Accounting Tinder:
Acquisition of Information with Uncertain Precision

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Abstract

We develop and analyze a model of information acquisition in capital markets. In the model, we assume that investors are uncertain about the precision of the private information *before* they acquire it. As a result, investors use prior prices and public information to estimate the precision and, therefore, value of the private information. We find that larger price to earnings deviations lead to more private information acquisition, higher future price volatility, and higher future trading volumes.

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1 Introduction

Active, as opposed to passive, investors seek out opportunities to allow them to profit from their investment research. Identifying those opportunities, however, is a challenging activity in and of itself. There are many, many possible securities to consider, and limited human resources to sift through them. Given these constraints, active investors often employ screens to identify investment opportunities, and those that pass through the screen are subject to more in-depth research. Those screens generally rely on accounting data, which is readily available in databases, and as a consequence, the accounting data effectively serves as the tinder that fuels investor interest in a security. Through an analytical modeling exercise, we explore why accounting based investment screens can identify favorable prospects, how wide spread use of them screens might alter trading activities across firms, and when they are likely to be effective.

The model we develop and analyze is grounded in a simple capital market with perfect competition. There are two firms and two periods of trade for the claims of each firm. Prior to the first round of trade for each firm, a public signal, which we call earnings, is released and speculators also observe some private information. The speculators, rational uninformed passive investors, and noise traders then trade in the shares of the firm. After the first round of trade, a new set of speculators arrive. Those new speculators are resource constrained and must choose which of the two firms to follow. After they make their choices, all speculators obtain the private information about the firm they follow and engage in a second round of trade with the passive investors and the noise traders. The critical assumption in our model is that the new speculators must make their choices with incomplete information about the quality of private information that can be obtained about each firm. Within the model that incomplete information includes past prices and earnings, which is intended to reflect the type of information available in actual databases employed for investment screens. Given the incomplete information, these investors employ an *endogenously* derived investment screen to decide which firm to follow.

Within the context of this model, the endogenous investment screen relies upon the first round price to earnings relation. Specifically, a deviation from the expected price to earnings relation is informative about the quality of private information that can be obtained for the firm because greater deviations suggest the presence of more private information. These endogenous screens arise because greater deviations from the expected price-to-earnings relation are likely to occur when informed investors in the first round of trade have traded more aggressively, which occurs when there is higher quality private information.

The new speculators who rely upon the price-to-earnings relation screen naturally gravitate to a greater extent to the firm that has the largest deviation from the normal pattern. As a consequence, the equilibrium allocation of speculators is greater for the firm with the largest deviation from the pricing norm. Hence, the model broadly predicts that firms with greater unexplained pricing patterns are more likely to attract the attention of active investors or speculators, which implies that those firms will exhibit greater price volatility and trading volume in subsequent periods of trade. In summary, we identify a mechanism through which available accounting information and current prices can impact the extent of private information gathering efforts.

Finally, because all public information does not end up in databases employed for investment screens (e.g., qualitative disclosures about firm specific events), we consider the impact of such information on the informativeness of screens. We find that such public information reduces the informativeness of the investment screens, which implies that active investors respond less to investment screens for firms that engage in more idiosyncratic disclosures that do not get incorporated into the variables employed for screens.

Our analysis relates to the literature on private information acquisition decisions in asset markets. Within that literature, various determinants of private information acquisition decisions have been studied, including the direct cost of acquiring the information (e.g., Grossman and Stiglitz (1980) or Verrecchia (1982)), the nature of available public information (e.g., Demski and Feltham (1994), Kim and Verrecchia (1994), or McNichols and Trueman

(1994)), the mechanisms for profiting from that information through trading or indirect sale (e.g., Garcia and Vanden (2009)), information processing biases (e.g., Ko and Huang (2007)), status concerns (e.g., Garcia and Strobl (2011)), and the nature of the information choice set and nature of information chosen by other investors (Froot, Scharfstein, and Stein (1992), Fischer and Verrecchia (1998), and Van Nieuwerburgh and Veldkamp (2009)). Our analysis is most related to the latter determinants in that we consider an information choice from some constrained set of information choices. In this work, the investors are generally selecting how much information to gather (i.e., how much variance to eliminate) or the degree to which one's information overlaps with others (i.e., the covariance of the private information). The attributes of the information that can be obtained, such as its precision, are assumed to be commonly known. The only uncertainty pertains to the realization of that information. In contrast, we assume the prospective informed investors are uncertain of the attributes of the information, the quality or precision in this case, and that they employ and investment screen to help them resolve that uncertainty.

The remainder of paper begins with an introduction of the model. After introducing the model, we characterize the equilibrium for the model. Following the characterization of the equilibrium for the model, we consider a couple of modifications for the model. Finally, we conclude.

2 Model

Consider a setting where the claims to two separate firms, a and b, are traded in two separate rounds of trade. The uncertain terminal value of firm $i \in \{a, b\}$ is $\tilde{v}_i = \tilde{e}_{i1} + \tilde{e}_{i2} + \tilde{e}_{i3}$, where $\tilde{e}_{it} = \tilde{e}_{it-1} + \tilde{\varepsilon}_{it}$, $\tilde{\varepsilon}_{it}\tilde{\ } N(0, s_i^2)$, $\tilde{\varepsilon}_{it}$ is independent of $\tilde{\varepsilon}_{j\tau}$ for all $j \in \{a, b\}$, $\tau \in \{1, 2, 3\}$ and $\{j, \tau\} \neq \{i, t\}$, and e_{i0} is normalized to 0. Prior to the first round of trade, the realizations for $\tilde{e}_{a1}$ and $\tilde{e}_{b1}$, e_{a1} and e_{b1}, are disclosed. After the first round of trade, but prior to the second round of trade, e_{a2} and e_{b2}, are disclosed.

The markets for the two sets of claims involve risk neutral speculators, noise traders, and risk neutral passive investors. In the first round of trade for each firm i's shares, the set of speculators has measure 1. In addition, there is one noise trader per speculator, and $m \to \infty$ passive investors per speculator. We let $m \to \infty$ to derive simple, risk-neutral pricing representations, although this assumption is not necessary for the model results. Finally, we normalize the supply of shares per speculator to 0 (that is, the cumulative holdings of speculators, noise traders, and passive investors have to equal zero in equilibrium).

Each of the speculators in the market for firm i privately observes the realization for $\tilde{x}_{i1}$, x_{i1}, prior to the first round of trade, where $\tilde{x}_{i1}$ is normally distributed with mean 0, variance $q_i^2 < s_i^2$, covariance with $\tilde{\varepsilon}_{i2}$ of q_i^2, and is independent of all other random variables in the model. Furthermore, each of speculator for firm i privately observes the realization for $\tilde{x}_{i2}$, x_{i2}, after the first round, but prior to the second round of trade, where $\tilde{x}_{i2}$ is normally distributed with mean 0, variance $q_i^2 < s_i^2$, covariance with $\tilde{\varepsilon}_{i3}$ of q_i^2, and is independent of all other random variables in the model. It follows from the information structure that q_i^2 represents the quality of the speculator's private information.

Prior to the second round of trade, a new set of risk neutral speculators arrive. We assume there is $\frac{1}{2}$ of a new speculator per existing speculator in the two markets combined (such that a measure of 1 new speculators arrive). The new speculators can follow only one of the two firms, and a speculator who follows firm i learns x_{i2} in addition to the public disclosures. We denote the proportion of new speculators who choose to follow firm i with π_i. If follows that if π_i of the new speculators follow firm i there is π_i new speculators per existing speculator in the market for firm i (and $1 - \pi_i$ new speculators in the market for firm j).

A speculator or passive investor in the market for firm i's shares chooses holdings d_{i1} and d_{i2} to maximize the expectation of

$$d_{i1}\left(P_{i2} - P_{i1}\right) - \frac{c}{2} d_{i1}^2 + d_{i2}\left(v_i - P_{i2}\right) - \frac{c}{2} d_{i2}^2. \tag{1}$$

The term $\frac{c}{2}d_{it}$ reflects some cost of holding a position after round of trade 1 (2) to round of trade 2 (terminal date), which crudely reflects the a cost of being exposed to the risks of holding i over that period. From a modeling perspective, the introduction of this cost is a parsimonious way to bound demands. Similarly, a new speculator who follows firm i in the second round of trade chooses holdings d_{i2} to maximize the expectation of

$$d_{i2}\left(v_i - P_{i2}\right) - \frac{c}{2}d_{i2}^2. \tag{2}$$

To introduce noise trade in a parsimonious manner, we assume that each noise trader in the market for firm i has period t demand influenced by the realization of the random variable $\tilde{n}_{it}$, which is normally distributed with mean 0 and variance σ_{it}^2, and is independent of all other random variables. The only investors who observe the realization of n_{it} are the noise traders in firm i,who observe that realization just prior to round of trade t. Given the realizations of these random variables each noise trader chooses holdings d_{i1} and d_{i2} to maximize the expectation of

$$d_{i1}n_{i1} - \frac{c}{2}d_{i2}^2 + d_{i2}n_{i2} - \frac{c}{2}d_{i2}^2. \tag{3}$$

The critical, and novel, assumption in our model concerns the knowledge the new speculators possess when they make the decision about which firm to follow prior to the second round of trade. We assume these investors know all of the model primitives expect for the values for q_a^2 and q_b^2. Their priors regarding q_a^2 and q_b^2 are that they are independent and identically distributed random variables with two equally likely outcomes, q_h^2 and q_l^2, where $q_h^2 > q_l^2$. In addition, these investors observe the disclosures of e_a and e_b, as well as the market clearing prices from the first round of trade, P_{a1} and P_{b1}. This information is consistent with the kind of data that actual investors could easily access prior to deciding where to focus their information gathering efforts. With this information, they try to assess which firm offers the greatest opportunity to acquire profitable information.

3 Equilibrium Derivation

An equilibrium is characterized by establishing equilibrium behavior for each of three stages: the first round of trade, the second round of trade, and the stage where the new second round speculators make a decision about which firm to follow. For the two stages involving trade, we restrict attention to noisy rational expectations equilibria as in Grossman and Stiglitz (1980). That is, we assume that the passive investors learn from price. The equilibrium condition for the firm following decisions simply requires that no new speculator would alter their firm following decision given their rational conjecture of the proportion of speculators following each firm.

3.1 Second Round Equilibrium Pricing

Assume that the proportion of new speculators who choose to follow firm i is given by the function $\pi_i(\Omega)$,where Ω is the outcomes from the first round of trade $\{e_{a1}, P_{a1}, e_{b1}, P_{b1}\}$. Taking $\pi_i(\Omega)$ as given, we determine the demands of each investor and then establish the second round price using a market clearing condition. The first order condition for a firm i original or new speculator's objective function yields an optimal demand for firm i claims of

$$d_{i2S} = \frac{E\left[\tilde{v}_i | e_{i1}, e_{i2}, x_{i2}\right] - P_{i2}}{c} = \frac{e_{i1} + 2e_{i2} + x_{i2} - P_{i2}}{c}. \tag{4}$$

Similarly, firm i passive investor demand is

$$\begin{aligned} d_{i2P} &= \frac{E\left[\tilde{v}_i | e_{i1}, e_{i2}, P_{i1}, P_{i2}\right] - P_{i2}}{c} \\ &= \frac{e_{i1} + 2e_{i2} + E\left[\tilde{x}_{i2} | e_{i1}, e_{i2}, P_{i1}, P_{i2}\right] - P_{i2}}{c}. \end{aligned} \tag{5}$$

Finally, noise trader demand is

$$d_{i2N} = \frac{n_{i2}}{c}. \tag{6}$$

The market clearing condition is

$$(1+\pi_i(\Omega))\, d_{i2I} + d_{i2N} + m d_{i2P} = 0.$$

Substituting in for the demands and rearranging terms implies that the market clearing price must satisfy:

$$P_{i2} = e_{i1} + 2e_{i2} + \frac{(1+\pi_i(\Omega))\, x_{i2}}{1+\pi_i(\Omega)+m} + \frac{n_{i2}}{1+\pi_i(\Omega)+m} + \frac{mE\left[\tilde{x}_{i2}|e_{i1}, e_{i2}, P_{i1}, P_{i2}\right]}{1+\pi_i(\Omega)+m}. \qquad (7)$$

As an aside, note that in the limit, as $m \to \infty$, equation (7) yields a second round price that is simply a passive investor's expectation of terminal value given the disclosures and prices

$$P_{i2} = e_{i1} + 2e_{i2} + E\left[\tilde{x}_{i2}|e_{i1}, e_{i2}, P_{i1}, P_{i2}\right]. \qquad (8)$$

In order to complete the characterization of equilibrium, we must determine $E\left[\tilde{x}_{i2}|e_{i1}, e_{i2}, P_{i1}, P_{i2}\right]$. Given the relationship between price and demands, knowledge of second round price allows the passive investors to infer the statistic, $y_{i2} = (1+\pi_i(\Omega))\, x_{i2} + n_{i2}$, which is a sufficient statistic for$\{e_{i1}, e_{i2}, P_{i1}, P_{i2}, y_{i2}\}$ with respect to $\tilde{x}_{i2}$: $E\left[\tilde{x}_{i2}|e_{i1}, e_{i2}, P_{i1}, P_{i2}, y_{i2}\right] = E\left[\tilde{x}_{i2}|y_{i2}\right]$. The statistic y_{i2} is the realization of a random variable with mean 0 variance $(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2$ and covariance with $\tilde{x}_{i2}$ of $(1+\pi_i(\Omega))\, q_i^2$. It follows that

$$\begin{aligned} E\left[\tilde{x}_{i2}|y_{i2}\right] &= \frac{(1+\pi_i(\Omega))\, q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2} y_{i2} \\ &= \frac{(1+\pi_i(\Omega))\, q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2} (1+\pi_i(\Omega))\, x_{i2} + \frac{(1+\pi_i(\Omega))\, q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2} n_{i2}. \qquad (9) \end{aligned}$$

Hence, the second round price can be written as the following linear function

$$\begin{aligned} P_{i2} &= e_{i1} + 2e_{i2} \\ &+ \left(1 + \frac{m\left(1+\pi_i\left(\Omega\right)\right) q_i^2}{\left(1+\pi_i\left(\Omega\right)\right)^2 q_i^2 + \sigma_{i2}^2}\right) \frac{1+\pi_i\left(\Omega\right)}{1+\pi_i\left(\Omega\right)+m} x_{i2} \\ &+ \left(1 + \frac{m\left(1+\pi_i\left(\Omega\right)\right) q_i^2}{\left(1+\pi_i\left(\Omega\right)\right)^2 q_i^2 + \sigma_{i2}^2}\right) \frac{1}{1+\pi_i\left(\Omega\right)+m} n_{i2}. \end{aligned} \tag{10}$$

Letting $m \to \infty$, yields the second round price, which is characterized in Observation 1.

Observation 1. Given any equilibrium $\pi_i(\Omega)$*, the second round price for firm* $i \in \{a, b\}$ *is uniquely characterized by a function of the form*

$$P_{i2} = e_{i1} + 2e_{i2} + \beta_{i2x}\left(\Omega\right) x_{i2} + \beta_{i2n}\left(\Omega\right) n_{i2},$$

where $\beta_{i2x}(\Omega) = \frac{(1+\pi_i(\Omega))^2 q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2}$ *and* $\beta_{i2n}(\Omega) = \frac{(1+\pi_i(\Omega)) q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2}$.

3.2 First Round Equilibrium Pricing

We derive the demands from the investors in our model in the first round analogously to those in the second round of trade. A feature of the model that greatly facilitates the derivations is that the speculators' and passive investors' expectation of $\frac{(1+\pi_i(\Omega))^2 q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2} \tilde{x}_{i2} + \frac{(1+\pi_i(\Omega)) q_i^2}{(1+\pi_i(\Omega))^2 q_i^2 + \sigma_{i2}^2} \tilde{n}_{i2}$ is always 0 regardless of how first round earnings and prices, Ω, determine $\pi_i(\Omega)$ in equilibrium. A firm i speculator's first round demand is

$$d_{i1S} = \frac{E\left[\tilde{P}_{i2} | e_{i1}, x_{i1}\right] - P_{i1}}{c} = \frac{3e_{i1} + 2x_{i1} - P_{i1}}{c}. \tag{11}$$

A passive investor's demand is

$$\begin{aligned} d_{i1P} &= \frac{E\left[\tilde{P}_{i2} | e_{i1}, P_{i1}\right] - P_{i1}}{c} \\ &= \frac{3e_{i1} + 2E\left[\tilde{x}_{i2} | e_{i1}, P_{i1}\right] - P_{i1}}{c}. \end{aligned} \tag{12}$$

Finally, a noise trader's demand is

$$d_{i1N} = \frac{n_{i1}}{c}. \tag{13}$$

The first round market clearing condition, requires that

$$P_{i1} = 3e_{i1} + \frac{1}{1+m}2x_{i1} + \frac{1}{1+m}n_{i1} + \frac{m}{1+m}2E\left[\tilde{x}_{i1}|e_{i1}, P_{i1}\right]. \tag{14}$$

Again, letting $m \to \infty$ yields a first round price that is a passive investor's expectation of second round price.

$$P_{i1} = 3e_{i1} + 2E\left[\tilde{x}_{i1}|e_{i1}, P_{i1}\right]. \tag{15}$$

To determine $E\left[\tilde{x}_{i1}|e_{i1}, P_{i1}\right]$, observe that the first round price allows the passive investors to infer the statistic $y_{i1} = 2x_{i1} + n_{i1}$, and y_{i1} is a sufficient statistic for$\{e_{i1}, P_{i1}, y_{i1}\}$ with respect to $\tilde{x}_{i1}$. It follows that

$$E\left[\tilde{x}_{i1}|e_{i1}, P_{i1}\right] = \frac{2q_i^2}{4q_i^2 + \sigma_{i1}^2}y_{i1} = \frac{4q_i^2}{4q_i^2 + \sigma_{i1}^2}x_{i1} + \frac{2q_i^2}{4q_i^2 + \sigma_{i1}^2}n_{i1}. \tag{16}$$

It follows that first round price can be written as

$$P_{i1} = 3e_{i1} + 2\left(1 + \frac{m2q_i^2}{4q_i^2 + \sigma_{i1}^2}\right)\frac{1}{1+m}x_{i1} + \left(1 + \frac{m2q_i^2}{2q_i^2 + \sigma_{i1}^2}\right)\frac{1}{1+m}n_{i1}. \tag{17}$$

Letting $m \to \infty$, yields the first round price, which is characterized in Observation 2.

Observation 2. In any equilibrium the first round price for firm $i \in \{a, b\}$ is uniquely characterized by a function of the form

$$P_{i1} = 3e_{i1} + \beta_{i1x}x_{i1} + \beta_{i1n}n_{i1},$$

where $\beta_{i1x} = 2\frac{2q_i^2}{4q_i^2+\sigma_{i1}^2}$ *and* $\beta_{i2n} = \frac{2q_i^2}{4q_i^2+\sigma_{i1}^2}$.

3.3 Expected Profits from Following Firm i

To facilitate the characterization of equilibrium, it is useful to compute the second round expected profits for a new speculator who follows firm i conditional upon the first round statistics available for making the firm following decision, Ω, and an equilibrium $\pi(\Omega)$. Given a second round pricing function of the form $P_{i2} = e_{i1} + 2e_{i2} + \beta_{i2x}(\Omega) x_{i2} + \beta_{i2n}(\Omega) n_{i2}$, which is characterized in Observation 1, a speculator who observes x_{i2} and experiences a price determined by n_{i2} has expected payoffs

$$d_{i2}\left(E\left[\tilde{v}_i|e_{i1}, e_{i2}, x_{i2}\right] - P_{i2}\right) - \frac{c}{2}d_{i2}^2 = \frac{1}{2c}\left(\left(1 - \beta_{i2x}(\Omega)\right) x_{i2} - \beta_{i2n}(\Omega) n_{i2}\right)^2. \qquad (18)$$

It follows that the expected payoffs prior to observing x_{i2} and n_{i2}, but with knowledge of q_i^2, σ_{i2}^2, and $\pi_i(\Omega)$, are

$$\begin{aligned} \Pi\left(q_i^2, \sigma_{i2}^2, \pi_i(\Omega)\right) &= \frac{1}{2c}\left(\left(1 - \beta_{i2x}(\Omega)\right)^2 q_i^2 + \beta_{i2n}(\Omega)^2 \sigma_{i2}^2\right) \\ &= \frac{1}{2c}\frac{q_i^2 \sigma_{2i}^2}{\left(1 + \pi_i(\Omega)\right)^2 q_i^2 + \sigma_{i2}^2}. \end{aligned} \qquad (19)$$

The expected payoffs have the intuitive properties in that they are increasing in the quality of the speculators' private information, $\frac{\partial \Pi}{\partial q_i^2} > 0$, and in the extent of noise trade, $\frac{\partial \Pi}{\partial \sigma_{i2}^2} > 0$, which serves to obfuscate the informed trading activity. In addition, and most importantly, it is decreasing in the proportion of new speculators who follow firm i, $\frac{\partial \Pi}{\partial \pi_i(\Omega)} < 0$.

When new speculators arrive, they do not know q_a^2 and q_b^2. Hence, they must assess their expected payoffs from following each firm given the probability that the quality of private information is high or low. These expected payoffs are

$$\begin{aligned} E\left[\Pi\left(q_i^2, \sigma_{i2}^2, \pi_i(\Omega)\right)|\Omega\right] &= \Pr\left(q_i^2 = q_h^2|\Omega\right)\Pi\left(q_h^2, \sigma_{i2}^2, \pi_i(\Omega)\right) + \Pr\left(q_i^2 = q_l^2|\Omega\right)\Pi\left(q_l^2, \sigma_{i2}^2, \pi_i(\Omega)\right). \\ &= \Pr\left(q_i^2 = q_h^2|\Omega\right)\Pi\left(q_h^2, \sigma_{i2}^2, \pi_i\right) + \left(1 - \Pr\left(q_i^2 = q_h^2|\Omega\right)\right)\Pi\left(q_l^2, \sigma_{i2}^2, \pi_i\right) \\ &= \Pr\left(q_i^2 = q_h^2|\Omega\right)\left(\Pi\left(q_h^2, \sigma_{i2}^2, \pi_i\right) - \Pi\left(q_l^2, \sigma_{i2}^2, \pi_i\right)\right) + \Pi\left(q_l^2, \sigma_{i2}^2, \pi_i\right) \end{aligned} \qquad (20)$$

where, for $j \in \{h, l\}$, $\Pr\left(q_i^2 = q_j^2|\Omega\right)$ is the probability $q_i^2 = q_j^2$ conditional on Ω.

Characterizing $\Pr\left(q_i^2 = q_h^2|\Omega\right)$ is necessary to assess how the expected payoffs from following firm i, $E\left[\Pi\left(q_i^2, \sigma_{i2}^2, \pi_i(\Omega)\right)|\Omega\right]$, are affected by the equilibrium $\pi_i(\Omega)$. To determine $\Pr\left(q_i^2 = q_h^2|\Omega\right)$, note first that the realizations e_{j1} and P_{j1}, $j \neq i$, have no relevant information content for assessing $\Pr\left(q_i^2 = q_h^2|\Omega\right)$. This is the case, because we assume that the information content of the private information is independent across the two firms. With that observation in hand, the probability that $q_i^2 = q_h^2$ is

$$\Pr\left(q_i^2 = q_h^2|\Omega\right) = \frac{\frac{1}{2} f\left(P_{i1} - 3e_{i1}|q_h^2\right)}{\frac{1}{2} f\left(P_{i1} - 3e_{i1}|q_h^2\right) + \frac{1}{2} f\left(P_{i1} - 3e_{i1}|q_l^2\right)}, \tag{21}$$

where $P_{i1} - 3e_{i1} = \beta_{i1x} x_{i1} + \beta_{i1n} n_{i1} = \frac{2q_i^2}{4q_i^2 + \sigma_{i1}^2}\left(2x_{i1} + n_{i1}\right)$ and $f\left(P_{i1} - 3e_{i1}|q_\tau^2\right)$ is the probability density function for $P_{i1} - 3e_{i1}$ conditional upon $q_i^2 = q_\tau^2$, $\tau \in \{l, h\}$. That density function is the density function of a normally distributed random variable with mean 0 and variance $\frac{4q_i^2}{4q_i^2 + \sigma_{i1}^2} q_i^2$. It follows that

$$\Pr\left(q_i^2 = q_h^2|\Omega\right) = \frac{1}{1 + \sqrt{\frac{V_h}{V_l}} \exp\left[-\left(P_{i1} - 3e_i\right)^2 \frac{V_h - V_l}{2V_h V_l}\right]}, \tag{22}$$

where $V_h = \frac{4q_h^2}{4q_h^2 + \sigma_{i1}^2} q_h^2$ and $V_l = \frac{4q_l^2}{4q_l^2 + \sigma_{i1}^2} q_l^2$, which implies that $\Pr\left(q_i^2 = q_h^2|\Omega\right)$ is not a function of the equilibrium $\pi_i(\Omega)$. Hence, $E\left[\Pi\left(q_i^2, \sigma_{i2}^2, \pi_i(\Omega)\right)|\Omega\right]$ is influence by $\pi_i(\Omega)$ solely through $\Pi\left(q_h^2, \sigma_{i2}^2, \pi_i(\Omega)\right)$ and $\Pi\left(q_l^2, \sigma_{i2}^2, \pi_i(\Omega)\right)$. Observation 3 naturally follows.

Observation 3. The expected profits of a new speculator who follows firm i decrease in the fraction of new speculators that follow firm i, $\pi_i(\Omega)$.

3.4 Firm Following Equilibrium

Observation 1 characterizes the unique equilibrium second round pricing functions given an equilibrium $\pi_a(\Omega)$ and $\pi_b(\Omega)$, and Observation 2 characterizes the unique first round pricing functions, which are not determined by $\pi_a(\Omega)$ and $\pi_b(\Omega)$. We complete the characterization of equilibrium by showing that there is a unique $\pi_a(\Omega)$ and $\pi_b(\Omega) = 1 - \pi_a(\Omega)$, $\pi_a^*(\Omega)$ and $\pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)$, such that no new speculators can strictly increase the expected profits by changing their firm following decision.

Formally, $\pi_a^*(\Omega)$ and $\pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)$ must satisfy

$$E\left[\Pi\left(q_a^2, \sigma_{a2}^2, \pi_a^*(\Omega)\right)|\Omega\right] = E\left[\Pi\left(q_b^2, \sigma_{b2}^2, \pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)\right)|\Omega\right], \tag{23}$$

if $\pi_a^*(\Omega) \in (0,1)$ and $\pi_b^*(\Omega) \in (0,1)$,

$$E\left[\Pi\left(q_a^2, \sigma_{a2}^2, \pi_a^*(\Omega)\right)|\Omega\right] \geq E\left[\Pi\left(q_b^2, \sigma_{b2}^2, \pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)\right)|\Omega\right], \tag{24}$$

if $\pi_a^*(\Omega) = 1$ and $\pi_b^*(\Omega) = 0$, and

$$E\left[\Pi\left(q_a^2, \sigma_{a2}^2, \pi_a^*(\Omega)\right)|\Omega\right] \leq E\left[\Pi\left(q_b^2, \sigma_{b2}^2, \pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)\right)|\Omega\right], \tag{25}$$

if $\pi_a^*(\Omega) = 1$ and $\pi_b^*(\Omega) = 0$. Observation 3 implies that $E[\Pi(q_a^2, \sigma_{a2}^2, \pi_a^*(\Omega))|\Omega]$ is decreasing in $\pi_a^*(\Omega)$ and $E[\Pi(q_b^2, \sigma_{b2}^2, \pi_b^*(\Omega) = 1 - \pi_a^*(\Omega))|\Omega]$ is increasing in $\pi_a^*(\Omega)$, which implies Observation 4.

Observation 4. For any $\Omega \equiv \{e_{a1}, P_{a1}, e_{b1}, P_{b1}\}$ there exists a unique $\pi_a^*(\Omega)$ and corresponding $\pi_b^*(\Omega) = 1 - \pi_a^*(\Omega)$ that equates the expected profits of each new speculator.

3.5 Equilibrium

Lemma 1 follows directly from Observations 1 to 4..

Lemma 1. There exists a unique equilibrium.

4 Firm Following and Price-to-Earnings

Within the context of our model, an investment screen tied to observed earnings and prices determines firm following decisions, which in turn, determines attributes of subsequent prices. As of this point, however, we have not discussed how the firm following investment screen is endogenously related to the observed earnings and prices, which underlies the main insights derived form the model.

The endogenous investment screen is formalized in Proposition 1.

Proposition 1. In the unique equilibrium, the number of informed investors following firm i weakly increases in the deviation of firm i's first round price from the expected price conditioned on earnings, $|P_{i1} - 3e_{i1}|$*, and weakly decreases in the deviation of firm j's first round price from the expected price conditioned on earnings,* $|P_{j1} - 3e_{j1}|$*, where* $i,j \in \{a,b\}$ *and* $j \neq i$.

The difference between price and the expected price conditioned upon earnings, which happens to be a simple multiple of earnings in our model, arises because greater deviations of price from the earnings based norm, three times earnings, suggests to potential informed investors that there is more private information being impounded into the price. The deviation from the pricing norm, however, could also be due to noise trade, so the presence of significant private information is not guaranteed. Furthermore, the deviation does not suggest whether the private information is good or bad news relative to what uninformed market participants (i.e., the passive investors) believe because the equilibrium price reflects a correct expectation given all of the public information. Additionally, we have set up the model such that all prior private information is made public before trading recommences in round 2. This implies that the variance of cash flows prior to the information acquisition decision is independent of the amount or quality of available private information. A larger deviation of price from the pricing norm just suggests that significant news is more likely to be obtained from engaging in information gathering activities.

Proposition 1 suggests that firms with prices that deviate from valuation norms should naturally attract more attention from speculative investors. In addition to that prediction, Proposition 1 also suggests predicted relations between deviations from valuation norms and properties of subsequent prices.

Conjecture 1. In the unique equilibrium, an increase in the deviation of firm i's first round price from the expected price conditioned on earnings, $|P_{i1} - 3e_{i1}|$, is *associated with an increase in the second round price variance and trading volume for firm i's claims and a decrease in second round price variance and trading volume in firm j's claims.*

The intuition underlying Conjecture 1, which is a conjecture due to a lack of a formal proof at this stage of the manuscript, is quite straight forward, a larger realization for $|P_{i1} - 3e_{i1}|$ attracts more informed speculators to the market for firm i claims and away from the market for firm j claims. As a consequence there is more (less) informed trade for firm i (j) claims, which leads to more (less) movement in prices and more (less) trading volume.

Proposition 1 and Conjecture 1 provide some empirically oriented observations that arise within the equilibrium for the model, as opposed to empirically oriented observations that arise from perturbing variables that are exogenous to the model. Noise trade variances are exogenous model parameters that also affect firm following decisions, although they are expected to affect those decisions in quite different ways.

Conjecture 2. Increases in the extent of second round noise trade for firm i, σ_{i2}^2*, increases firm i's following and decreases firm j's following. Increases in the first round noise trade for firm i,* σ_{i1}^2*, moderates the impact of deviations of price from the expected price expected price conditioned on earnings,* $|P_{i1} - 3e_i|$*, on firm i and firm j following.*

The observations in Conjecture 2, are driven by two different intuitions. Consider first an increase in σ_{i2}^2, which has the more straightforward intuition. As discussed previously, increases in round 2 noise trade in the market for firm i claims increases informed trading profits, which serves to attract additional speculators from the market for firm j claims. Consider next increases in round 1 noise trade in the market for firm i claims. This noise trade has no direct effect on the round 2 trading profits but adds noise to the inference drawn from $|P_{i1} - 3e_i|$ about the quality of private information, q_i^2, available to firm i speculators. Consequently, the impact of changes in $|P_{i1} - 3e_i|$ on firm following is muted.

5 Other Public Information

In reality, there are likely many sources of public information available to investors in a firm. However, it may be the case that speculators which consider to enter the market do not use *all* public information to do so (as this effectively would be information acquisition). Instead, the investment screen may be based on some summary measure, such as annual earnings. To this point, we have assumed that the informed investors who enter the market prior to the second round of trade know all of the public disclosure that has occurred in the initial round of trade, which is confined in the model to earnings $\{e_{a1}, e_{b1}\}$. If we include more dimensions to the round 1 public disclosure without affecting the amount of private information obtained (e.g., the public disclosure does not reduce an informed speculators information advantage) and assume that the second round informed investors observe all of that information prior to making the choice about what firm to follow, the results do not change in any substantive manner. In particular, all that would change is that the pricing norm (i.e., the expected price conditional upon the public information) would be altered to reflect the additional public information. The deviation of observed price from the pricing norm, however, would still convey the same information about the quality of private information that can be obtained if the firm is followed.

In this extension, we assume that not all dimensions of public disclosure are incorporated into the databases employed for screening activities. For example, income statement line items, balance sheet line items, and management forecasts are reflected in databases employed for screening activities, but more qualitative and situation specific quantitative public disclosures are unlikely to be in those databases. Given that all public disclosures might not be accessible to prospective investors engaging in screening activities, we consider an extension of the model in which there is additional public information made available in the initial round of trade and that information is not available to the new informed investors when they decide which firm to follow, but is accessed for the firm that is followed.

To extend the model to introduce additional public information, assume that an additional public disclosure for firm i prior to round of trade 1 is $\tilde{\omega}_{i1}$, where $\tilde{\omega}_{i1}$ is normally distributed with mean 0 and variance $\varpi_i^2 < s_i^2 - q_h$. The covariance between $\tilde{\omega}_{i1}$ and $\tilde{\varepsilon}_{i2}$ is ϖ_i^2, and $\tilde{\omega}_i$ is independent of all other random variables. Hence, the expectation of terminal value prior to the first round of trade is $3e_{i1} + 2\omega_{i1}$, as opposed to the just $3e_{i1}$ in our primary model.

In this extension, the second round pricing function characterized in Observation 1 is unchanged. Observation 2 is nearly identical, with the only change being that a term is added to the first round pricing function,

$$P_{i1} = 3e_{i1} + 2\omega_{i1} + \beta_{i1x}x_{i1} + \beta_{i1n}n_{i1}, \tag{26}$$

where $\beta_{i1x} = 2\frac{2q_i^2}{4q_i^2+\sigma_{i1}^2}$ and $\beta_{i2n} = \frac{2q_i^2}{4q_i^2+\sigma_{i1}^2}$. Finally Observations 3 and 4, and Lemma 1 continue to hold as written.

While little is affected by the additional disclosure in the first round of trade, it does affect how the new speculators respond to the price-to-earnings screen in equilibrium

Conjecture 3. Assume that there is the additional public disclosure of $\{\omega_{a1}, \omega_{b1}\}$ *prior to the initial round of trade, which does not affect the information advantage of speculators. If the informed investors in the second round of trade do not observe that public disclosure before deciding which firm to follow, the increase (decrease) in the number of informed investors following firm* i *arising from an an increase (decrease) in the deviation of price from the expected price conditioned on earnings* $|P_{i1} - 3e_i|$ $(|P_{j1} - 3e_j|)$ *is moderated by increases in* ϖ_i^2 (ϖ_j^2)*.*

Conjecture 3, assuming it holds, suggests that the presence of the additional public information that is not easily accessed by the informed investors prior to their firm following decision reduces their responses to the screen.

To understand why Conjecture 3 is expected to hold, it is useful to examine the informed investors' assessment of $\Pr\left(q_i^2 = q_h^2|\Omega\right)$ at the time they decide which firm to follow:

$$\Pr\left(q_i^2 = q_h^2|\Omega\right) = \frac{1}{1 + \sqrt{\frac{V_h^2 + 4\varpi_i^2}{V_l^2 + 4\varpi_i^2}} \exp\left[-\left(P_{i1} - 3e_i\right)^2 \frac{V_h^2 - V_l^2}{2\left(V_h^2 + 4\varpi_i^2\right)\left(V_l^2 + 4\varpi_i^2\right)}\right]}. \tag{27}$$

When $\varpi_i^2 = 0$, $\Pr\left(q_i^2 = q_h^2|\Omega\right)$ collapses to the expectation in the primary model. As ϖ_i^2 approaches infinity, $\Pr\left(q_i^2 = q_h^2|\Omega\right)$ approaches the prior of $\frac{1}{2}$, which means that the deviations from the expected price-to-earnings relation conveys no incremental information about the quality of the private information. Effectively, then, $\tilde{\omega}_{i1}$ simply adds noise to the price-to-earnings screen because the deviation of price from the norm is now due to the additional, but unknown, public information. Hence, the impact of the deviation of price from expected price conditioned on earnings has a smaller influence on perceived information quality and, accordingly, firm following.

6 Conclusion

We develop and analyze a model where speculators can only gather private information about one of two firms, and that they observe past prices and earnings before making the decision about which of the two firms to follow. When these speculators are uncertain about the quality (i.e., profitability) of the private information that can be garnered by following a firm, they are more inclined to follow firms that have a larger deviation between price and the expectation of price conditioned on earnings, which happens to be a simple multiple. They do so because a larger deviation suggests that there is more private information in the marketplace.

Our analysis broadly suggests that firms whose prices deviate from database driven pricing norms are more likely to attract the attention of speculators. Given that observation, our analysis also implies that firms whose prices deviate from database driven pricing norms will be more incline to experience increased price volatility and trading activity. Finally, our analysis suggests that the relation between price deviations from pricing norms will be muted when firms provide public disclosures that are not tracked in databases, which might include qualitative narratives or quantitative disclosures about situation specific events.

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