# Information Disclosure, Growth, and the Cost of Capital

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**Abstract**

This paper studies how information disclosure affects the cost of equity capital (i.e., risk premium) and investors welfare in a dynamic setting with overlapping generations of investors. Our analysis demonstrates that a firm's cost of capital decreases (increases) in the precision of public disclosure if the firm's growth rate is below (above) a certain threshold. The threshold growth rate is higher for firms with more persistent cash flows. We find that while current shareholders always prefer maximum public disclosure, future shareholders' welfare decreases (increases) in the precision of public disclosure if the firm's growth rate is below (above) the threshold. Our results extend to multi-firm large economies in which a firm's risk premium depend only on its systematic risk. We find that a firm's threshold growth rate below which its risk premium declines in disclosure quality is lower (higher) when other firms in the economy are growing at fast (slow) rates. We also examine a production economy in which information disclosure has real effects on firms' internal investment choices.

*JEL classification*: G12, D90, M45.

*Keywords*: Cost of capital; Information disclosure; Risk premium; Overlapping generations model.

# 1 Introduction

The link between information disclosure and the cost of equity capital is of fundamental interest to academicians and regulators alike.[1] In an influential paper, Easley and O'Hara (2004) examine the relationship between the quality of firms' public disclosures and their cost of equity capital. A central finding of their analysis is that the cost of capital decreases in the quality of public disclosures, since the informed investors are better able to shift their portfolio weights to reflect new information.[2] However, they derive this result of a negative relation between disclosure quality and cost of capital in a static (single period) pure exchange setting. Consequently, there is a need to investigate whether and how the prediction of a negative relationship between information disclosure and cost of capital will apply to dynamic multiperiod settings in both pure exchange and production economies.

This paper develops such a dynamic model of an infinitely lived firm owned by overlapping generations of investors. At the end of each period, the firm publicly discloses a signal that provides information about the cash flows to be realized in the next period. The precision of the firm's public signals is governed by exogenously enforced accounting standards and reporting requirements. Each generation of investors expect to earn returns in the form of cash dividends and capital gains resulting from the sale of their shares to the investors of the next generation. The risk premium in each period is therefore determined by the sum of the premium that the investors demand for bearing *dividend* risk and the premium associated with resale *price* risk. A more informative disclosure regime reduces the conditional variance of the forthcoming cash flows and hence lowers the risk premium demanded by the current shareholders for bearing the dividend risk.[3] However, a more precise disclosure also makes the resale price more volatile, which results in an increase in the risk premium associated with the price component of the shareholders' payoffs. Therefore, the equilibrium relationship between disclosure quality and risk premia depends on the relative strength of these two countervailing effects.

In particular, our analysis shows that the relationship between risk premium and information depends on the firm's growth trajectory. When the firm is growing at a rate slower than

[1] Referring to this link, Arthur Levitt, the former Chairman of the Securities and Exchange Commission, suggests that "high quality accounting standards...reduce capital costs".

[2] Hughes et al. (2007) and Lambert et al. (2007) observe that this line of argument holds only when information disclosure pertains to systematic risk.

[3] Initially, we examine a single firm setting in which all risk is systematic and priced as such. We later verify that our results extend to multi-asset economies with both idiosyncratic and systematic risks.

a certain threshold, a more informative public disclosure system results in lower risk premium. On the other hand, the risk premium *increases* in the precision of public disclosures for a firm in relatively fast growth phase (i.e., when its growth rate exceeds the threshold growth rate). We note that price (dividend) risk represents uncertainty associated with the firm's more immediate (distant) future cash flows. A fast growing firm's cash flows are riskier in more distant future periods because they reflect payoffs from the firm's more recent, and hence larger, investments. As a consequence, investors are more concerned about the price risk component, and hence overall risk premium increases in the informativeness of public disclosures or such firms. On the other hand, dividend risk is the dominant determinant of overall risk premium for low growth firms, and hence the risk premium decreases in the precision of public information for such firms. Therefore, in contrast to Easley and O'Hara (2004), who predict an unambiguously negative relationship between the quality of public disclosure and the risk premium, our analysis shows that the nature of this relationship crucially hinges on a firm's growth trajectory in dynamic settings.

If the firm's cash flows are uncorrelated, then the threshold growth rate (above which the risk premium increases in the precision of public information) is simply equal to the risk-free interest rate. We demonstrate that the threshold rate monotonically increases in the degree of correlation among periodic cash flows. With autocorrelated cash flows, the one-period ahead market price (i.e., the resale price for the current generation of buyers) will vary with (i) the firm's public disclosure about its future dividends, as well as (ii) the realized dividends in that period. While a policy of higher quality disclosures has the effect of increasing the first component of the price risk, it lowers the price variability due the second component. As a consequence, the resale price risk is relatively less sensitive to the quality of public disclosures, and hence the overall risk premium decreases in the precision of public information for a larger range of growth rates.

With respect to investors welfare, we find that disclosure preferences of the firm's current shareholders often diverge from those of future shareholders. While the existing shareholders unequivocally prefer the most precise disclosure policy, the expected utilities of future generation of investors *increase* in the periodic risk premium during their investment horizon.[4] Consequently, preferences of future generations of investors for the amount of public

[4]This is a consequence of the observation that investors generally prefer access to riskier assets, since they can earn more surplus for bearing risks associated with these assets. See Kurlat and Veldkamp (2015) for a similar result.

information depend on growth during their investment horizons. An implication of these results is that the net impact of public information on total social welfare depends on how one weighs the utilities of different generations in the overall social welfare function.

We show that our results readily extend to settings with multiple risky firms in which each firm's periodic cash flows are subject to both systematic (i.e., market-wide) and idiosyncratic (i.e., firm-specific) shocks. Each firm releases a public signal that contains information on both the systematic and idiosyncratic components of its future cash flows.[5] In our dynamic setting, the equilibrium risk premium again consists of the dividend and price risk components. In a multi-firm economy, however, these risk premium components are determined by the conditional *covariances* of a firm's dividends and resale prices with the corresponding variables for the market as a whole.

Confirming the standard intuition from static settings, we find that a firm's risk premium in large multi-firm economies depend only on systematic risk, since investors can eliminate their exposure to idiosyncratic risk by holding well-diversified portfolios. As in the single firm setting, a firm's risk premium decreases (increases) in the informativeness of disclosure system if the firm's growth rate is below (above) a certain threshold. We find, however, that the threshold growth rate decreases in the average growth rate in the economy. In equilibrium, each investor holds a share of the market portfolio. Consequently, we find that future shareholders' welfare decreases (increases) in the precision of public information when the aggregate investment in the economy is expected to grow slower (faster) than the risk-free interest rate during their investment horizon. As before, current shareholders' welfare is always maximized by the most informative disclosure policy.

While most of our paper focuses on a pure exchange setting in which firms' investment choices are exogenous, we also consider a production economy in which each generation of shareholders is in charge of choosing the firm's investments during its period of ownership. In such a setting, public disclosure of information has not only intergenerational risk allocation effects as identified in our pure exchange settings, but also real effects on the firm's internal investment choices. As before, we find that more informative public disclosures result in lower (higher) risk premium when the firm is growing at a rate lower (higher) than a certain threshold. However, the threshold growth rate above which the risk premium increases in

[5] In a large economy, each firm's disclosure would contain relatively small amount of information on systematic risk. However, we find that aggregated firm disclosures can still have a large effect on risk premium.

the quality of public information is lower in the endogenous investment setting than that in the pure exchange setting. Furthermore, in contrast to our finding in the pure exchange setting that the firm's existing shareholders always prefer maximum level of public disclosure, we find plausible circumstances under which current shareholders' welfare is maximized at an intermediate level of disclosure. The reason is that future shareholders invest too much relative to the preferred amount of investment from the perspective of the firm's existing shareholders, and the level of this overinvestment increases in quality of public disclosures.

Our theory provides guidance for empirical studies that seek to examine the link between disclosure quality and the cost of equity capital (see, for example, Botosan 1997, Botosan and Plumlee 2002, Easley et al. 2002, and Francis et al. 2008). Specifically, our analysis predicts that the relationship between cost of capital and quality of accounting disclosures should depend on a firm's growth trajectory. Without sorting on investment (or, cash flow) growth rates, the average relation between quality of disclosure and cost of capital would reflect the relative mix of high and low growth firms in the economy. This might explain some of the earlier mixed findings in this area. More generally, our analysis suggests a need for sorting on growth rate in future empirical investigations of the link between information disclosure and cost of capital. Another potentially testable implication of our theory is that the relation between cost of capital and disclosure is more likely to be negative for firms with more persistent cash flows. Furthermore, our multi-asset results are relevant for empirical studies that seek to examine the link between the aggregate information environment and the market cost of capital (e.g., Bhattacharya et al. 2003; Bhattacharya and Daouk 2002; Jain 2005).

In disclosure regulation contexts, a lower cost of capital is frequently cited as a justification for improving disclosure quality. For instance, Foster (2003) writes: "*Less uncertainty results in less risk and a consequent lower premium being demanded. In the context of financial information, the end result is that better disclosure results in a lower cost of capital.*" However, our analysis of the effects of public information on investors' equilibrium expected utilities demonstrates that the cost of capital is generally not an appropriate metric to rank welfare implications of alternative public disclosure policies. More generally, we find that public information has an ambiguous effect on total social welfare because its impact on current and future shareholders' welfare is dependent on growth and technology.

In terms of the basic modeling framework, our paper is related to the asset pricing literature based on infinite horizon overlapping generations models with the CARA-Normal structure (e.g., Albagli 2015, Bachheta 2006, Banerjee 2011, De Long et al. 1990, Spiegel 1998, and Watanabe 2008). For modeling the large economy limit in our multi-firm setting, we follow Hughes et al. (2007). A number of papers (e.g., Christensen et al. 2010, Easley and O'Hara 2004, Hughes et al. 2007, Lambert et al. 2007) investigate the relationship between information disclosure and the cost of capital. However, these papers model static pure exchange settings, and hence do not investigate how endogenous and exogenous growth affects the link between disclosure quality, risk premium, and investors' welfare. Kurlat and Veldkamp (2015) show that regulations that require more information disclosure can negatively affect investor welfare. Our paper demonstrates that the disclosure preferences of the current and future shareholders are generally different and dependent on the firm's technology and growth trajectory. Understanding the preferences of different generations of shareholders is important in light of the stated objective of the Financial Accounting Standards Board (FASB) to provide information "useful to existing and potential investors, lenders, and other creditors".[6]

The rest of the paper is organized as follows. Section 2 describes the basic setting. Section 3 develops a model of information disclosure in a single firm setting and characterizes the equilibrium relationships among information disclosure, risk premium, and investors welfare when cash flows are serially uncorrelated. Section 4 considers a setting with serially correlated cash flows. We examine multi-asset and production economies in Section 5. Section 6 concludes the paper.

# 2 Model Setup

We consider an economy where shares of a single risky firm and a risk-free asset are traded among overlapping generations of identical risk-averse investors. While the firm is an infinitely lived entity, investors live only for a finite time. Specifically, generation $t$ investors buy the shares of the firm from the previous generation at date $t-1$ and sell them to the next generation at date $t$. The investors of each generation have homogenous prior beliefs and symmetric information about the firm's future cash flows. The firm's shares are traded in a perfectly competitive market. We assume that the risk-free asset is in unlimited supply

[6] See the FASB Conceptual Framework for Financial Reporting (FASB 2010).

and yields a rate of return of $r > 0$. Let $\gamma \equiv \frac{1}{1+r}$ be the corresponding risk-free discount factor.

The firm undertakes a sequence of overlapping projects each with a useful life of two periods. The firm's history of investments is known to all future generations of investors. Let $I_t$ denote the scale of the project implemented at date $t$ with its cost of investment denoted by $c(I_t)$. This project generates uncertain cash flows of $X_{t+2}$ dollars at date $t + 2$:

$$X_{t+2} = I_t \cdot (x_{t+2} + m_{t+2}),$$

where $x_{t+2}$ is the random component of investment productivity in period $t+2$ and $m_{t+2} > 0$ is its unconditional mean. Initially, we focus on a setting in which the random variables $\{x_t\}$ are *iid* normal with mean zero and variance $\sigma^2$. In Section 4, we examine the case of serially correlated project cash flows.

Therefore, during each period $(t-1, t)$, the firm has two projects in progress: one that will deliver cash flows at the end of the current period, $X_t$, and another one that will deliver cash flows at the end of the next period, $X_{t+1}$. Consistent with much of the earlier literature in this area (Christensen et al. 2010, Easley and O'Hara 2004, Hughes et al. 2007, Lambert et. al. (2007), and Suijs 2008) , we initially focus on a pure exchange economy in which the investment levels $(I_1, I_2, \dots)$, and hence the distributions of future cash flows, are exogenous to the model. To ensure that the firm does not grow without bound and the expected firm price remains finite, we assume that the investment level is asymptotically bounded from above by some $K$.[7] We endogenize the firm's investment choices in Section 5.2 below.

At date $t$, the firm publicly discloses a signal, e.g. an accounting report, that conveys information about the cash flows to be realized in the next period, $X_{t+1}$. The signal is denoted by $S_t$, and the informativeness of the reporting system refers to the extent to which $S_t$ reveals $X_{t+1}$. Specifically, we assume that

$$S_t \equiv I_{t-1} \cdot (s_t + m_t),$$

where $s_t$ is a signal about the random component of investment productivity in the next

[7]If each investment's NPV (calculated net of its risk related costs) is non-negative, the firm's expected price will also be non-negative at each date. We will assume this to be the case in the exogenous investment setting. A sufficient condition is that $\gamma^2(m_t K - \sigma^2 K^2) - c(K) \geq 0$ for each $t$.

period, $x_{t+1}$. The scaled public signal $s_t$ measures $x_{t+1}$ with noise:

$$s_t = x_{t+1} + \eta_t.$$

The noise terms $\eta_t$ are serially uncorrelated, drawn from identical normal distributions with mean zero and variance $\sigma_\eta^2$, and independent of all other random variables. We note that $\frac{1}{\sigma_\eta^2}$ measures the informativeness of the signal.

The investments are irreversible, i.e., $I_{t-1}$ cannot be changed at date $t$ if signal $S_t$ turns out to be low. Subsequent to the firm's public release of signal $S_t$, the market for the firm's shares opens and the current shareholders sell their stock to the investors of next generation. At date $t$, the sequence of events is as depicted in the timeline below.

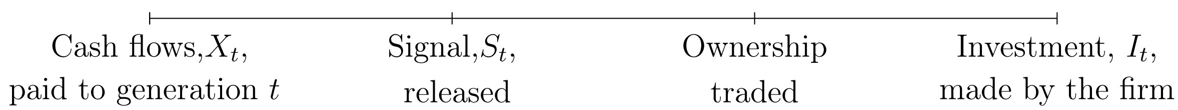


Figure 1: **Sequence of events at date** $t$

We assume that the firm does not retain any cash, and hence $X_t$ is distributed immediately as dividends to the firm's generation $t$ shareholders. The assumption that the firm does not carry any cash is without loss of generality, since dividend policy is irrelevant in our symmetric information setting. The firm raises the cash needed for new investment, $c(I_t)$, from generation $t+1$ investors through a seasoned equity offering (SEO). Specifically, we assume that the total supply of shares at date $t$ consists of shares sold by generation $t$ and the new shares issued by the firm. Let $z_t$ denote the number of new shares issued for each existing share, and let $P_t$ be the total price of all shares outstanding at date $t$ after the SEO. Then, the total payoff to generation $t$ shareholders is given by $X_t + \delta_t \cdot P_t$, where $\delta_t \equiv \frac{1}{1+z_t}$. After observing signal $S_t$, the firm decides how many new shares need to be issued to finance the new project, so that in equilibrium $(1-\delta_t) \cdot P_t = c(I_t)$. Hence, generation $t$ shareholders receive

$$\delta_t \cdot P_t = P_t - c(I_t)$$

when they resell their shares to the next generation. For future reference, we note that $Var_{t-1}(\delta_t \cdot P_t) = Var_{t-1}(P_t)$, since investment cost $c(I_t)$ is non-stochastic. Here $Var_{t-1}(\cdot)$ denotes the variance operator conditional on date $t-1$ information.

Each generation consists of a continuum of investors (with unit mass) who act as price takers in the stock market. Since investors are identical and have symmetric information, it is without loss of generality to represent each generation by a single representative investor. The representative investor of generation $t$ seeks to maximize the expected utility of his consumption (terminal wealth) at the end of period $t$, $\omega_t$. We assume that the preferences of the representative investor of generation $t$ can be described by an exponential utility function with a coefficient of constant absolute risk aversion (CARA) $\rho$:

$$U_t(\omega_t) = -exp[-\rho \cdot \omega_t].$$

Without loss of generality, we normalize initial wealth of each generation of investors to zero.

# 3 Disclosure, Risk Premium, and Investor Welfare

We first examine a setting in which cash flows are serially uncorrelated. Our primary objective is to examine how the precision of public information and investment growth jointly affect the risk premium that the rational investors demand for holding the firm. Let $\phi_t \equiv (X_t, S_t)$ denote the information that is publicly released at date $t$. From the perspective of predicting the distributions of future cash flows, the current public signal $S_t$ is a sufficient statistic for the entire history of information $(\phi_1, \ldots, \phi_t)$ available at date $t$, since the investment payoffs $X_t$ and signals $S_t$ are both serially uncorrelated.

Generation $t+1$ buys the firm's stock at price $P_t$ at date $t$, gets a dividend of $X_{t+1}$ at date $t+1$, and sells its stock for a price of $\delta_{t+1}P_{t+1} = P_{t+1} - c(I_{t+1})$ to the next generation. The risk premium in period $t+1$ can then be written as:

$$RP_{t+1} = E_t[X_{t+1} + P_{t+1}] - c(I_{t+1}) - (1+r) \cdot P_t,$$

where $E_t(\cdot)$ denotes the expectation operator conditional on the information available at date $t$. We note that in our CARA-Normal framework, the risk premium is independent of the realized value of date $t$ signal $s_t$.[8]

Let

$$\sigma_p^2 \equiv Var_t(x_{t+1}) = Var(x_{t+1}|s_t)$$

[8]We verify this in the proofs of Lemmas 1-3.

denote the posterior variance of $x_{t+1}$. It can be easily checked that $\sigma_p^2 = (1-k)\cdot\sigma^2$, where $k \equiv \frac{\sigma^2}{\sigma^2+\sigma_\eta^2}$. We also note that conditional on date $t$ information $s_t$, the posterior mean of $x_{t+1}$ is given by $E_t(x_{t+1}) = k \cdot s_t$. From an ex-ante perspective, this posterior mean is a normally distributed random variable with variance $\sigma_a^2$, where

$$\sigma_a^2 \equiv Var_{t-1}[E_t(x_{t+1})].$$

It is easily verified that $\sigma_a^2 = k \cdot \sigma^2$, and the law of total variance holds; i.e., $\sigma_a^2 + \sigma_p^2 = \sigma^2$. As expected, $\sigma_p^2$ decreases in the precision of public information (i.e., $\frac{1}{\sigma_\eta^2}$), while $\sigma_a^2$ increases as the information becomes more precise. Therefore we will sometimes use $\sigma_a^2 \in [0, \sigma^2]$ as a measure of the informativeness of the public disclosure system. While $\sigma_a^2 = 0$ corresponds to the limiting case of no disclosure, $\sigma_a^2 = \sigma^2$ represents the most precise disclosure system.

**Lemma 1.** *The risk premium in period* $t+1$ *is given by:*

$$RP_{t+1} = \rho \cdot \left(I_{t-1}^2 \cdot \sigma_p^2 + \gamma^2 \cdot I_t^2 \cdot \sigma_a^2\right) \tag{1}$$

*for all* $t$.

When the investors of the current generation buy the firm's stock at date $t$, they expect to receive two different forms of payoffs at date $t+1$: (i) the dividends in the amount of $X_{t+1}$, and (ii) $P_{t+1} - c(I_{t+1})$, the resale price at which they sell their shares to the next generation. A standard result in the CARA-Normal framework is that the equilibrium price for uncertain payoffs is given by $p = \gamma[E(y) - \rho Var(y)]$. Consistent with this result, the proof of Lemma 1 shows that date $t$ market price of the firm must satisfy the following condition:

$$P_t = \gamma \cdot \left[E_t(X_{t+1} + P_{t+1}) - c(I_{t+1}) - \rho \cdot Var_t(X_{t+1} + P_{t+1})\right].$$

This implies that the equilibrium amount of risk premium earned by generation $t+1$ investors is given by $\rho \cdot Var_t(X_{t+1} + P_{t+1})$.[9] Since investment payoffs are serially uncorrelated, the cash flows in period $t+1$, $X_{t+1}$, are uncorrelated with the one-period ahead market price, $P_{t+1}$. The expression for the risk premium can thus be written as:

$$RP_{t+1} = \rho \cdot Var_t(X_{t+1}) + \rho \cdot Var_t(P_{t+1}).$$

[9] In this single risky asset setting, any risk is systematic and priced as such. We examine a multi-asset extension of our model in Section 5.1 below.

That is, the equilibrium risk premium, $RP_{t+1}$, is the sum of the investors' compensation for bearing (i) a *dividend* risk as measured by $Var_t(X_{t+1}) = I_{t-1}^2 \cdot \sigma_p^2$, and (ii) a resale *price* risk as measured by $Var_t(P_{t+1})$. The dividend risk for generation $t+1$ investors is determined by the uncertainty of payoffs from project $I_{t-1}$, since they receive these payoffs as dividends.

To calculate the resale price risk as measured by $Var(P_{t+1})$, we note from the proof of Lemma 1 that the equilibrium market price of the firm is equal to the discounted sum of expected future cash flows net of periodic risk premia and investment costs; i.e.,

$$P_{t+1} = \sum_{\tau=1}^{\infty} \left[ E_{t+1}(X_{t+\tau+1}) - c(I_{t+\tau+1}) - RP_{t+\tau+1} \right]. \tag{2}$$

Since $E_{t+1}(X_{t+\tau+1})$ are non-stochastic for all $\tau \geq 2$, the above expression can be written as $P_{t+1} = \gamma \cdot E_{t+1}(X_{t+2}) + constant$. This implies that the resale price risk is given by:

$$Var_t(P_{t+1}) = \gamma^2 \cdot Var_t \left[ E_{t+1}(X_{t+2}) \right] = \gamma^2 \cdot I_t^2 \cdot \sigma_a^2.$$

To summarize, equation (1) demonstrates that the investors buying the firm at date $t$ are exposed: (i) to uncertainty of the payoffs from project $I_{t-1}$, since these payoffs directly affect their dividends, and (ii) to uncertainty of the payoff from project $I_t$ indirectly through these payoffs' effect on the firm's resale price at date $t+1$. The risk premium term corresponding to project $I_t$ is discounted by $\gamma^2$ because $P_{t+1}$ reflects the (one-period) discounted value of $X_{t+2}$, and the risk premium is proportional to the variance in our mean-variance framework.

To formulate our first proposition, let $\mu_t$ denote the investment growth rate in period $t$; i.e.,

$$I_t = (1 + \mu_t) \cdot I_{t-1}.$$

**Proposition 1.** *The risk premium in period $t+1$ decreases (increases) in the informativeness of public disclosure if $\mu_t < r$ ($\mu_t > r$).*

This result highlights that the equilibrium relationship between risk premium and the precision of public information depends on the firm's growth trajectory. When investments are growing relatively slowly (i.e., $\mu_t < r$), a higher quality disclosure system results in a lower risk premium. On the other hand, the risk premium *increases* in the precision of public information for firms in relatively fast growth phase (i.e., $\mu_t > r$). The investors of each generation are subject to a dividend risk, which is proportional to $Var_t(x_{t+1}) \equiv \sigma_p^2$, and a (resale) price risk, as measured by $Var_t[E_{t+1}(x_{t+2})] \equiv \sigma_a^2$. While a more informative disclosure regime reduces the dividend risk, it also makes the resale price more volatile by increasing $\sigma_a^2$. By the law of total variance, $\sigma_p^2 + \sigma_a^2 = \sigma^2$, and hence the risk premium in period $t+1$ can be written as:

$$RP_{t+1} = \rho \cdot [I_{t-1}^2 \cdot \sigma^2 + (\gamma^2 \cdot I_t^2 - I_{t-1}^2) \cdot \sigma_a^2].$$

Therefore, the net effect of the precision of public disclosure on the overall risk premium depends on the weights assigned to the dividend and price risk components (i.e., $I_{t-1}$ and $\gamma \cdot I_t$, respectively). For a fast growing firm, the investors rationally assign more weight to the price risk; that is the risk associated with the payoffs from more recent, and hence larger, projects. As a result, the overall risk premium for a fast growing firm increases in the precision of public information. On the other hand, the dividend risk is the dominant determinant of the overall risk premium for low growth firms, and hence the risk premium decreases in the informativeness of public disclosures for such firms.

These results provide a potential explanation for the mixed empirical evidence on the relation between firms' disclosure qualities and their costs of equity capital (see, for example, Botosan 1997, Botosan and Plumlee 2002, Easley et al. 2002, and Francis et al. 2008). Specifically, Proposition 1 highlights that the relationship between cost of capital and quality of accounting disclosures crucially depends on a firm's growth trajectory. Without sorting on investment growth rates, our analysis predicts that the average relation between quality of disclosure and cost of capital would reflect the relative mix of high and low growth firms in the economy. In future empirical studies, therefore, it might be helpful to investigate the link between disclosure and cost of capital after sorting on growth rates.

Easley and O'Hara (2004) also investigate the link between the precision of public information and risk premium in symmetric information settings within the class of Normal-CARA models. In contrast to our findings, their model predicts an unambiguously negative relationship between risk premium and disclosure quality. This difference in the results arises because while we consider an infinite horizon model with growth, Easley and O'Hara (2004) examine a static one period setting. Consistent with their result, Proposition 1 shows that the risk premium unambiguously decreases in the quality of public disclosure for the special case of a no growth firm (i.e., $\mu = 0$).

In another related paper, Christensen et al. (2010) consider a two-period model in which investors can trade before, as well as after, public disclosure. They find that the reduction in the ex-post risk premium following a more informative disclosure is precisely offset by the increase in the risk premium for the period before disclosure, and hence the total risk premium remains unchanged. In contrast, our analysis shows that the risk premium generally varies with the precision of public disclosure. It can be readily verified that the overall risk premium decreases in the quality of public information even when our overlapping generations model is reduced to a two-period setting with a single terminal payoff (which corresponds to the setting in Christensen et al.). This difference in the results arises from different assumptions about the investors' planning horizons in the two papers. In Christensen et al. (2010), the investors care only about the uncertainty of the terminal payoff, since they can hold the firm for its entire duration of two periods. In our overlapping generations model, however, the shareholders are concerned about the risk associated with the intermediate price.

While a lower cost of capital is frequently cited as a justification for improving disclosure quality, our analysis allows us to explicitly characterize the impact of public information on investors' equilibrium expected utilities. The following Proposition characterizes how a change in quality of public information affects welfare of the firm's *existing* shareholders, as well as that of *future* generations of shareholders. The distinction between existing and future shareholders is relevant for this welfare analysis, since existing shareholders already own the firm (i.e., the price they have paid for the firm, $P_t$, is a sunk cost for them). Hence, they are only concerned with how a shift in the future disclosure requirements will affect their resale price. In contrast, any change to the disclosure regime will affect both purchase and resale prices for future generations of shareholders.

To characterize welfare implications of alternative disclosure regimes, suppose a new disclosure policy (i.e., a new value of precision for all future disclosures) takes effect between dates $t-1$ and date $t$ when the firm is owned by generation $t$ investors.

**Proposition 2.**

*i. Welfare of future investors of generation $t+\tau$ for all $\tau \geq 1$ decreases (increases) in the informativeness of public disclosure if $\mu_{t+\tau-1} < r$ ($\mu_{t+\tau-1} > r$).*

*ii. Welfare of the firm's current shareholders increases in the precision of public disclosure.*

The first part of the above result shows that *future* investors' welfare and risk premium during the period in which they plan to hold the firm are *positively* associated. To understand the intuition, note that the price of the firm must satisfy the following equilibrium condition:

$$P_t = \gamma \cdot [E_t(Y_{t+1}) - \rho \cdot Var_t(Y_{t+1})],$$

where $Y_{t+1} \equiv X_{t+1} + P_{t+1} - c(I_{t+1})$ denotes the firm's cum-dividend price at date $t+1$. The certainty equivalent of generation $t+1$ representative investor is given by

$$CE_{t+1} = E_t[Y_{t+1}] - (1+r) \cdot P_t - \frac{\rho}{2} \cdot Var_t(Y_{t+1}). \tag{3}$$

As $Var_t(Y_{t+1})$ increases by one unit, the equilibrium price of the firm drops by $\gamma \cdot \rho$ units. This implies that the expected return of holding the risky asset, $E_t[Y_{t+1}] - (1+r) \cdot P_t$, increases by $\rho \cdot \gamma \cdot (1+r) = \rho$ units. This is the indirect effect of increased variance on the investor's certainty equivalent as given by the first two terms on the right-hand side of (3). An increase of one unit of variance also has the direct effect, as captured by the second term on the right hand side of (3), of reducing the investor's certainty equivalent by $\frac{\rho}{2}$ units. In the Normal-CARA framework, therefore, the indirect effect of increased expected returns dominates and the investor's expected utility is increasing in $Var_t(Y_{t+1})$ and hence in the risk premium.[10] The result then follows from Proposition 1.

In contrast, the second part of Proposition 2 shows that the firm's existing shareholders, who are not concerned with the purchase price since they already own the firm, unambiguously prefer the most precise disclosure regime. To understand why, recall from (2) that firm's equilibrium market price at each date is given by the discounted sum of expected future cash flows net of periodic risk premia; i.e.,

$$P_t = \sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot [E_t(X_{t+\tau}) - c\,(I_{t+\tau}) - RP_{t+\tau}].$$

Consider welfare of the firm's original owners who sell the firm to the first generation of investors at price $P_0$. Their welfare is maximized by a disclosure policy that minimizes the discounted sum of periodic risk premia $\sum_{t=1}^{\infty} \gamma^t \cdot RP_t$. We note that public signal $s_t$ shifts some of the risk associated with period $t+1$ cash flows $X_{t+1}$ from generation $t+1$ to generation $t$. For instance, relative to a policy of no disclosure, a policy of complete disclosure ($\sigma_\eta^2 = 0$)

[10]This intuition is based on a similar finding in Kurlat and Veldkamp (2015), who also argue that this result (i.e., investors prefer riskier payoffs) is likely to remain valid in settings beyond the Normal-CARA framework. Dye (1990) and Gao (2010) provide a similar argument.

effectively advances the risk associated with each project by one period. This has the effect of increasing the present value of future risk premia by a factor of $(1+r)=\gamma^{-1}$. However, full disclosure also implies that each generation is exposed to the corresponding project risk only through the resale price. Since price risk is equal to the overall project risk discounted by $\gamma^2$, the present value of future periodic risk premia decreases in the informativeness of public disclosure. Consequently, unlike future generations of investors whose preferences for public information depend on the firm's growth rate during their investment horizons, the original owners' welfare unambiguously increases in the precision of public disclosure. A similar argument shows that the welfare of the firm's *existing* shareholders of any generation increases in the quality of *future* public disclosures.

Proposition 2 shows that the impact of public disclosure on total social welfare will generally depend on how one weighs the utilities of different generations in the overall social welfare function. This analysis also highlights that the periodic risk premium (e. g., cost of capital) is generally not an appropriate summary metric to rank welfare implications of alternative public disclosure policies.

# 4 Correlated Cash Flows

We have thus far assumed that the firm's cash flows are serially uncorrelated. In this section, we investigate an extension of our basic model in which investment payoffs are positively correlated across periods. Specifically, suppose that the random component of investment productivity, $x_t$, evolves according to the following stochastic process:

$$x_t = w \cdot x_{t-1} + \varepsilon_t,$$

where $w$ is a commonly known persistence parameter between zero and one. The innovation terms $\varepsilon_t$ are serially uncorrelated and follow a joint normal distribution with mean zero and variance $\sigma^2$. When $w<1$, the investment productivity parameters $x_t$ evolve according to a mean-reverting AR(1) process. The polar case of $w=1$ represents a setting when the investment productivity parameters $x_t$ follow a random walk.

The total gross cash flow in period $t$ is again given by $I_{t-2} \cdot x_t$ and the public signal $S_t = I_{t-1} \cdot s_t$ provides information about date $t+1$ cash flows $X_{t+1}$. As before, $s_t$ is a noisy measure of the random component of investment productivity in the next period,

$x_{t+1}$. With uncorrelated cash flows, the current public signal $s_t$ was sufficient for the entire history of information for the purpose of predicting future cash flows. In contrast, when cash flows are autocorrelated, the current cash flow parameter $x_t$ also provides useful information for predicting future cash flows. With autocorrelated cash flows, $\psi_t = (x_t, s_t)$ constitutes a sufficient statistic for the history of information $(\psi_1, \ldots, \psi_t)$. It will be convenient to normalize the public signal to $\hat{s}_t = s_t - E[x_{t+1}|x_t]$. We note that $(x_t, \hat{s}_t)$ is informationally equivalent to $(x_t, s_t)$ and

$$\hat{s}_t = \varepsilon_{t+1} + \eta_t.$$

Since cash flows are serially correlated, the realized value of cash flow in the current period is informative about all future cash flows. It can be easily checked that:

$$E_t[x_{t+\tau}] = w^{\tau} \cdot x_t + w^{\tau-1} \cdot E_t(\varepsilon_{t+1}), \tag{4}$$

where $E_t(\varepsilon_{t+1}) = k \cdot \hat{s}_t$ with $k = \frac{\sigma^2}{\sigma^2 + \sigma_\eta^2}$.

To provide an expression for the equilibrium risk premium in the correlated cash flows setting, it will be convenient to define $Q_t \equiv \sum_{\tau=0}^{\infty} (\gamma \cdot w)^{\tau} \cdot I_{t+\tau}$.

**Lemma 2.** *Suppose that the cash flows are autocorrelated. The risk premium in period $t+1$ is given by:*

$$RP_{t+1} = \rho \cdot \left( Q_{t-1}^2 \cdot \sigma_p^2 + \gamma^2 \cdot Q_t^2 \cdot \sigma_a^2 \right). \tag{5}$$

*It decreases in the precision of public disclosure if $I_{t-1} > \gamma(1-w)Q_t$, and increases otherwise.*

A comparison of expression (5) for the risk premium with the corresponding expression in the uncorrelated case reveals that $I_{t-1}$ and $I_t$ in (1) need to be replaced with $Q_{t-1}$ and $Q_t$, respectively, to account for serial correlation in the project cash flows. To understand why, we note that the risk premium is again given by:

$$RP_{t+1} = \rho \cdot Var_t[X_{t+1} + P_{t+1}].$$

However, dividends and resale prices are no longer independent because current cash flow news $x_t$ is informative about *all* future cash flows in the correlated setting. Specifically, equation (4) implies that conditional on date $t+1$ information $(x_{t+1}, \hat{s}_{t+1})$, the present value of future expected cash flows, $\sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot E_{t+1}(X_{t+\tau+1})$, increases (i) in $x_{t+1}$, at the rate of $\gamma \cdot w \cdot Q_t$, and (ii) in $\hat{s}_{t+1}$ at the rate of $\gamma \cdot k \cdot Q_t$. Consequently, the market price at date

$t + 1$ can be written as:[11]

$$P_{t+1} = \gamma \cdot Q_t \cdot [w \cdot x_{t+1} + k \cdot \hat{s}_{t+1})] + const. \tag{6}$$

Equation (6) implies that $X_{t+1} + P_{t+1} = Q_{t-1}x_{t+1} + \gamma Q_t k \hat{s}_{t+1} + \beta_{t+1}$, and hence expression (5) for the risk premium in Lemma 2.

As before, while a more informative disclosure results in a more volatile resale price, it also leads to a lower level of dividend risk and a lower covariance between contemporaneous dividends and price. However, Lemma 2 shows that the risk premium unambiguously decreases in the informativeness of public disclosure if the productivity parameters follow a random walk, since inequality $I_t > \gamma(1 - w)Q_t$ always holds for $w = 1$. For $w < 1$, the relation between the risk premium and public information depends on the investment growth rates. Unlike the uncorrelated cash flows case, however, the link between risk premia and the precision of public information generally depends on all future growth rates.

To characterize how growth affects the relationship between public disclosure and risk premium, we examine a setting in which the firm initially grows at a constant rate of $\mu$ until it achieves a steady state size at some future date $T$. That is, $I_t = (1+\mu) \cdot I_{t-1}$ for $t \leq T$ and $I_t = I_T$ for all $t > T$. In the steady state phase, $Q_{t-1} = Q_t$ for all $t > T$, and hence it follows from expression (5) that the periodic risk premium unambiguously decreases in the quality of public information. The result below characterizes the relation between risk premia and public information during the firm's growth phase.

**Proposition 3.** *Suppose $w \in (0, 1)$ and the firm grows at a constant rate until it reaches a steady state size.*

*i. There exists a $\hat{\mu} \in \left(r, \frac{r+w}{1-w}\right)$ such that the periodic risk premium decreases (increases) in the precision of public information if the growth rate is less (more) than $\hat{\mu}*$.*

*ii. The threshold growth rate $\hat{\mu}$ increases in the persistence parameter $w$ and approaches infinity as $w \to 1$.*

When the project payoffs are serially uncorrelated (i.e., $w = 0$), the periodic risk premium decreases (increases) in the precision of public disclosure if the current growth rate is below (above) the risk-free interest rate $r$. Proposition 3 shows that a similar relationship between

[11]See the proof of Proposition 3 for details.

growth and risk premium holds when project payoffs are autocorrelated. However, the threshold growth rate $\hat{\mu}$ in the correlated case is higher than the one in the uncorrelated case (i.e., $\hat{\mu} > r$).

To understand why, notice from equation (6) that the market price in the correlated case varies not only with the forward-looking information $\hat{s}_{t+1}$, but also with the current cash flow news $x_{t+1}$. While a policy of higher quality disclosures has the effect of increasing the first component of the price risk, it lowers the price variability due to the second component. As a consequence, the resale price risk is less sensitive to the quality of public disclosures, and hence the overall risk premium decreases in the precision of public information for a larger range of growth rates for firms with serially correlated cash flows.

The threshold growth rate $\hat{\mu}$ increases in the persistence parameter $w$ because the price risk becomes increasingly less sensitive to public information as $w$ increases and periodic cash flows become more highly autocorrelated. As discussed before, in the polar case when the investment cash flows follow a random walk (i.e., $w = 1$), the overall risk premium decreases in the precision of public information regardless of the growth rate (i.e., $\hat{\mu} \to \infty$).

Similar to the welfare result in Proposition 2, it can be verified that while the firm's existing shareholders always prefer the most precise disclosure policy, disclosure preferences of future investors depend on the firm's growth rate. Specifically, as before, welfare of future investors is positively associated with the risk premium in the period when they plan to hold the firm. Consequently, they would prefer less (more) disclosure for low (high) growth firms and the threshold growth rate would be higher for firms with more persistent cash flows.

The result in Proposition 3 identifies cash flows persistence as another key determinant of the link between disclosure quality and cost of capital. In particular, this result predicts that a negative relation between cost of capital and disclosure quality will hold for larger range of growth rates for firms with more persistent cash flows. Thus, an implication of our analysis is that even after controlling for growth rates, the relation between equity cost of capital and disclosure quality would crucially depend on the degree of cash flow persistence. In particular, we predict that, all else equal, the relation between cost of capital and disclosure is more likely to be negative for firms with more persistent cash flows.

# 5 Extensions

Our analysis has thus far assumed that the economy consists of a single risky firm. Furthermore, up to this point we have focused on a setting in which the firm's investment choices, and hence the distributions of its future cash flows, were exogenously fixed. In this extension section, we relax these assumptions. Section 5.1 extends our model to a multi-firm economy. Section 5.2 considers a setting where the firm's investment policy is endogenously determined. For tractability, we now revert back to a setting in which investment payoffs are serially uncorrelated.

## 5.1 Multi-Firm Economy

Our analysis has thus far focused on a single firm setting in which all risk is systematic and priced as such in the market. We now investigate an extended model with multiple risky firms in which each firm's periodic cash flows are subject to both systematic (i.e., market-wide) and idiosyncratic (i.e., firm-specific) shocks. We show that the qualitative nature of our main results in Section 3 readily extends to such multi-asset economies.

We examine a pure exchange economy consisting of $J$ firms with indefinite lives. Each overlapping generation of investors consists of $N$ identical risk-averse investors with CARA risk preferences. We seek to characterize the equilibrium relationship between information disclosure and cost of capital in large economies (i.e., $J \to \infty$ and $N \to \infty$). As before, generation $t$ investors buy the firms at date $t-1$ and sell them to the next generation at date $t$.

Firm $j$'s period $t$ cash flows are given by:

$$X_t^j = I_{t-2}^j \cdot (x_t^j + m_t^j),$$

where $I_{t-2}^j$ is firm $j$'s investment in period $t-2$. The marginal product of $I_{t-2}^j$ is equal to the sum of a non-random component $m_t^j > 0$ and a random component $x_t^j$. As before, we assume that each firm $j$ raises the needed amount of capital for investment, $c(I_t^j)$, through a secondary offering in each $t$.

The random investment productivity parameters $x_t^j$ are subject to both systematic and idiosyncratic risks. Specifically, we assume that

$$x_t^j = x_t + \theta_t^j,$$

where $x_t \sim N(0, \sigma_x^2)$ is a common risk factor that affects all the firms in the economy and $\theta_t^j \sim N(0, \sigma_\theta^2)$ represents the firm-specific idiosyncratic shock to period $t$ cash flows. We assume that $x_t$ and $\theta_t^j$ are serially and mutually independent.

At the end of each period, the firms issue public reports (forecasts) of their next period cash flows. Specifically, as before, we assume that firm $j$ releases signal $S_t^j = I_{t-1}^j(s_t^j + m_t^j)$ at date $t$, where $s_t^j$ is a noisy measure of the random component of its investment productivity $x_{t+1}^j$:

$$s_t^j = x_{t+1}^j + \eta_t^j.$$

Noise terms $\eta_t^j$ are drawn from identical normal distributions with zero mean and variance $Var(\eta)$, and are independently distributed. We also assume that noise terms $\eta_t^j$ are independent of all other random variables. For each firm, we note that signal $s_t^j$ contains information on the common risk factor $x_t$ as well as the idiosyncratic productivity shock $\theta_t^j$.

Let $X_t^m$ denote the market cash flows at date $t$; i.e., $X_t^m = \sum_{i=1}^J X_t^i$. Similarly, let $P_t^m = \sum_{i=1}^J P_t^i$ denote the equilibrium price of the market portfolio at date $t$. We verify in the Appendix that firm $j$'s equilibrium market price satisfies

$$P_t^j = \gamma \cdot [E_t(X_{t+1}^j + P_{t+1}^j) - c(I_{t+1}^j) - RP_{t+1}^j],$$

and risk premium $RP_{t+1}^j$ is given by

$$RP_{t+1}^j = \frac{\rho}{N} \cdot \left[Cov_t\left(X_{t+1}^j, X_{t+1}^m\right) + Cov_t\left(P_{t+1}^j, P_{t+1}^m\right)\right]. \tag{7}$$

In our dynamic setting, the equilibrium risk premium again consists of (i) a *dividend* risk component as given by the first term inside the square bracket of the above expression, and (ii) a *price* risk component represented by the second term inside the square bracket. However, in multi-firm settings, these risk premium components are determined by the conditional *covariances* of a firm's dividends and resale prices with the corresponding variables for the whole market.

We seek to investigate how information disclosure affects risk premia when the economy is large in the sense that both the number of firms $J$ and the number of investors $N$ go to infinity. For evaluating the large economy limit, we follow the approach in Hughes et al. (2007) and require that the number of investors $N$ grows at the same rate as the number of firms $J$ so that $\frac{J}{N}$ approaches a constant (normalized to one without loss of generality).[12] This restriction rules out unrealistic scenarios of zero risk premium, which would happen if $N$ grew faster than $J$, and infinite risk premium, which would occur if $J$ were to expand faster than $N$. In the following analysis, we will refer to the limiting case of $J = N \to \infty$ as the *large economy*.

With information disclosure of the form modeled in our paper, another related issue is that date $t$ public information $s_t \equiv (s_t^1, \cdots, s_t^J)$ would perfectly reveal systematic risk factor $x_{t+1}$ in the large economy i.e., $Var_t(x_{t+1}) \to 0$ as $J \to \infty$. This sounds unrealistic because one would expect residual uncertainty about the market-wide factors even when each firm's accounting report contain some information about the economy. To avoid this unrealistic scenario, we assume that each firm's disclosure becomes less informative as the number of firms $J$ increases. Specifically, following Hughes et al. (2007), we assume that $Var(\eta) = J\sigma_\eta^2$. This assumption captures the intuition that a firm's accounting report becomes increasingly limited in its information content about the overall economy as the firm becomes an increasingly small part of that economy. It ensures that the aggregate information released by the firms in the economy reduces, but does not entirely eliminate, systematic uncertainty about future cash flows. Specifically, it can be verified that as $J \to \infty$, $Var_t(x_{t+1}) \to \sigma_{xp}^2$, where

$$\sigma_{xp}^2 = \frac{\sigma_x^2 \sigma_\eta^2}{\sigma_x^2 + \sigma_\eta^2}.$$

We note that $\frac{1}{\sigma_\eta^2}$ is again a measure of the precision of public disclosure, since the above posterior variance declines in $\frac{1}{\sigma_\eta^2}$. To state our next result, let $\bar{I}_t \equiv \frac{\sum_{j=1}^J I_t^j}{J}$ denote the *average* investment in period $t$.[13]

**Lemma 3.** *In the large economy, firm $j$'s risk premium in period $t+1$ is given by*

$$RP_{t+1}^j = \rho \left[ I_{t-1}^j \cdot \bar{I}_{t-1} \cdot \sigma_{xp}^2 + \gamma^2 \cdot I_t^j \cdot \bar{I}_t \cdot \sigma_{xa}^2 \right], \tag{8}$$

[12] Lambert et al. (2007) and Ou-Yang (2005) place similar restrictions in their investigations of large economies.

[13] We assume that the average investment amount $\bar{I}_t$ does not become arbitrarily small as $J \to \infty$. As before, we also assume that $I_t^j$ is bounded from above by some $K_j$ for each $j \in \{1, \cdots, J\}$.

*where* $\sigma_{xa}^2 = \sigma_x^2 - \sigma_{xp}^2$.

As before, the first term on the right-hand side of (8) captures dividend risk, while the second term reflects price risk. It is instructive to compare the above expression for the risk premium with the corresponding expression in (1) for the single firm setting. While the price risk is proportional to the square of the firm's own investment (i.e., $I_t^2$) in the single firm setting, the price risk in the multi-firm setting is proportional to the product of the firm's own investment (i.e., $I_t^j$) and that of the average investment during that period (i.e., $\bar{I}_t$). This is a consequence of the fact that in multi-firm settings, the price risk is determined by the conditional covariance of the firm's resale price with the resale price of the market portfolio. The same argument applies for why the dividend risk is proportional to $I_{t-1}^j \cdot \bar{I}_{t-1}$.

Equation (8) also reveals that a firm's risk premium in the multi-firm large economy depends only on systematic risk, which is parameterized by $\sigma_x^2$ in our model. This confirms the standard intuition that idiosyncratic risk is not priced, since investors can eliminate their exposure to idiosyncratic risk by holding well-diversified portfolios. The proof of Lemma 3 confirms that each investor holds a share of equally-weighted market portfolio in equilibrium. Lemma 3 also shows that even an infinitesimally small amount of information on systematic risk factor for each firm has a finite effect on risk premium.

Our next result shows that the relationship between a firm's risk premium and information disclosure again depends on the firm's growth rate. To state this result, let $\bar{\mu}_t$ denote the market-wide growth rate in period $t$; i.e, $\bar{I}_t = (1 + \bar{\mu}_t)\bar{I}_{t-1}$.

**Proposition 4.** *In the large economy, firm $j$'s risk premium in period $t+1$ decreases (increases) in the precision of public information if its investment growth rate $\mu_t^j$ is below (above) a threshold $\hat{\mu}_t^j$, which is given by*

$$\hat{\mu}_t^j = \frac{(1+r)^2}{1+\bar{\mu}_t} - 1. \tag{9}$$

As in the single firm setting, a firm's risk premium decreases (increases) in the informativeness of disclosure system if its growth rate is below (above) a certain threshold. However, equation (9) shows that the threshold growth rate now depends on the average growth rate in the economy. This is a consequence of the fact that in multi-firm settings, the dividend and risk components of a firm's risk premium are determined by conditional covariances of the firm's own cash flows with those of other firms. For instance, firm $j$'s dividend risk premium component is a multiple of $Cov_t(X^j_{t+1}, X^m_{t+1})$, which is, in turn, proportional to $I^j_{t-1} \cdot \bar{I}_{t-1}$.

Equation (9) shows that the threshold rate $\hat{\mu}^j_t$ *decreases* in the average growth rate in the economy $\bar{\mu}_t$. For example, it can be verified that if the average growth rate in the economy were equal to $r(2+r)$, the risk premium would increase in disclosure quality even for steady state firms. This is intuitive because when the economy is growing relatively fast, the price risk component of a firm's risk premium, as determined by the covariance between the firm's own cash flows with those of other firms, becomes the dominant driver of the firm's overall risk premium, and hence the risk premium increases in the precision of public disclosure.

These results support our earlier suggestion that future empirical studies can generate a cleaner test of the link between cost of capital and information disclosure by sorting on growth rate. Our multi-asset results are also relevant for empirical studies that seek to examine the link between the aggregate information environment and the market cost of capital (see, for example, Bhattacharya et al. 2003; Bhattacharya and Daouk 2002; Jain 2005). For instance, equation (8) implies that the risk premium for each share (i.e., share $\frac{1}{N}$) of the market portfolio is given by

$$\overline{RP}_t = \rho \left[ \bar{I}^2_{t-1} \cdot \sigma^2_{xp} + \bar{I}^2_t \cdot \sigma^2_{xa} \right].$$

It can then be easily verified that the market cost of capital (i.e., $\overline{RP}_t$) decreases (increases) in the informativeness of disclosure system when the market growth rate $\bar{\mu}_t$ is below (above) $r$. Thus, a potentially testable implication of our theory is that whether the market cost of capital decreases or increases in quality of information disclosure depends on the market-wide growth rate. Specifically, we predict that the market cost of capital should be lower (higher) for high quality disclosure regimes when the average growth rate in the economy is relatively low (high).

The result below characterizes how a change in the quality of public information affects welfare of current shareholders as well as future investors. As before, suppose the new disclosure policy takes effect between dates $t-1$ and date $t$ when the firms are owned by generation $t$ investors.

**Proposition 5.** *In the large economy,*

*i. welfare of future investors of generation $t + \tau$ for all $\tau \geq 1$ decreases (increases) in the informativeness of public disclosure if $\bar{\mu}_{t+\tau-1} < r$ ($\bar{\mu}_{t+\tau-1} > r$),*

*ii. welfare of current shareholders increases in the precision of public disclosure.*

Unlike potential future shareholders, existing shareholders are only concerned with the effect of information disclosure on the resale prices of their portfolios. Therefore, as before, their welfare is always maximized by the most informative disclosure policy.

The first part of the above result follows from the fact that each investor holds a fraction of the market portfolio, and therefore her overall risk exposure is determined by the market portfolio risk. As before, in equilibrium, potential investors receive higher utilities when they have access to riskier investment opportunities, and hence future shareholders' welfare *increases* in the market risk premium. As discussed earlier, the market risk premium declines (increases) in disclosure quality when the aggregate growth rate $\bar{\mu}_t$ is lower (higher) than the risk-free interest rate $r$. Consequently, future shareholders' welfare decreases (increases) in the precision of public information when the market is expected to grow slower (faster) than $r$ during their investment horizon.

## 5.2 Endogenous Investments

This subsection extends our earlier analysis to an endogenous investment setting in which each generation of shareholders is in charge of choosing the firm's investment policy during its period of ownership. In such a setting, public disclosure of information would have not only the intergenerational risk allocation effects as characterized in the previous section, but also real effects on the firm's investment choices. We seek to characterize how these real effects alter the equilibrium relationships among information disclosure, risk premia, and investors welfare.

To characterize the firm's endogenous investment choices, we assume that the cost of investment takes the following quadratic form:

$$c(I_t) = b \cdot I_t^2$$

with $b > 0$.[14] The firm chooses its investment level at date $t$ so as to maximize the expected utility of its current (generation $t+1$) shareholders. Specifically, the existing shareholders would choose the investment level $I_t$ so as to maximize the certainty equivalent of[15]

$$X_{t+1} + \delta_{t+1} \cdot P_{t+1} - (1+r) \cdot c(I_t).$$

After substituting $\delta_{t+1}P_{t+1} = P_{t+1} - c(I^*_{t+1})$ and (2) for $P_{t+1}$ into the certainty equivalent of the above expression and dropping the terms unrelated to $I_t$, the shareholders' optimization problem can be expressed as follows:[16]

$$\max_{I_t} \; V(I_t, \sigma_a^2) \equiv \gamma m_{t+2} I_t - (1+r) \cdot c(I_t) - \gamma \rho I_t^2 \sigma_p^2 - \frac{\gamma^2}{2} \rho I_t^2 \sigma_a^2. \tag{10}$$

The first term of objective function $V(I_t, \sigma_a^2)$ reflects the present value of expected gross payoffs from the investment undertaken in the current period. The second term in (10) captures the direct cost of investment $c(I_t) \equiv bI_t^2$. The third term in (10) reflects that a higher level of investment in the current period makes future cash flows riskier, which lowers the *expected* value of the selling price at date $t+1$. Lastly, a higher level of investment also makes $P_{t+1}$ more volatile lowering the current owners' certainty equivalent by the amount of $\frac{\rho}{2} Var_t(P_{t+1})$. This risk cost is captured by the last term of (10).

The optimal investment level $I_t^*$ is given by the following first-order condition to the above maximization problem:

$$I_t^* = \frac{\gamma^2}{2b + \rho\gamma^2(2\sigma_p^2 + \gamma\sigma_a^2)} \cdot m_{t+2}. \tag{11}$$

Equation (11) implies that the optimal investment level $I_t^*$ increases in the quality of information disclosure (i.e., $\frac{\partial I_t^*}{\partial \sigma_a^2} > 0$). Intuitively, a more precise public disclosure lowers the risk-related marginal cost of investments as represented by the last two terms of the objective function in (10). Consistent with the standard intuition, it can be checked that the optimal

[14] It can be verified that the qualitative nature of our main results remain unchanged under more general assumptions on investment costs.

[15] We assume that the firm chooses the number of new shares issued at date $t$, $n_t$, anticipating the optimal investment size that would be chosen by the new shareholders, i.e., so that $(1-\delta_t)P_t = c(I_t^*)$. If this were not the case, the incoming generation of investors could immediately raise more cash by issuing a new SEO at date $t$ and make the optimal investment $I_t^*$.

[16] To ensure a finite market price for the firm, we assume that $\sum_{\tau=1}^{\infty} \gamma^\tau m_{t+\tau+2}^2 < \infty$ for each $t$. This condition will be satisfied, for example, when the asymptotic growth rate of the investment productivity parameters $\{m_t\}$ does not exceed $\sqrt{1+r}$.

investment level is more sensitive to the precision of public disclosure (i.e., $\frac{\partial I_t^*}{\partial \sigma_a^2}$ is higher) when the marginal cost parameter $b$ is small, or the expected marginal benefit $m_{t+2}$ is large.

As before, the risk premium in period $t+1$ is given by

$$RP_{t+1} = \rho \left[ {I_{t-1}^*}^2 \cdot \sigma_p^2 + \gamma^2 \cdot I_t^{*2} \cdot \sigma_a^2 \right].$$

The result below characterizes how endogenous investments change the equilibrium relationship between risk premium and information disclosure.

**Proposition 6.** *With endogenous investments, the risk premium in period $t+\tau$ increases in the informativeness of public disclosure if*

$$\frac{m_{t+\tau+1}^2}{m_{t+\tau}^2} \geq (1+r)^2 - l(\sigma_a), \tag{12}$$

*where $l(\sigma_a) > 0$. The risk premium in period $t+\tau$ decreases in the informativeness of public disclosure if the opposite inequality holds.*

Since the optimal investment level $I_{t+\tau-1}^*$ is proportional to the productivity parameter $m_{t+\tau}$, the inequality in the above result can be equivalently expressed in terms of the endogenous growth rate; i.e., $\mu_{t+\tau} = \frac{I_{t+\tau}^*}{I_{t+\tau-1}^*} - 1$. Analogous to our finding in Proposition 1, this result shows that the equilibrium relationship between risk premium and the quality of public disclosure depends on the firm's growth trajectory. For instance, it shows that the risk premium in period $t+1$ increases in the informativeness of public information if the endogenous growth rate $\mu_t$ exceeds a certain threshold. However, since $l > 0$, Proposition 6 shows that the threshold growth rate is lower than $r$, the threshold for the exogenous investment setting. With endogenous investments, a more precise public disclosure affects not only allocation of the total project risk between its dividend and price components, but also results in higher optimal investment levels, which leads to higher project risk. It is because of this real effect of public disclosure that the threshold growth rate is lower in the endogenous investment setting.

Next we investigate the relationship between information disclosure and investors welfare with endogenous investments. As before, suppose a new disclosure policy (i.e., a new value of precision for all future disclosures) takes effect between dates $t-1$ and date $t$ when the firm is owned by generation $t$ investors.

**Proposition 7.**

*i For all $\tau \geq 1$, welfare of future investors of generation $t + \tau$ increases (decreases) in the precision of public disclosure if the inequality in (12) holds (does not hold).*

*ii. Welfare of the existing shareholders is maximized at an intermediate level of public disclosure if future investments are sufficiently sensitive to information disclosure.*[17]

The first part of the above proposition follows from Proposition 6 because, as discussed in connection with Proposition 2, the expected utility of *future* potential investors increases in the risk premium in the period during which they plan to hold the firm. The second part of the above result contrasts with the corresponding finding in Propositions 2 and 5 which show that current shareholders unambiguously prefer the most informative public disclosure regime in pure exchange settings. In contrast, when investments are endogenously chosen and sufficiently sensitive to the precision of public disclosure, the current shareholders' welfare is maximized at an intermediate level of disclosure. This result implies that even if the shareholders could increase the precision of public disclosures *costlessly*, they might still prefer financial disclosure regimes that require less than full disclosure.

With endogenous investments, the current shareholders' expected utility varies with the amount of public information directly through its effect on the total risk premium (for *fixed* investment levels), as well as indirectly through the effect of public disclosures on the firm's optimal investment choices. While the direct effect of information disclosure on the current shareholders' welfare is always positive (see Propositions 2 and 5), the indirect effect is detrimental to the original shareholders' welfare. Intuitively, future generations of shareholders overinvest relative to the preferred investment levels from the perspective of current shareholders, and the amount of overinvestment increases in the precision of public disclosure.[18]

For large values of cost parameter $b$, the optimal investment levels are relatively insensitive to the precision of public disclosure and hence the direct beneficial effect dominates, and the welfare of the current shareholders increases in the informativeness of public disclosures. Similarly, when the current investment is large relative to future investments, the current

[17]Future optimal investments are more sensitive to the precision of public information for low values of $b$ and high values of $\{m_{t+\tau}\}$. See the proof of Proposition 7 for a precise set of sufficient conditions.

[18]Specifically, it can be shown that while future investors will choose $I_{t+\tau}$ to maximize $V(I_{t+\tau}, \sigma_a^2)$, the current shareholders would prefer them to maximize $V(I_{t+\tau}, \sigma_a^2) - \frac{\gamma^2}{2}\rho I_{t+\tau}^2 \sigma_a^2$.

shareholders' welfare is primarily determined by their expected utility from the payoffs related to the current project; i.e., $V(I_{t-1}, \sigma_a^2)$, which monotonically increases in the quality of public information. In all other cases, the current shareholders' welfare is maximized at an intermediate level of disclosure.

# 6 Conclusion

We have investigated how information disclosure affects risk premia and investors' welfare in a dynamic setting with overlapping generations of investors. Our analysis demonstrates that the relationship between a firm's cost of capital and quality of its public disclosures crucially depends on the firm's growth trajectory. In particular, we find that the risk premium decreases (increases) in the quality of information disclosure when the firm's growth rate is lower (higher) than a threshold. Our analysis also demonstrates that serial correlation among periodic cash flows plays a critical role in determining the nature of this relationship.

We demonstrate that our results extend to multi-firm economies in which each firm's cash flows are subject to both systematic and idiosyncratic risks. In the large economy limit, a firm's risk premium depend only on its exposure to systematic risk. As in the single firm setting, we find that a firm's risk premium decreases (increases) in information disclosure if the firm's growth rate is below (above) a certain threshold. The threshold growth rate for a firm is lower (higher) when other firms in the economy are growing at higher (lower) rates.

With regard to the effect of public disclosure on investors welfare, we find that disclosure preferences of current and future shareholders are not always aligned. The expected utilities of future generation of investors *increase* in the periodic risk premium during their investment horizon. On the other hand, current shareholders unambiguously prefer the most precise public disclosures in pure exchange settings. Our analysis of the production economy, however, shows that current shareholders' welfare may be maximized at an intermediate level of disclosure if endogenous investments are sufficiently sensitive to information disclosure.

# Appendix

**Proof of Lemma 1:**

We will first prove that the equilibrium market price of the firm at date $t$ is given by

$$P_t = \sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot \left[E_t\left(X_{t+\tau}\right) - c(I_{t+\tau}) - RP_{t+\tau}\right], \tag{13}$$

where

$$RP_{t+\tau} = \rho \cdot \left(I_{t+\tau-2}^2 \cdot \sigma_p^2 + \gamma^2 \cdot I_{t-\tau-1}^2 \cdot \sigma_a^2\right)$$

denotes the risk premium in period $t+\tau$. Since signal $s_t$ is uninformative about $x_{t+\tau}$ for all $\tau > 1$, $E_t(x_{t+\tau}) = m_{t+\tau+1}$ for all $\tau > 1$. The formula for the conditional expectations of normal random variables gives $E_t(x_{t+1}) = k \cdot s_t$ where $k \equiv \frac{\sigma^2}{\sigma_2 + \sigma_\eta^2}$. Hence, the pricing function in (13) can be expressed as follows:

$$P_t = \beta_t + I_{t-1} \cdot k \cdot s_t, \tag{14}$$

where $\beta_t$ is a constant. Since signal $s_t$ is normally distributed, equation (14) implies that $P_t$ is also normal from the perspective of date $t-1$.

We will now verify that the pricing function in (13) satisfies the market clearing condition at each $t$. Consider the portfolio choice problem of the representative investor of generation $t$. Without loss of generality, we normalize the investor's initial wealth to zero and assume that the investor pays for the purchase cost of shares by borrowing at the risk-free rate of $r$. If the representative investor of generation $t-1$ buys $\alpha$ fraction of the firm's shares outstanding (including the SEO) at date $t-1$, her date $t$ wealth (consumption) is given by

$$\omega_t = \alpha \cdot \left[X_t + \delta_t \cdot P_t - (1+r) \cdot P_{t-1}\right].$$

Taking price $P_{t-1}$ as given, the investor chooses $\alpha$ to maximize his expected utility of wealth $\omega_t$. Since $P_t$ as conjectured in (13) is normal, the investors's terminal wealth $\omega_t$ is also normal. Hence, maximizing expected utility is equivalent to maximizing the following certainty equivalent expression:

$$CE_{t-1}(\alpha) = \alpha \cdot \left[E_{t-1}\left(X_t + P_t\right) - c(I_t) - (1+r) \cdot P_{t-1}\right] - \frac{\rho}{2} \cdot \alpha^2 \cdot Var_{t-1}\left(X_t + P_t\right),$$

where we have used the fact that $\delta_t P_t = P_t - c(I_t)$. Therefore, the optimal $\alpha$ is determined by the following first-order condition:

$$E_{t-1}\left(X_t + P_t\right) - c(I_t) - (1+r)\cdot P_{t-1} - \rho\cdot\alpha\cdot Var_{t-1}\left(X_t + P_t\right) = 0.$$

Imposing the market clearing condition $\alpha = 1$ gives

$$P_{t-1} = \gamma\cdot\left[E_{t-1}\left(X_t + P_t\right) - c(I_t) - \rho\cdot Var_{t-1}\left(X_t + P_t\right)\right]. \tag{15}$$

Equation (15) implies that the equilibrium risk premium in period $t$, $RP_t$, is given by $\rho\cdot Var_{t-1}(X_t + P_t)$. We note that $Var_{t-1}(X_t) = I_{t-2}^2\cdot\sigma_p^2$ and equation (14) implies $Var_t(P_t) = \gamma^2\cdot I_{t-1}^2\cdot\sigma_a^2$. Since $P_t$, as conjectured in equation (13), is independent of $X_t$, it follows that

$$\begin{aligned} RP_t &= \rho\cdot\left[Var_{t-1}\left(X_t\right) + Var_{t-1}\left(P_t\right)\right] \\ &= \rho\cdot\left[I_{t-2}^2\cdot\sigma_p^2 + \gamma^2\cdot I_{t-1}^2\cdot\sigma_a^2\right]. \end{aligned}$$

We can now verify that if the prices are given by equation (13), the market clearing condition (15) holds at all dates. To show this, we note that equation (13) implies

$$P_{t-1} = \sum_{\tau=1}^{\infty}\gamma^\tau\left[E_{t-1}\left(X_{t+\tau-1}\right) - c(I_{t+\tau-1}) - RP_{t+\tau-1}\right],$$

which can be written as

$$\begin{aligned} P_{t-1} &= \gamma\cdot\left[E_{t-1}\left(X_t\right) - c\left(I_t\right) - RP_t\right] + \gamma\cdot\sum_{\tau=1}^{\infty}\gamma^\tau\left[E_{t-1}\left(X_{t+\tau}\right) - c(I_{t+\tau}) - RP_{t+\tau}\right] \\ &= \gamma\cdot\left[E_{t-1}(X_t + P_t) - c(I_t) - RP_t\right]. \end{aligned}$$

Thus, the pricing function in (13) satisfies the market clearing condition in (15). $\square$

**Proof of Proposition 1:**

The proof of Lemma 1 implies that the equilibrium risk premium in period $t+1$ is given by

$$RP_{t+1} = \rho\cdot\left[I_{t-1}^2\cdot\sigma_p^2 + \gamma^2\cdot I_t^2\cdot\sigma_a^2\right]. \tag{16}$$

By the law of total variance, $\sigma_p^2 = \sigma^2 - \sigma_a^2$. Hence, equation (16) yields

$$RP_{t+1} = \rho I_{t-1}^2 \sigma^2 + \rho I_{t-1}^2 \left[\gamma^2 \cdot (1+\mu_t)^2 - 1\right] \sigma_a^2.$$

Since the precision of the public disclosure is proportional to $\sigma_a^2$, the risk premium increases (decreases) in the informativeness of public disclosure if $\gamma^2 \cdot (1+\mu_t)^2 - 1$ is positive (negative), which is equivalent to the condition that growth rate $\mu_t$ is more (less) than $r$.

$\square$

**Proof of Proposition 2:**

**Proof of part i**

We will prove the result for welfare of generation $t+1$ investors, since the same argument applies for all other future generations. The equilibrium expected utility of generation $t+1$ investor is monotonically increasing in following certainty equivalent expression:

$$CE_{t+1} = E_t[X_{t+1} + P_{t+1}] - c(I_{t+1}) - (1+r) \cdot P_t - \frac{1}{2} \cdot \rho \cdot Var_t[X_{t+1} + P_{t+1}].$$

Substituting for $P_t$ from (15) yields

$$CE_{t+1} = \frac{1}{2} \cdot \rho \cdot Var_t[X_{t+1} + P_{t+1}].$$

The proof of Lemma 1 shows that $RP_{t+1} = \rho \cdot Var_t[X_{t+1} + P_{t+1}]$, and hence

$$CE_{t+1} = \frac{1}{2} \cdot RP_{t+1}.$$

It thus follows from Proposition 1 that generation $t+1$ investor's expected utility decreases (increases) in the precision of public disclosure if $\mu_t < r$ ($\mu_t > r$).

**Proof of part ii**

The expected utility of the existing shareholders of generation $t$ can be represented by the following certainty equivalent expression:

$$CE_t = E_{t-1}(P_t) - \frac{\rho}{2} \cdot Var_{t-1}(P_t) + \beta_t, \tag{17}$$

where $\beta_t \equiv E_{t-1}(X_t) - c(I_t) - (1+r) \cdot P_t - \frac{\rho}{2} \cdot Var_{t-1}(X_t)$ does not depend on the precision of future disclosures. Using the law of iterated expectations, equation (13) yields

$$E_{t-1}(P_t) = \sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot [m_{t+\tau} \cdot I_{t+\tau-2} - c(I_{t+\tau}) - RP_{t+\tau}] \,.$$

Moreover, we note that $Var_{t-1}(P_t) = \gamma^2 \cdot I_{t-1}^2 \cdot \sigma_a^2$. Substituting these into (17) and denoting the terms independent of the precision of future disclosures by $A_t$ yield

$$CE_t = A_t - \frac{\rho}{2} \cdot \gamma^2 \cdot I_{t-1}^2 \cdot \sigma_a^2 - \sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot RP_{t+\tau}.$$

Since

$$\begin{aligned}\sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot RP_{t+\tau} &= \rho \sum_{\tau=1}^{\infty} \gamma^{\tau} \left[I_{t+\tau-2}^2 \sigma_p^2 + \gamma^2 I_{t+\tau-1}^2 \sigma_a^2\right] \\ &= \rho \gamma I_{t-1}^2 \sigma_p^2 + \rho \sum_{\tau=1}^{\infty} \gamma^{\tau+1} I_{t+\tau-1}^2 (\sigma_p^2 + \gamma \sigma_a^2),\end{aligned}$$

it follows that

$$CE_t = A_t - \rho \gamma I_{t-1}^2 \left[\sigma_p^2 + \frac{\gamma}{2} \sigma_a^2\right] - \rho \sum_{\tau=1}^{\infty} \gamma^{\tau+1} I_{t+\tau-1}^2 (\sigma_p^2 + \gamma \sigma_a^2).$$

By the law of total variance, $\sigma_p^2 + \gamma\sigma_a^2 = \sigma^2 - (1-\gamma)\sigma_a^2$ and $\sigma_p^2 + \frac{\gamma}{2}\sigma_a^2 = \sigma^2 - (1-\frac{\gamma}{2})\sigma_a^2$. Making these substitutions and differentiating with respect to $\sigma_a^2$ gives

$$\frac{\partial CE_t}{\partial \sigma_a^2} = \rho\gamma \left(1 - \frac{\gamma}{2}\right) I_{t-1}^2 + \rho\gamma\,(1-\gamma) \sum_{\tau=1}^{\infty} \gamma^{\tau} I_{t+\tau-1}^2, \tag{18}$$

which is positive.

□

**Proof of Lemma 2:**

We first prove that the equilibrium market price of the firm as a function of date $t$ information $(x_t, s_t)$ is given by

$$P_t = \gamma \cdot Q_{t-1} \cdot [w \cdot x_t + k \cdot \hat{s}_t] + \sum_{\tau=1}^{\infty} \gamma^{\tau} \cdot [m_{t+\tau} \cdot I_{t+\tau-2} - c\,(I_{t+\tau}) - RP_{t+\tau}] \tag{19}$$

where

$$RP_{t+\tau} = \rho \cdot \left[\sigma_p^2 \cdot Q_{t+\tau-2}^2 + \gamma^2 \cdot \sigma_a^2 \cdot Q_{t+\tau-1}^2\right]$$

denotes the risk premium in period $t$.

We now proceed to verify that the pricing function in (19) satisfies the market clearing condition at each $t$. As before, market price $P_{t-1}$ must satisfy the following market-clearing condition:

$$P_{t-1} = \gamma \cdot [E_{t-1}(X_t + P_t) - c(I_t) - \rho \cdot Var_{t-1}(X_t + P_t)], \tag{20}$$

where we have used the fact that $\delta_t P_t = P_t - c(I_t)$. Equation (20) implies that the equilibrium risk premium in period $t+1$ is given by

$$RP_t = Var_{t-1}(X_t + P_t).$$

Substituting for $P_t$ from (19) and simplifying yield

$$X_t + P_t = Q_{t-2} \cdot x_t + \gamma \cdot k \cdot Q_{t-1} \cdot \hat{s}_t + I_{t-2} m_t + \sum_{\tau=1}^{\infty} \gamma^\tau \cdot [m_{t+\tau} I_{t+\tau-2} - c\,(I_{t+\tau}) - RP_{t+\tau}],$$

where we have used the facts that $X_t = I_{t-2}(x_t + m_t)$ and $Q_{t-2} = I_{t-2} + \gamma \cdot w \cdot Q_{t-1}$. Since $E_{t-1}(x_t) = w \cdot x_{t-1} + k \cdot \hat{s}_{t-1}$ and $E_{t-1}(\hat{s}_t) = 0$, it follows that

$$E_{t-1}[X_t + P_t] = Q_{t-2} \cdot [w \cdot x_{t-1} + k \cdot \hat{s}_{t-1}] + I_{t-2} m_t + \sum_{\tau=1}^{\infty} \gamma^\tau \cdot [m_{t+\tau} I_{t+\tau-2} - c\,(I_{t+\tau}) - RP_{t+\tau}].$$

Since $Var_{t-1}(x_t) = \sigma_p^2$ and $Var_{t-1}(k \cdot \sigma_a^2) = \sigma_a^2$, we get

$$Var_{t-1}(X_t + P_t) = \sigma_p^2 \cdot Q_{t-2}^2 + \gamma^2 \cdot \sigma_a^2 \cdot Q_{t-1}^2 = RP_t.$$

Substituting the above expressions for the conditional mean and variance into (20) yields

$$\begin{aligned} P_{t-1} &= \gamma \cdot [Q_{t-2} \cdot (w \cdot x_{t-1} + k \cdot \hat{s}_{t-1}) + I_{t+2} m_t - c(I_t) - RP_t] \\ &\quad + \sum_{\tau=1}^{\infty} \gamma^\tau \cdot [m_{t+\tau} I_{t+\tau-2} - c\,(I_{t+\tau}) - RP_{t+\tau}] \\ &= \gamma \cdot Q_{t-2} \cdot [w \cdot x_{t-1} + k \cdot \hat{s}_{t-1}] + \sum_{\tau=1}^{\infty} \gamma^\tau \cdot [m_{t+\tau-1} I_{t+\tau-1} - c\,(I_{t+\tau-1}) - RP_{t+\tau-1}]. \end{aligned}$$

We have thus verified that the market clearing condition in (20) holds for all $t$ if the market price is given by (19).

The risk premium in period $t+1$ is given by

$$RP_{t+1} = \rho \cdot \left[\sigma_p^2 \cdot Q_{t-1}^2 + \gamma^2 \cdot \ \sigma_a^2 \cdot Q_t^2\right].$$

Substituting $\sigma^2 = \sigma_p^2 + \sigma_a^2$ yields

$$RP_{t+1} = \rho \cdot \sigma^2 \cdot Q_{t-1}^2 + \rho \cdot \sigma_a^2 \cdot [\gamma^2 \cdot Q_t^2 - Q_{t-1}^2].$$

Since $\sigma_a^2$ increases in the precision of public disclosure, the risk premium decreases (increases) in the informativeness of public disclosure when $\gamma \cdot Q_t - Q_{t-1}$ is negative (positive). Since $Q_{t-1} = I_{t-1} + \gamma \cdot w \cdot Q_t$, we get

$$\gamma \cdot Q_t - Q_{t-1} = \gamma \cdot (1-w) \cdot Q_t - I_{t-1}. \tag{21}$$

It thus follows that the risk premium decreases (increases) in the disclosure quality when $I_{t-1}$ is more (less) than $\gamma \cdot (1-w) \cdot Q_t$.

□

**Proof of Proposition 3:**

We recall from the proof of Lemma 2 that the risk premium decreases (increases) in the precision of public information if $\gamma \cdot Q_t - Q_{t-1}$ is negative (positive), where

$$Q_t \equiv \sum_{\tau=0}^{\infty} (\gamma \cdot w)^\tau \cdot I_{t+\tau}.$$

For brevity, let us define

$$q \equiv \gamma \cdot (1+\mu).$$

Since the firm grows at a constant rate of $\mu$ until it reaches the steady state size of $I_{t-1} \cdot (1+\mu)^{T-t+1}$ at date $T$, it follows that

$$Q_\tau = I_\tau \cdot \left( \frac{1-(w \cdot q)^{T-\tau}}{1-w \cdot q} + \frac{(w \cdot q)^{T-\tau}}{1-\gamma \cdot w} \right) \text{ for } w \cdot q \neq 1$$

and $Q_\tau = I_\tau \cdot \left(T - \tau + \frac{1}{1-\gamma \cdot w}\right)$ for $w \cdot q = 1$

Using the above expression for $Q_\tau$, it can be verified that for $w \cdot q \neq 1$[19]

$$\gamma \cdot Q_t - Q_{t-1} = \frac{I_{t-1} \cdot \Gamma(q)}{(1 - w \cdot q) \cdot (1 - w \cdot \gamma)}, \tag{22}$$

where

$$\Gamma(q) \equiv (1 - w \cdot \gamma) \cdot (q - 1) - (q \cdot w)^{T-t+1} \cdot (1 - w) \cdot (q - \gamma). \tag{23}$$

We note that $\Gamma(q) < 0$ for all $q \leq 1$. This implies that $\gamma \cdot Q_t - Q_{t-1}$ is negative for all $q \leq 1$ (i.e., all $\mu \leq r$). It thus follows that the risk premium decreases in the precision of the disclosure system for all $\mu \leq r$.

To derive an upper bound on the growth rate above which the risk premium increases in the precision of information, we substitute $Q_t = I_t + \gamma \cdot w \cdot Q_{t+1}$ and $I_t = (1 + \mu) \cdot I_{t-1}$ in equation (21) to obtain

$$\gamma \cdot Q_t - Q_{t-1} = I_{t-1} \cdot [q \cdot (1 - w) - 1] + \gamma^2 \cdot w \cdot (1 - w) \cdot Q_{t+1}.$$

Hence, a sufficient condition for $\gamma \cdot Q_t - Q_{t-1}$ to be positive is that

$$q \geq \frac{1}{1 - w},$$

which is equivalent to

$$\mu \geq \frac{r + w}{1 - w}.$$

Therefore the risk premium increases in the precision of public information for all $\mu \geq \frac{r+w}{1-w}$.

To prove the existence of a unique threshold growth rate $\hat{\mu}$, we investigate the sign of $\gamma \cdot Q_t - Q_{t-1}$ for the values of $q$ greater than 1. Using the definition in (23), it can be verified that function $\Gamma(\cdot)$ is strictly concave, $\Gamma(1) < 0$, and $\Gamma(w^{-1}) = 0$. Since $w^{-1} > 1$, these facts imply that either:

(i) $\Gamma(q)$ initially increases, takes its maximum value at some unique $q^* > 1$ (with $\Gamma(q^*) \geq 0$), and then decreases, or

(ii) $\Gamma(q)$ is monotonically increasing in $q$ with $\Gamma(q) < 0$ for $q < w^{-1}$ and $\Gamma(q) > 0$ for all

[19]We note that $\gamma \cdot Q_t - Q_{t-1} = I_{t-1} \cdot \left[\frac{(1-w)\cdot[(T-t)(1-\gamma w)+1]}{w(1-\gamma w)} - 1\right]$ for $w \cdot q = 1$, and hence $\gamma \cdot Q_t - Q_{t-1}$ is a continuous and differentiable function of $q$.

$q > w^{-1}$.

Equation (22) implies that $\gamma \cdot Q_t - Q_{t-1}$ and $\Gamma(q)$ have the same (opposite) signs when $q < w^{-1}$ ($q > w^{-1}$). This implies that if case (ii) above were to hold, then $\gamma \cdot Q_t - Q_{t-1} < 0$ for all $q \geq 1$. This, however, contradicts the above result that the risk premium increases in the precision of information (i.e., $\gamma \cdot Q_t - Q_{t-1} > 0$) for all $q > \frac{1}{1-w}$. It therefore follows that case (i) above must apply and $\Gamma(\cdot)$ is a single peaked function.

We need to consider two possibilities for the maximizer of $\Gamma(q)$: (i) $q^* < w^{-1}$, and (ii) $q^* \geq w^{-1}$. In the first case, the function $\Gamma(q)$ achieves its maximum at some point below $w^{-1}$. As a consequence, there exists a $\hat{q} \in (1, q^*)$ such that (i) $\Gamma(\hat{q}) = \Gamma(w^{-1}) = 0$, (ii) $\Gamma(q) > 0$ for all $q \in (\hat{q}, w^{-1})$, and (iii) $\Gamma(q) < 0$ for all $q \notin (\hat{q}, w^{-1})$. It then follows from equation (22) that $\gamma \cdot Q_t - Q_{t-1}$ is negative for all $q < hatq$, and positive for all $q > \hat{q}$. Define

$$\hat{\mu} \equiv (1 + r) \cdot \hat{q} - 1.$$

It then follows that the risk premium decreases (increases) in the precision of public information if the firm's growth rate is less (more) than $\hat{\mu}$, where $\hat{\mu} \in (r, \frac{r+w}{1-w})$.

Consider now the second possibility that $q^* \geq w^{-1}$. In this case, there exists a $\hat{q} \geq w^{-1}$ such that $\Gamma(q)$ is negative for $q \in [1, w^{-1}]$, positive for $q \in (w^{-1}, \hat{q})$, and again negative for $q \geq \hat{q}$. Therefore, it again follows from (**??**) that there exists a unique $\hat{\mu}$ such that $\gamma \cdot Q_t - Q_{t-1}$ is negative for $\mu < \hat{\mu}$, and positive for $\mu > \hat{\mu}$.

We have thus proven the existence of a unique threshold growth rate $\hat{\mu} \in (r, \frac{r+w}{1-w})$. To show that $\hat{\mu}$ increases in the persistence parameter $w$, it suffices to show that $q^*$ increases in $w$. We note that $\Gamma(q^0) = 0$ for $q^0 \in \{q^*, w^{-1}\}$. That is, $\hat{q}$ is given by the solution to the following equation:

$$(1 - w \cdot \gamma) \cdot (q^0 - 1) - (q^0 \cdot w)^{T-t+1} \cdot (1 - w) \cdot (q^0 - \gamma) = 0,$$

with $q^0 \neq w^{-1}$. Implicitly differentiating the above equation with respect to $w$ and using the fact that $\Gamma(q^0) = 0$ yield (for both values of $q^0$)

$$sgn\left[\frac{dq^0}{dw}\right] = sgn\left[\frac{-w\,(1-\gamma) + (T-t+1)\,(1-w\gamma)\,(1-w)}{H(q^0)}\right], \tag{24}$$

where

$$H(q^0) \equiv \left[q^0(1-\gamma) - (T-t+1)(q^0-\gamma)(q^0-1)\right].$$

It can be easily checked that

$$H(w^{-1}) = -\frac{1}{w^2}\left[-w\left(1-\gamma\right) + \left(T-t+1\right)\left(1-w\gamma\right)\left(1-w\right)\right].$$

Since function $\Gamma(q)$ is concave and its graph crosses the horizontal axis at $q = \hat{q}$ and $q = w^{-1}$, its derivative $\Gamma'(q)$ must be of the opposite signs at these two values of $q$. Furthermore, it is easy to verify that $sgn\left[H(q^0)\right] = sgn\left[\Gamma'(q^0)\right]$. It therefore follows that

$$\begin{aligned} sgn[H(\hat{q})] &= -sgn[H(w^{-1})] \\ &= sgn\left[\frac{1}{w^2}\left[-w\left(1-\gamma\right) + \left(T-t+1\right)\left(1-w\gamma\right)\left(1-w\right)\right]\right]. \end{aligned}$$

Substituting this in (24) yields

$$sgn\left[\frac{d\hat{q}}{dw}\right] = sgn\left[\frac{1}{w^2}\right],$$

and thus $\frac{d\hat{q}}{dw} > 0$. This proves that $\hat{q}$ increases in $w$.

□

**Proof of Lemma 3:**

We will first show that for each $j$ and each $t$, the equilibrium market price of firm $j$ at date $t$ is given by

$$P_t^j = \sum_{\tau=1}^{\infty} \gamma^\tau \cdot \left[E_t\left(X_{t+\tau}^j\right) - c(I_{t+\tau}^j) - RP_{t+\tau}^j\right], \tag{25}$$

where

$$RP_{t+\tau}^j = \frac{\rho}{N} \cdot \left[Cov_{t+\tau-1}(X_{t+\tau}^j, X_{t+\tau}^m) + Cov_{t+\tau-1}(P_{t+\tau}^j, P_{t+\tau}^m)\right]$$

denotes the risk premium, $X_t^m \equiv \sum_{i=1}^J X_t^i$, and $P_t^m \equiv \sum_{i=1}^J P_t^i$. From an ex-ante perspective, we note that $P_t^j$ is a normally distributed random variable because (i) $E_t(X_{t+1}^j)$ is a linear function of $s_t \equiv (s_t^1, \cdots, s_t^J)$, (ii) $E_t(X_{t+\tau}^j)$ are non-stochastic for all $\tau \geq 2$, and (ii) $c(I_{t+\tau}^j)$ and $RP_{t+\tau}^j$ are non-stochastic for all $t+\tau$.

Consider the portfolio choice problems of generation $t$ investors. If investor $n$ of generation $t$ buys $\alpha_{tn}^j$ fraction of firm $j$'s shares outstanding (including the SEO) at date $t-1$, her date

$t$ wealth is given by

$$\omega_{tn} = \sum_{i=1}^{J} \alpha_{tn}^i \cdot \left[X_t^i + P_t^i - c(I_t^i) + (1+r) \cdot P_{t-1}^i\right],$$

where we have used $\delta_t^i P_t^i = P_t^i - c(I_t^i)$ for all $i$. Note that $\omega_{tn}$ is normally distributed since the conjectured equilibrium price $P_t^i$ is normal for each $i$. This implies that investor $n$ of generation $t$ will choose $\alpha_{tn} \equiv (\alpha_{tn}^1, \cdots, \alpha_{tn}^J)$ to maximize the following certainty equivalent expression:

$$\begin{aligned} CE_{tn} &= \sum_{i=1}^{J} \alpha_{tn}^i \cdot \left[E_{t-1}(X_t^i + P_t^i) - c(I_t^i) - (1+r) \cdot P_{t-1}^i\right] \\ &- \frac{\rho}{2} \cdot \sum_{i,k=1}^{J} \alpha_{nt}^i \cdot \alpha_{tn}^k \left[Cov_{t-1}(X_t^i, X_t^k) + Cov_{t-1}(P_t^i, P_t^m)\right], \end{aligned} \tag{26}$$

where we have used the fact that $\{X_t^i\}_{i=1}^J$ and $\{P_t^i\}_{i=1}^J$ are all independent of each other. Therefore, investor $n$'s first-order condition with respect to her choice of $\alpha_{tn}^j$ yields

$$E_{t-1}\left(X_t^j + P_t^j\right) - c(I_t^j) - (1+r) \cdot P_{t-1}^j - \rho \cdot \sum_{i=1}^{J} \alpha_{tn}^i \left[Cov_{t-1}(X_t^j, X_t^i) + Cov_{t-1}(P_t^j, P_t^i)\right] = 0.$$

The above condition implies that the optimal value of $\alpha_{tn}^j$ is the same for each investor; i.e., $\alpha_{tn}^j = \alpha_t^j$ for all $n$. Imposing the market-clearing condition $\sum_{n=1}^N \alpha_{tn}^j = 1$ yields that each investor buys fraction $\frac{1}{N}$ of firm $j$ for each $j \in \{1, \cdots, J\}$. Summing over both sides of the above equation with respect to $n$ and substituting $\sum_{n=1}^N \alpha_{tn}^j = 1$ give

$$N\left[E_{t-1}\left(X_t^j + P_t^j\right) - c(I_t^j) - (1+r) \cdot P_{t-1}^j\right] - \rho \cdot \left[Cov_{t-1}(X_t^j, X_t^m) + Cov_{t-1}(P_t^j, P_t^m)\right] = 0.$$

This, in turn, implies that the market-clearing price for firm $j$ is given by

$$P_{t-1}^j = \gamma\left[E_{t-1}\left(X_t^j + P_t^j\right) - c(I_t^j) - RP_t^j\right],$$

where

$$RP_t^j \equiv \frac{\rho}{N} \cdot \left[Cov_{t-1}(X_t^j, X_t^m) + Cov_{t-1}(P_t^j, P_t^m)\right] \tag{27}$$

is the risk premium in period $t$. Substituting for $P_t^j$ from (25) into the above expression for

$P^j_{t-1}$, it can be checked that

$$P^j_{t-1} = \sum_{\tau=1}^{\infty} \gamma^\tau \cdot \left[ E_{t-1}\left(X^j_{t+\tau-1}\right) - c(I^j_{t+\tau-1}) - RP^j_{t+\tau-1} \right].$$

We have thus verified that the pricing function in (25) satisfies the market clearing conditions for all $t$.

From equation (27), the risk premium for firm $j$ in period $t+1$ is given by

$$RP^j_{t+1} = \frac{\rho}{N} \cdot \left[ Cov_t(X^j_{t+1}, X^m_{t+1}) + Cov_t(P^j_{t+1}, P^m_{t+1}) \right] \tag{28}$$

To calculate the conditional covariances in (28), it will be convenient to denote the $1 \times 2$ vector $(X^j_{t+1}, X^m_{t+1})$ by $X_{t+1}$. Letting $\sigma^2 \equiv \sigma^2_x + \sigma^2_\theta$, it can be verified that

$$Var(X_{t+1}) \equiv \Sigma_{xx} = \begin{bmatrix} (I^j_{t-1})^2\sigma^2 & I^j_{t-1}I^m_{t-1}\sigma^2_x + (I^j_{t-1})^2\sigma^2_\theta \\ I^j_{t-1}I^m_{t-1}\sigma^2_x + (I^j_{t-1})^2\sigma^2_\theta & (I^m_{t-1})^2\sigma^2_x + \sum_{i=1}^J (I^i_{t-1})^2\sigma^2_\theta \end{bmatrix},$$

$$Var(s_t) \equiv \Sigma_{ss} = \begin{bmatrix} \sigma^2 + J\sigma^2_\eta & \sigma^2_x & \cdots & \sigma^2_x \\ \sigma^2_x & \sigma^2 + J\sigma^2_\eta & \cdots & \sigma^2_x \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^2_x & \sigma^2_x & \cdots & \sigma^2 + J\sigma^2_\eta \end{bmatrix},$$

and

$$Cov(s_t, X_{t+1}) \equiv \Sigma_{sx} = \begin{bmatrix} I^j_{t-1}\sigma^2_x & \cdots & I^j_{t-1}\sigma^2 & \cdots & I^j_{t-1}\sigma^2_x \\ I^m_{t-1}\sigma^2_x + I^1_{t-1}\sigma^2_\theta & \cdots & \cdots & \cdots & I^m_{t-1}\sigma^2_x + I^J_{t-1}\sigma^2_\theta \end{bmatrix}.$$

It follows from the properties of normal random variables that

$$Var_t(X_{t+1}) \equiv Var(X_{t+1}|s_t) = \Sigma_{xx} - \Sigma_{xs}\Sigma^{-1}_{ss}\Sigma_{sx},$$

and $Cov_t(X^j_{t+1}, X^m_{t+1})$ is given by the off-diagonal element of the above $2 \times 2$ matrix. Calculating $Var(X_{t+1}|s_t)$ shows that

$$Cov_t(X^j_{t+1}, X^m_{t+1}) = \frac{I^j_{t-1}J\sigma^2_\eta \left[ I^m_t J\sigma^2_x\sigma^2_\eta + I^j_{t-1}(J\sigma^2_\eta + \sigma^2_\theta) + \sigma^2_\theta(J\sigma^2_x + \sigma^2_\theta) \right]}{(J\sigma^2_\eta + \sigma^2_\theta)(J\sigma^2_x + J\sigma^2_\eta + \sigma^2_\theta)}. \tag{29}$$

From equation (25), we note that

$$P_{t+1}^j = \gamma \cdot E_{t+1}(X_{t+2}^j) + const.$$

It thus follows that

$$Cov_t(P_{t+1}^j, P_{t+1}^m) = \gamma^2 \cdot Cov_t\left[E_{t+1}(X_{t+2}^j), E_{t+1}(X_{t+2}^m)\right].$$

To calculate the conditional covariance term above, we apply the law of total covariance to obtain

$$Cov_t[E_{t+1}(X_{t+2}^j), E_{t+1}(X_{t+2}^m)] = Cov_t(X_{t+2}^j, X_{t+2}^m) - Cov_{t+1}(X_{t+2}^j, X_{t+2}^m).$$

We can calculate $Cov_{t+1}(X_{t+2}^j, X_{t+2}^m)$ from equation (29). Moreover, $Cov_t(X_{t+2}^j, X_{t+2}^m)$ is equal to $I_t^j(I_t^m \sigma_x^2 + I_t^j \sigma_\theta^2)$, the unconditional covariance between $X_{t+2}^j$ and $X_{t+2}^m$. Substituting these into the above equation yield

$$\begin{aligned} & Cov_t[E_{t+1}(X_{t+2}^j), E_{t+1}(X_{t+2}^m)] \\ & \quad = \frac{I_t^j\left[I_t^m \sigma_x^2\left(J\sigma_\eta^2\sigma_\theta^2 + (J\sigma_\eta^2 + \sigma_\theta^2)(J\sigma_x^2 + \sigma_\theta^2)\right) + I_t^j \sigma_\theta^4 (J\sigma_x^2 + J\sigma_\eta^2 + \sigma_\theta^2)\right]}{(J\sigma_\eta^2 + \sigma_\theta^2)(J\sigma_x^2 + J\sigma_\eta^2 + \sigma_\theta^2)}. \end{aligned}$$

Substituting the above expressions for the conditional covariances into expression (28) and taking the large economy limit (i.e., $J = N \to \infty$), we get that

$$RP_{t+1}^j = \rho\left[I_{t-1}^j \cdot \bar{I}_{t-1} \cdot \sigma_{xp}^2 + \gamma^2 \cdot I_t^j \cdot \bar{I}_t \cdot \sigma_{xa}^2\right],$$

where $\sigma_{xa}^2 = \sigma_x^2 - \sigma_{xp}^2$, $\sigma_{xp}^2 = \frac{\sigma_x^2\sigma_\eta^2}{\sigma_x^2+\sigma_\eta^2}$, and $\bar{I}_\tau \equiv \frac{I_\tau^m}{J}$ denotes the average investment in period $\tau$.

$\square$

**Proof of Proposition 4:**

Define

$$h \equiv \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\eta^2}.$$

We note that $h$ increases in the precision of public disclosure, $\frac{1}{\sigma_\eta^2}$. The risk premium for firm

$j$ in period $t+1$ (in the large economy limit) can then be written as:

$$RP^j_{t+1} = \rho\sigma^2_x \left[I^j_{t-1} \cdot \bar{I}_{t-1} \cdot (1-h) + \gamma^2 \cdot I^j_t \cdot \bar{I}_t \cdot h\right]. \tag{30}$$

Substituting $I^j_t = I^j_{t-1}(1+\mu^j_t)$ and $\bar{I}_t = \bar{I}_{t-1}(1+\bar{\mu}_t)$ into (30) and simplifying give

$$RP^j_{t+1} = I^j_{t-1}\bar{I}_{t-1}\rho\sigma^2_x \left[1 + h\left(\gamma^2(1+\mu^j_t)(1+\bar{\mu}_t) - 1\right)\right]$$

It thus follows that $RP^j_{t+1}$ increases (decreases) in the precision of information disclosure $h$ if firm $j$'s investment growth rate $\mu^j_t$ is more (less) than the following threshold:

$$\hat{\mu}^j_t = \frac{(1+r)^2}{1+\bar{\mu}_t} - 1.$$

□

**Proof of Proposition 5:**

**Part i:**

As before, we prove the result for welfare of generation $t+1$ investors. The same argument applies for welfare of all other future generations.

The proof of Lemma 3 shows that each identical investor will buy fraction $\frac{1}{N}$ of the aggregate market portfolio; that is, $\alpha^j_{tn} = \frac{1}{N}$ for each $n \in \{1, \cdots, N\}$ and each $j \in \{1, \cdots, J\}$. Substituting $\alpha^j_{tn} = \frac{1}{N}$ in the expression for investor $n$'s certainty equivalent as implied by equation (26), the representative investor's certainty equivalent can be expressed as:

$$\begin{aligned} CE_{t+1} \quad = \quad & \frac{1}{N}\sum_{i=1}^{J}[E_t(X^i_{t+1} + P^i_{t+1}) - c(I^i_{t+1}) - (1+r)\cdot P^i_t] \\ & -\frac{\rho}{2N^2} \cdot \sum_{i=1}^{J}\left(\sum_{k=1}^{J}\left[Cov_t(X^i_{t+1}, X^k_{t+1}) + Cov_t(P^i_{t+1}, P^k_{t+1})\right]\right). \end{aligned}$$

The term inside the summation sign in the first line of the above expression is equal to $RP^i_{t+1}$. Furthermore, the definition of risk premium in connection with equation (25) implies that

$$\sum_{k=1}^{J}\left[Cov_t(X^i_{t+1}, X^k_{t+1}) + Cov_t(P^i_{t+1}, P^k_{t+1})\right] = \frac{N}{\rho}RP^i_{t+1}.$$

It thus follows that

$$CE_{t+1} = \frac{1}{2N}\sum_{i=1}^{J} RP^i_{t+1}.$$

Using expression (30) for the risk premium, the expected utility of the representative investor of generation $t+1$ in the large economy can be represented as follows:

$$\begin{aligned} CE_{t+1} &= \frac{\rho\sigma_x^2}{2}\left[(\bar{I}_{t-1})^2 \cdot (1-h) + \gamma^2 \cdot (\bar{I}_t)^2 \cdot h\right] \\ &= \frac{\rho\sigma_x^2(\bar{I}_{t-1})^2}{2}\left[1 + \left(\gamma^2\bar{\mu}_t^2 - 1\right)h\right]. \end{aligned}$$

It thus follows from the last expression that generation $t+1$ investors' welfare decreases (increases) in the precision of public disclosure $h$ when $\bar{\mu}_t$ is less (more) than $r$.

**Part ii:**

Since each investor holds fraction $\frac{1}{N}$ of the market portfolio, the expected utility of the *existing* shareholders of generation $t$ can be represented as follows:

$$CE_t = E_{t-1}(\overline{P}_t) - \frac{\rho}{2}Var_{t-1}(\overline{P}_t) + \beta_t,$$

where $\beta_t$ is a term independent of future disclosures and $\overline{P}_t \equiv \frac{P_t^m}{N}$ denotes the price of $\frac{1}{N}$ share of the market portfolio. In the large economy,

$$\overline{P}_t = \sum_{\tau=1}^{\infty}\gamma^\tau \cdot \left[E_{t-1}(\overline{X}_{t+\tau}) - \overline{C}_{t+\tau} - \overline{RP}_{t+\tau}\right],$$

where $\overline{X}_{t+\tau} = \frac{X^m_{t+\tau}}{J}$, $\overline{C}_\tau = \frac{\sum_{j=1}^J c(I^j_\tau)}{J}$, and $\overline{RP}_\tau = \rho\sigma_x^2[(\bar{I}_{\tau-2})^2(1-h) + \gamma^2(\bar{I}_{\tau-1})^2 h]$. As in the proof of Proposition 2, the expected utility of the existing shareholders of generation $t$ can therefore be written as:

$$CE_t = A_t + B_t \cdot h,$$

where $B_t \equiv \rho\gamma\sigma_x^2\left[(\bar{I}_{t-1})^2\left(1-\frac{\gamma}{2}\right) + \sum_{\tau=1}^{\infty}\gamma^\tau(\bar{I}_{t+\tau-1})^2(1-\gamma)\right]$ is positive and $A_t$ is independent of $h$. It thus follows that the expected utility of the existing shareholders of generation $t$ unambiguously increases in the precision of public disclosure $h$.

□

**Proof of Proposition 6:** For given investment levels $I^*_{t+\tau-2}$ and $I^*_{t+\tau-1}$, the risk premium in period $t+\tau$ is given by

$$RP_{t+\tau} = \rho[(I^*_{t+\tau-2})^2 \cdot \sigma^2_p + \gamma^2 \cdot (I^*_{t+\tau-1})^2 \cdot \sigma^2_a].$$

Substituting for the optimal investments from (11) and $\sigma^2_p = \sigma^2 - \sigma^2_a$ yields

$$RP_{t+\tau} = \frac{\rho\gamma^4\left[\left(\sigma^2-\sigma^2_a\right)m^2_{t+\tau} + \gamma^2\sigma^2_a m^2_{t+\tau+1}\right]}{\left[2\rho\gamma^2\left(\sigma^2-\sigma^2_a\right)+\rho\gamma^3\sigma^2_a+2b\right]^2}.$$

Differentiating with respect to $\sigma^2_a$ reveals

$$sgn\left[\frac{\partial RP_{t+}}{\partial\sigma^2_a}\right] = sgn\left[\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} - \frac{2b-2\left(1-\gamma\right)\rho\gamma^2\sigma^2+\left(2-\gamma\right)\gamma^2\rho\sigma^2_a}{\gamma^2\left(2b+2\rho\gamma^2\sigma^2+\left(2-\gamma\right)\rho\gamma^2\sigma^2_a\right)}\right]$$

Therefore, $\frac{\partial RP_{t+\tau}}{\partial\sigma^2_a} \geq 0$ if and only if

$$\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} \geq \frac{2b-2\left(1-\gamma\right)\rho\gamma^2\sigma^2+\left(2-\gamma\right)\gamma^2\rho\sigma^2_a}{\gamma^2\left(2b+2\rho\gamma^2\sigma^2+\left(2-\gamma\right)\rho\gamma^2\sigma^2_a\right)}.$$

The inequality above can be simplified as follows:

$$\frac{m^2_{t+\tau+1}}{m^2_{t+\tau}} \geq (1+r)^2 - l(\sigma_a),$$

where

$$l(\sigma_a) = \frac{\left(4-2\gamma\right)\rho\sigma^2}{2b+\gamma^2\left(2\rho\sigma^2+2\rho\sigma^2_a-\gamma\rho\sigma^2_a\right)}.$$

□

**Proof of Proposition 7:** The first part of follows from Proposition 6 because, as discussed in connection with Proposition 2, the expected utility of *future* potential investors increases in the risk premium in the period during which they plan to hold the firm.

The expected utility of the existing shareholders of generation $t$ can be represented by the following certainty equivalent expression:

$$CE_t = B + E_{t-1}\left(P_t\right) - c(I^*_t) - \frac{\rho}{2}Var_{t-1}\left(P_t\right),$$

where $B \equiv E_{t-1}(X_t) - \frac{\rho}{2}Var_{t-1}(X_t) - (1+r)P_{t-1}$ does not depend on future disclosure policies. Substituting for the equilibrium price at date $t$ from equation (**??**) and rearranging terms, it can be verified that

$$CE_t = B + V(I_{t-1}, \sigma_a^2) + \sum_{\tau=1}^{\infty} \gamma^\tau \left[ V(I^*_{t+\tau-1}, \sigma_a^2) - \frac{\gamma^2}{2} \rho (I^*_{t+\tau-1})^2 \sigma_a^2 \right],$$

where $V(I^*_\tau, \sigma_a^2) \equiv \gamma m_{\tau+2} I^*_\tau - (1+r) b (I^*_\tau)^2 - \gamma\rho(I^*_\tau)^2\sigma_p^2 - \frac{\gamma^2}{2}\rho(I^*_\tau)^2\sigma_a^2$ denotes the maximized value of the firm's period $\tau$ objective function, as defined in (10). To emphasize that date $t-1$ investment does not vary with the precision of future disclosures, we do not use any superscript on $I_{t-1}$. Differentiating with respect to $\sigma_a^2$ and applying the Envelope Theorem yield

$$\frac{dCE_t}{d\sigma_a^2} = \frac{\partial CE_t}{\partial \sigma_a^2} - \rho\gamma^2\sigma_a^2 \cdot \sum_{\tau=1}^{\infty} \gamma^\tau I^*_{t+\tau-1} \frac{\partial I^*_{t+\tau-1}}{\partial \sigma_a^2}.$$

We note that $\frac{dCE_t}{d\sigma_a^2} > 0$ at $\sigma_a^2 = 0$ because (i) equation (18) implies $\frac{\partial CE_t}{\partial \sigma_a^2} \geq \rho\gamma(1-\frac{\gamma}{2})I_{t-1}^2 > 0$ for all $\sigma_a^2 \in [0, \sigma^2]$, and (ii) the second term on the right hand side of the above expression is zero for $\sigma_a^2 = 0$. It thus follows from continuity that there exists a $\sigma_L \in (0, \sigma^2]$ such that the existing shareholders' welfare increases in $\sigma_a^2$ for all $\sigma_a^2 \in [0, \sigma_L]$.

Substituting $\frac{\partial CE_t}{\partial \sigma_a^2} = \rho\gamma\left(1-\frac{\gamma}{2}\right)(I_{t-1})^2 + \rho\gamma(1-\gamma)\sum_{\tau=1}^{\infty}\gamma^\tau (I^*_{t+\tau-1})^2$ from equation (18), the optimal investments $I^*_{t+\tau}$ from (11), and simplifying reveal that

$$\left.\frac{dCE_t}{d\sigma_a^2}\right|_{\sigma_a^2=\sigma^2} = \frac{\rho\gamma(2-\gamma)}{2} I_{t-1}^2 - \frac{\rho\gamma^5[\rho\gamma^3\sigma^2 - 2(1-\gamma)b]}{[2b+\rho\gamma^3\sigma^2]^3} \sum_{\tau=1}^{\infty} \gamma^\tau m^2_{t+\tau+1}.$$

The above equation implies that $\left.\frac{dCE_t}{d\sigma_a^2}\right|_{\sigma_a^2=\sigma^2} < 0$ if

$$2(1-\gamma)b < \rho\gamma^3\sigma^2 \tag{31}$$

and

$$\sum_{\tau=1}^{\infty} \gamma^\tau m^2_{t+\tau+1} > \frac{(2-\gamma)[2b+\rho\gamma^3\sigma^2]^3}{2\gamma^4[\rho\gamma^3\sigma^2 - 2(1-\gamma)b]} \cdot I_{t-1}^2. \tag{32}$$

It then follows from continuity that if the inequalities in (31-32) hold, there exists a $\sigma_H \in (\sigma_L, \sigma^2)$ such that $CE_t$ decreases in $\sigma_a^2$ for all $\sigma_a^2 \in [\sigma_H, \sigma^2]$. This proves that when (31-32) hold, the existing shareholders' welfare is maximized at some $\sigma_a^2 \in [\sigma_L, \sigma_H]$.

□

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