swap optimally, his certainty equivalent is:*

$$CE(b_p|c_p, q, l) \equiv \begin{cases} c_p + \frac{1}{2}\left(\frac{q^2}{r\sigma_y^2} + \frac{(b_p(1-l)w^2k - \theta^{FB}q)^2}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\right), \text{ if } q < q^{res}(b_p|l); \text{ and} \\ c_p + \frac{1}{2}\left(\frac{(b_p(1-l)w^2k)^2}{(1-2l)w^2k + r\sigma_v^2}\right), \text{ if } q \geq q^{res}(b_p|l). \end{cases} \tag{19}$$

The expression (19) is useful for determining the CEO's optimal base salary. Given any value for the PFPS $b_p > 0$, a rational firm will employ (19) to adjust the CEO's base pay so that his IR constraint binds. Recalling the normalization $\bar{U} = 0$, it follows that the firm will adjust the CEO's base salary $c_p = c_p(b_p|q, l)$ so that:

$$c_p(b_p|q, l) \equiv -CE(b_p|0, q, l). \tag{20}$$

So, the firm's expected profits from giving the CEO the contract $s_p(v) = c_p(b_p|q, l) + b_p v$ are:

$$\begin{aligned} \pi(b_p|q, l) &\equiv E[\tilde{v} - s_p(\tilde{v})|a^*(b_p - \beta^*(l|b_p, q))] \\ &= (1 - b_p)E[\tilde{v}|a^*(b_p - \beta^*(l|b_p, q))] - c_p(b_p|q, l) \\ &= (1 - b_p)w^2k(b_p - \beta^*(l|b_p, q)) - c_p(b_p|q, l). \end{aligned} \tag{21}$$

Optimizing (21) with respect to b_p determines the PFPS of the optimal contract that solves Program 1. We let the optimizing value of b_p be denoted by $b_p^*(q, l)$, and the value of Program 1 at its optimum be denoted by $V_A(q, l)$. Theorem 4 specifies the optimal values for $b_p^*(q, l)$ and $V_A(q, l)$. (In the theorem, $q^{*res}(l)$ is the CEO's reservation price for the PH, given that the CEO is compensated by the firm with a contract whose PFPS is given by $b_p^*(q^{*res}(l), l)$.)[32]

Theorem 4 *The solution to Program 1, $s_p^*(v) = c_p^*(q, l) + b_p^*(q, l)v$, is as follows*:

[32] Recalling the demand curve for the hedge $\hat{\theta}^*(q|b_p, l)$ in (17) above, for all l in (7), $q^{*res}(l)$ is uniquely defined as that price that satisfies the recursive identity $\hat{\theta}^*(q^{*res}(l)|b_p^*(q^{*res}(l), l), l) \equiv 0$.

(a) the optimal PFPS $b_p^*(q,l)$ is:

$$b_p^*(q,l) = \begin{cases} \frac{1}{1+l} + \frac{l\theta^{FB}q}{w^2k(1-l^2)}, \text{ if } q < q^{*res}(l) \equiv \frac{w^2k(1-l)\theta^{FB}r\sigma_y^2}{(\theta^{FB})^2r\sigma_y^2+(1+l)((1-2l)w^2k+r\sigma_v^2(1-\rho^2))}; \text{ and} \\ \frac{1}{1+l}, \text{ if } q \geq q^{*res}(l), \end{cases} \tag{22}$$

(b) the optimal value for the base salary, $c_p^*(q,l)$, is given by (20) above, with $b_p^*(q,l)$ replacing b_p; and

(c) the value of the firm (21) when evaluated at the optimal contract is:

$$V_A(q,l) \equiv \begin{cases} \frac{1}{2}\left(\frac{(w^2k(1-l)-\theta^{FB}q)^2}{(1-l^2)((1-2l)w^2k+r\sigma_v^2(1-\rho^2))} + \frac{q^2}{r\sigma_y^2}\right), \text{ if } q < q^{*res}(l); \text{ and} \\ \frac{1}{2}\left(\frac{(w^2k)^2(1-l)}{(1+l)((1-2l)w^2k+r\sigma_v^2)}\right), \text{ if } q \geq q^{*res}(l). \end{cases} \tag{23}$$

We comment on part (a) of this theorem through the next corollary and the discussion that follows it. (We defer discussion of the economics underlying part (c) of the theorem to the next section.[33])

Corollary 3 *(The comparative statics of the optimal contract and firm value). For any $l > 0$ (in (7)):*

(a) the optimal PFPS $b_p^*(q,l)$ is strictly increasing in q over the interval $q \in [0, q^{*res}(l))$;

(b) the optimal PFPS $b_p^*(q,l)$ is constant over the interval $q \in [q^{*res}(l), \infty)$ with $b_p^*(q,l) = \frac{1}{1+l} = b_p^*(0,l)$;

(c) for any $q \in [0, q^{*res}(l))$, $b_p^*(q,l)$ is strictly decreasing in both w and k;

(d) $b_p^*(q,l)$ is strictly decreasing in l for all $q \geq 0$ provided $\rho^2 < \frac{(w^2k - r\sigma_v^2)^2}{w^2kr\sigma_v^2}$;

(e) $V_A(q,l)$, is strictly decreasing in q for all $q < q^{*res}(l)$; and

(f) for ρ^2 sufficiently large and $l \leq .5$, $V_A(q,l)$ is strictly increasing in l for all $q < q^{*res}(l)$.

Understanding the economic factors that influence the CEO's preferred RRI in the firm's contract in (18) is key to obtaining intuition about the behavior of the optimal PFPS $b_p^*(q,l)$ described in this corollary. (18) above shows that increases in the price q of the PH, for $q \in [0, q^{*res}(l))$, lead the CEO to reduce his RRI in the firm's contract. This is explained as follows: through his choice of RRI in the original contract, the CEO deliberately exposes himself to compensation risk which he can partially hedge by purchasing the PH. As the price q of this hedge increases, the CEO optimally reduces the risk he exposes himself to by reducing his RRI in the original contract. This, in turn, reduces his demand for this (now more expensive) insurance. It follows that an increase in the price q the CEO pays to acquire the PH drags down the incentives of the CEO to work hard, because it reduces the CEO's preferred RRI in the original contract. For all fees in the region ($q \in (0, q^{*res}(l))$),

[33] Part (b) of the Theorem is included only for completeness and will not be discussed further.

the firm optimally offsets this change in the CEO's behavior as q increases by increasing the PFPS $b_p^*(q,l)$ of the optimal contract.

When the PH is free ($q = 0$), obviously the "fee" for the PH in that case creates no incentive drag on the CEO. When $q \geq q^{res}(b_p|l)$, (16) implies that

$$b_p - \beta^*(l|b_p, q) = \frac{b_p(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2}. \tag{24}$$

It is clear from (24) that the fee q again imposes no incentive drag on the CEO when $q \geq q^{*res}(l)$, since for such high fees, the CEO does not buy the PH. Therefore, for such high fees, the firm chooses the same b_p as when $q = 0$. This explains why the PFPS of the optimal contract increases in q up to (but not including) $q = q^{*res}(l)$, and then discontinuously drops at $q = q^{*res}(l)$, and then remains at the same level as when $q = 0$ for all $q \geq q^{*res}(l)$, as Corollary 3(a) and 3(b) report.

Corollary 3(c) reports that for intermediate values of the fee q for acquiring the PH (i.e., for $q \in (0, q^{*res}(l))$), increasing either the CEO's productivity parameter w or decreasing the CEO's disutility of effort results in the optimal PFPS declining. This is also explained by examining the CEO's RRI in the original contract: we already noted in the discussion of Corollary 1(e) and 2(e) that, for a given PFPS b_p, as w or k increase, the CEO optimally increases his RRI in the original contract. Were the contract's PFPS to increase as w or k increases, this would load an excessive amount of risk on the CEO, and so, as Corollary 3(c) reports, the firm optimally responds to an increase in w or k by reducing the PFPS b_p.[34]

Corollary 3(d) establishes broad conditions under which the optimal PFPS is everywhere declining in the haircut l. This is surprising when juxtaposed to Corollary 3(a), since the latter result establishes general conditions under which the PFPS increases in q. But it makes economic sense. The higher the PFPS of the firm's original contract, the more the CEO makes use of the swap (Corollary 2(c)). Given a fixed contract, we know that it is beneficial to the CEO to make less use of the swap as the haircut l increases by Corollary 2(a); it is also beneficial to the firm to have the CEO make less use of the swap too, because ultimately, the firm pays for the CEO's use of the swap (through the CEO's IR constraint). So, the firm self-interestedly gets the CEO to use less of the swap as l increases by reducing the PFPS of the optimal contract he awards the CEO.

The conclusion in Corollary 3(e) that firm value decreases in the price q the IB charges the CEO for

[34] This result merits contrast to how the firm changes the optimal contract's PFPS in response to an increase in w or k in the conventional principal-agent problem (where the CEO retains his original contract and does not transact with an IB). In the conventional problem, it is easy to show that the firm optimally increases the contract's PFPS as w or k increases. This occurs in the conventional problem because, on his own, the CEO cannot make any adjustment to his incentives as w or k change, and so it is left to the firm to modify the contract it gives the CEO to exploit the improved work environment brought about by an increase in w or k.

the PH when the CEO is allowed to transact with the IB is not surprising if one views the result from the "more costly input" perspective, i.e., that firm value should be expected to decline as the price of any input (such as the price of the PH) the firm uses increases. The conclusion in Corollary 3(f) that firm value increases in the size (or "price") of the haircut l is not surprising if one views the result from the "agency-theoretic perspective"that anything that deters an agent from taking an action the firm does not want the agent to take (such as buying more of the swap from the IB) is desirable because it makes it less costly for the firm to satisfy the incentive compatibility constraint related to the action the firm *does* want the agent to take. But, what remains to be explained is why the "more costly input" perspective is the appropriate one for understanding the effects of increasing the price q of the PH whereas the "agency theoretic perspective" is the appropriate one for understanding the effects of increasing the price or haircut l of the swap. The "more costly input" perspective is applicable to the PH, because easier access to the PH improves the contracting relationship between the firm and its CEO by improving risk-sharing; the "agency theoretic perspective" is applicable to the swap because easier access to the swap worsens the contracting relationship between the firm and its CEO by making the CEO pay for something (reduced PFPS) that the firm could have supplied to the CEO for free.

Some of the comparative statics presented in Corollary 3 are illustrated in the figures 2 and 3. In both figures, we set $w = 5$, $k = 1, r = 1.5$, $\sigma_y^2 = 4$ and $\sigma_v^2 = 25$. Figure 2 illustrates Corollaries 3(a) and 3(b). It shows that the optimal contract's PFPS, $b_p^*(q, l)$, is strictly increasing in q over the interval $q \in [0, q^{*res}(l))$ and is constant on the interval $q \in [q^{*res}(l), \infty)$ with $b_p^*(q, l) = \frac{1}{1+l}$.[35] Figure 2 also illustrates Corollary 3(d) by demonstrating that $b_p^*(q, l)$ is strictly decreasing in l for all $q \geq 0$. In figure 2, we set $\sigma_{vy} = -3.5$, so we satisfy the condition in Corollary 3(d) that $\rho^2 = 0.1225 < \frac{(w^2k - r\sigma_v^2)^2}{w^2kr\sigma_v^2} = 0.167$.

Figure 3 illustrates Corollaries 3(e) and 3(f). For this figure, we set $\sigma_{vy} = -9.8$, so that $\rho^2 = 0.96$, which is sufficiently large to meet the condition in Corollary 3(f). Consistent with Corollary 3(f), figure 3 shows that the optimal firm value, $V_A(q, l)$, is strictly increasing in l for all $q < q^{*res}(l)$. Further, consistent with Corollary 3(e), figure 3 illustrates that the optimal firm value, $V_A(q, l)$, decreases in q for all $q < q^{*res}(l)$.

[35] In Figures 2 and 3 below, the white (separating) space represents the graph of those (q, l) pairs for which $q = q^{*res}(l)$, so that the "meshed" area to the left of the white (separating) space entails the case where $q < q^{*res}(l)$ and the area marked with horizontal lines to the right of the white separating space reflects the case where $q > q^{*res}(l)$.

6 The Value of Transacting with the IB

In this section, we determine when firm value is higher by allowing the CEO to transact with the IB or by prohibiting the CEO from transacting with the IB and, instead, having the firm itself transact directly with the IB.

When the firm itself transacts directly with the IB and prohibits the CEO from doing so, the firm expands the potential space of contracting variables from those that depend just on the performance measure $\tilde{v}$ to those that depend on both $\tilde{v}$ and $\tilde{y}$. That is, the space of contracts expands to those of the form $s_p(v, y) = c_p + b_p v + d_p \bar{\theta} y$, where $\bar{\theta}$ is the quantity of the PH the firm buys from the IB.[36] (This is the only relevant expansion in the contracting space when the firm transacts directly with the IB: the firm would never buy a swap from the IB because the firm can replicate the effects of any swap with the IB simply by reducing the PFPS of the contract it offers the CEO and avoid incurring the haircut l.)

Observe that when the firm itself transacts with the IB, the firm will pay only an infinitesimal amount to the IB for the PH regardless of the price $\$q$. This is clear since if we start with the contract $s_p(v, y) \equiv c_p + b_p v + d_p \bar{\theta} y$ with $\bar{\theta} > 0$, then this contract yields exactly the same certainty equivalent to the CEO under all circumstances as does the contract $s'_p(v, y) \equiv c_p + b_p v + d'_p \bar{\theta}' y$, where $d'_p = 2d_p$ and $\bar{\theta}' = .5\bar{\theta}$. Of course, the latter contract entails having the firm purchase half as much of the PH from the IB as does the former contract, and hence involves only half the outlay to the IB. Since this is true for any quantity $\bar{\theta} > 0$ of the PH, the observation follows that the firm's optimal quantity of, and expenditures on, the PH is infinitesimal. This conclusion obtains because, unlike the case of the (risk-averse) CEO who derives utility from the actual cash flows produced by the PH (because those cash flows help to reduce the total variance in the CEO's hedge-inclusive compensation), the only value a risk-neutral firm derives from acquiring the PH is its *informational* value as (a) the latter facilitates contracting with the CEO; and (b) the firm is indifferent as to the amount of variability in the value of its residual claim, and hence, is unconcerned with the total amount of cash flows "thrown off" by the PH.

Thus, when the firm itself transacts with the IB to obtain the PH $\tilde{y}$ for contracting purposes, the optimal linear contract the firm offers the CEO is in the limit (as the quantity of the PH goes to zero) the one that solves Program 2.[37]

[36] The notation $\bar{\theta}$ here is used to distinguish the quantity of the PH purchased by the firm when it transacts directly with the IB from the quantity $\hat{\theta} = (b_p - \beta)\,\theta$ of the PH purchased by the CEO when the CEO is allowed to transact with the IB.

[37] We do not index the optimal contract that solves Program 2 by either of the "prices" q or l. As was noted prior

Program 2 (Optimal contracting when the firm transacts directly with the IB and the firm forbids the CEO from transacting with the IB)

$$\max_{a_p^*, \bar{\theta},\ s_p = c_p + b_p v + d_p \bar{\theta} \tilde{y}} E[\tilde{v} - s_p(\tilde{v}, \tilde{y}) | a_p^*]$$

$$\text{subject to } a_p^* = b_p wk; \text{ and} \tag{25}$$

$$E[s_p(\tilde{v}, \tilde{y})|a_p^*] - .5(a_p^{*2}/k + rvar(s_p(\tilde{v}, \tilde{y})) \geq 0. \tag{26}$$

The constraints in this program are standard: (25) is the CEO's incentive compatibility constraint and (26) is the CEO's individual rationality constraint. This program does "double duty" in so far when it is solved for $\rho = 0$, it yields the solution to the firm's contracting problem when neither the firm nor its CEO engages in any transactions with the IB.[38]

The following Proposition records the solution to Program 2.[39] In the statement of the proposition, we let the optimizing value of b_p, i.e., the PFPS of the CEO's optimal linear contract when evaluated at the solution to Program 2, be denoted by b_F^*, where the subscript "F" reminds us that the firm is transacting directly with IB (and the CEO is prohibited from transacting with the IB). Sometimes, when we wish to emphasize the dependence of b_F^* on the correlation ρ, we write $b_F^*(\rho)$ in place of b_F^*. Also, we let the value of Program 2 at its optimum, i.e., the value of the firm, be denoted by V_F (or $V_F(\rho)$, as appropriate).

Proposition 1 *(Firm value and the form of optimal contracts when the firm contracts directly with the IB and the firm forbids the CEO from contracting with the IB)*

The contract that solves Program 2 is given by $s_F^*(v, y) = c_F^* + b_F^* v + d_F^* \bar{\theta}^* y$, *with*

$$b_F^* = \frac{kw^2}{kw^2 + r\sigma_v^2(1 - \rho^2)}; \tag{27}$$

$$d_F^* = b_F^*;\ \bar{\theta}^* = \theta^{FB};\ \text{and}$$

$$c_F^* = .5(b_F^*)^2(r\sigma_v^2(1 - \rho^2) - w^2 k), \tag{28}$$

and firm value at the optimum equals:

$$V_F(\rho) = 0.5 \frac{k^2 w^4}{kw^2 + r\sigma_v^2(1 - \rho^2)}. \tag{29}$$

to the statement of the proposition, when the firm employs the optimal contract, the firm makes only an infinitesimal purchase of the PH. Thus, the price q is irrelevant. Also, since it never purchases a swap, the size of the haircut l is irrelevant too.

[38] As the firm will not make any use of the PH $\tilde{y}$ in contracts of the form $s_p = c_p + b_p v + d_p \tilde{y}$ unless the correlation ρ is nonzero, and hence, the case of $\rho = 0$ is tantamount to neither the firm nor the CEO transacting with the IB.

[39] The proof is straightforward and omitted.

Proposition 1 identifies both the solution to Program 2 and the value of the firm when evaluated at that solution. From Proposition 1, we can immediately conclude that both the value of the firm and the optimal contract's PFPS are increasing in the effectiveness of the PH, as measured by ρ^2. In addition, Proposition 1 establishes that both firm value and the optimal contract's PFPS when the firm transacts directly with the IB but the firm prohibits the CEO from transacting with the IB are higher than when no transactions at all with the IB take place. This last claim follows from Proposition 1 in conjunction with the observation preceding Program 2's statement that Program 2 does "double duty" by also applying to the case where neither the firm nor the CEO transact with the IB. Hence, when no PH is available, the PFPS of the CEO's optimal contract shrinks to

$$b^*_{notrans} \equiv b^*_F(\rho = 0) = \frac{kw^2}{kw^2 + r\sigma_v^2} \tag{30}$$

and the expected value of the firm shrinks to

$$V_{notrans} \equiv V_F(\rho = 0) = 0.5\frac{k^2w^4}{kw^2 + r\sigma_v^2}, \tag{31}$$

and so the two inequalities $0.5\frac{k^2w^4}{kw^2+r\sigma_v^2(1-\rho^2)} > 0.5\frac{k^2w^4}{kw^2+r\sigma_v^2}$ and $\frac{kw^2}{kw^2+r\sigma_v^2(1-\rho^2)} > \frac{kw^2}{kw^2+r\sigma_v^2}$ prove these claims.

At this point, it is appropriate to discuss what is socially optimal for the agency given that both hedging instruments are available for contracting, while disregarding the distribution of rents among the firm, the CEO, and the IB, i.e., while disregarding the prices q and l, and the level of the IR constraint $\bar{U}$. Based on the analysis we have already performed, it is clear that making the swap available for contracting is of no social value to the agency, since as we previously noted, the firm can replicate the effects of any swap simply by reducing the CEO's contract's PFPS suitably. It is also clear that the socially optimal use of the PH to the agency is to use the PH at the first-best level $\bar{\theta}^* = \theta^{FB}$ in exactly the same way the firm optimally uses the PH when it alone contracts with the IB as was concluded in Proposition 1 above.

Given the preceding observation that swaps are not socially value enhancing, one might ask why swaps, or other similar hedges that effectively duplicate swaps, are available in the marketplace.[40] This has an easy answer: in the marketplace, it is commonplace to see transactions or activities which are not socially valuable but which are demanded by a subsegment of the population. This is particularly true of transactions or activities whose principal function is redistributional. Besides swaps, examples of such activities or transactions include tax motivated structures that are unproductive but reduce the tax payers' tax burden, earnings management activities that decrease a firm's long-term cash flows

[40] We include these remarks at the suggestion of one of the referees.

but temporarily boost its reported earnings, etc. When such activities cannot be costlessly stopped or costlessly committed to not being engaged in, then some subsegments of society - but not society as a whole - will benefit from, and hence seek to engage in, these activities. While we do not mean to discount the possibility of there being other reasons besides their role in redistribution that help to explain the demand for swaps, their redistributive role is more than sufficient reason to predict the marketplace's offering of swaps for sale.

In the following corollary, by comparing the expressions for firm value derived in Proposition 1 and Theorem 4, we obtain further conclusions about which of the following generates highest firm value: allowing the CEO to transact with the IB, having the firm transact directly with the IB and prohibiting the CEO from transacting with the IB, or having neither party transaction with the IB.[41]

Corollary 4 *(A comparison of firm value under alternative contracting scenarios)*

(a) Firm value is the same when the firm acquires hedges directly from the IB and the CEO is prohibited from contracting with the IB as when the CEO is allowed to transact with the IB, provided the IB charges actuarially fair prices for both the PH and the swap, i.e.,

$$V_F = V_A(q = 0, l = 0).$$

(b) Firm value is higher when the firm acquires hedges directly from the IB and the CEO is prohibited from contracting with the IB than when the CEO is allowed to transact with the IB, provided the IB charges fees that exceed the actuarially fair prices for either the PH or the swap, i.e.,

$$V_F > V_A(q, l) \text{ if either one or both of } q \text{ and } l \text{ are positive.}$$

(c) Firm value is greater when the CEO is allowed to contract with the IB than when no one (either firm or CEO) transacts with the IB, provided the prices q and l are sufficiently low, i.e.,

$$V_{notrans} < V_A(q, l) \text{ if both } q \text{ and } l \text{ are sufficiently low.}$$

(d) Firm value is higher when no one transacts with the IB than when the CEO is allowed to transact with the IB for any positive haircut ($l > 0$ in (7)) the IB charges, provided the price q the IB charges is sufficiently close to (but below) the reservation price $q^{*res}(l)$, i.e.,

$$V_{notrans} > V_A(q, l) \text{ for any } l > 0, \text{ provided } q \text{ is sufficiently near (but below) } q^{*res}(l).$$

[41] When reading this proposition, recall that the notation $V_A(q, l)$ refers to firm value when the CEO transacts directly with the IB, as in Theorem 4 above; the notation V_F refers to firm value when the firm transacts directly with the IB and prohibits the CEO from transacting with the IB as in Lemma 2 above, and the notation $V_{notrans}$ in expression (31) refers to firm value when neither the firm nor the CEO transacts with the IB.

The intuition underlying Corollary 4(a) is simple. If $q = l = 0$, then according to (18) above, given any contract with bonus coefficient b_p, if the firm allows the CEO to transact with an IB, the CEO's optimal RRI in the original contract is

$$b_p - \beta^*(b_p|q=0, l=0) = b_p \times \frac{w^2 k}{w^2 k + r\sigma_v^2(1-\rho^2)}.$$

That is, through his transactions with an IB, the CEO "knocks down" the PFPS of his original contract by the fraction $f \equiv \frac{w^2 k}{w^2 k + r\sigma_v^2(1-\rho^2)}$. So, the firm can restore the incentives of his original contract by "scaling up" the bonus coefficient by $\frac{1}{f}$. This can be done in particular for the optimal contract obtained by having the firm transact directly with the IB (and prohibiting the CEO from doing so), and so the firm is neither better off nor worse off having the CEO transact with the IB than having the firm transact with the IB directly and forbidding the CEO from transacting with the IB.

The preceding is an intuitive result. What is perhaps less intuitive is that the same result does not obtain when either q or l is positive. While the firm will still adjust the contract it offers to account for the CEO's propensity to "knock down" the incentive effects of that contract through transactions with the IB, the firm will not simply "boost" the incentive effects of the contract it offers to offset fully the CEO's "knocking down" those incentive effects. To see this, suppose, for example, that q is so large that the CEO does not purchase any of the PH from the IB, then - as we demonstrated in (24) above - when the firm gives the CEO a contract with PFPS b_p, the fraction of the PFPS the CEO retains is given by $\frac{b_p - \beta^*(b_p|q,l)}{b_p} = \frac{(1-l)w^2 k}{(1-2l)w^2 k + r\sigma_v^2}$. So, were the firm just interested in retaining the incentive effects of (27), the firm would give the CEO a contract with PFPS:

$$b_p = \frac{(1-2l)w^2 k + r\sigma_v^2}{(1-l)w^2 k} \times \frac{kw^2}{kw^2 + r\sigma_v^2(1-\rho^2)} = \frac{(1-2l)w^2 k + r\sigma_v^2}{(1-l)(kw^2 + r\sigma_v^2(1-\rho^2))}. \tag{32}$$

But, the firm does not do that. Instead, as (22) shows, in these circumstances, the firm optimally offers the CEO a contract with PFPS $b_p^*(q,l) = \frac{1}{1+l}$, which is easily shown to be smaller than (32) when $l > 0$ belongs to the interval (7). Consequently, when $l > 0$, the firm does not find it optimal to fully restore the CEO's incentives to work hard even when he anticipates the CEO's propensity to transact with the IB. The intuition for this follows from the discussion of Corollary 3(d) above: the haircut the IB charges the CEO for the swap is ultimately paid for by the firm, since the firm has to ensure the satisfaction of the CEO's constraint. Hence, to economize on these costs, the firm reduces the PFPS of the contract it offers the CEO to limit the CEO's propensity to transact with the IB and incur the haircut.[42]

[42] It is possible, if no other restrictions are added to Program 1, that the contract that solves this program may entail the CEO's base pay c being negative and so the CEO will have to pay his firm (rather than vice versa) if the

Regarding Corollary 4(b): one might expect that if the price of the PH q equals zero, then firm value should be the same whether or not $l = 0$, under the reasoning that as long as the CEO has access to the PH for free, it should not matter what the size of the haircut is, since the "important" service the IB is providing is access to the PH "for free." The IB's offering of the fixed-for-variable swap to the CEO, in contrast, would not seem to be an "important" service, because the firm always has a ready - in fact, perfect - substitute for this latter service: the firm can simply provide the CEO with a contract that has lower powered incentives, i.e., with a smaller PFPS b_p. So, this reasoning goes, if the CEO could have the opportunity to buy PH at actuarially fair prices from the IB, it should not matter whether the IB imposes a haircut for swapping compensation risk with the CEO, because the firm has a perfect substitute for this latter service.

But, the preceding reasoning turns out to be wrong, as Corollary 4(b) reports: as we originally noted in the discussion following Theorem 1, a risk-averse CEO always finds it irresistible to swap some compensation risk with the IB, even when those swaps take place at actuarially unfair rates, and the expectation that the CEO will engage in such transactions with the IB leads to a reduction in the firm's value, even when the IB sells the CEO the PH at actuarially fair rates.

Corollary 4(c) reports that even if the IB exercises his market power by charging above actuarially fair prices for the PH and for the service of swapping compensation risk with the CEO and even if the CEO's transactions with the IB reduce the incentive effects of the contract the firm originally offers him, firm value can still be higher by allowing the CEO to transact with the IB as compared to having neither the firm nor the CEO hedge through the IB, provided the premiums the IB charges the CEO for these transactions are not too large. This result follows because having access to the PH is valuable provided the IB does not charge excessively high fees for that access.

Corollary 4(d) indicates that firm value when the CEO hedges his compensation risk by transacting

realized value of $\tilde{v}$ (and, hence, $b\tilde{v}$) is small. In the event the CEO is wealth-constrained, a contract with such "reverse payments" may be infeasible. While we do not pursue the study of this possibility (of manager's limited wealth) in the text, in this footnote, we briefly consider this situation. In particular, we suppose here that the CEO has no outside wealth and so the CEO's compensation must always be nonnegative. We also suppose that $\tilde{v}$ is also always nonnegative (which would be true (and compatible with other assumptions we have already made if, for example, when the CEO takes action a, $\tilde{v}$ is distributed uniformly on $[0, 2aw]$). Combined with our other maintained assumptions (which include the specification $\bar{U} = 0$), we conclude in this case that the CEO's base pay c optimally will be set equal to zero. That is, all of the CEO's compensation will consist of just variable pay, $b_p\tilde{v}$. The optimal b_p is then computed by first substituting the optimal value for the CEO's retained residual interest $b_p - \beta^*(l|b_p, q)$ into the profit function (21), where $\beta^*(l|b_p, q)$ is as reported in Theorem 3, and then maximizing with respect to the choice of b_p. When q is below the CEO's reservation price, it is easy to show that the optimal b_p is given by $b_p^* = \frac{1}{2} + \frac{\theta^{FB} q}{2(1-l)w^2 k}$. It is clear from this last expression that this value for b_p^* is strictly increasing in l, and so the stated value of the PFPS will be increasing in the haircut when the IR constraint does not bind. In contrast, if q is above the CEO's reservation price, it is easy to show (by using the same procedure as above) that the optimal value for b_p is $b_p = \frac{1}{2}$ independent of the size of the haircut. Thus, whether the firm will boost the PFPS of the contract it gives its CEO, as well as how this boosting changes with the haircut l, depends at least sometimes on whether the IR constraint for the CEO is binding.

We wish to thank an anonymous referee for suggesting that we undertake the preceding analysis.

with the IB can be lower than firm value when no transactions with the IB take place, even when the IB charges the CEO a trivially small, but positive, haircut, as long as the price q the IB charges the CEO for the PH is sufficiently near the CEO's reservation price for the PH. While it might be thought that the CEO has an incentive to eschew transacting with an IB when such transactions will result in the firm's value dropping below what the firm's value would have been if all transactions with the IB were prohibited, this is not true. In other words, Corollary 4(d) identifies a "hold-up problem" (see, e.g., Aghion and Bolton [1992]) for the firm that is so severe that, even though the IB has a resource (the PH) that is valuable to the agency, and even though the firm anticipates the CEO's transacting with an IB when designing the contract, the firm is worse off than it would have been had all transactions with the IB been eliminated. However, unlike the results in traditional "hold-up problems," Corollary 4(c) demonstrates that the CEO's opportunity to engage in ex post hedging does not always destroy firm value. Thus, whether the manager's opportunity to engage in hedging is value-destroying hinges on the magnitudes of the prices q and l, as described in Corollary 4(c) and (d).

We conclude this section by comparing the optimal contract's PFPS when hedges are available to the optimal contract's PFPS were there no hedges. When hedging opportunities exist, we refer to the expression $b_p^*(q,l)$ for the optimal PFPS in Theorem 4 as the optimal *stated* PFPS, and the expression $b_p - \beta^*(l|b_p,q)$ for the CEO's optimal retained residual interests in (18) and (24) above evaluated at the optimal PFPS $b_p = b_p^*(q,l)$ as the optimal contract's *effective* PFPS. (When there are no hedging opportunities, then of course the stated and effective PFPS are the same.) We have the following result.

Corollary 5 *For all l in (7) and all $q \geq 0$, when the firm allows the CEO to transact with an IB:*

(a) the stated PFPS of the optimal contract that solves Program 1 always weakly exceeds the PFPS of the optimal contract when no hedging opportunities exist;

(b) the effective PFPS of the optimal contract that solves Program 1 is always weakly less than the PFPS of the optimal contract when no hedging opportunities exist.

This last corollary demonstrates that were one to formulate hypotheses about the effect of hedging on the design of compensation contracts, it is important to be specific about what particular PFPS one is referring to. Because the firm anticipates the CEO will "knock down" a portion of his incentive pay by off-loading it onto an IB, the firm optimally responds by boosting the stated PFPS of the contract he gives the CEO to an amount that always weakly exceeds the PFPS associated with the optimal contract were no hedges available. But, the net effect of the CEO knocking back down the PFPS of the firm's original contract is to give the CEO lower powered incentives than had no hedging possibilities existed, as this corollary reports. This phenomenon is explored further in the next section.

7 When the CEO has superior private information about PH's effectiveness

In this section, we extend the base model by considering the situation where the CEO has better knowledge of the PH's effectiveness, as measured by the correlation between the PH $\tilde{y}$ and the performance measure $\tilde{v}$, than the firm. This is a practically realistic and important extension, in so far as managers are likely to know many operational details about the firms they are hired to run not known by the firm's owners, and in particular, managers are likely to know more about the joint distribution between $\tilde{v}$ and $\tilde{y}$ than the firms' owners. Through this extension, we can determine how information asymmetry between a firm and its CEO about a hedge's effectiveness affects each of: the firm's value, the contract the firm gives the CEO, and the CEO's optimal contract transformation choices.

The assumptions underlying the model of this section are the same as those of the base model except that now we assume that: (a) the PH's effectiveness is state-dependent and the CEO knows the state before working for the firm, but the firm never acquires this information about the state; (b) the firm wants to hire the CEO regardless of the PH's effectiveness, i.e., regardless of the state; and (c) the firm does not make use of "communication based contracts," that is, the firm gives the CEO the same contract regardless of the hedge's effectiveness.[43]

Now onto the formal model. We consider a two state model where, in state 1, the correlation between the performance measure $\tilde{v}$ and the PH $\tilde{y}$ is zero, and in state 2 this correlation ρ is nonzero (and is the same ρ as in the base model). All aspects of the joint distribution of $\tilde{v}$ and $\tilde{y}$ other than the correlation are assumed to be independent of state: in particular, the means $E[\tilde{y}] = 0$ and $E[\tilde{v}|a] = wa$ of $\tilde{y}$ and $\tilde{v}$ are the same in both states, as are the variances σ_y^2 and σ_v^2, etc.

[43] It was common in the agency literature of the 1980s to posit that, when a CEO had private information not possessed by his firm, the firm could and would extinguish this information asymmetry by offering the CEO a menu of contracts, one for each possible realization of the CEO's private information, resulting in the CEO revealing his private information to the firm through his selection from among the menu of contracts. (Christensen [1981], among others, is representative of this approach.)

There are many potential difficulties with this "menu of contracts" approach. First, in view of the breadth of the CEO's possible private information, it is difficult to imagine that all information asymmetry between the firm and its CEO could be eliminated in real life by the "menu of contracts" approach. Second and related, there may be significant hurdles in getting the CEO to articulate fully his private information to the firm (for reasons related either to Williamson's [2002] "information impactedness" observations or for reasons related to Demski and Sappington's [1987] "delegated expertise" arguments). Finally, there may be several other reasons including complexity related costs, the practical difficulties of drafting the menu of contracts, and the lack of robustness of these "communication based" contracts (the design of such contracts can be quite sensitive to the details of the contracting environment) that inhibit their use in practice.

For all of the preceding reasons, and also for analytical simplicity, we do not proceed with the "menu of contracts" approach in this section.

State i occurs with probability $p_i \in (0,1)$. When the firm allows the CEO to directly transact with the IB and state i occurs, we denote the CEO's certainty equivalent after receiving the contract $s_p(v) = c_p + b_p v$ and optimally transacting with the IB by $CE_i(b_p|c_p,q,l)$. When state 2 occurs ($i=2$), this certainty equivalent is the same as what we previously referred to as $CE(b_p|c_p,q,l)$ in (19) with the reservation price $q^{res}(b_p|l)$ being the same as it appears in (16). When state 1 occurs ($i=1$), the certainty equivalent $CE_1(b_p|c_p,q,l)$ corresponds to $CE(b_p|c_p,q,l)$ as defined in (19) above provided the correlation ρ that appears in (19) is replaced by 0; the reservation price for the PH in state 1 is zero.

When the firm allows the CEO to transact with the IB, the sequence of events in this extended model runs as follows. First, the firm offers the CEO the contract $c_p + b_p v$. The CEO's certainty equivalent under this contract must meet or exceed the CEO's IR constraint in each state. The CEO then observes the state determining the PH's effectiveness and decides whether to work for the firm. Then the CEO transacts with the IB. Thereafter, the CEO makes his effort choice. Finally, all random variables assume their realized values; the CEO is paid by the firm according to the terms of the contract; and the CEO settles any transactions he engaged in with the IB. This sequence of events is summarized in the time-line below.

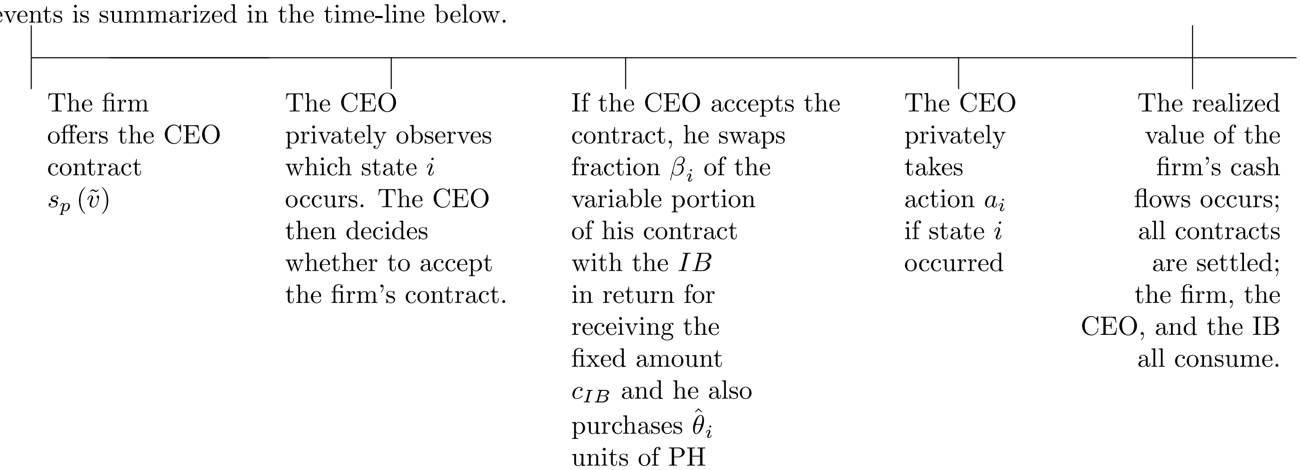

Figure 4 : Time line when the CEO has superior private information

There are other possible time lines corresponding to other possible cases. E.g., when the firm prohibits the CEO from transacting with the IB, but the firm itself transacts with the IB. In this case, the firm directly incorporates the PH into the contract with the CEO, and the sequence of events remains as described in the time line above, except now the contract takes the form $c_p + b_p v + d_p \bar{\theta} y$ (and, of course, the CEO does not transact with the IB).

The multi-state counterpart to Program 1 above that describes the firm's contracting problem when the CEO is allowed to transact with the IB is described below in Program 3. (The notation $E_i[\cdot|a_i^*]$ or $E_i[\cdot]$ refers to expected values when state i occurs; also, the CEO's optimal choice of the quantity of the swap and the PH if state i occurs is denoted by $\beta_i^*(b_p|q,l)$ and $\hat{\theta}_i^*(q|b_p,l)$ respectively.)

Program 3 (Optimal contracting when the CEO is allowed to transact with the IB and the CEO privately knows the effectiveness of the hedge)

$$\max_{s_p(\tilde{v})=c_p+b_p\tilde{v},a_i^*(b_p,\beta_i),\ \beta_i^*(b_p|q,l),\ \hat{\theta}_i^*(q|b_p,l)} p_1E_1[\tilde{v}-s_p(\tilde{v})|a_1^*(b_p,\beta_1^*(b_p|q,l))]+p_2E_2[\tilde{v}-s_p(\tilde{v})|a_2^*(b_p,\beta_2^*(b_p|q,l))]$$

subject to

for each b_p and β_i and state i, $a_i^* = a_i^*(b_p,\beta_i)$ as in (4) above;

for each b_p and β_i and state i, $c_{IBi}(\beta_i|b_p)$ is as in (6) above (with $E_i[\cdot|a_i^*]$ replacing $E[\cdot|a^*]$); (33)

when state i occurs, $\beta_i^*(b_p|q,l),\ \hat{\theta}_i^*(q|b_p,l) \in \arg\max_{b_p\geq\beta_i\geq 0,\ \hat{\theta}\geq 0} CE_i(T(\tilde{v},\tilde{y}|c_{IB_i}(\beta_i|b_p),\beta_i,\hat{\theta}_i,s_p),a_i^*(b_p,\beta_i^*(b_p|q,l)));$

and

for both state $i=1$ and $i=2$, $CE_i(T(\tilde{v},\tilde{y}|c_{IB_i}(\beta_i^*(b_p|q,l)),\beta_i^*(b_p|q,l),\hat{\theta}_i^*(q|b_p,l),s_p),a_i^*(b_p,\beta_i^*(b_p|q,l)) \geq \bar{U}.$

The meanings of these constraints are the same as the meanings of the corresponding constraints in Program 1 above except that expectations now are indexed by the state. It is worthwhile remarking, in reference to constraint (33) which details the constant payment the CEO gets from the IB in the swap, that since the CEO's preferred action choice is determined by the CEO's RRI in the original contract, and the IB infers the CEO's RRI correctly in equilibrium, it makes no difference whether the IB sees the state when he transacts with the CEO.[44]

The next theorem describes features of the solution to Program 3. This theorem makes use of the following notation: $\gamma \equiv \frac{(1-2l)w^2k+r\sigma_v^2}{(1-2l)w^2k+r\sigma_v^2(1-\rho^2)}$, $b_A^{*multistate}$, and $V_A^{multistate}$, where γ is a fraction useful for expressing the PFPS $b_A^{*multistate}$ of the optimal linear contract in this multi-state case, and $V_A^{multistate}$ refers to the value of the firm in this multi-state case; and as above, the subscript A indicates that the firm is allowing the CEO to transact with the IB.[45]

Theorem 5 *The solution to Program 3 has the following features:*

(a) if $q < q^{*res}(l)$, then:

[44] Expanding on this point: since the CEO's preferred action choice is dictated entirely by the expected value of the CEO's compensation and the disutility of the CEO's action, that action, and hence the expected value of the variable portion of the swap transferred to the IB, is affected by the state only to the extent that the CEO's optimal RRI varies with the state.

[45] We do not present the proofs of the theorems and other results of this section. These proofs are available from the authors.

(ai) the optimal contract's PFPS is given by:

$$b_A^{*multistate} = \frac{p_1 + p_2\gamma + p_2\gamma\theta_2^{FB} q/(1-l)w^2k}{2p_1 - (1-l) + 2p_2\gamma};$$

(aii) $\hat{\theta}_2^*(q|b_p, l)$ *is given by* $\hat{\theta}^*(q|b_p, l)$ *in (17) above with* $b_p = b_A^{*multistate}$; $\hat{\theta}_1^*(q|b_A^{*multistate}, l)$ *is equal to zero;*

(aiii) $\beta_2^*(b_p|q, l)$ *is given by* $\beta^*(b_p|q, l)$ *in (16) above with* $b_p = b_A^{*multistate}$*, and* $\beta_1^*(b_A^{*multistate}|q, l) = \frac{b_A^{*multistate}(r\sigma_v^2 - lw^2k)}{(1-2l)w^2k + r\sigma_v^2}$*; and*

(aiv) the value of the firm is given by:

$$V_A^{multistate} = \frac{\frac{1}{2}w^4k^2(1-l)\frac{\{p_1+p_2\gamma+p_2\gamma\theta_2^{FB}q/(1-l)w^2k\}^2}{1+l+2p_2(\gamma-1)} - p_2w^2k\gamma\theta_2^{FB}q}{(1-2l)w^2k + r\sigma_v^2}.$$

(b) When $q \geq q^{*res}(l)$*, the optimal contract's PFPS is* $b_A^{*multistate} = \frac{1}{1+l}$ *and the value of the firm is* $V_A^{multistate}(q, l) = \frac{1}{2}\left(\frac{(w^2k)^2(1-l)}{(1+l)((1-2l)w^2k + r\sigma_v^2)}\right)$.

When $q < q^{*res}(l)$, the solution to Program 3 depends on all of: how probable the two states are, the prices l and q of the hedges, and the effectiveness of the PH, as measured by the correlation ρ (note that ρ appears implicitly in both θ^{FB} and γ). The state-specific effectiveness of the PH affects the CEO's interactions with the IB, as the CEO buys the PH from the IB only in state 2 when the CEO knows the PH is effective. More subtly, since the PH and the swap are known to be substitutes (recall Corollaries 1 and 2 above), the CEO's preferred RRI in the original contract also varies by state, with the CEO purchasing more of the swap when he knows that the PH is not effective, even though the effectiveness of the swap is not state-specific.

When $q \geq q^{*res}(l)$, the solution to Program 3 is the same as the solution to Program 1 in the previous section where the effectiveness of the PH was commonly known to both the firm and the CEO and the price of the hedge is $q \geq q^{*res}(l)$. This follows because, when $q \geq q^{*res}(l)$, then regardless of whether the PH is effective, the PH is so expensive that the CEO never purchases any of it. Thus, fluctuations across states in the effectiveness of the PH are irrelevant, and that case devolves into the "one state" case previously studied.

The multi-state counterpart to Program 2 that describes the firm's contracting problem when the firm transacts directly with the IB (and the firm forbids the CEO from transacting with the IB) is presented in Program 4. The notation $var_i(s_p(\tilde{v}, \tilde{y}))$ in this program refers to the variance of the contract $s_p(\tilde{v}, \tilde{y})$ when state i occurs.[46]

[46] The constraint $\bar{\theta} \geq 0$ must be included in this program in order to make the potential bargaining power of the IB as manifest in the IB charging the CEO or the firm a price $q > 0$ per unit of the PH (which exceeds the IB's marginal cost of zero in supplying the PH) sensible, as we expounded upon in footnote 25 above.

Program 4 (Optimal contracting when the firm transacts directly with the IB, the firm forbids the CEO from transacting with the IB, and the CEO has private information about the state)

$$\max_{a_p^*,\ \bar{\theta}, s_p(\tilde{v},\tilde{y})=c_p+b_p\tilde{v}+d_p\bar{\theta}\tilde{y}} p_1 E_1[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*] + p_2 E_2[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*] - \bar{\theta}q$$

subject to:

$$a_p^* = b_p w k \tag{34}$$

and

$$\text{for each state } i,\ E_i[s_p(\tilde{v},\tilde{y})|a_p^*] - .5(a_p^{*2}/k + rvar_i(s_p(\tilde{v},\tilde{y})) \geq 0. \tag{35}$$

In stating the program this way, we have implicitly assumed that the firm never opts to obtain swaps from the IB. This assumption is without loss of generality, as the fixed-for-variable swaps are of no value to the firm (recall the discussion in Section 6). Another implicit feature of writing the program in this way is that the firm selects the same quantity of hedge in both states. This is necessarily a feature of having the firm contracting directly with the IB, since the firm itself does not see the state.[47] Because of this, and also because the CEO is prohibited from transacting with the IB in the present case, the CEO retains the same RRI in the original contract in both states, and hence takes the same action in both states, as we report in constraint (34). However, since the CEO's expected utility varies by state, we must present two IR constraints, one for each state, as in (35). Written out, the IR constraints for states 1 and 2 are respectively given by:

$$c_p + .5(b_p w)^2 k - .5r(b_p^2\sigma_v^2 + d_p^2\bar{\theta}^2\sigma_y^2) \geq 0;\ \text{and} \tag{36}$$

$$c_p + .5(b_p w)^2 k - .5r(b_p^2\sigma_v^2 + 2b_p d_p \rho\sigma_y\sigma_v\theta + d_p^2\bar{\theta}^2\sigma_y^2) \geq 0. \tag{37}$$

The constraint (36) (resp., (37)) reflects the absence (resp., presence) of a nonzero correlation between the PH $\tilde{y}$ and the performance measure $\tilde{v}$ in state 1 (resp., state 2).

The key to solving Program 4 is to conjecture that the IR constraint for state 2 will not bind, based on the intuition that any contract designed so that the CEO is willing to work for the firm when no effective hedge is available (in state 1) should also make the CEO willing to work for the firm when an effective hedge is available (in state 2). We formally solve Program 4 in the accompanying footnote, and we state the principal implications of the solution to Program 4 as the following lemma.[48]

[47] Or have the state communicated to it by the CEO in view of it being presumed that the firm does not offer the CEO "communication based contracts."

[48] To prove this lemma, first consider the program derived from Program 4 with the IR constraint for state 2, inequality (37), dropped. We first claim that for whatever $b_p > 0$ the firm chooses, the optimal choice for $\bar{\theta}$ for this less constrained

Lemma 2 *In the solution to Program 4, where the firm, but not the CEO, can transact with the IB, the optimal linear contract* $s_p^*(v,y) = c_p^* + b_p^* v + d_p^* \bar{\theta}^* y$ *has the following features:*

$$b_F^{multistate*} = \frac{w^2 k}{w^2 k + r\sigma_v^2};\ \bar{\theta}^* = 0; \tag{38}$$

and firm value is given by

$$V_F^{multistate} = V_{notrans} = .5 \times \frac{\left(w^2 k\right)^2}{w^2 k + r\sigma_v^2}.$$

The lemma reports that the firm optimally makes no use of the PH as part of the optimal solution to Program 4, and that this is true regardless of the probability p_2 that state 2 occurs, regardless of the effectiveness of the hedge as measured by the correlation ρ in state 2, and regardless of the price q of the PH. The core economic reason for this is that the PH does not help the firm in reducing the cost of hiring the CEO when state 1 occurs, and - as intimated in the discussion above - only the IR constraint when the hedge is ineffective, i.e., in state 1, is binding at the optimum. While the manager would benefit personally from a positive amount of hedging when state 2 occurs, none of that benefit can get conferred to the firm here in the form of a lower base salary c_p because lowering the base salary would result in a violation of the manager's IR constraint in state 1 (where the hedge is ineffective). This result is driven by the maintained assumption that a single linear contract must

version of Program 4 is $\bar{\theta} = 0$: to verify this, take any positive choice for $\bar{\theta}$, and note that in order to satisfy the (only) remaining IR constraint, (36), the base salary c_p can be reduced – as compared to the size of the base salary needed to satisfy this constraint were $\bar{\theta}$ set at $\bar{\theta} > 0$. Also notice that since the unconditional expected value of $\tilde{y}$ is zero, the expected value of the objective function is affected by changing $\bar{\theta}$ only to the extent that the base salary c_p has to be adjusted to meet the IR constraint. Further, since b_p is being held fixed in this discussion, changing $\bar{\theta}$ has no effect on the incentive constraint (34). Thus, reducing $\bar{\theta}$ to zero leads to an increase in the value of the firm's objective function. This proves the first claim.

Next, we claim that the objective function of Program 4 when $\bar{\theta}$ is set equal to zero collapses to $E[\tilde{v} - c_p - b_p\tilde{v}|a_p^*]$ (here, it does not matter whether the expectation $E[\cdot]$ is conditional on state 1 or state 2, since $\tilde{y}$ does not appear in the expectation). This follows because when $\bar{\theta} = 0 : E_1[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*] = E_1[\tilde{v} - (c_p + b_p\tilde{v} + d_p\bar{\theta}\tilde{y})|a_p^*] = E_1[\tilde{v} - (c_p + b_p\tilde{v})|a_p^*] = E_2[\tilde{v} - (c_p + b_p\tilde{v})|a_p^*] = E_2[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*]$; that is, both expectations $E_1[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*]$, $E_2[\tilde{v} - s_p(\tilde{v},\tilde{y})|a_p^*]$ are the same and independent of the state, so these two expectations can be combined together; from this observation it also follows that the two probabilities p_1 and p_2 disappear from the objective function.

Finally, notice that the relaxed version of Program 4 with constraint (36) deleted and the variable $\bar{\theta}$ set equal to zero is the standard linear contracting problem

$$\max_{a_p^*,\ \bar{\theta}\geq 0,\ s_p = c_p + b_p\tilde{v} + d_p\bar{\theta}\tilde{y}} E[\tilde{v} - c_p - b_p\tilde{v}|a_p^*] \text{ subject to } a_p^* = b_p wk \text{ and } c_p + .5(b_p w)^2 k - .5 r b_p^2 \sigma_v^2 \geq 0.$$

As is usual, to solve this last program, observe that for any candidate optimal PFPS $b_p > 0$, the base salary c_p optimally will be adjusted so that the IR constraint binds. Identifying the optimal (b_p- contingent) c_p in this way and substituting it into the objective function yields: $b_p w^2 k (1 - b_p) + .5(b_p w)^2 k - .5 r b_p^2 \sigma_v^2$. Maximizing this last expression yields the first-order condition for the optimal b_p:

$$w^2 k (1 - 2b_p) + b_p(w^2 k - r\sigma_v^2) = 0,$$

which implies $b_p = \frac{w^2 k}{w^2 k + r\sigma_v^2}$. Substituting this value of b_p, and the associated (b_p- contingent) c_p into the IR constraint (37) reveals that this IR constraint is satisfied, so we were justified in dropping that constraint. Finally, we note that when these choices of b_p and c_p are substituted back into the objective function, we discover that the firm's value is given by $.5\left(\frac{(w^2 k)^2}{w^2 k + r\sigma_v^2}\right)$.

satisfy the IR constraints in both states, which prevents the firm from sharing in the social benefits of hedging even if state 2 (where hedging is effective) occurs with very high probability.

We now get to the main results of this section by comparing the optimal firm values reported in the last lemma with Theorem 5. This comparison allows us to determine the desirability of allowing the CEO to transact directly with the IB versus prohibiting the CEO from transacting with the IB when the CEO has better information about the effectiveness of the PH than his employer.

Theorem 6 *(i) If the haircut l on the swap is sufficiently small and the price q of the PH is any price $q < q^{*res}(l)$, then*

$$V_A^{multistate} > V_F^{multistate}; \text{ and}$$

(ii) if the haircut on the swap l is positive (and in the interval (7)) and the price q of the PH is any price $q \geq q^{res*}(l)$, then

$$V_A^{multistate} < V_F^{multistate}.$$

This theorem addresses the classic question of when is it desirable to "co-locate" decision rights with information related to the decision (see, e.g., Jensen and Meckling [1992]). In this section, the CEO is better informed about the effectiveness of the PH than is the firm, and so to take advantage of that better information, it would seem that the CEO should be granted control of making the hedging decision. Since that control only has consequences if the CEO is permitted to transact with the IB, it would further seem to follow that the firm always would be better off letting the CEO transact with the IB. Part (ii) of the theorem shows that this conjecture is not always correct: notwithstanding the CEO's superior information about hedging, it can be beneficial not to give the CEO control of the hedging decision and transact directly with the IB. Specifically, when the PH is priced above the CEO's reservation price $q^{res*}(l)$, it is not desirable to let the CEO transact with the IB for any positive level of the haircut on the swap because letting the CEO transact with the IB in this circumstance results in the agency bearing a deadweight loss (due to the haircut) with no offsetting benefit (because the CEO never buys any units of the PH from the IB in either state).[49]

But, when the price of the PH is low enough so that the CEO would purchase a positive quantity of the PH through the IB when the PH is effective (if the firm allowed him to transact with the IB), then as part (i) reports, the value of the firm is positively affected by allowing the CEO to transact with the IB (relative to prohibiting the CEO from transacting with the IB). In this case, the CEO's superior information about the hedge's effectiveness can be exploited to the firm's benefit, as long as

[49] The haircut is a deadweight loss to the agency, since as noted above, there is no aspect of the swap that the firm cannot itself replicate by reducing the PFPS coefficient of the CEO's contract.

the deadweight loss due to the haircut on the swap is low enough. Further, this expected benefit for the firm exceeds the expected rents earned by the CEO in state 2, regardless of the values of p_1 and p_2.

This theorem yields a price-based explanation for why some firms allow their managers to transact with IBs at the same time other firms prohibit their managers from engaging in such transactions, even when all IBs charge all parties the same prices for all hedging services. To see this, note that each firm's CEO's reservation price $q^{res*}(l)$ varies with each of following firm-and-CEO specific parameters: the unhedged variance of the firm's performance measure (σ_v^2), the effectiveness of the PH (ρ), the marginal productivity of the CEO's effort (w), the CEO's risk aversion (r), and the disutility of CEO's effort (k). Thus, the firm-and-CEO specific reservation price $q^{*res}(l)$ can be above the (common) price q IBs charge for the PH at the same time this firm-and-CEO specific reservation price is below the price q for other firm/CEO pairs. Hence, for sufficiently small haircuts l, depending on the values of these parameter values for individual firms and CEOs, either the first part or the second part of Theorem 6 may apply.

Finally, by combining the results of Theorem 6 and Lemma 2, we have an explanation why, when firms do not permit their managers to transact directly with IBs, the firms need not find it desirable to hedge on behalf of their executives. That is, we explain how circumstances can arise in which the inequality $V_A^{multistate} < V_F^{multistate}$ holds at the same time $V_F^{multistate} = V_{notrans}$.

8 Conclusions

Recently, there has been substantial concern about CEOs hedging their compensation risk by engaging in transactions with investment banks. This article addresses some of these concerns by developing a principal-agent model of the contracting process in which we evaluate some of the economic effects of allowing CEOs to engage in these transactions. Our analysis emphasizes how, by transacting with investment banks, the CEOs transform the compensation contracts their firms give them into effectively different contracts. We show that some firms can benefit from allowing their CEOs to engage in these contract transformations at the same time other firms are made worse off. Central to our results are the combined effects of: first, the ability of the investment banks to synthesize performance hedges that reduce the compensation risk CEOs are subject to without compromising the pay-for-performance sensitivities of the firms' original contracts, thereby improving the principal-agent contracting process; second, the ability of investment banks to offer fixed-for-variable swaps which hedging device reduces the PFPS of the CEOs' original contracts and thereby reduce firms' control over their CEOs' compensation, and hence worsen the principal-agent contracting relationship; and third, the prices investment banks charge for hedging services, with increases in the price of performance hedges generally leading to decreases in firm value, and increases in the price of swaps often leading to increases in firm value. Our analysis suggests that different forms of hedging have different effects on managers' contract transformation choices and also their firms' market values.

Another general finding of our analysis is that by allowing CEOs to engage in compensation hedging, the PFPS of the contracts the firms *give* their CEOs need not be the same as the PFPS of the contracts the CEOs effectively *receive;* that is, stated or de jure PFPS are not the same as effective or de facto PFPS. This difference between stated and effective PFPS requires that researchers, investors, and others interested in assessing how strong CEOs' incentives are to work hard must not rely on the stated terms of CEO incentive compensation when formulating their assessments if the CEOs can hedge the risk in their compensation contracts. Moreover, we show that when firms are designing their CEOs' compensation contracts, their recognition of the discrepancy between stated and effective PFPS causes the firms to revise the stated PFPS of their CEOs' compensation contracts always to be strictly higher than what the stated PFPS would have been were compensation hedging forbidden, but - even with these adjustments - the effective PFPS of their CEOs' compensation contracts are always weakly lower than what they would have been had compensation hedging been forbidden.

8.1 Appendix: Selected Proofs

Proof of Theorems 2 and 3

The start of this proof derives the CEO's demand for the PH in the general case (applicable to Theorem 3) where the IB offers for sale both PH and swaps to the CEO. Then, we specialize to the case where the IB only sells the PH (applicable to Theorem 2).

We begin by showing that, if we take β as given, then among all $\hat{\theta} \geq 0$, the CEO's optimal value for $\hat{\theta} = \hat{\theta}(q|b_p, \beta) = (b_p - \beta)\theta$ is given by:

$$\hat{\theta}(q|b_p, \beta) = \begin{cases} (b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2}, \text{ if } (b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2} > 0 \\ 0, \text{ if } (b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2} \leq 0. \end{cases} \tag{39}$$

To see this, note that when $\theta > 0$ is optimal, the first-order condition associated with maximizing (15) with respect to θ is given by:

$$-(b_p - \beta)q - r(b_p - \beta)^2(\theta\sigma_y^2 + \sigma_{vy}) = 0.$$

(Observe that the second-order condition $-r(b_p - \beta)^2\sigma_y^2 < 0$ is satisfied as long as the CEO retains some of the bonus.) So, the optimal value for θ is given by:

$$\theta^* = -\frac{q}{r(b_p-\beta)\sigma_y^2} - \frac{\sigma_{vy}}{\sigma_y^2} = -\frac{q}{r(b_p-\beta)\sigma_y^2} + \theta^{FB} \equiv \theta^{FB} - H, \tag{40}$$

where $H \equiv \frac{q}{r(b_p-\beta)\sigma_y^2}$. If θ^* in (40) is zero or negative, then the optimal value of θ is $\theta^* = 0$. This proves (39).

Now, suppose q is so small that $\theta^* > 0$ and hence that $\hat{\theta} = (b_p - \beta)\theta^* > 0$.[50] Using (40) and (13), we can write $var(\tilde{v} + \theta^*\tilde{y})$ as:

$$\sigma_v^2 + \theta^{*2}\sigma_y^2 + 2\theta^*\sigma_{vy} = \sigma_v^2(1-\rho^2) + H^2\sigma_y^2.$$

Therefore, we can reduce $-(b_p-\beta)\theta^* q - \frac{r}{2}(b_p-\beta)^2 var(\tilde{v}+\theta^*\tilde{y})$ as:

$$-(b_p-\beta)\theta^{FB} q + \frac{q^2}{2r\sigma_y^2} - \frac{r}{2}(b_p-\beta)^2\sigma_v^2(1-\rho^2).$$

It follows that, by substituting this last expression for $-(b_p-\beta)\theta^* q - \frac{r}{2}(b_p-\beta)^2 var(\tilde{v}+\theta^*\tilde{y})$ into the certainty equivalent (15), the CEO's certainty equivalent can be written as

$$c_p + (1-l)(b_p-\beta)\beta w^2 k - (b_p-\beta)\theta^{FB} q + \frac{q^2}{2r\sigma_y^2} + .5(b_p-\beta)^2(w^2k - r\sigma_v^2(1-\rho^2)), \text{ if } \theta > 0. \tag{41}$$

Then by maximizing (41) we obtain the first-order condition for β:

$$(1-l)w^2k(b_p - 2\beta) + \theta^{FB} q - (b_p-\beta)(w^2k - r\sigma_v^2(1-\rho^2)) = 0, \text{ or equivalently,}$$

$$(1-l)w^2kb_p + \theta^{FB} q - b_p(w^2k - r\sigma_v^2(1-\rho^2)) = \{2(1-l)w^2k - (w^2k - r\sigma_v^2(1-\rho^2))\}\beta. \tag{42}$$

Solving this last equation for β shows that $\beta^*(l|b_p,q)$, which we temporarily write as $\beta_+^*(b_p|q,l)$ (the "+" indicating that we are dealing with the case where $\theta > 0$) is given by:

$$\beta_+^*(b_p|q,l) = \frac{b_p(r\sigma_v^2(1-\rho^2) - lw^2k) + \theta^{FB} q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}, \text{ if } \theta > 0, \tag{43}$$

which in turn implies that the CEO's RRI in the original contract is given by:

$$b_p - \beta_+^*(b_p|q,l) = \frac{b_p(1-l)w^2k - \theta^{FB} q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}, \text{ if } \theta > 0. \tag{44}$$

[All of the current paragraph is devoted to the special case of Theorem 2, i.e., where the IB is presumed to sell only has the PH to the CEO and the CEO optimally buys a positive amount of the PH, the CEO's CE is obtained from (41) and (39) by setting $\beta = l = 0$ in (41)). In this special case, the CE becomes:

$$CE = c_p - b_p\theta^{FB} q + \frac{q^2}{2r\sigma_y^2} + .5b_p^2(w^2k - r\sigma_v^2(1-\rho^2)), \text{ when } \theta > 0, \text{ i.e., when } b_p\theta^{FB} > \frac{q}{r\sigma_y^2}$$

[50] We shall show exactly how large q can be while the CEO maintains a positive demand for the swap below.

This expression for the CE applies when $q < q^{res}(b_p) \equiv -\sigma_{vy} r b_p$ (since by appealing to (39), we see that when $\beta = 0$, then $(b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2} = 0$ iff $b_p \theta^{FB} r\sigma_y^2 = q$, i.e., iff $-\sigma_{vy} r b_p = q$. Thus, when $q < q^{res}(b_p)$, since the constant c_p will be set to make the IR constraint bind, and since the CEO's preferred action when no swaps are available is given by $a^* = b_p wk$, the firm's net expected profits upon giving the CEO the contract $s_p(\tilde{v}) = c_p + b_p \tilde{v}$ and allowing the CEO to transact with the IB are given by

$$\begin{aligned} E[\tilde{v} - s_p(\tilde{v})|a^*] &= (1-b_p)E[\tilde{v}|a^*] - c_p = (1-b_p)b_p w^2 k - c_p \\ &= b_p(w^2k - \theta^{FB} q) + \frac{q^2}{2r\sigma_y^2} - .5b_p^2(w^2k + r\sigma_v^2(1-\rho^2)). \end{aligned}$$

It follows that the first-order condition b_p when $q < q^{res}(b_p)$ is given by:

$$w^2k - \theta^{FB} q - b_p(w^2k + r\sigma_v^2(1-\rho^2)) = 0.$$

Hence, the optimal PFPS b_p^* is given by

$$b_p^* = \frac{w^2k - \theta^{FB} q}{w^2k + r\sigma_v^2(1-\rho^2)}.$$

Plugging this expression for b_p^* into the firm's objective function, we get:

$$.5\frac{(w^2k - \theta^{FB} q)^2}{w^2k + r\sigma_v^2(1-\rho^2)} + \frac{q^2}{2r\sigma_y^2}.$$

Since $q^{res}(b_p)$ itself depends on q, we must be more careful in determining what range of prices q to which the preceding applies. This applies iff $q < q(b_p^*) = -\sigma_{vy} r \frac{w^2k - \theta^{FB} q}{w^2k + r\sigma_v^2(1-\rho^2)}$,which reduces to, iff

$$q < \frac{-\sigma_{vy} r w^2 k}{w^2k + r\sigma_v^2} \equiv q^{*res}.$$

For all $q \geq q^{*res}$, the CEO buys no PH from the IB, in which case the CEO's certainty equivalent is given by $c_p + .5b_p^2(w^2k - r\sigma_v^2)$ (as we can see directly from (15) by substituting $b = \theta = 0$), and so the firm's objective function becomes:

$$\max_{b_p} \ (1-b_p)b_p w^2 k + .5b_p^2(w^2k - r\sigma_v^2) = \max_{b_p} \ b_p w^2 k - .5b_p^2(w^2k + r\sigma_v^2).$$

It is easy to check that the firm's preferred choice for b_p in this case is given by $b_p = \frac{w^2k}{w^2k + r\sigma_v^2}$ and that the value of the objective function evaluated at this b_p is $.5\frac{(w^2k)^2}{w^2k + r\sigma_v^2}$.

Returning to the general case of Theorem 3. Let $Q_+ \equiv \{q \geq 0|$ the CEO's demand for the PH at price q per unit is positive$\}$. Our next goal is to establish that $Q_+ = [0, q^{res}(b_p|l))$, where $q^{res}(b_p|l)$ defined as in (14), uniquely solves the equation $\hat{\theta}(q^{res}(b_p|l)|b_p, \beta) = 0$.

We know from (39) that, if $q \in Q_+$, the CEO's demand $\hat{\theta}$ for the PH is given by $\hat{\theta} = \hat{\theta}(q|b_p, \beta) = (b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2}$, and otherwise, demand for the PH is zero. Thus, we know:

$$\text{for all } q \geq 0: \ \hat{\theta}(q|b_p, \beta) = \max\{(b_p - \beta)\theta^{FB} - \frac{q}{r\sigma_y^2}, 0\}. \tag{45}$$

Since the CEO's preferred choice of β is given by $\beta_+^*(b_p|q, l)$ when the demand for the PH is positive, we can refine the statement (45) to:

$$\text{for all } q \geq 0: \ \hat{\theta}(q|b_p, \beta) = \max\{(b_p - \beta_+^*(b_p|q, l))\theta^{FB} - \frac{q}{r\sigma_y^2}, 0\}.$$

Thus, in view of (43) and (44), we can restate this last assertion as:

$$\text{for all } q \geq 0: \ \hat{\theta}(q|b_p, \beta) = \max\{\frac{b_p(1-l)w^2k - \theta^{FB}q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q}{r\sigma_y^2}, 0\}. \tag{46}$$

To complete the proof, all that has to be done is verify that, with $q^{res}(b_p|l)$ defined as in (14), for all $q < q^{res}(b_p|l)$, demand for the PH is positive, whereas for all $q \geq q^{res}(b_p|l)$ demand for the PH is zero. Equivalently, it sufficient to show that $q^{res}(b_p|l)$ uniquely solves the equation $\hat{\theta}(q^{res}(b_p|l)|b_p, \beta) = 0$, i.e.,

$$\frac{b_p(1-l)w^2k - \theta^{FB}q^{res}(b_p|l)}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q^{res}(b_p|l)}{r\sigma_y^2} = 0,$$

and hence, that $\frac{b_p(1-l)w^2k - \theta^{FB}q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q}{r\sigma_y^2} \gtrless 0$ as $q \lessgtr q^{res}(b_p|l)$. This (easy) computation is omitted.

Next, suppose q is so big that $\hat{\theta} = \theta = 0$. In that case, $-(b_p - \beta)\theta^* q - \frac{r}{2}(b_p - \beta)^2 var(\tilde{v} + \theta^* \tilde{y})$ obviously simplifies to $-\frac{r}{2}(b_p - \beta)^2 var(\tilde{v})$, so the CEO's certainty equivalent (15) reduces to:

$$c_p + (1-l)(b_p - \beta)\beta w^2 k + .5(b_p - \beta)^2(w^2k - r\sigma_v^2), \text{ if } \theta = 0. \tag{47}$$

Maximizing this last expression with respect to β leads to the optimal $\beta(\equiv \beta_0^*(b_p|q, l))$ in the case where $\theta = 0$. We now write as $\beta_0^*(b_p|q, l)$ (the "0" indicating that we are dealing with the case where $\theta = 0$). We obtain:

$$\beta_0^*(b_p|q, l) = \frac{b_p(r\sigma_v^2 - lw^2k)}{(1-2l)w^2k + r\sigma_v^2}, \text{ if } \theta = 0 \tag{48}$$

which in turn implies that the CEO's RRI in the original contract takes the form:

$$b_p - \beta_0^*(b_p|q, l) = \frac{b_p(1-l)w^2k}{(1-2l)w^2k + r\sigma_v^2}, \text{ if } \theta = 0. \tag{49}$$

The proof of (16) now follows directly from (17), along with the derivations of $\beta_+^*(b_p|q, l)$ in (43) and $\beta_0^*(b_p|q, l)$ in (48) above. ■

Proof of Theorem 4

In the text, at (21), we showed that, given the contract $s_p'(\tilde{v}) = c_p(b_p|q,l) + b_p v$, the firm's expected profits are given by:

$$E[\tilde{x} - s_p'(\tilde{v})|a^*(\beta|q)|b_p)] = (1-b_p)w^2k(b_p - \beta) - c_p(b_p|q,l). \tag{50}$$

If q is sufficiently small so that the optimal value for θ is positive,[51] then the CEO's preferred choice of β is given by $\beta_+^*(b_p|q,l)$ in (43), and so $b_p - \beta$ is given by $b_p - \beta_+^*(b_p|q,l)$ in (44), and $c_p(b_p|q,l) = -\frac{1}{2}\left(\frac{q^2}{r\sigma_y^2} + \frac{(b_p(1-l)w^2k - \theta^{FB}q)^2}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\right)$ (as given in (20)). Thus, the firm's expected profits, (50), in this case are given by:

$$(1-b_p)w^2k\left(\frac{b_p(1-l)w^2k - \theta^{FB}q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\right) + \frac{1}{2}\left(\frac{q^2}{r\sigma_y^2} + \frac{(b_p(1-l)w^2k - \theta^{FB}q)^2}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\right). \tag{51}$$

Maximizing (51) with respect to b_p results in the same first-order condition as does maximizing:

$$(1-b_p)w^2k(b_p(1-l)w^2k - \theta^{FB}q) + .5(b_p(1-l)w^2k - \theta^{FB}q)^2.$$

The first-order condition for the latter objective function is:

$$(1-b_p)w^2k(1-l)w^2k - w^2k(b_p(1-l)w^2k - \theta^{FB}q) + (b_p(1-l)w^2k - \theta^{FB}q)(1-l)w^2k = 0,$$

which yields the result that

$$b_p = \frac{w^2k(1-l) + l\theta^{FB}q}{w^2k(1-l^2)} \tag{52}$$

Define the function $b_p^+(q,l) \equiv \frac{w^2k(1-l)+l\theta^{FB}q}{w^2k(1-l^2)}$, which we have just shown to be the firm's optimal choice of the PFPS when the price q of the PH is sufficiently small that CEO's demand for the PH is positive. Recall the function $\hat{\theta}^*(q|b_p,l)$ from (17), which defines the CEO's demand for the PH, given the prices q and l and given the PFPS b_p. Construct the composite function $f_+(q,l) \equiv \hat{\theta}^*(q|b_p^+(q,l),l)$. Then, $f_+(q,l)$ is the CEO's demand for the PH, given the (low) price q and given l and given that the firm (optimally) offers the CEO a contract with PFPS $b_p^+(q,l)$. Since the CEO's demand for the PH is always nonnegative, we can define the CEO's demand for the PH given *any* $q \geq 0$, given any l satisfying (7), and given the PFPS $b_p^+(q,l)$ by the function: $\max\{f_+(q,l), 0\}$.

We next claim that the price $q^{*res}(l)$ defined by:

$$q^{*res}(l) \equiv \frac{w^2k(1-l)\theta^{FB}r\sigma_y^2}{(\theta^{FB})^2r\sigma_y^2 + (1+l)((1-2l)w^2k + r\sigma_v^2(1-\rho^2))} \tag{53}$$

is the unique positive price such that

$$f_+(q,l) \gtrless 0 \text{ as } q \lesseqgtr q^{*res}(l), \tag{54}$$

[51] We show exactly how large q can be while θ remains positive in (54) below.

i.e., $q^{*res}(l)$ is the CEO's reservation price for the PH when the firm gives the CEO a contract with PFPS $b_p^+(q^{*res}(l), l)$.

We will show this by showing that $q^{*res}(l)$ uniquely satisfies the equation $f_+(q, l) = 0$. Notice some algebra yields the following equations:

$$\begin{aligned} f_+(q^{*res}(l), l) &= \hat{\theta}^*(q^{*res}(l)|b_p^+(q^{*res}(l), l), l) = \frac{b_p^+(q^{*res}(l), l)(1-l)w^2k - \theta^{FB} q^{*res}(l)}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\theta^{FB} - \frac{q^{*res}(l)}{r\sigma_y^2} \\ &= \frac{w^2k(1-l)\theta^{FB} - (\theta^{FB})^2 q^{*res}(l)}{(1+l)((1-2l)w^2k + r\sigma_v^2(1-\rho^2))} - \frac{q^{*res}(l)}{r\sigma_y^2} \qquad (55) \\ &= 0. \end{aligned}$$

The uniqueness of $q^{*res}(l)$ is apparent, for example, from line (55) of the preceding, which establishes that the composite function $f_+(q^{*res}(l), l)$ is a decreasing function of $q^{*res}(l)$, and so there cannot be more than one value of $q^{*res}(l)$ that satisfies $f_+(q^{*res}(l), l) = 0$. That $q^{*res}(l)$ is positive is obvious.

A similar analysis applies in the case where q is so large that the optimal value for θ is zero. We omit the details. This proves part (a) of the proposition.

Part (b) is immediate from the discussion in the text. To prove part (c), in case $q < q^{*res}(l)$, just substitute (52) into (51) to get:

$$\begin{aligned} &(1-b_p)w^2k\left(\frac{b_p(1-l)w^2k - \theta^{FB}q}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)}\right) + \frac{q^2}{2r\sigma_y^2} + \frac{1}{2}\frac{(b_p(1-l)w^2k - \theta^{FB}q)^2}{(1-2l)w^2k + r\sigma_v^2(1-\rho^2)} \\ = \;& \frac{1}{2}\left(\frac{(w^2k(1-l) - \theta^{FB}q)^2}{(1-l^2)((1-2l)w^2k + r\sigma_v^2(1-\rho^2))} + \frac{q^2}{r\sigma_y^2}\right) \qquad (56) \end{aligned}$$

where the last equation follows from some algebraic manipulation.

When $q \geq q^{*res}(l)$, similar calculations show that the maximum expected profits of the firm are given by:

$$\frac{1}{2}\left(\frac{(w^2k)^2(1-l)}{(1+l)((1-2l)w^2k + r\sigma_v^2)}\right). \blacksquare$$

9 References

Aghion, P., and P. Bolton. 1992. An incomplete contracts' approach to financial contracting. Review of Economic Studies 59: 473-494.

Antle, R., and A. Smith. 1986. An empirical investigation of the relative performance evaluation of corporate executives. Journal of Accounting Research 24 (2): 1-39.

Arya, A., J. Glover, and S. Sunder. 1998. Earnings management and the revelation principle. Review of Accounting Studies 3: 7-34.

Baker, G., R. Gibbons, and K. J. Murphy. 1994. Subjective performance measures in optimal incentive contracts. The Quarterly Journal of Economics 27 (109): 1125-1156.

Bar-Gill, O., and L. Bebchuk. 2003. The Costs of Permitting Managers to Sell Shares. Boston, MA: Harvard University Press.

Bebchuk, L., and J. Fried. 2004. Pay Without Performance: The Unfulfilled Promise of Executive Compensation. Boston, MA: Harvard University Press.

Bettis, J. C., J. Bizjak, and S. Kalpathy. 2010. Why do insiders hedge their ownership? An empirical examination. Working paper, Arizona State University.

Bettis, J. C., J. Bizjak, and M. Lemmon. 2001. Managerial ownership, incentive contracting and the use of zero-cost collars and equity swaps by corporate insiders. Journal of Financial and Quantitative Analysis 36 (3): 345-370.

Bharath, S., S. Dahiya, A. Saunders, and A. Srinivasan. 2007. So what do I get? The bank's view of lending relationships. Journal of Financial Economics 85 (3): 368-419.

Bushman, R., and R. Indjejikian. 1993. Accounting income, stock price, and managerial compensation. Journal of Accounting and Economics 16: 3-23.

Dash, E. 2011. Stock-hedging lets bankers skirt efforts to overhaul pay. The New York Times (February 5).

Datar, S., S. Kulp, and R. Lambert. 2001. Balancing performance measures. Journal of Accounting Research 39 (1): 75-92.

Demarzo, P., and D. Duffie. 1995. Corporate incentives for hedging and hedge accounting. Review of Financial Studies 8: 743-771.

Demski, J. S. 1998. Performance measure manipulation. Contemporary Accounting Research 15 (3): 261-285.

Demski, J. S., and D. Sappington. 1987. Delegated expertise. Journal of Accounting Research 25 (1): 68-89.

Dye, R. 1992. Relative performance evaluation and project selection. Journal of Accounting Research 30 (1): 27-52.

Dye, R. 1988. Earnings management in an overlapping generations model. Journal of Accounting Research 26 (2): 195-235.

Feltham, G. A., and J. Xie. 1994. Performance measure congruity and diversity in multi-task principal/agent relations. The Accounting Review 69 (3): 429-453.

Froot, K., D. S. Scharfstein, and J. Stein. 1993. Risk management: Coordinating corporate investment and financing policies. Journal of Finance 48: 1629-1658.

Froot, K., D. S. Scharfstein, and J. Stein. 1994. A framework for risk management. Harvard Business Review 72 (6): 59-71.

Gao., H. 2010. Optimal compensation contracts when managers can hedge. Journal of Financial Economics 97: 218-238.

Holmstrom, B. 1979. Moral hazard and observability. Bell Journal of Economics 10: 74-91.

Holmstrom, B., and J. Ricarti-Costa. 1986. Managerial incentives and capital management. Quarterly Journal of Economics 101 (4): 835-860.

Hull, J. 2012. Options, Futures, and Other Derivatives. 8th (revised). Upper Saddle River, NJ: Prentice Hall.

Internal Revenue Service (IRS). 2008. Variable Prepaid Forward Contracts Incorporating Share Lending Arrangements. Coordinated Issue Paper. LMSB-04-1207-077. Washington, D.C.: IRS.

Jagolinzer, A., S. Matsunaga, and E. P. Yeung. 2007. An analysis of insiders' use of prepaid variable forward transactions. Journal of Accounting Research 45 (5): 1055-1079.

Jensen, M., and W. Meckling. 1992. Specific and general knowledge and organizational structure. Contract Economics: 251-274.

Jorgensen, B. 1999. Essays on Accounting, Capital Structure, and Risk Management. Ph.D. dissertation, Northwestern University.

Lambert, R. A. 2001. Contracting theory and accounting. Journal of Accounting and Economics 32 (1-3): 3-87.

Lazear, E. P., and S. Rosen. 1981. Rank-order tournaments as optimum labor contracts. Journal of Political Economy 89 (5): 841-864.

Laux, V. 2010. On the benefits of allowing CEOs to time their stock option exercises. RAND Journal of Economics 41: 118-138.

Melumad, N., G. Weyns, and A. Ziv. 1999. Comparing alternative hedge accounting standards: Shareholders' perspective. Review of Accounting Studies 4: 265-292.

Sapra, H. 2002. Do mandatory hedge disclosures discourage or encourage excessive speculation. Journal of Accounting Research 30 (3): 933-964.

Schenone, C. 2010. Lending relationships and information rents: Do banks exploit their information advantage. The Review of Financial Studies 23 (3): 1149-1199.

Smith, Clifford and R. Stulz. 1985. The determinants of firms' hedging policies. Journal of Financial and Quantitative Analysis 20 (4): 391-405.

Wang, L. 2014. Recognizing the Best: The Performance and Sabotage Effects of Relative Performance Recognition. Working paper, The University of Texas at Austin.

Williamson, Oliver E. 2002. The theory of the firm as governance structure: From choice to contract. Journal of Economic Perspectives 16 (3): 171-195.